Shakedown and Fatigue of Pavements with Granular Bases

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Performance prediction of pavements requires the proper assessment of permanent deformations and fatigue of the structure under applied traffic loads. Of particular importance in this case is whether a given pavement structure will experience progressive accumulation of plastic strains or whether the increase in plastic strains will cease to occur, thereby leading to a stable response or shakedown. A numerical method for predicting shakedown of pavements is developed in this paper. The proposed numerical approach involves discretization of the pavement structure using the finite element method. An iterative scheme is implemented that satisfies shakedown conditions, together with the nonlinear resilient load-deformation characteristics of the granular and subgrade layers. Convergence is attained when a limiting or shakedown load could be determined for which the stress-resilient strain relations are satisfied, and a time-independent residual stress field exists for which equilibrium conditions, boundary conditions, and yield conditions (i.e., Mohr-Coulomb yield criterion in this case) are fulfilled. The proposed method is applied to study the shakedown behavior of pavements with granular layers. Specifically, the influence of strength of the granular layer in terms of cohesion and friction is investigated. In this case, the results of a limited number of laboratory triaxial tests showing the effect of aggregate interlock, percent fines, and compaction water content on the cohesion and friction parameters are used. The influence of other factors (such as initial stresses induced by compaction and overburden pressure) is illustrated. Shakedown behavior is then compared with fatigue of the surface layer in an attempt to develop a better understanding of pavement performance.

Pavement structures are generally designed to resist repeated load applications over a given design period. In many rational design procedures, limiting values in the critical response parameters have been proposed as a means of achieving satisfactory pavement performance. In three-layer pavements consisting of asphalt concrete surface, granular base, and subgrade, critical response parameters could include surface deflections, tensile strains on the underside of the asphalt concrete surface course, and normal stresses and strains on top of the subgrade layer. The influence of strength and resilient properties of granular bases on the performance of pavement structures has been recognized by many investigators (*1–3*). Although pavement response parameters could be determined within reasonable accuracy limits using finite element techniques (*4,5*), performance models fall short of predicting the stability of pavement systems under long-term repeated loading. Of particular significance in this case is whether such systems will exhibit progressive accumulation of plastic strains, or whether the accumulation of plastic strains will cease and a stable response or a shakedown condition is attained.

The shakedown theory, which was originally developed by Melan (*6*), has been applied numerically to discrete structures (*7,8*) and more recently to pavements (*9,10*). According to the theory, a pavement would exhibit progressive or increased accumulation of plastic strains under repeated load applications if the magnitude of these loads exceeded a limiting value defined as the shakedown load. In this case, the pavement is said to exhibit an incremental failure mode or incremental collapse, which is physically reflected in the gradual accumulation of permanent deformations followed possibly by material breakdown of the pavement structure.

On the other hand, if the applied loads were smaller than the shakedown load, the accumulation of plastic strains will eventually cease, and the pavement is said to have attained a state of adaptation or shakedown, whereby the pavement response will be elastic under additional load applications. The magnitude of the shakedown load predicted using available numerical algorithms (*9,10*) depends on the thickness, shear strength, and elastic properties of the individual pavement layers. These algorithms, however, do not consider the nonlinear stress-dependent resilient properties of granular and subgrade layers in pavement structures.

In this paper, a numerical method using the shakedown theory and incorporating the stress-dependent resilient properties of granular and subgrade layers in pavements will be introduced. The proposed method will be used to investigate the shakedown behavior of pavements. Specifically, the influence of compaction stresses, strength, and load-deformation characteristics of the granular base on shakedown capacity will be assessed. Moreover, shakedown and fatigue predictions will be compared for the purpose of developing improved pavement performance models.

PROPOSED NUMERICAL MODEL

In the proposed method, the pavement is discretized into a series of rectangular elements, each with four external primary nodes. The displacement functions used are complete to the second degree and satisfy compatibility conditions. The material is assumed to be initially elastic-ideally plastic with convex yield surface and applicable normality condition. A quasi-static analysis is used assuming negligible viscous and inertia effects. If stress states σ^o, σ^s, and σ^a correspond, respectively, to body forces P^o, statically applied forces f^s, and repeated loads f^a, then the system will shake down—provided a time-independent stress increment $\Delta\sigma$ can be found

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such that equilibrium conditions, boundary conditions, and yield relations are satisfied. In this case, plane strain finite element analysis is used to determine the stresses in the system.

The determination of the shakedown load is then reduced to an optimization problem, as suggested by Raad et al. (*10*), and is stated as follows.

Minimize

$$Q = -\alpha + \sum_{i=1}^{NP} (S_{xi})^2 + \sum_{i=1}^{NP} (S_{yi})^2 \qquad (1)$$

subject to the following constraints:

$$\alpha > 0 \qquad (2)$$

$$f(\sigma) \leq 0 \qquad (3)$$

$$\sigma_3 \geq -2c \tan (45 - \phi/2) \qquad (4)$$

where

NP = number of nodal points,
α = load multiplier associated with repeated loads f^a,
$\sigma = (\sigma_{ij})_o + (\sigma_{ij})_s + \alpha (\sigma_{ij})_a + \Delta\sigma_{ij}$ (5)
$(\sigma_{ij})_o, (\sigma_{ij})_s, (\sigma_{ij})_a$ = stresses at the center of a given element due to P^o, f^s, and f^a, respectively,
$\Delta\sigma_{ij}$ = arbitrary stress increment applied at the center of each element,
S_{xi}, S_{yi} = resultant forces in the x and y directions at a nodal point due to $\Delta\sigma_{ij}$ with respect to a global set of coordinates (x,y), and
f = Mohr-Coulomb failure criterion with failure occurring for $f \geq 0$.

f is given by

$$f = \sigma_1 - \sigma_3 \tan^2 (45 + \phi/2) - 2c \tan (45 + \phi/2) \qquad (6)$$

where σ_1 and σ_3 are major and minor principal stresses, and c and ϕ are equal to the cohesion and angle of friction.

Minimizing Q subject to the indicated constraints would yield a maximum value for the load multiplier (α), while satisfying equilibrium conditions, boundary conditions, and yield conditions in a weak sense. Because Q is quadratic with nonlinear constraints, quadratic optimization techniques are not feasible. Instead, a pattern search algorithm is developed based on the original work by Hooke and Jeeves (*11*). The method could be summarized in the following steps.

1. Determine the stresses resulting from P^o, f^s, and an intially applied repeated load f^a.
2. Find a load multiplier (α_{st}) such that ($\alpha_{st} f^a$) would cause yielding of the most critically stressed element in the system. This will shift the search to the vicinity of the region of interest.
3. The search starts by determining Q for α_{st} and a set of $\Delta\sigma_{ij}$ that satisfy the constraint conditions of Equations 2, 3, and 4.
4. During a given exploratory sequence, the variable (α) is allowed one disturbance in the direction of decreasing Q. Each of the stress variables ($\Delta\sigma_{ij}$) is allowed as many disturbances, each equal to its step size and in the same direction as long as the objective function (Q) decreases and the imposed constraints are satisfied. Otherwise, the exploratory sequence is rated a failure.
5. A new search is initiated about the last base point determined in Step 4, using smaller step sizes. The algorithm terminates when the values of the step sizes are reduced to a certain preassigned value. In this case, the shakedown load will be equal to ($\alpha_{st} \cdot \alpha \cdot f^a$).

To improve predictions of the shakedown capacity of pavements, more realistic modeling of material properties should be incorporated in the analysis. Specifically, the nonlinear stress-dependent resilient moduli for granular and subgrade layers should be used. For granular layers, the resilient modulus is generally expressed as

$$M_R = K_1 \theta^{K_2} \qquad (7)$$

where $\theta = \sigma_1 + \sigma_2 + \sigma_3$ is the sum of principal stresses, and K_1 and K_2 are coefficients derived experimentally.

For fine-grained soils, a typical representation of resilient modulus (M_R) as a function of repeated deviator stresses ($\sigma_1 - \sigma_3$) has been proposed by Figueroa (*5*) and is illustrated in Figure 1.

Numerically, the shakedown capacity could be obtained using a series of iterative steps. It is assumed in this case that the response under a given repeated state of stress at a given point in the pavement stabilizes and remains elastic so long as these stresses do not exceed the strength as defined by the Mohr-Coulomb yield criterion. Such behavior is illustrated schematically in Figure 2. A series of iterations using finite element analysis is conducted so that the stresses at the center of each element satisfy the stress-dependent modulus relationship.

A new shakedown pressure acting on the pavement surface is then calculated by using the newly determined moduli at the center of elements and following the approach previously summarized in Steps 1 through 5. The shakedown pressure (P_i) for a given iterative step—i in this case—will be equal to $\alpha_{st}\alpha_t P_{i-1}$, where α_{st} is the load multiplier associated with initiation of failure, α_t is the maximum load multiplier obtained from Equation 1, and P_{i-1} is the shakedown pressure obtained for the previous iterative step ($i-1$).

The procedure is repeated until convergence is attained, whereby the shakedown pressure in two consecutive steps reaches essentially the same value, and hence shakedown conditions are satisfied simultaneously with the stress-dependent moduli relations. The proposed method is shown in Figure 3.

The convergence pattern of the proposed numerical approach is illustrated for two examples in Figure 4. Material properties used in the analysis are summarized in Table 1.

The variation of $\alpha_{st}\alpha_t$ and the ratio (P_i/P_{SD}) of the shakedown pressure (P_i) obtained for iterative step i to the final shakedown pressure (P_{SD}), with number of iterative steps, indicates that convergence is essentially attained after six iterations.

MATERIALS CHARACTERIZATION

The determination of shakedown behavior requires the proper assessment of the load-deformation properties and strength of the pavement materials. Although the stress-dependent resilient properties of the granular layers used in this paper are expressed in terms of $M_R = K\theta^n$, other more refined models developed recently could be incorporated (*12, 13*). Of major importance in this case are the strength parameters of the pavement components under repeated states of stress.

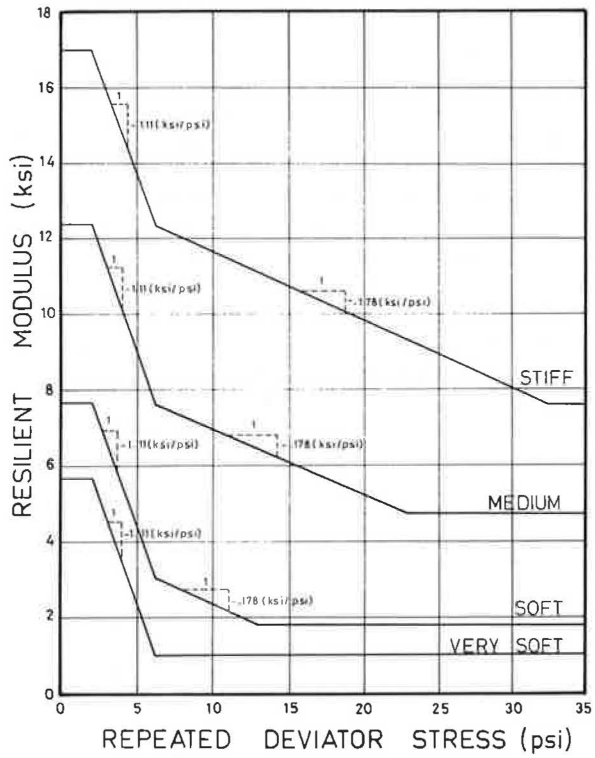

FIGURE 1 Resilient properties of subgrade soils (*5*).

Repeated load tests on fine-grained subgrade soils show that a stress level might exist below which no sudden increase in deformation would occur, irrespective of the number of stress repetitions. The rate of accumulation of plastic strains with a number of stress applications below this critical condition would eventually approach zero. Larew and Leonards (*14*) reported values of the ratio of repeated deviator stress to the conventional strength obtained from triaxial tests on a compacted silty clay in the range of 0.80 to 0.90. Similar tests by Brown et al. (*15*) on a silty clay consolidated to different overconsolidation ratios yielded values between 0.82 and 0.93.

Repeated load triaxial data on granular soils (*16, 17*) strongly indicate the existence of a critical deviator stress for a given confining pressure below which the rate of accumulation of plastic strains tended to decrease as the number of load applications increased. For repeated deviator stresses greater than the critical value, the rate of accumulation of plastic strains would increase, leading eventually to failure of the specimen. If it is hypothesized that such a critical stress value would exist for a given confining pressure, then the envelope of principal stresses in this case would define a yield criterion that could be used in shakedown analysis.

In an attempt to determine the strength parameters under repeated loads, repeated load triaxial data were used for a number of granular soils presented by Barksdale (*16, 18*). In this case, the critical repeated deviator stress corresponding to 10^5 repetitions was estimated. The resulting data were interpreted using a Mohr-Coulomb yield representation. Values of cohesion ranged between zero and 10 psi, whereas the friction angle varied between 26 and 40 degrees.

Recent studies on the behavior of granular layers under repeated loads show a significant influence of cohesion on their shakedown capacity (*19*). Cohesion of granular soils could depend on interlock between aggregates, density, moisture content, and fines content. To illustrate this, a limited number of triaxial tests were conducted on a uniform sand (S1), a crushed limestone with no fines (CLS1), a crushed limestone with 5 percent fines (CLS2), and a crushed limestone with 12 percent fines (CLS3). The crushed limestone specimens were compacted in cylindrical 4 × 8 in. molds,

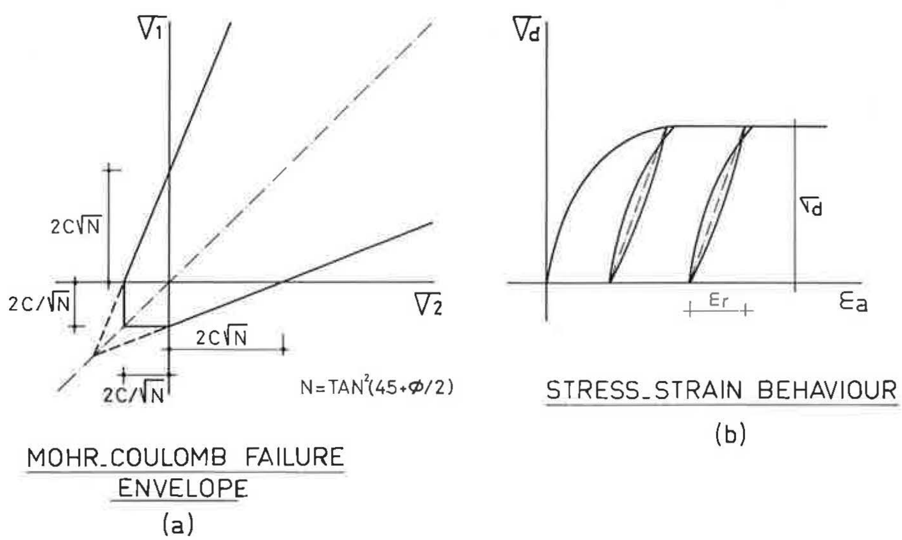

FIGURE 2 Schematic representation of stress-strain behavior and yield for a typical soil under repeated loading.

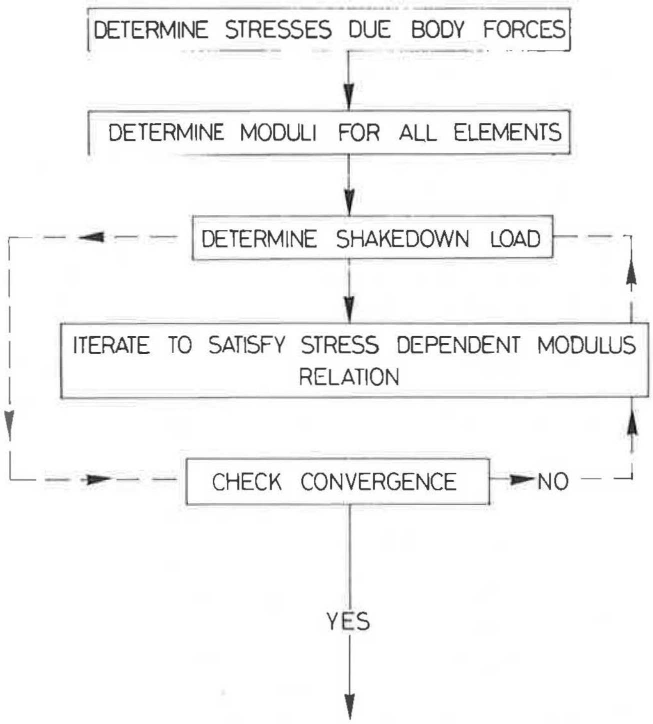

FIGURE 3 Flowchart of the proposed numerical method.

whereas 1.5 × 3 in. molds were used for the sand specimens. All specimens were compacted using modified AASHTO compaction energy (ASTM D1557-66T) at the corresponding optimum moisture content. A summary of material properties and the results of triaxial strength tests are presented in Tables 2 and 3. Results indicate that the tested granular soils exhibited varying degrees of cohesion, induced probably by pore pressure suction in the fines matrix and by aggregate interlock. The most significant factor influencing the cohesion seemed to be the percent of fines (i.e., percent passing No. 200 sieve). An increase in fines content from 0.3 to 11 percent would increase the cohesion from 1.6 to 13 psi.

APPLICATIONS

The proposed numerical approach was used to investigate the influence of granular base characteristics on shakedown behavior of pavement structures. The effects of compaction stresses, and friction and cohesion of the base on shakedown capacity were illustrated. Moreover, fatigue and shakedown predictions for bases with different strength and resilient properties were compared.

Three-layer pavements consisting of an asphalt concrete surface with a granular base over a clay subgrade were considered. The pavements were analyzed under an applied sur-

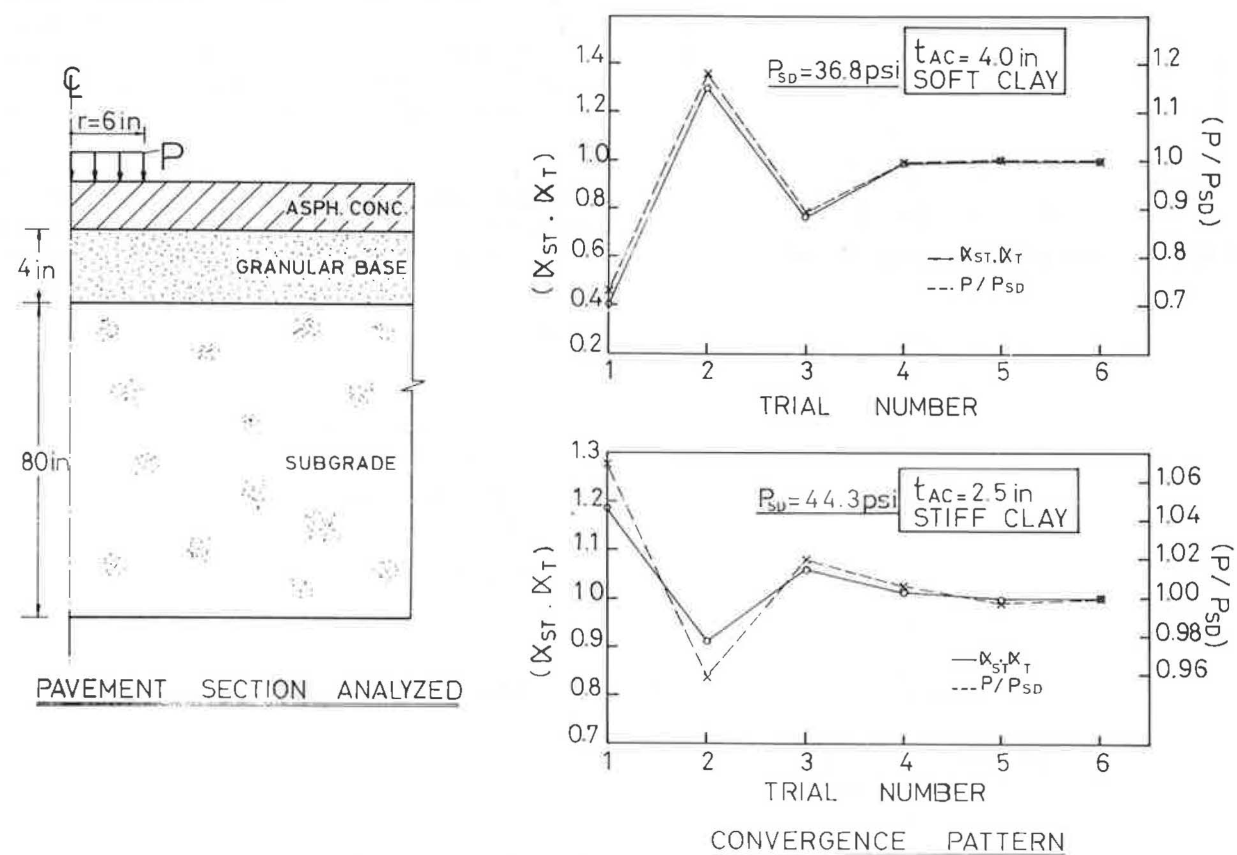

FIGURE 4 Convergence pattern of the proposed numerical method.

TABLE 1 PAVEMENT MATERIALS PROPERTIES

Asphalt Concrete Layer	Granular Base	Subgrade
E_{ac} = 5.0 x 10^5 psi	M_R = 5000$\theta^{0.50}$ (psi)	Soft, C_s = 3 psi; ϕ_s = 0°
ν_{ac} = 0.40	M_R = 2000$\theta^{0.60}$ (psi)	Stiff, C_s = 12 psi; ϕ_s = 0°
γ_{ac} = 140 lb/cu ft	ν_b = 0.35	ν_s = 0.47
K_o = 0.50	γ_b = 120 lb/cu ft.	γ_s = 110 lb/cu ft
ϕ_{ac} = 37°		K_o = 0.50
C_{ac} = 650 psi		

Notes

E_{ac} is modulus of asphalt concrete layer.

γ_{ac}, ϕ_{ac}, C_{ac} are density, friction angle, and cohesion of asphalt concrete mix.

γ_b, ϕ_b, C_b are density, friction angle, and cohesion of granular base.

γ_s, ϕ_s, C_s are density, friction angle and cohesion of the subgrade.

ν_{ac}, ν_b, ν_s are Poisson's ratio for asphalt concrete, granular base, and subgrade.

K_o is the coefficient of at rest pressure.

M_R = 5000$\theta^{0.5}$ was used for all bases studied, except for comparing fatigue and shakedown, where M_R = 2000$\theta^{0.6}$ was also used.

Subgrade resilient moduli relations used for all cases studied are shown in Fig. 1.

face pressure of radius equal to 6 in., assuming plane strain conditions. Properties of pavement materials used in the analyses are summarized in Table 1.

Influence of Compaction

Compaction of granular layers induces soil stresses, which could influence the mechanical properties of these layers. Measured compaction-induced stresses from a number of laboratory and full-scale field studies indicate that the process of load application and removal can result in significant increases in residual lateral earth pressures. These may exceed the theoretical at-rest values and may approach the limit dictated by passive earth pressure (*20–23*).

Residual lateral compaction stresses are also affected by the number of load cycles applied. In a study conducted by Stewart et al. (*24*), residual lateral stresses were determined experimentally for a ballast material under tie contact pressure equivalent to that of a 32-kip wheel load from a train. Results show that residual horizontal stresses increased up to 100 cycles, after which they became constant. Maximum values for lateral earth pressure coefficient in the ballast ranged between 2 and 11.

Prediction of such compaction effects is, thus, necessary to predict properly the response and performance of pavement structures. Recently, a hysteretic model for predicting compaction stresses was proposed by Duncan and Seed (*25*). This model was used to determine the compaction stresses in the granular base for all cases considered in this paper. Plane strain conditions were assumed and the residual lateral stress buildup was determined under the multiple passes of a 4-kip compactor with a width of 4 ft. The influence of the cohesion and angle of friction of the base on compaction stresses is illustrated in Figure 5. The residual lateral compaction stress (σ_{HC}) is greater than the vertical stress (σ_V) and increases with depth below the surface of the granular base under a 3-in. thick asphalt concrete surface. Moreover, compaction stresses increase with increase in friction angle and cohesion, but they reach a maximum value for cohesion of about 1 psi, with a corresponding average value of lateral earth pressure coefficient essentially equal to 7.

The variation of shakedown load with thickness of granular base (M_R) = 5000 $\theta^{0.5}$, ϕ = 35°) was determined for a pavement with a 3-in. asphalt concrete surface. The influence of compaction stresses was investigated by comparing the shakedown capacity for the base where the coefficient of lateral pressure (K_o) was equal to 0.5, with the case where the compaction stresses were included. Results are shown in Figures 6 and 7. Compaction stresses would increase the predicted shakedown loads for both pavements with soft and stiff subgrade conditions. The increase, in general, is most significant for granular bases with low cohesion and for bases overlying stiff subgrades.

Limited field data on the variation of compaction-induced stresses with time due to stress relaxation effects show that

TABLE 2 PROPERTIES OF TESTED GRANULAR SOILS

Material	% Passing						Maximum Dry Density (lb/cu ft)	Optimum Moisture Content (%)	AASHTO Classification
	No. 4	No.10	No.40	No.60	No.100	No.200			
S1	-	-	82.2	16.5	0.9	0.2	94.1	5.2	A-3
CLS1	100	63.7	22.0	11.9	5.8	0.3	112.0	4.8	A-1-b
CLS2	100	67.1	25.4	13.8	8.0	5.0	128.0	7.3	A-1-b
	(100)	(76.6)	(35.1)	(25.6)	(19.9)	(13.8)			
CLS3	100	58.2	27.2	18.8	14.5	11.8	131.8	7.2	A-1-b
	(100)	(68.0)	(35.7)	(27.1)	(21.0)	(15.9)			

Notes

Maximum dry density and optimum moisture content correspond to Modified AASHTO Compaction Energy (ASTM D1557-66T).

Quantities in parentheses correspond to grain size distribution after compaction.

Fines (percent passing No. 200) are nonplastic.

for cohesionless soils, the compaction stresses are essentially locked in and do not change with time. For cohesive soils, on the other hand, a reduction up to 30 percent is observed to occur after 24 hours following compaction (*23*). Moreover, other field observations illustrate the effect of increased lateral structural deflections on compaction stresses (*20–22, 26*). In this case, increased lateral strains seem to reduce lateral compaction pressures. This could be particularly significant in pavements. For example, increasing moisture conditions due to spring thaw could reduce the shear strength at the base/subgrade interface to a near-zero condition, thereby leading to increased lateral strains under wheel load applications and potential decrease or loss of lateral compaction stresses.

Influence of Cohesion and Friction of Granular Base

Analyses were performed to assess the influence of the strength of the granular base on the shakedown behavior of pavement structures. The pavement considered for this purpose consisted of a 3-in. asphalt concrete surface and a 12-in. granular base ($M_R = 5{,}000\ \theta^{0.5}$) overlying soft or stiff subgrades. Results are presented in Figures 8 and 9. For a given angle of friction, increasing the cohesion of the granular base would significantly increase the shakedown capacity of the pavement. Moreover, for values of cohesion less that 5 psi, in the case of soft subgrade, and less than 17 psi, in the case of stiff subgrade, an increase in the friction angle would enhance the shakedown capacity of the system.

However, cohesion of the base seems to be a more significant parameter on shakedown capacity than the friction angle. For cohesion values greater than 5 psi, in the case of soft subgrade, and 17 psi, in the case of stiff subgrade, an increase in the angle of friction reduces shakedown capacity. It should be noted that, for a given base cohesion, an increase in the angle of friction would not only reduce the tensile strength of the base but would increase its shear strength. It seems, therefore, that for high values of cohesion, the relative effect on shakedown capacity associated with a decrease in the tensile strength of the base due to an increase in its friction angle surpasses the effect of the corresponding increase in its shear strength. This could probably be a result of increased mobilization of the shear resistance of the base prior to the initiation of tensile failure on its underside.

A comparison between the applied pressure on the pavement surface required to initiate failure (P_{si}) and the shake-

TABLE 3 TRIAXIAL TEST RESULTS

Material	Deviator Stress at Failure (psi) σ_3 0psi	σ_3 7.2psi	σ_3 14.4psi	σ_3 21.6psi	Dry Density (lb/cu ft)	Moisture Content (%)	Cohesion (psi)	Angle of Friction (degrees)
S1	4.70	21.1	38.2	55.8	93.9	4.90	2.90	30
	5.20	21.7	38.0	48.4	93.2	4.94	2.87	29
CLS1	5.26	44.1	61.9	78.7	109.7	4.20	1.97	40
	4.31	36.4	50.5	79.3	107.0	5.13	1.32	41
CLS2	34.7	79.3	110.0	154.1	128.8	7.10	17.30	47
	45.5	124.4	156.4	203.7	130.3	6.50	12.9	49
CLS3	47.1	123.2	158.3	201.0	133.7	6.80	12.3	50
	60.0	126.0	159.4	195.4	134.6	6.90	13.5	48

Notes

σ_3 is the applied confining pressure.

Triaxial tests were conducted at a constant rate of strain equal to 2% per minute

down pressure (P_{sD}) is presented in Figure 10. Results indicate that the ratio (P_{st}/P_{sD}) is essentially a function of base cohesion. The variation of friction angle between 30 and 45 degrees will not influence the ratio of (P_{st}/P_{sD}) by more than about 5 percent. It can be concluded that the shakedown pressure is larger than the surface pressure required to initiate failure in the base. It follows that the pavement system would shake down even though some elements of the granular base are in a state of yield. In this case, the yield zone is contained, and the pavement would still stabilize under long-term repeated loading. It is interesting to note that, for values of cohesion less than 5 psi, the ratio (P_{st}/P_{sD}) will increase and could reach 0.97 for a value of cohesion equal to 0.10 psi.

Fatigue and Shakedown Behavior

Prediction of fatigue and shakedown is required for the proper assessment of pavement performance. For example, a pavement designed to carry a number of load applications in fatigue could actually be carrying a load greater than its shakedown capacity and would, therefore, exhibit incremental collapse due to the continuous accumulation of plastic strains. Moreover, if the magnitude of the wheel loads exceeds the shakedown capacity of the pavement after application of maintenance and rehabilitation procedures, then the accumulation of plastic strains will continue and incremental collapse could occur.

Analyses were conducted to investigate the influence of strength and resilient properties of the granular base on fatigue and shakedown predictions. Fatigue resistance of the asphalt concrete surface was determined using a fatigue failure criterion proposed by Monismith et al. (*27*). This criterion expresses limiting tensile strains as a function of number of load repetitions required to cause fatigue failure for different moduli values of the asphalt concrete layer (Figure 11). The limiting tensile strains on the underside of the asphalt concrete layer were determined for plane strain loading conditions using the stress-dependent resilient properties of the granular base and subgrade.

The influence of cohesion of granular base (M_R = 5,000 $\theta^{0.5}$, ϕ = 40°) on shakedown and fatigue behavior is illustrated in Figure 12. Fatigue resistance seems to be essentially unaffected by the cohesion of the granular base. On the other hand, shakedown behavior is significantly affected by base cohesion. For example, a reduction in cohesion from 5 to 0.1 psi would reduce the shakedown capacity of a pavement with an 8-in. base from 50 to 15 psi. In this case, a pavement designed to satisfy acceptable fatigue and shakedown conditions could be unsafe with respect to shakedown, if loss in base cohesion reduces the shakedown limit below the magnitude of applied traffic loads. This could eventually cause

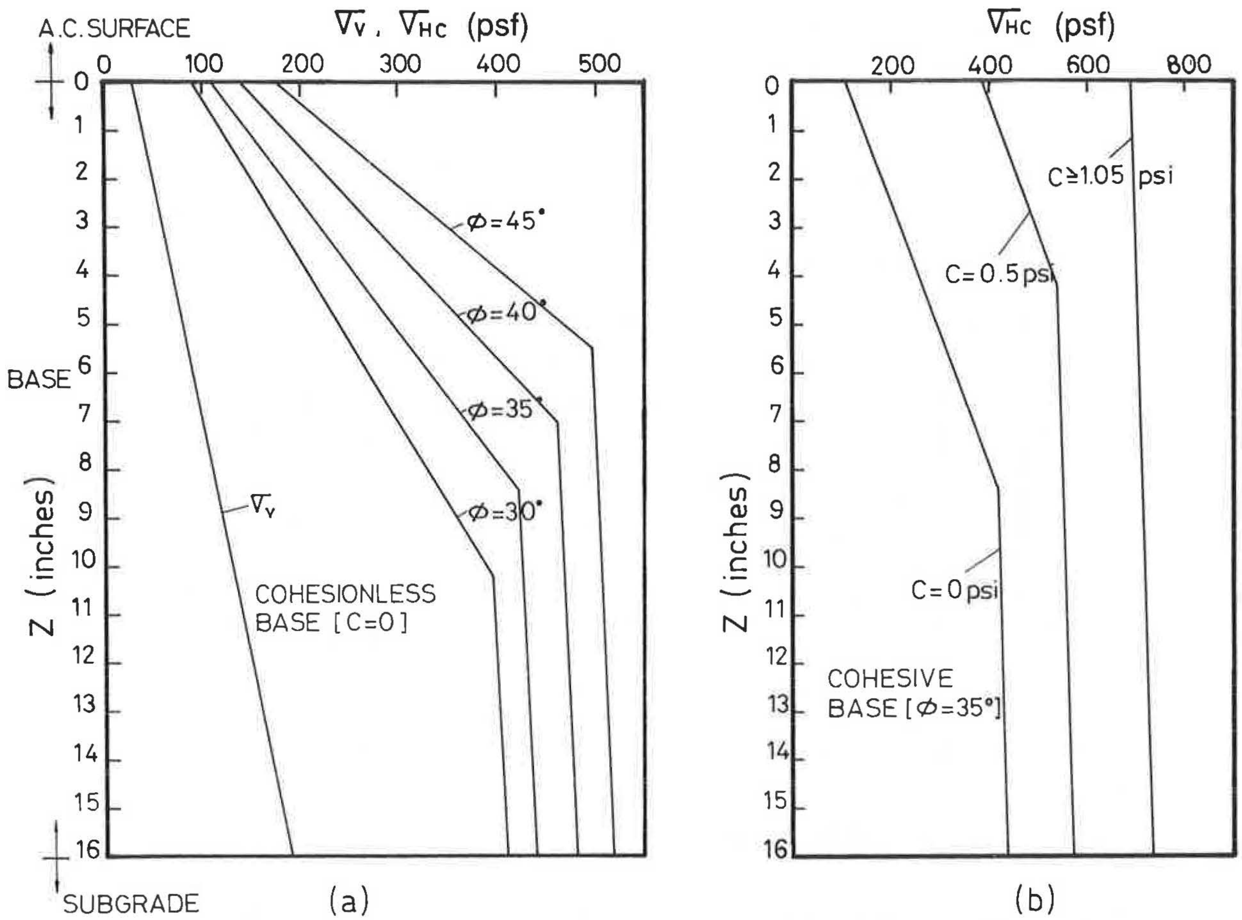

FIGURE 5 Compaction stresses for cohesionless and cohesive base.

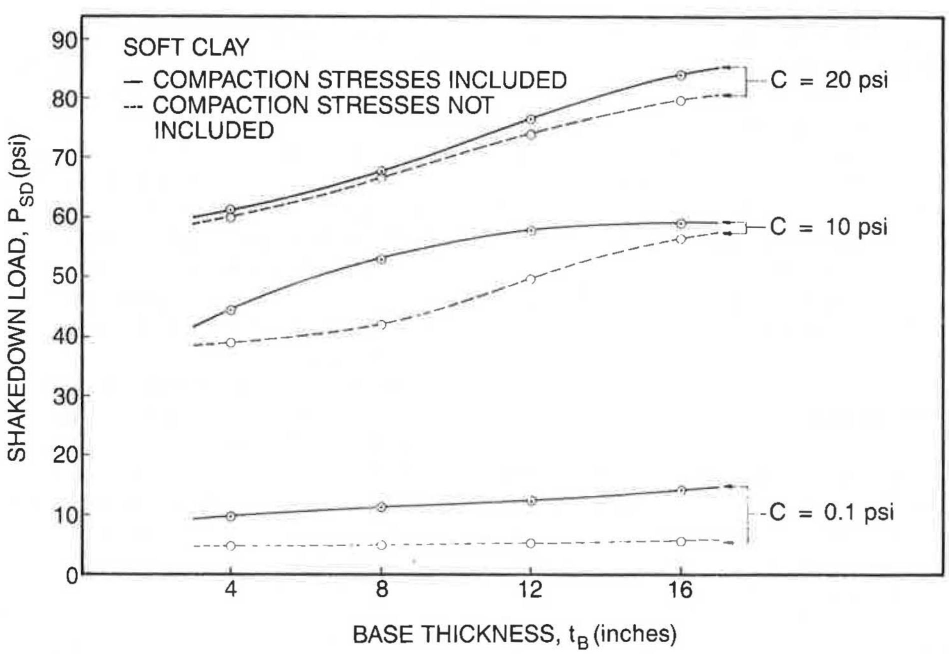

FIGURE 6 Influence of compaction stresses on shakedown for soft subgrade conditions.

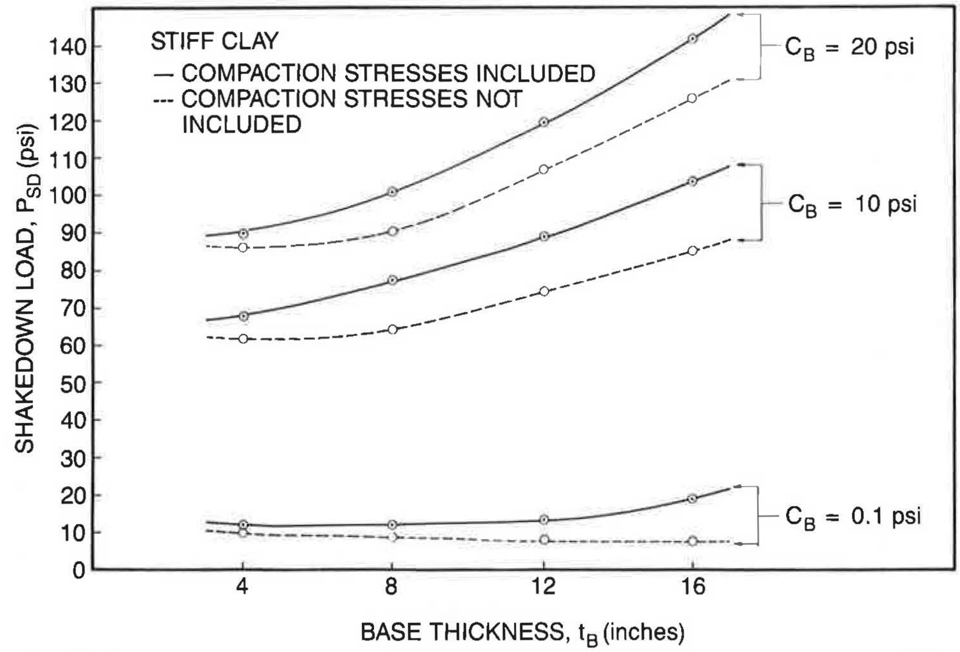

FIGURE 7 Influence of compaction stresses on shakedown for stiff subgrade conditions.

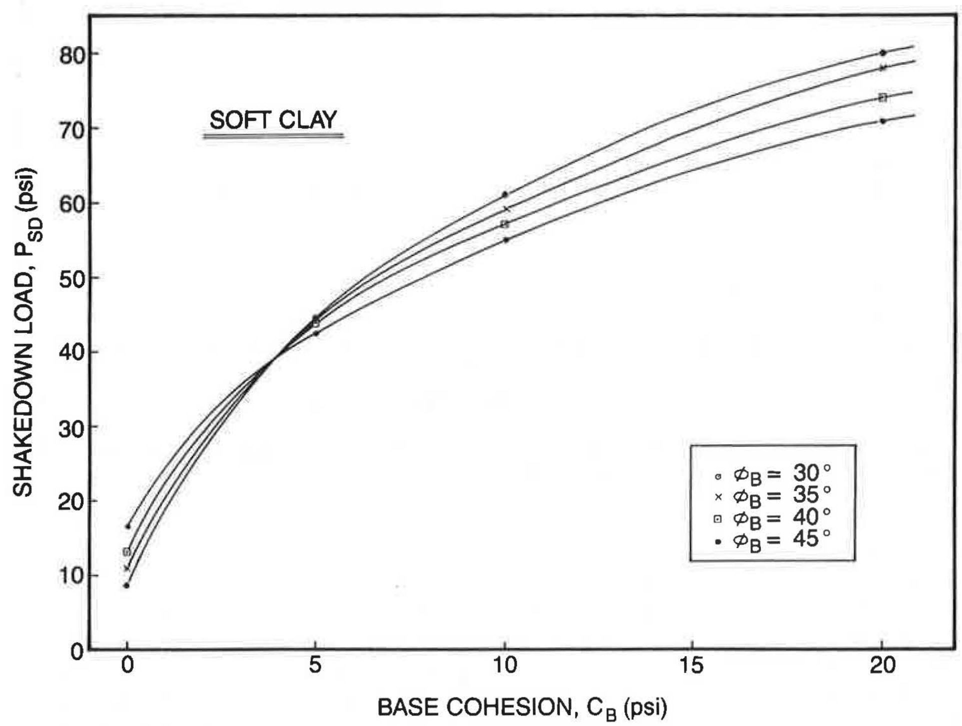

FIGURE 8 Influence of base cohesion and friction on shakedown for soft subgrade conditions.

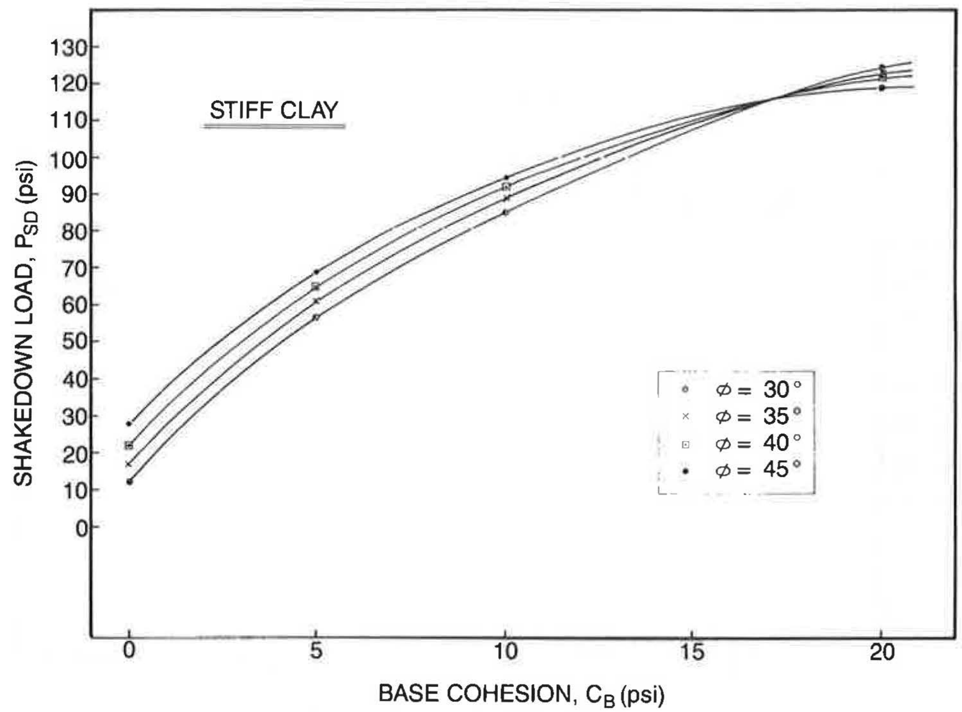

FIGURE 9 Influence of base cohesion and friction on shakedown for stiff subgrade conditions.

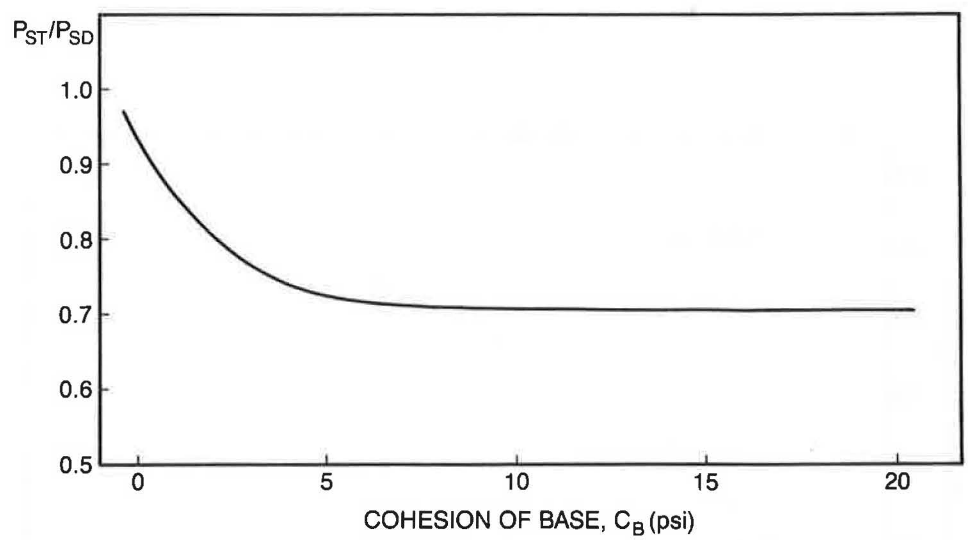

FIGURE 10 Variation of the ratio of failure initiation-pressure (P_{st}) applied on the pavement surface to the shakedown pressure (P_{sD}) with base cohesion.

sudden increase in the rate of accumulation of permanent deformations, thereby leading to incremental collapse.

Analyses were also conducted to compare fatigue and shakedown behavior of pavement with low cohesion-high friction granular base (M_R = 5,000 $\theta^{0.5}$, ϕ = 40°, C = 5 psi) with pavements having high cohesion-low friction granular base (M_R = 2,000 $\theta^{0.6}$, ϕ = 20°, C = 15 psi), assuming stiff subgrade conditions (Figures 13 and 14). Results indicate that pavements with low cohesion-high friction base exhibit better fatigue properties and lower shakedown capacity than pavements having high cohesion-low friction base. Moreover, in both cases, the increase in base thickness did not affect significantly the fatigue resistance of the asphalt concrete surface layer for large numbers of load applications. This agrees well with findings from other procedures (*28*). Pavement loading conditions represented by points below the shakedown curve would result in a stable pavement response, provided proper maintenance procedures were implemented as soon as fatigue cracking appeared on the pavement surface.

Although pavements with high cohesion-low friction base have larger shakedown capacity, they would probably exhibit larger permanent deformations before shakedown and stability conditions were attained. Moreover, these bases were more susceptible to environmental changes (such as increases

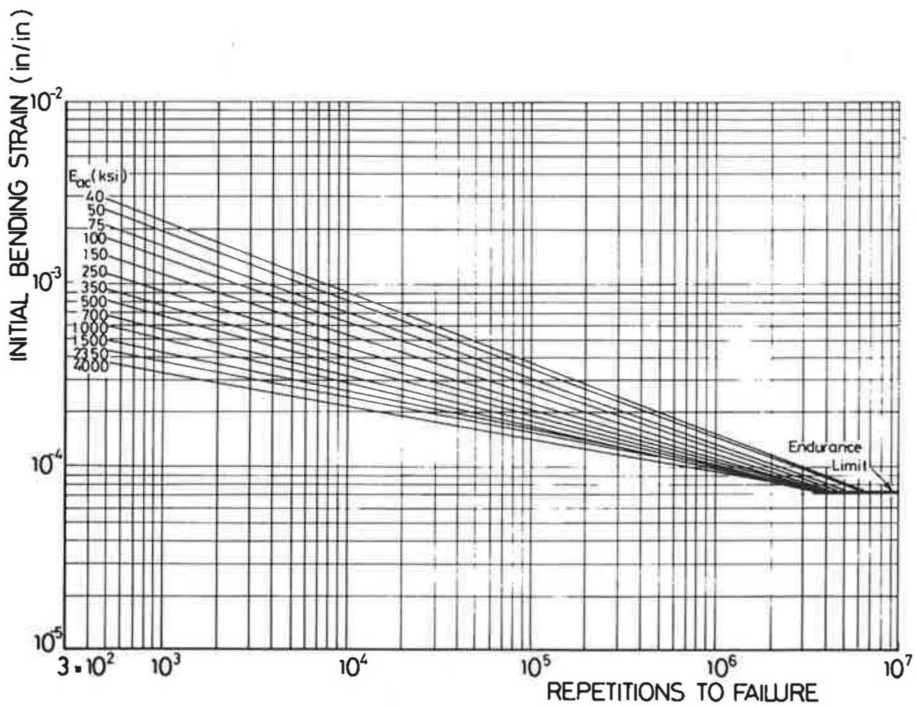

FIGURE 11 **Fatigue failure criterion for asphalt concrete mixtures (*27*).**

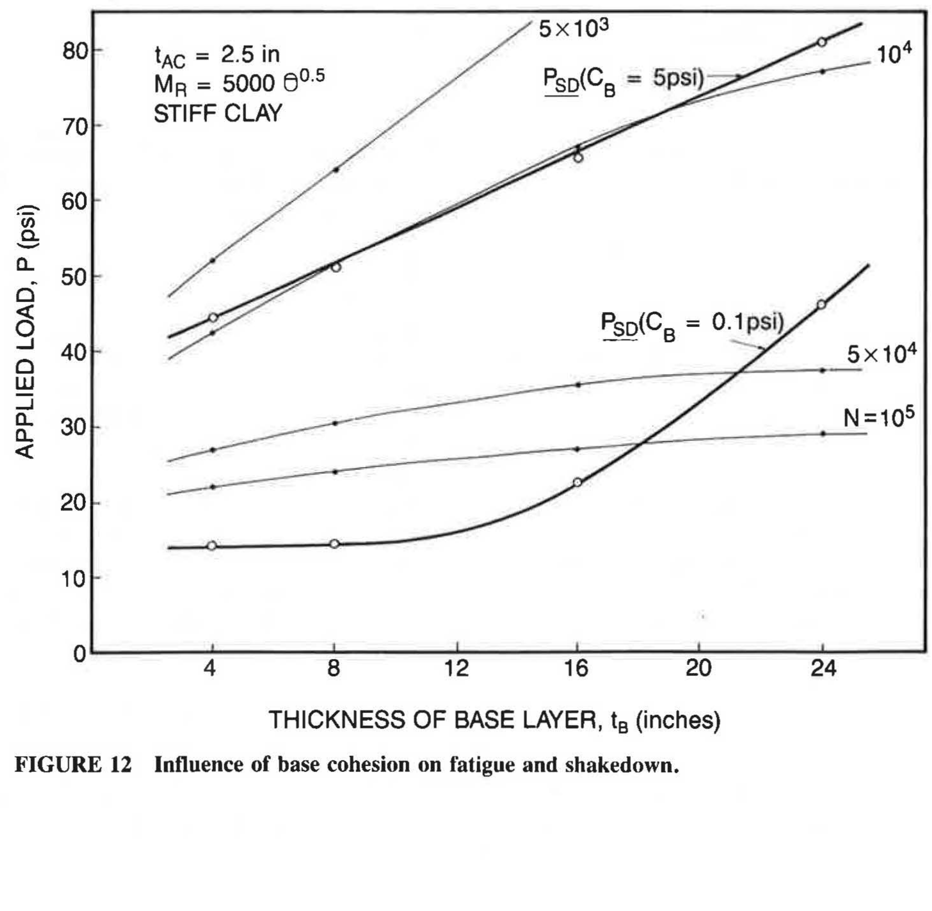

FIGURE 12 **Influence of base cohesion on fatigue and shakedown.**

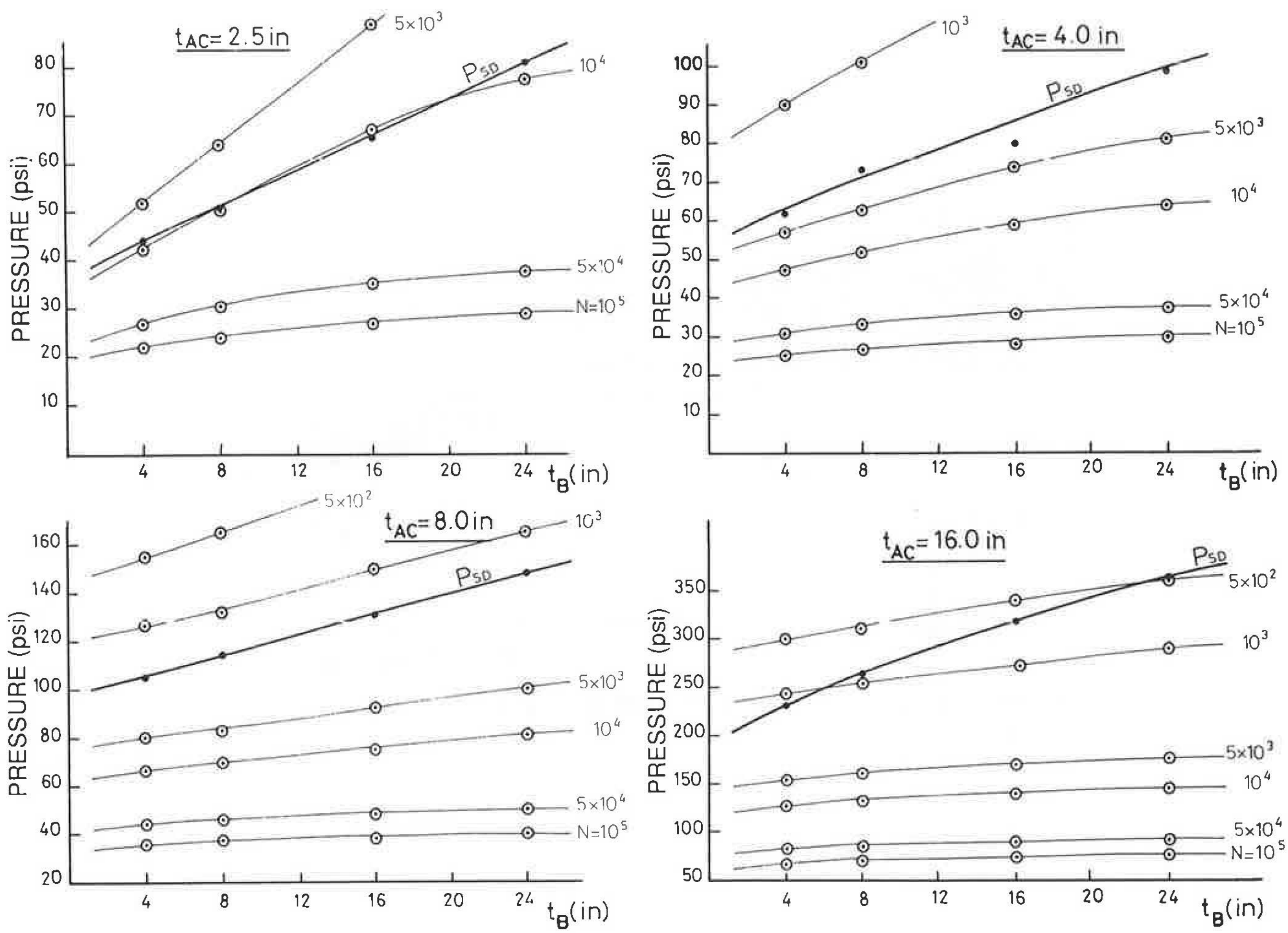

FIGURE 13 Fatigue and shakedown behavior for pavements with a low cohesion–high friction base.

in base moisture and freezing-thawing), which would reduce base cohesion and lead to a significant decrease in shakedown capacity.

SUMMARY AND CONCLUSIONS

A numerical method for applying the shakedown theory in pavements was proposed. The method incorporates the stress-dependent resilient properties of the granular and subgrade layers in the pavement structure. The method was used to investigate the influence of properties of granular base on the shakedown behavior of three-layer pavement systems consisting of an asphalt concrete surface, a granular base, and a clay subgrade. Specifically, the effects of compaction stresses, and cohesion and friction on the shakedown capacity of the pavement were assessed. Fatigue and shakedown predictions for pavements with low cohesion-high friction and high cohesion-low friciton bases were compared.

Compaction stresses in the granular base improved the shakedown capacity of pavements and should, therefore, be considered for the proper assessment of shakedown behavior. The shakedown capacity of pavements was affected by the cohesion and, to a lesser extent, by the angle of friction of the granular base. A loss of base cohesion could result in a significant loss of shakedown capacity, thereby leading to increased accumulation of plastic strains and eventual incremental collapse.

Results of analyses also indicate that fatigue and shakedown criteria should be considered in the design and evaluation of pavement structures. A pavement designed to resist a number of repeated load applications in fatigue would exhibit increased accumulation of plastic strains if the applied load exceeded the shakedown capacity, but the pavement would reach a stable response if the applied load were smaller than the shakedown capacity.

Pavements with high cohesion-low friction granular bases had lower fatigue resistance and higher shakedown capacity than those with low cohesion-high friction granular bases. However, the shakedown capacity in the case of high cohesion-low friction base could be reduced significantly by adverse environmental factors that would cause a loss of base cohesion.

In addition to providing a basis for analyzing shakedown behavior for the conditions considered herein, improved versions of the proposed numerical model would provide a basis for analyzing pavements under more general and more complex loading and environmental conditions.

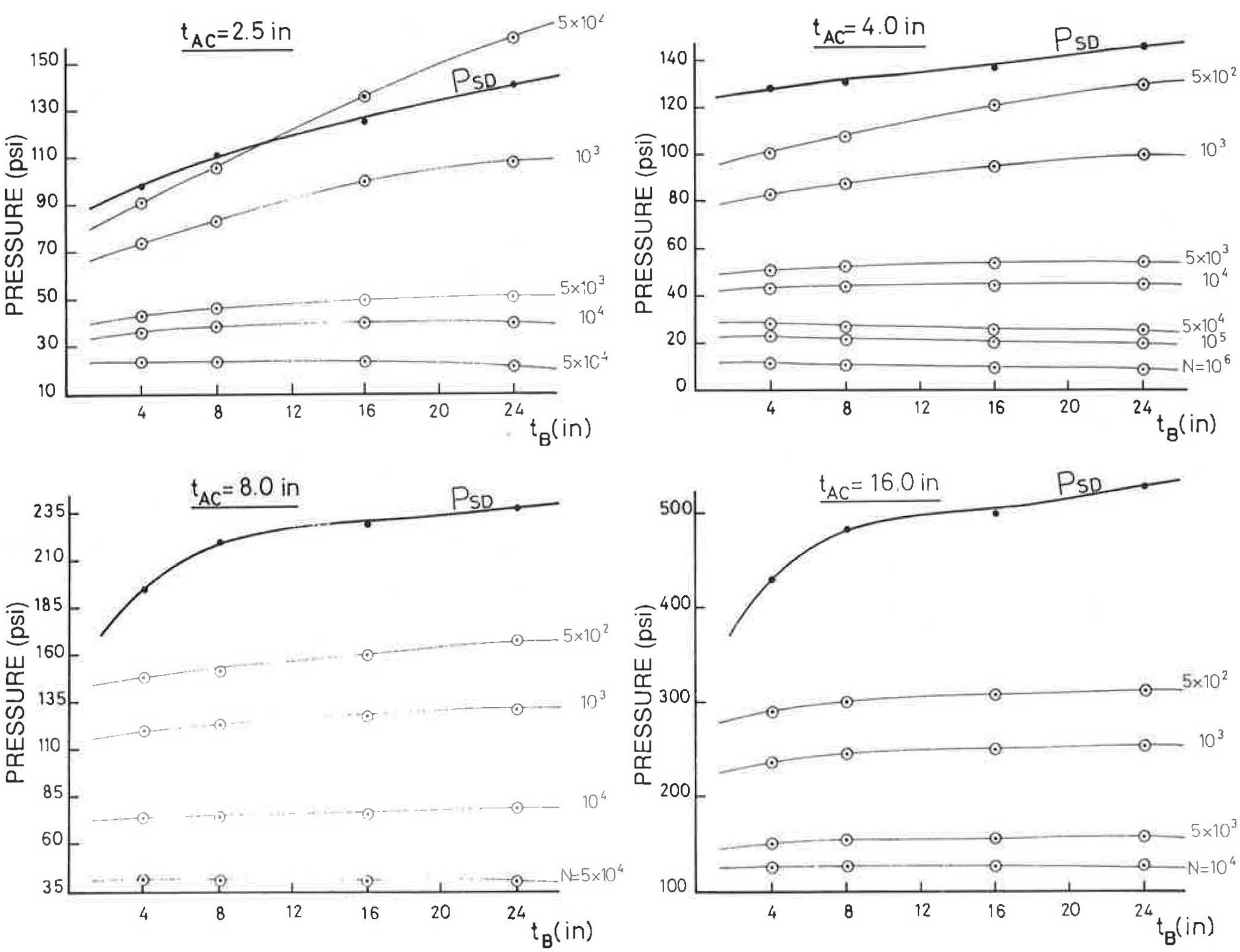

FIGURE 14 Fatigue and shakedown behavior for pavements with a high cohesion–low friction base.

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