Evaluation of Speed Estimates Made with Single-Detector Data from Freeway Traffic Management Systems

Fred L. Hall and Bhagwant N. Persaud

Freeway management systems that rely on single-detector data acquisition generally use a simple equation to calculate speeds. In this paper, the validity of that equation is tested using data from two locations in Ontario, collected using paired-detector speed traps. The results show that the equation gives biased estimates of speeds over a major portion of the range of operating conditions. Discussion of possible causes demonstrates that at least two key assumptions underlying the equation are not met by actual traffic. This result has important implications not only for operation and design of freeway traffic management systems, but also for theoretical work, such as that on speed-flow relationships.

Freeway traffic management systems (FTMSs) acquire data from the roadway and process these data to identify and respond to problems and to notify motorists of those problems. If some aspects of the data are unreliable, then the response decisions and the information given motorists may well be faulty. This paper investigates speed, one variable produced by most FTMSs.

The reason for focusing on speed is that even though speed is an important variable, not all systems measure it directly. Its importance lies in the fact that it is both a potential indicator of problems on the roadway and a good measure of system effectiveness in terms of travel times across a section of road. Further, if there is any intention of informing motorists of travel times across particular sections of a road, accurate speed data are desirable.

The FTMS data acquisition systems are based on the use of vehicle detectors in the roadway, with stations perhaps 0.8 km apart. Some systems and locations use closely spaced (e.g., 6-m separation) pairs of detectors that are capable of calculating speeds on the basis of the time taken to cross the gap between the detectors. Other systems or locations consist of only single detectors at a station. The single-detector stations are able to measure flow rates and the percentage of time the detector is occupied by a vehicle (occupancy), but not speed. Speed must be calculated on the basis of the measured variables. In these cases, one needs to be sure that the calculation procedures are reliable.

In this paper, the accuracy and reliability of the normal calculation procedures are investigated, using data obtained from two systems with paired detectors, which therefore provide direct measurement of speeds. First, the source of the data that will be used for these analyses is described; then, the current procedure for calculating speed is identified and tested with the data. Because problems with the current procedures will be identified by the analysis, possible explanations are examined, including a discussion of a new interpretation of freeway traffic flow, based on catastrophe theory. The implications of these results for traffic flow theory will be considered; and finally, conclusions will be presented.

DATA SOURCE

The data were obtained from two separate FTMSs along Queen Elizabeth Way (QEW) in Ontario, Canada. Both systems recorded volume counts, occupancies, and speeds every 30 sec, 24 hr a day.

The first system is at the Burlington Skyway, a portion of the QEW that goes over the entrance to Hamilton Harbour. Because of the need to allow shipping to clear the skyway, it consists of a 3-percent grade for roughly 1.2 km, symmetrically about the shipping canal. At the time these data were recorded, the system was collecting data at each of only six southbound and six northbound stations, although the full FTMS will incorporate more stations. Data have been used from three stations with different grades, but the bulk of the analysis has been done with data from a level station (NB7) just before the beginning of the skyway structure. It is worth noting that extended congestion arises on the skyway only during incidents.

The second system from which data were used is the Mississauga FTMS, eastbound on the QEW approaching Toronto. This is a relatively flat section of roadway on which there is recurrent daily congestion from commuter traffic because of several heavily used entrance ramps (which are metered as part of the FTMS).

TEST OF SPEED CALCULATION PROCEDURES

Calculation of Speeds

In the absence of pairs of closely spaced detectors to collect speeds directly, speed is calculated on the basis of flow and occupancy:

\[ \text{speed} = \frac{\text{flow}}{(\text{occupancy} \times g)} \]  

(1)

where \( g \) is a constant to convert the units to their proper values and is related to mean vehicle length plus detector size.
This procedure is used, for example, on some of the Los Angeles area freeways, where g is apparently calibrated during free-flow conditions when speeds can safely be assumed to be known (personal communications from California Department of Transportation, District 7, August 1987 and May 1988). Equivalent procedures are identified by Courage et al. (1) and by Mikhalkin et al. (2).

Given the data obtained from the QEW, it is possible to test the validity of this approach. The obvious way to do this is to compare the estimated speeds (found using Equation 1) with those actually obtained by the detector pairs. There are two possible flaws: (a) a consistent difference, suggesting the wrong g value had been used, and (b) a systematic change in the difference, suggesting that in fact g is not a constant. The first flaw is easily corrected, and therefore not of much interest. Mikhalkin et al. (2) note that Equation 1 gives a biased estimate, but the magnitude of the bias (0.6 mph or 1.0 km/hr) is small compared to the variation in the data. The second flaw is the more important one. If the purpose is to test whether g is indeed constant across the range of operations, this test can be done directly with values of g calculated from the data. Both speed and occupancy are indicators of traffic conditions, but because speed is the item at issue here, occupancy has been used as the variable against which to inspect whether in fact the "constant" g varies.

The "Constant" g as a Function of Occupancy

Station NB7 on the skyway, just before the beginning of the upgrade of the bridge, northbound, was selected for the detailed part of this analysis. The observed values of speed, flow, and occupancy were used to calculate g as shown in Equation 1 for data drawn from several days, including six incidents that caused congestion. The results, as calculated separately for each of the 30-sec intervals, are displayed in Figure 1. There is considerable scatter in the data, particularly for the extreme values of occupancy.

However, there is a trend in the results, and this trend is perhaps more easily seen by looking simply at the mean values of g at each occupancy value, as shown in Figure 2. No attempt was made to fit a regression line, largely because there is no theory to suggest what shape such a line should have. For occupancies from 1 to 37, most sample sizes were large enough to permit good estimation of the mean at each occupancy value. For occupancies above 37, means have been calculated for intervals of 2 or 3 percent because the number of observations at each occupancy is smaller; and for occupancies above 60 percent, for an interval of 5 percent. The figure also shows an approximation of the 95 percent confidence limits for the mean at each observation. (Because sample sizes at each occupancy varied considerably, rather than introduce a table of t-statistics into the calculation, the value for a sample size of 10 was used to simplify the estimation. This gives a conservative—i.e., wider than actual—confidence interval in most cases. The square root of the actual sample size was, however, used.)

One of the two extreme points in Figure 2, that at 1 percent occupancy, may be spurious, in that at this low occupancy most of the flows will be based on a single observed vehicle. Thus round-off error in either occupancy or speed, or the discreteness of the volume counts, will contribute considerably to the calculation of g. At the other extreme, high occupancies, the confidence intervals are wider, in part because of the smaller sample sizes. However, the fact that there is a consistent trend over all of the observations for these higher occupancies overcomes those wider intervals and increases confidence in the result.

Despite the problems at the extremes, however, these results appear to support three important points: (a) for most of the range of uncongested occupancies (roughly the 8 to the mid-20 percent range), the variation in calculated g is minimal (Figure 1), and for most of this same range (8 to 20 percent) the mean value of g appears not to change appreciably with occupancy (Figure 2); (b) for higher occupancies, all of which are associated with congested operations, g is subject to considerably more scatter, and the mean value appears to decline with increasing occupancy; and (c) for very low occupancies, as occupancy decreases the range of g values increases and the mean value also increases. In short, the ratio of flow rates to the product of speed and occupancy is not constant, but decreases in a regular fashion as occupancy increases, through two portions of the range.

In an effort to confirm this result, data from three other locations were also analyzed. The first two are additional stations at the skyway FTMS; the third is one station at the Mississauga FTMS. The first is Station SB7, opposite Station
NB7 at the downhill end of the skyway grade. The results (Figure 3) show considerably less regularity than did the NB7 data, both in the presence of wider confidence limits (the total sample was only half the size) and in the absence of a region of roughly constant \( g \). Nonetheless, the main conclusion from the NB7 analysis is clearly supported by these data as well: \( g \) is not constant over the range of occupancies; rather, it tends to decrease as occupancy increases. An earlier analysis also found that the mean value for the 8 to mid-20 percent range is a bit higher than at NB7 (3), which was attributed to the higher mean speeds found at the foot of an extended downgrade compared to those on a level roadway.

The second additional station is SB5 on the skyway, located two-thirds of the way up the grade (Figure 4). Here, the mean of \( g \) behaves in very similar fashion to that at NB7, in that there is a range over which the value seems to be fairly constant. (Note that in this range, it is lower than at either SB7 or NB7.) Station EB16 of the Mississauga FTMS (Figure 5) is a station upstream of the main bottlenecks for the commuting traffic, so is a reflection of the effects of recurrent congestion, rather than of incident-caused congestion. Nevertheless the pattern is the same as that originally found at NB7: as occupancy increases from very low values, \( g \) declines steeply, briefly; then levels off and remains constant until congestion begins; at which point \( g \) decreases again, although perhaps not so steeply.

**DISCUSSION OF RESULTS**

The conclusion from the preceding analyses is that the use of Equation 1 with a single value of \( g \) will not produce good estimates of speed. This conclusion is clearly the case from these results for any single station across a range of operating conditions. It is also the case, shown incidentally here and in more detail in an earlier paper (3), that a single value of \( g \) is not appropriate across several stations, at least if there are grade changes from station to station. The obvious next question is why. According to conventional traffic flow theory, such results should not arise. Three possible explanations are discussed here: the first looks simply at possible measurement errors; the second looks at some of the assumptions in conventional theory behind Equation 1 and the extent to which they are contradicted in practice; and the third is a summary of a new model of freeway traffic flow, based on the mathematical approach called catastrophe theory.

**Measurement Error**

The simplest possibility is that measurement errors in the data acquisition have caused these results. The problem with this explanation is that it needs to account for the changing nature of the error as traffic conditions change as well as for the changes across the different stations. Although this explanation can account relatively easily for changes across stations, it does not easily explain the variations at a single station. There would have to be a systematic error in the data acquisition that increases with decreasing speed (especially during congestion and at very low flows) to produce the results described above.

To investigate this possible source of error, we used the closed circuit television (CCTV), which is part of the skyway FTMS, to record travel across a known distance (roughly 110 m), just upstream of the NB7 detectors. Only a limited amount of timing of vehicles was done from the tape, covering a total...

![Figure 3: The mean of \( g \) and its 95 percent confidence interval versus occupancy for skyway Station SB7 data.](image)

![Figure 4: The mean of \( g \) and its 95 percent confidence interval versus occupancy for skyway Station SB5 data.](image)

![Figure 5: The mean of \( g \) and its 95 percent confidence interval versus occupancy for QEW Mississauga Station EB16.](image)
of 317 vehicles over 27 30-sec intervals. There was a minor problem in matching the VCR times against the detector timing because the time recorded on the videotape was not precisely synchronized with the computer clock, but a close match was found.

Over the range of occupancies from 4 to 17 percent, all speed differences for the 30-sec intervals were less than 6 km/hr. For those occupations with multiple observations, the averages of the differences were all less than 2 km/hr. Given the time offset, this seemed a very good match. At the lowest occupancies (4 to 7 percent), the match between VCR and detector speeds was so close that this result alone is enough to refute the hypothesis that there is a systematic error in detector-based speed measurement that would lead to underestimation of speeds and consequent inflation of the value of the "constant" $g$. At high (congested) occupations, there were only two data points (at 34 and 57 percent occupancies). Although the detector-based speeds were higher than the VCR speeds in both cases, the difference was less than 10 km/hr, which is not high enough to account for the change in the value of $g$ at higher occupancies. Further, the congested speeds would be most affected by the slight difference in location of the VCR speed trap from that of the FTMS detectors. On the whole, then, measurement error does not seem to be able to account for the change in the mean value of $g$ as occupancy changes.

**Assumptions Behind Equation 1**

To calculate speed from flow and occupancy information, Equation 1 relies on two major assumptions. The first is the so-called fundamental equation of traffic flow:

$$\text{flow} = \text{speed} \times \text{density} \tag{2}$$

The second assumption is that occupancy and density are linearly related:

$$\text{occupancy} = c \times \text{density} \tag{3}$$

As the following discussion shows, neither of these assumptions is met by actual traffic across the full range of operations.

The fundamental equation (Equation 2) assumes that traffic flow is uniform (i.e., that there are constant vehicle speeds and spacing), at least within substreams of the traffic. In congested conditions, this assumption clearly is not met. Individual vehicle speeds change frequently, with irregular acceleration and deceleration. The spacing between vehicles also changes rapidly, as queues alternate compress and relax. It is not clear whether Equation 2 should be expected to hold for very low flows. When there are only a few vehicles on a freeway, the notion of substreams with constant spacing makes no sense. Nor is the full traffic stream one of uniform flow. As the Highway Capacity Manual expresses it (5, pp. 1-3), "Individual users are virtually unaffected by the presence of others in the traffic stream. Freedom to select desired speeds and to maneuver within the traffic stream is extremely high."

As a result, not only are speeds of different vehicles unrelated, but at these low flow conditions the spacing between vehicles is random rather than regular. Hence the fundamental equation may not be valid for very light traffic and is clearly not valid for congested operations—the very conditions under which the speed $g$ does not behave as expected. Note, however, that conditions approximating uniform flow clearly do occur for high uncongested flows, such as say from 1,500 veh/hr in a lane up to capacity. Judging by Figures 1 to 5, it may in fact be a good approximation for operations down to perhaps 8 percent occupancy.

Likewise, the assumption that occupancy is a constant multiple of density is valid only under limited conditions, the most important of which are that vehicle lengths and speeds are constant. This dependence on the assumptions can be shown when these possibilities are introduced into Athol's original derivation (6) of the relationship between occupancy and density, as has been done earlier by one of the authors (7), as follows. Occupancy is the ratio of the sum of time taken by all vehicles to cross a detector (which includes not only the time to cross the detector, but also the time the vehicle covers the detector) to the total time of measurement. Let

- $k$ = density
- $u$ = space mean speed for vehicles passing in $T$
- $q$ = flow rate in vehicles/hour, expanded from time $T$
- $u_i$ = speed of vehicle $i$
- $x_i$ = length of vehicle $i$
- $d$ = effective detector length

Then

$$\text{occupancy} = \frac{(\text{sum} (x_i + d)u_i)/T}{x_i u_i/T + \text{sum} (d u_i)/T} \tag{4}$$

Following Athol, it is helpful to multiply the second term by $n \times (1/n)$:

$$\text{occupancy} = \frac{(\text{sum} (x_i/u_i)/T + d \times (1/n) \text{sum} (1/u_i)*n/T)}{x_i/u_i/T + d \times u^{-1} \times q} \tag{5}$$

Assuming that the fundamental equation holds, this becomes:

$$\text{occupancy} = \frac{\text{sum} (x_i/u_i)/T + d \times k}{x_i u_i/T} \tag{6}$$

Noting that $T$ is simply the sum of the individual vehicle headways, $h_i$, and multiplying top and bottom by $1/n$ gives

$$\text{occupancy} = \frac{\text{mean} x_i/u_i/\text{mean headway} + d \times k}{x_i/u_i/\text{mean headway}} \tag{7}$$

Athol assumed uniform vehicle length $(x)$, which gives

$$\text{occupancy} = x \times \text{mean} (1/u_i)/\text{mean headway} + d \times k \tag{8}$$

but since the inverse of mean headway is the flow rate, this becomes

$$\text{occupancy} = x \times u^{-1} \times q + d \times k = (x + d) \times k \tag{9}$$

Thus for a uniform vehicle length (and at a single detector location), occupancy is a constant multiple of density. Likewise, for uniform vehicle speeds Equation 9 is still valid, if $x$ is taken to be the mean of the vehicle lengths. However, if
both vehicle lengths and speeds vary, then Equation 9 is not strictly correct.

The analyses in this paper were restricted to the median lane in part to limit the variation in both speeds and vehicle lengths. (Trucks are prohibited from that lane in both FTMS sections.) Nevertheless, there is obviously some variation in both, and this undoubtedly accounts for a large part of the scatter in the data. In addition, it is worth pointing out explicitly that the relationship in Equation 9 depends at several steps on the fundamental equation, which holds true over only a single part of the range of occupancies.

As a result of the violation of these key assumptions under actual operating conditions, one should perhaps have expected Equation 1 to be correct only under limited conditions. This is in fact what has been found. The good news is that those conditions cover a wider range of occupancies than might have been expected.

An Alternative Model

The conventional understanding of traffic flow theory is, then, inadequate for explaining why g in Equation 1 varies. The fact that key assumptions are not met explains why the conventional understanding is not adequate, but leaves one looking for a better theoretical understanding. One recently proposed model (8,9), based on the mathematics of catastrophe theory, offers some promise in this context and is therefore worth a brief discussion here.

The first point to note about this new model is that, in contrast to the standard treatment of traffic flow theory, it uses occupancy rather than density. There are two reasons for using occupancy: first, occupancy is used in FTMS logic, so it makes sense to build occupancy into theory as well as practice; and second, density is difficult to obtain accurately. Three methods have been used, but all have their shortcomings. Density can be measured directly, but such measurement is much too expensive to do on a regular basis. Even when measurement is done, density must be measured over a large space whereas speed and flow are commonly point (or very short distance) measures, which leads to incommensurate data. (Occupancy on the other hand is relatively easily obtained and is commensurate with the speed and flow measures.) The previous section discussed the flaws in the other two methods: calculation from the fundamental equation and from the presumed constant relationship with occupancy.

Even with density replaced by occupancy, the standard depiction of relationships among the three key variables (5, Figure 1-1), when considered in a three-dimensional context, implies something like a horseshoe, located at an angle to the orientation of the three axes (speed, flow, occupancy). The catastrophe theory model on the other hand represents operations as taking place on a partly folded (or split) surface. The original derivation of this surface mathematically comes from work by Thom (10), explained subsequently by Zeeman (11) and Saunders (12) among others. This model has led to a new logic for incident detection with FTMS data, which has proven remarkably robust in preliminary trials (9).

Recent work by Gilchrist (13,14) has provided some very strong support for the model, including the feature that is a key one for explaining the failure of Equation 2 in congested data: that the congested and uncongested data lie on different planes, which meet at an angle and which do not both correspond to the surface described by the fundamental equation. Gilchrist has worked with the data in a three-dimensional graphical representation, for Station EB16 on the QEW in Mississauga, and then has rotated that representation to get a better picture of how the data actually occur. One consequence of this work has been to confirm the planar nature of the bulk of the uncongested data. It is clear from his work that all of the scatter within the uncongested data lies on a single plane, and that the congested data do not lie on that same plane. This observation is entirely consistent with the catastrophe theory model and is not accommodated by the conventional theory.

Summary

Three possible explanations have been discussed for the failure of Equation 1 to calculate speeds accurately across the full range of operations. It seems clear that measurement error is not the source of the problem. The speeds calculated from the detector data have been verified by CCTV videotaping. Hence the problem is in Equation 1. It turns out that two key assumptions underlying the equation are not in fact met by normal freeway operations. Because at least one of those assumptions is fundamental to conventional traffic flow theory, another possible model has been considered briefly. This model is consistent with the findings about speed estimation: uncongested data appear to lie in a different plane than do the congested data. Thus this discussion has shown clearly that the results of the analysis in this paper are not only reasonable but perhaps even to be expected.

IMPLICATIONS

Three practical implications follow from these results. The first two should be of concern to those responsible for FTMS; the third is important for traffic flow theory.

Estimation of Speeds Using g

Many systems have only single-loop detectors, yet still wish to obtain estimates of speeds. The question in the past for such systems has been simply what value of g to use. One practice has apparently been to calibrate g when traffic approximates free-flow conditions on the grounds that speeds can be reliably estimated then, whereas they cannot be reliably estimated under other operating conditions. The results of the current analysis suggest that if this type of calibration is done for occupancies of around 10 percent, the resulting value of g is probably a reasonable one for most uncongested conditions. However, if the calibration is done for lower occupancies, there would appear to be a good chance that g has been overestimated. For example, Figures 2 through 5 suggest that the mean g for occupancies of 4 to 7 percent is 8 to 10 percent higher than the value for occupancies of 10 to 20 percent. Hence if g was calculated for the lower occupancies,
speeds during those higher occupancies would tend to be underestimated by a similar 8 to 10 percent.

Even if \( g \) is calculated using data for occupancies of 10 to 20 percent, there will be a systematic bias in calculating speeds during congestion. In Figure 6, measured and estimated speeds for such value of \( g \) are compared. (The mean value of \( g \) for occupancies from 5 to 25 percent has been used to calculate speeds for the full range of data for skyway Station NB7.) On first glance, this figure suggests the estimates are not bad, but a closer look at high and low values of estimated speeds shows the problems. The magnitude of the error can be seen more easily in Figure 7. For low speeds (i.e., those during congestion), the negative errors show that the estimated speed is consistently lower than the observed speed. At high speeds, there is a consistent overestimation of speeds. If the value of \( g \) were taken from some other range of occupancies, the location of the points in Figure 7 would just be shifted up or down relative to zero error.

One unusual aspect of Figure 7 that merits comment is the vertical set of data at 81.4 km/hr, as well as the small ranges of excluded values of estimated speeds either side of these data. The vertical array at 81.4 km/hr arises because 120/1.475 is 81.4; 1.475 is the value of \( g \) used to calculate the estimated speeds, and 120 is the expansion factor used to obtain hourly flow rates from 30-sec volume counts, so all of these observations arise when occupancy is identical to the 30-sec volume count. The excluded ranges arise because flow and occupancy are in fact not independent variables. The pairs of values that would result in speeds in these ranges simply do not occur in the data.

**Estimation of Vehicle Lengths**

When speed, flow, occupancy, and detector length are all known, vehicle lengths are calculated by some FTMSs (15) as

\[
x = \frac{(u \times \text{occupancy})}{q} - d
\]  

(10)

This equation, however, is derived from Equation 9 (i.e., the assertion of a linear relationship between occupancy and density). Because it has been shown that this relationship is approximately true over only part of the range of operations, calculation of vehicle lengths is likely to be reasonable only over that same range. In practice, the “constant” necessary to correct the units in this calculation will undoubtedly behave very much as \( g \) has in the above analyses.

**Speed-Flow Diagrams**

One intriguing question that these results raise, but to which we do not have a clear answer, is the extent to which earlier and ongoing work on speed-flow relationships was and is based on speed data calculated using Equation 1. Certainly if a value of \( g \) calculated from very low occupancies were used, the resulting calculated speeds would seem reasonable and would suggest a relationship that is more parabolic than the data presented in more recent papers.

An example of this is shown in Figures 8 and 9, using the Mississauga EB16 data. For these speed calculations, a value
The safest procedure to follow for single-detector systems would appear to be to do without speed estimates. Although speeds are probably the clearest indicator of a breakdown in operations, flow and occupancy are equally important variables and are more reliably obtained. Incident management identification has worked quite well in the past using only these two, so there may be no need to calculate speeds.

On the other hand, speed estimates are valuable and are worth some effort to approximate well. If the constant for a given detector location can be estimated for occupancies of 10 to 20 percent, then the results in this paper suggest that the speed estimates for most uncongested flow will be quite reasonable on average. Figure 7 can provide an estimate of the correction that would need to be made for very high or low estimated speeds to bring them back to a more likely value. Alternatively, a variable value of $g$ can be used for very low occupancies and for occupancies during congestion. In Figure 2, the general nature of the variation is suggested. With either approach, a larger sample from more locations would be necessary before definitive correction factors can be provided. It is important to note, however, that these estimates would not be good enough for incident detection or any other application requiring accurate short-duration speed estimates. The estimates would only be reasonable on average.

A better approach in the longer term is to develop a new relationship among the three variables. Our own work along these lines builds on the catastrophe theory model of traffic operations, but other approaches may also be productive. It is too early to offer any good answer to this issue.

The main conclusion has important implications for the design of new FTMSs. Unless a reliable set of sliding values for $g$ or a new equation can be identified, single-detector data acquisition should not be used if knowledge of vehicle speeds is thought to be at all important. The apparent cost savings from single-detector versus paired-detector stations represent a false economy in that such systems probably cannot provide good indications of vehicle speeds. Particularly if speed is to be used in an incident-detection algorithm, reliable speed data are essential. The current approach using data from single-detector stations cannot provide reliable speed data.

CONCLUSIONS

The most important conclusion from this paper is that calculating speed as

$$\text{speed} = \frac{\text{flow}}{(\text{occupancy} \times g)}$$

gives biased results. The particular results describing the bias as a function of occupancy are based on limited data. More data are needed before specific proposals can be made for a way to modify that equation (or the value of $g$) to overcome this effect. The general conclusion, however, is supported not only by those data but also by the discussion of the reasons for these results. It is clear that one of the assumptions used to derive this particular equation is not valid in congested traffic, and that the other is not strictly correct when both vehicle lengths and speeds vary. Hence this important conclusion is stronger than the somewhat limited data used in the first instance to test it.


Presented at the 1988 Transportation Research Board Annual Meeting.


Publication of this paper sponsored by Committee on Freeway Operations.