

# Bearing Capacity Approach to Railway Design Using Subgrade Matric Suction

PAMELA SATTLER, D. G. FREDLUND, M. J. KLASSEN, AND W. G. ROWAN

A bearing capacity type design procedure for assessing the stability of the track subgrade is presented in this paper. The current phase of the research concentrates on the incorporation of the matric suction term into a bearing capacity design procedure. This permits the design procedure to use the additional strength of the soil resulting from the matric suction in the subgrade. A previous research program has enabled the measurement of the matric suction in the subgrade of a tie-track system. Design charts have been produced for various train loads, subballast thicknesses, soil types, and design matric suction values. The design charts give a factor of safety against bearing capacity failure as a function of the above parameters. Stresses in the subgrade are predicted using the computer program GEOTRACK for various design train loads, subballast thicknesses, and soil parameters. The ultimate bearing capacity is determined using bearing capacity theories that have been modified to accommodate the layered track system and also incorporate the additional strength of the soil resulting from matric suction. A comparison of predicted stresses and the bearing capacity defines the factor of safety against a bearing capacity failure.

The design of an adequate railway system has developed over a long period of time. In fact, railways were built before the advent of modern soil mechanics. It is not surprising, therefore, that the technology associated with railway design has remained largely empirical, from a soil mechanics standpoint. Although this approach has served quite well, Canadian Pacific Railways, Canadian National Railways, and the Transportation Development Center have embarked on research programs at the University of Saskatchewan that give consideration to the benefits that could accrue from the application of modern soil mechanics knowledge to railway design.

The present research program has concentrated on the effect of soil suction in the subgrade of the track structure. Soil suction is defined as negative pore-water pressure referenced to the pore-air pressure. The pore-water pressure above the groundwater table is negative and when referenced to the pore-air pressure becomes a variable that varies in response to the surrounding microclimate. Figure 1 illustrates typical negative pore-water pressure profiles in the upper layers of a soil.

Previous research conducted at the University of Saskatchewan for the railway companies has enabled the measurement

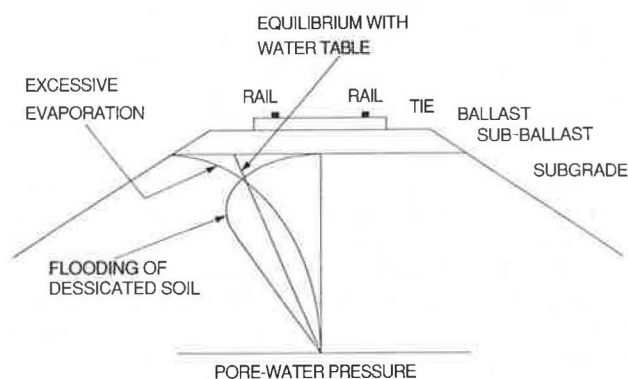


FIGURE 1 Idealized pore-water pressure profiles beneath a railroad track.

of soil suction in the subgrade of a tie-track system (1). The objective of a recent research program was to develop a bearing capacity design procedure for the railway ballast-subballast-subgrade system, incorporating soil suction into the design procedure. This permits the design procedure to use the additional strength of the soil resulting from soil suction. Further details on the bearing capacity design procedure that has been developed are discussed in a report by Sattler and Fredlund (2).

The design procedure that has been developed is a bearing capacity approach to the problem. Previously, railway design has concentrated on minimizing stresses in the rails, in the ties, and in the subgrade. The bearing capacity approach provides a measure against which the subgrade stresses can be compared. The strength of the subgrade is computed using a conventional bearing capacity equation with the incorporation of the soil suction term, whereas the stresses in the subgrade are estimated using a computer stress model. A comparison of subgrade strength and subgrade stresses provides bearing capacity factors of safety for various subgrade properties and loading conditions. It is the development of this bearing capacity approach that is given primary consideration in this paper.

## HISTORY OF BEARING CAPACITY APPROACH TO RAILWAY DESIGN

An examination of the literature reveals two facets to the railway design problem: (a) the predictions of stress distributions beneath the tie-track structure and (b) the evolution

P. Sattler and D. G. Fredlund, Department of Civil Engineering, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 0W0. M. J. Klassen, Canadian Pacific Railways, Windsor Station, P.O. Box 6042, Station A, Montreal, PQ, Canada H3C 3E4. W. G. Rowan, Transportation Development Center, 200 Dorchester Boulevard West, Suite 601 West Tower, Montreal, PQ, Canada H2Z 1X4.

of the bearing capacity equation to include layered systems and subgrade soil suction.

The most notable contributions to railway design are those of Talbot (3–9). Talbot assumed that the rail-tie system acted as a beam continuously supported on a homogeneous, elastic medium. This became known as the beam-on-elastic-foundation approach to railway design. At the same time that railway design approaches were being developed, Boussinesq was developing similar procedures for computing stress distributions beneath a loaded structure for modern soil mechanics (10,11). The first contribution to the development of theories of stress distribution beneath a layered system was presented by Burmister in his analyses of airport runways (12). The advancement of the digital computer in the 1970s resulted in the development of numerous models to predict the stresses beneath the complex tie-track structure.

A model combining Burmister's three-dimensional elasticity solution and a structural analysis model that solves for the tie-ballast reaction was proposed by Kennedy and Prause in 1978 (13). Their MULTA model predicts reasonable values for tie loads and tie-ballast pressures; however, accurate modeling of the ballast and subgrade system becomes more difficult. Developers of the GEOTRACK model (14) chose to adopt the MULTA model's representation of the tie and rail components because the structural analysis model provided accurate results. However, modeling of the ballast-subgrade system was replaced with a model representing a series of continuous plates (14). The GEOTRACK model was chosen for predictions of stress distribution and is later discussed in more detail.

Characterization of soil strength beneath a loaded structure has conventionally been expressed as a bearing capacity. The first equations for bearing capacity by Terzaghi (15) were later extended by Meyerhof to include the effects of foundation shape, eccentric loading, base roughness, and varying groundwater conditions (16–18). The extension of bearing capacity theories to multilayered systems was first attempted by Broms in 1965 for applications to highway systems (19). More recently, work by Hanna and Meyerhof (20) has extended the conventional bearing capacity equation to layered soils treating the upper granular material as a continuation of the footing that punches into the weaker subgrade material, in many ways like a driven pile.

The extension of conventional soil mechanics to unsaturated soil mechanics has opened the way for determining the bearing capacity of railway subgrades in a less empirical manner. The soil suction term can be incorporated into the bearing capacity equation much like shear strength extensions of unsaturated soil mechanics. The result is that the ultimate bearing capacity can now be reasonably estimated from measurements of the subgrade shear strength parameters and the subgrade soil suction. Stress distributions can now be predicted with reasonable reliability using computer models. A comparison of predicted stress distributions and estimated bearing capacity provides a measure of the factor of safety against failure of the railway subgrade.

## PREDICTION OF STRESS DISTRIBUTIONS

The prediction of the three-dimensional stress state beneath a loaded tie-track structure is accomplished through the use

of the GEOTRACK computer model (14). The GEOTRACK model emphasizes the geotechnical behavior of the tie-track structure and provides a reasonable representation of the soil layers. Figure 2 illustrates that the rails are modeled as elastic beams supported by ties that are also represented as elastic beams. The ties are divided into 10 segments, each segment capable of transmitting a load to the ballast surface. Each soil layer is characterized by a flexible plate of a given modulus of elasticity, Poisson's ratio, and stress-dependent modulus equation. Reasonable predictions for the stress distributions beneath the loaded tie-track structure are provided by the GEOTRACK computer model.

The first step in using the GEOTRACK model for predictions of stress distributions beneath the loaded tie-track structure was to perform a sensitivity analysis on the computer generated output to the variations in input parameters. Stewart and Selig (21) published the results of a sensitivity analysis for the GEOTRACK model. The sensitivity analysis performed at the University of Saskatchewan confirmed the results of Stewart and Selig.

The parameters considered were (a) axle load, (b) ballast E modulus, (c) subgrade E modulus, (d) granular depth, (e) tie spacing, and (f) tie modulus. Although significant sensitivity was determined for tie spacing and tie modulus, the tie spacing was fixed at 0.508 m (20 in.), and the tie modulus was fixed at  $1.65 \times 10^7$  kPa ( $2.4 \times 10^6$  psi). Therefore, the design procedure was documented so that the user could change parameters that would vary from the standards assumed. The vertical stresses were used for comparison to the bearing capacity results. Therefore, all sensitivity analyses used the vertical stress distribution as a guide to determine parametric sensitivity.

The sensitivity analysis determined which parameters should be varied for the purposes of developing design charts. It was decided to keep the ballast modulus constant at 241 MPa, to vary the subballast modulus from 103.5 to 241 MPa, and to

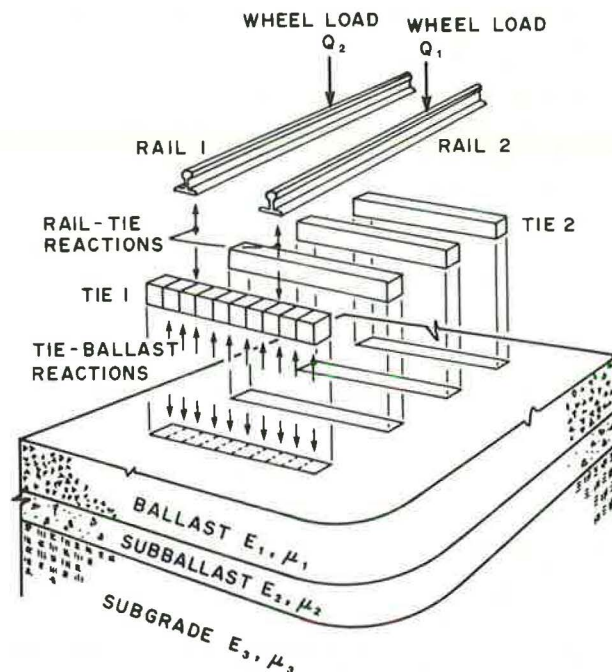


FIGURE 2 Forces and elements in the GEOTRACK model.

vary the subgrade modulus from 6.9 to 103.5 MPa. Two ballast depths of 203 and 305 mm were used. Four subballast depths, 203, 406, 610, and 813 mm, were used for each case. Complete documentation on the parameters input to the GEOTRACK model can be found in a report by Sattler and Fredlund (2). A total of 64 computer runs were generated for each of six different axle loads making a total of 384 GEOTRACK computer runs.

Following the data generation process for all of the varying parameters, it was necessary to reduce the data to values that could be compared to the ultimate bearing capacity. Figure 3 illustrates typical vertical stress output from the GEOTRACK computer model for a subgrade modulus of 69.0 kPa, subballast modulus of 241.5 MPa, a total granular depth of 610 mm, and an axle load of 184 kN. A postprocessing graphics package was written for the GEOTRACK model for the purpose of plotting the output. Figure 3 represents a typical plot produced by the postprocessing package GEOPLOT.

Two geometries must be considered for bearing capacity analysis: (a) geometry parallel to the track and (b) geometry perpendicular to the track.

A sectional view of the geometry perpendicular to the track can be used to represent the two most critical failure conditions: (a) the failure of the complete tie-track system for the length of the track as shown in Figure 4 and (b) the failure

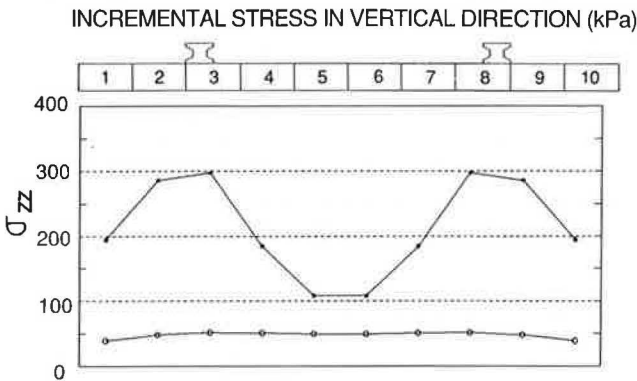


FIGURE 3 Typical vertical stress output from GEOTRACK and GEOPLOT.

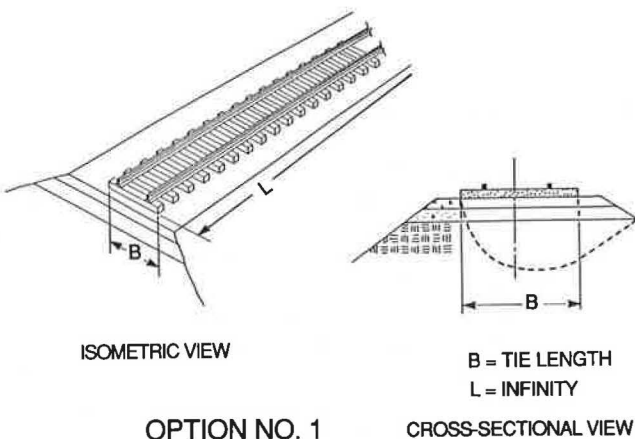


FIGURE 4 Failure geometry for the complete tie-track system for the length of the track.

of a portion of the track beneath the heavily stressed outer third of the tie and the distance of a double truck loading as shown in Figure 5. The failure conditions represented by Figure 5 are often recognized in the field where bearing capacity failures occur. The failure geometry depicted in Figure 4 is usually of lesser significance and is better analyzed using slope stability methods of analysis.

A sectional view of the geometry parallel to the track can be used to represent two failure conditions: (a) the local failure of an individual tie punching into the subgrade as shown in Figure 6 and (b) the failure of a portion of the track corresponding to the loading that occurs beneath a double truck arrangement between two cars as depicted in Figure 7. Field experience suggests that the failures represented by Figures 6 and 7 may occur in the absence of subballast for weak clay subgrades. The failure geometry depicted in Figure 7, in which the track is pushed up in a wave ahead of the moving cars, is a dynamic effect that could be important particularly where braking occurs on curves. These two failure mechanisms are presented, but emphasis is placed on the more critical failure geometries represented by Figures 4 and 5.

The failure geometry depicted in Figure 6 is analyzed by the Hanna and Meyerhof approach (20) to bearing capacity,

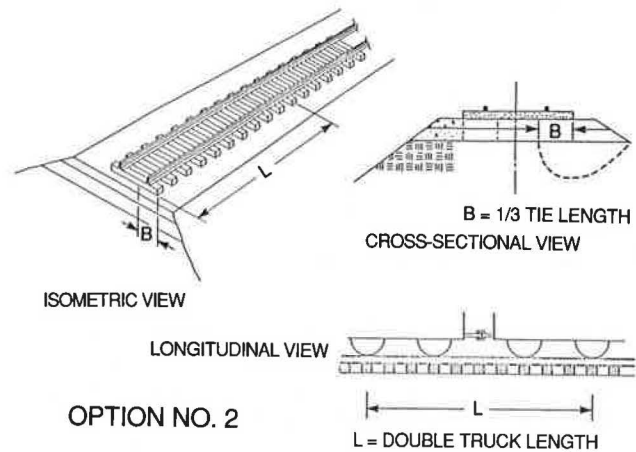


FIGURE 5 Failure geometry for the heavily stressed outer third of the tie and the distance of a double truck loading.

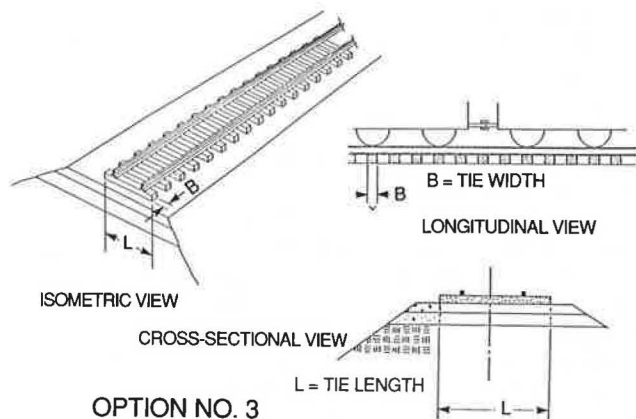


FIGURE 6 Failure geometry for the local failure of an individual tie punching into a weak subgrade.

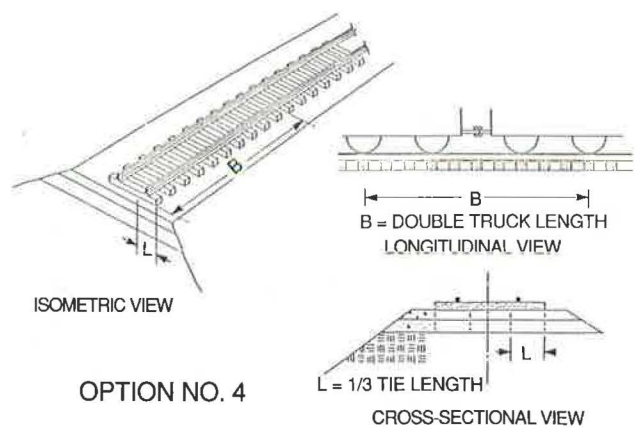


FIGURE 7 Failure geometry for a portion of the track beneath a double truck arrangement for weak subgrades.

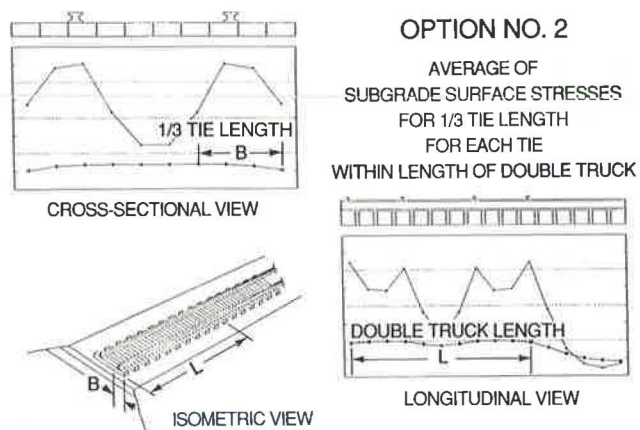


FIGURE 8 Option 2 Stress Averaging Technique.

whereas the geometries illustrated in Figures 4, 5, and 7 are all analyzed by the Broms approach (19) to bearing capacity.

The necessity to create a program that averages the stress distribution comes from the fact that a bearing capacity analysis produces a unique value to represent the strength of the soil, whereas a stress analysis program, like the computer program GEOTRACK, creates a stress distribution (i.e., several values of stress beneath several different locations of the railway track structure). Because comparison of one value for the bearing capacity to several values representing the stress distribution would result in confusion, it was decided to average the stress distribution for comparison to the bearing capacity.

There are four averaging techniques corresponding to each of the four bearing capacity options. Only the Option 2 Averaging Technique is presented in the figures. Each option corresponding to an averaging technique also corresponds to a bearing capacity loading configuration. In other words, the Option 2 Averaging Technique presented in Figure 8 must be compared to the Option 2 Bearing Capacity Loading Configuration presented in Figure 5.

Figure 8 shows the averaging technique for Option 2. The stress distribution beneath the most heavily loaded outer one-third of the tie is averaged for a distance beneath the track equal to the length of two closest trucks. For bearing capacity computations, the width,  $B$ , is equal to one-third of the tie

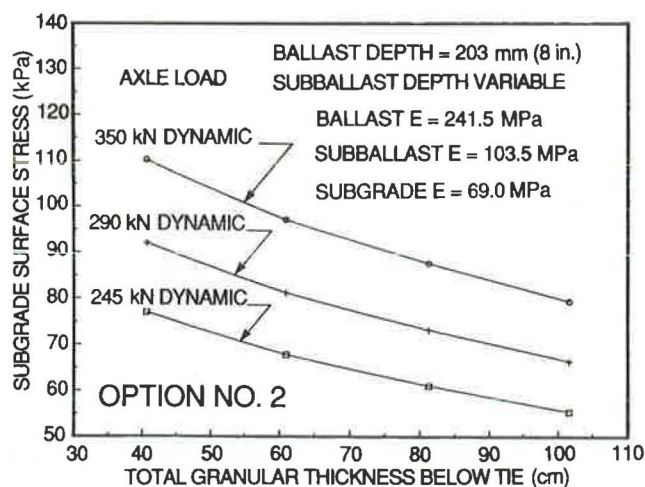


FIGURE 9 Averaged subgrade surface stress from several GEOTRACK computer runs for comparison to the Broms approach.

length and the length,  $L$ , is equal to the length of a double truck loading. The bearing capacity option corresponding to Option 2 of the Stress Averaging Technique is a Broms approach and is presented in Figure 5.

Figure 9 illustrates the results of 24 separate GEOTRACK computer runs on one set of ballast, subballast, and subgrade moduli for which the subgrade stresses have been averaged for later comparison to the Broms bearing capacity values. The three axle loads (245, 290, and 350 kN) shown in the charts correspond to (a) a 100,000-kg (220,000-lb) car, (b) a 119,000-kg (263,000-lb) car, and (c) a 143,000-kg (315,000-lb) car, respectively. For dynamic loading conditions, the axle load was increased by 50 percent to account for wheel speed and impact loadings. (In other words, a 245-kN dynamic load has been modeled as a 367-kN static load.)

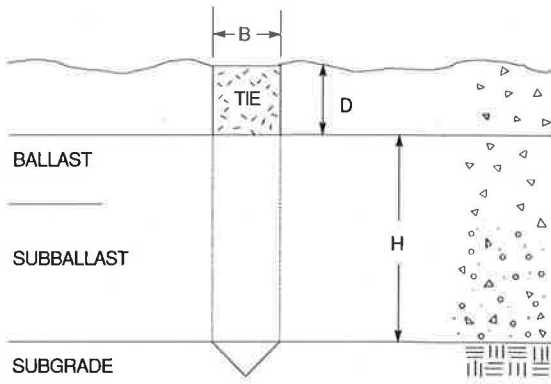
## ULTIMATE BEARING CAPACITY

Two methods are used for the estimation of the bearing capacity beneath a loaded track structure. The Hanna and Meyerhof approach applies only to weak clay subgrades, whereas the Broms approach applies to the stronger clays, glacial tills, silt, and sand subgrades. The soil suction term is incorporated into both procedures.

### Hanna and Meyerhof Bearing Capacity Approach

Figure 10 illustrates the bearing capacity equation for the Hanna and Meyerhof approach. The failure mechanism assumes that a soil mass of the upper sand layer of approximately pyramidal shape is pushed into the lower clay layer. At the point of limiting equilibrium, the sum of forces in the vertical direction yields an equation for the ultimate bearing capacity. Laboratory data collected by Hanna and Meyerhof reveal an appropriate equation for the passive earth resistance based on an assumed coefficient of punching shear (20). The resulting bearing capacity equation is as follows:

$$q_u = cN_c + \gamma_1 H^2 (1 + 2D/H) K_s \tan \phi'/B - \gamma_1 H \quad (1)$$



$$q_u = c N_c + \gamma_1 H^2 (1 + 2D/H) K_s \tan \phi' / B - \gamma_1 H$$

FIGURE 10 Illustration of the bearing capacity equation for the Hanna and Meyerhof approach.

where

- $c$  = total cohesion of the clay subgrade,
- $N_c$  = bearing capacity factor,
- $\gamma_1$  = unit weight of overlying dense sand (or ballast and sub-ballast),
- $H$  = thickness of dense sand below the bottom of the footing (or railway tie),
- $D$  = depth of embedment of the footing (or railway tie),
- $K_s$  = coefficient of punching shear resistance (determined from published charts),
- $\phi'$  = friction angle of the upper dense sand (or ballast and sub-ballast), and
- $B$  = width of footing (or railway tie, depending on the geometry considered).

The soil suction term is incorporated into the Hanna and Meyerhof equation by replacing the undrained shear strength of the clay with the more rigorous  $c'$  and  $\phi^b$  terms, illustrated as follows:

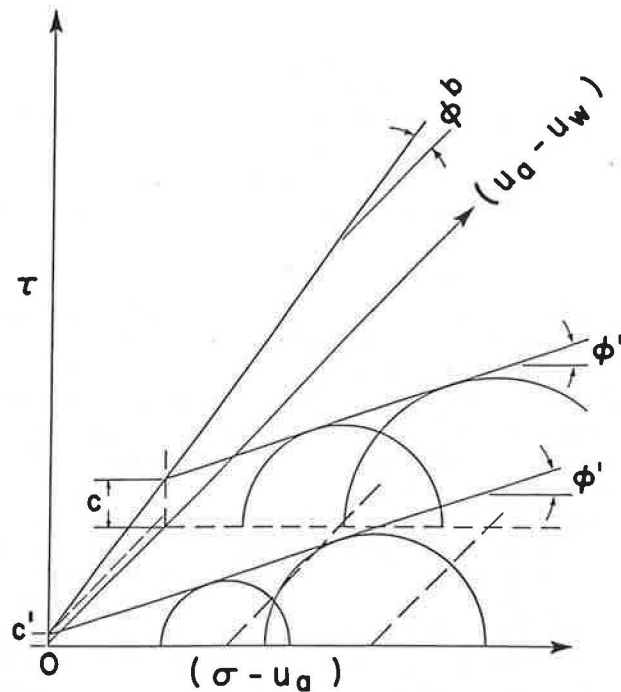
$$c = c' + (u_a - u_w) \tan \phi^b \tag{2}$$

where

- $c'$  = the effective cohesion of the subgrade,
- $(u_a - u_w)$  = design suction value for the subgrade, and
- $\phi^b$  = rate of increase in shear strength with respect to soil suction.

Figure 11 illustrates the components of the total cohesion term,  $c$ .

Kraft and Helfrich (22) suggest that the Hanna and Meyerhof approach to computing the bearing capacity is quite accurate for shallow footings. If an individual tie can be assumed to act in the same manner, then the results should also apply to the case of the geometry perpendicular to the track (Figure 6). The replacement of the cohesive strength term with unsaturated soil parameters renders the computations more rigorous because the strength can be related to microclimatic conditions. Kraft and Helfrich suggested that the Hanna and Meyerhof approach cannot be extended to stronger deposits while still maintaining reliable predictions.



$$c = c' + (u_a - u_w) \tan \phi^b$$

FIGURE 11 Components of the total cohesion term.

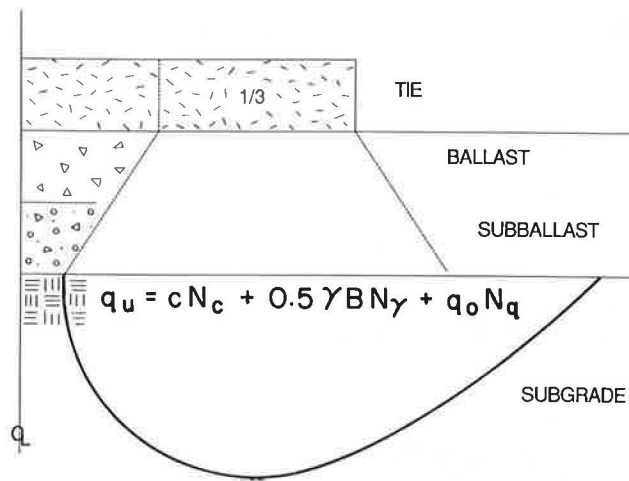


FIGURE 12 The Broms approach to bearing capacity for highway pavements.

### Broms Bearing Capacity Approach

The Broms approach to computing bearing capacity is illustrated in Figure 12. When the rail-tie system is considered as a contiguous unit placed on the ballast and subgrade, the Broms approach provides a reasonable estimate of the general bearing capacity failures that occur under field conditions. The equation for bearing capacity bears the same form as the conventional bearing capacity equation:

$$q_u = c N_c + 0.5 \gamma B N_\gamma + q_o N_q \tag{3}$$

where

- $N_c, N_\gamma, N_q$  = bearing capacity factors,
- $c$  = total cohesion,
- $\gamma$  = unit weight of the overlying ballast and sub-ballast material,
- $B$  = width of the footing (or railway tie loaded area, depending on the geometry considered), and
- $q_o$  = surcharge loading.

Again the equation can be modified to replace the undrained shear strength parameter with the unsaturated soil parameters. Broms suggested incorporation of the  $\chi$  parameter to account for the degree of saturation of the subgrade (19). The present formulation will treat the stress state variables in an independent manner, thereby eliminating the need for the  $\chi$  parameter.

The subgrade bearing capacity for various total cohesion values of a clay subgrade, as computed from the Broms approach, is illustrated in Figure 13. Figure 13 presents typical bearing capacity values for a clay with a total cohesion ranging from 0 to 100 kPa and a  $\phi'$  angle ranging from 10 to 25 degrees. The second term in the bearing capacity equation becomes negligible for the small  $\phi$  angles, and, hence, the width assumed for the tie-track system becomes insignificant to the computations.

**Bearing Capacity Factors**

The bearing capacity factors are computed from the following equations:

$$N_q = e^{(\pi \tan \phi')} \left( \frac{1 + \sin \phi'}{1 - \sin \phi'} \right) \tag{4}$$

$$N_c = (N_q - 1) \cot \phi' \tag{5}$$

$$N_\gamma = 1.5 (N_q - 1) \tan \phi' \tag{6}$$

where  $\phi'$  is the friction angle of the subgrade material. The equations are derived from Brinch-Hansen (23) and

Vesic (24). Other published equations for the bearing capacity factors could also be used.

**PREPARATION OF BEARING CAPACITY DESIGN CHARTS**

A comparison of the predicted stresses from the computer model to the ultimate bearing capacity computed from the above methods provides a factor of safety against failure.

A bearing capacity factor of safety greater than one does not imply that a bearing capacity failure could not occur. This results from the fact that the bearing capacity approach produces a single value to represent the strength of a heterogeneous subgrade system in which planes of weakness may exist. In addition, the choice of design values for the soil parameters, track structure, and loading configuration is subject to error, which may be cumulative for each of the many design values that enter into the analysis.

A bearing capacity factor of safety 2.5 to 3.0 is often used for shallow foundations. Current research and implementation of the design procedure suggest that smaller bearing capacity factors of safety be used in railway design. At present, it is suggested that the bearing capacity factor of safety should be in the range of 2.0 to 2.5. It may be necessary to change this value after more experience is obtained with its use.

Design charts for various geometries, train loads, soil parameters, and stress-dependent moduli can be produced. It is not the purpose of the authors in this paper to present the design charts, but rather to present the procedure from which charts can be produced. A typical design chart for a clay subgrade is illustrated in Figure 14. Figure 14 was produced from a comparison of the stress distributions in Figure 9 to the bearing capacity for a clay in Figure 13.

The design charts are produced from comparisons of the stress distributions for the GEOTRACK computer program to the bearing capacity estimates. The bearing capacity factor of safety (BCF) is defined as:

$$BCF = \frac{\text{Ultimate bearing capacity}}{\text{Predicted average stress}} \tag{7}$$

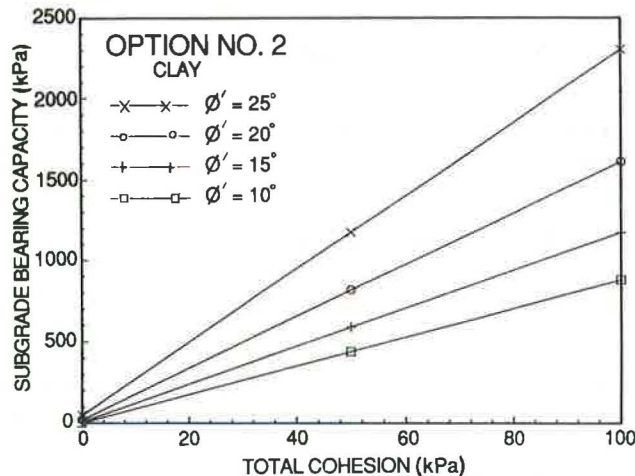


FIGURE 13 Subgrade bearing capacity as a function of total cohesion for a clay subgrade by the Broms approach.

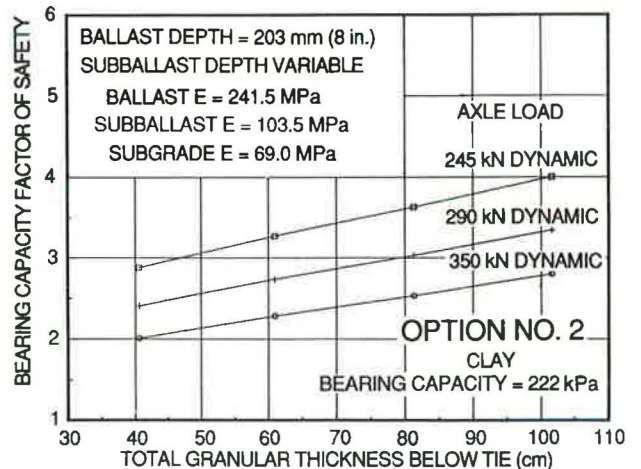


FIGURE 14 Typical bearing capacity design chart for a clay subgrade.

Computer programs have been written to perform the computations for each of the procedures used in the final production of the design charts. Complete documentation for the computer programs is presented in a report by Sattler and Fredlund (25).

## SUMMARY

In this paper, the authors illustrate a proposed bearing capacity approach to railway design that can be used with present design methods as a complementary design tool. Soil suction is incorporated into bearing capacity theories for layered systems to arrive at the bearing capacity of the subgrade below the tie-track system. Design charts can be produced for various train loads, subballast thicknesses, soil types, and design suction values. The procedure that can be used to develop the design charts is presented. The implementation of the design charts for an example location will be presented in a future publication.

## ACKNOWLEDGMENTS

The authors wish to acknowledge the financial contributions provided by Canadian National Railways (CN Rail), Canadian Pacific Railways, and the Transportation Development Center of Transport Canada. Their support has made possible this research. Sincere appreciation is expressed to W. E. Jubien of CN Rail for his contributions toward this study.

## REFERENCES

1. P. van der Raadt, D. G. Fredlund, A. W. Clifton, M. J. Klassen, and W. E. Jubien. Soil Suction Measurements at Several Sites in Western Canada. Presented to the Session on Geotechnical Problems in Arid Regions, Committee A2L06, TRB, National Research Council, Washington, D.C., 1987.
2. P. J. Sattler and D. G. Fredlund. *The Development of Bearing Capacity Design Charts for Track Systems*. Document TP9521E; Final Report. The Transportation Development Center, Transport Canada, Montreal, Quebec, 1988.
3. A. N. Talbot. First Progress Report on the Special Committee to Report on Stresses in Railroad Track. *Transactions, ASCE*, Vol. 82, or Proceedings A.R.E.A., Vol. 19, 1918.
4. A. N. Talbot. Second Progress Report. *Transactions, ASCE*, Vol. 83, or Proceedings A.R.E.A., Vol. 21, 1920.
5. A. N. Talbot. Third Progress Report. *Transactions, ASCE*, Vol. 86, or Proceedings A.R.E.A., Vol. 24, 1923.
6. A. N. Talbot. Fourth Progress Report. *Transactions, ASCE*, Vol. 88, or Proceedings A.R.E.A., Vol. 26, 1925.
7. A. N. Talbot. Fifth Progress Report. *Transactions, ASCE*, Vol. 93, or Proceedings A.R.E.A., Vol. 31, 1930.
8. A. N. Talbot. Sixth Progress Report. *Transactions, ASCE*, Vol. 97, or Proceedings A.R.E.A., Vol. 35, 1934.
9. A. N. Talbot. Seventh Progress Report. *Transactions, ASCE*, Vol. 104, or Proceedings A.R.E.A., Vol. 42, 1941.
10. K. Terzaghi and R. B. Peck. *Soil Mechanics in Engineering Practice*. John Wiley & Sons, Inc., New York, 1948.
11. H. O. Ireland. Railroad Subgrade Stresses. *Proc., American Railway Engineering Association*, Bulletin 641, 1973, pp. 382-386.
12. D. M. Burmister. The Theory of Stresses and Displacements in Layered Systems and Applications to Design of Airport Runways. *Proc., 23rd Annual Meeting of the Highway Research Board, Chicago, Ill.*, Vol. 23, HRB, National Research Council, Washington, D.C., 1943, pp. 126-148.
13. J. C. Kennedy, Jr. and R. H. Prause. Development of Multilayer Analysis Model for Tie-Ballast Track Structures. In *Transportation Research Record 694*, TRB, National Research Council, Washington, D.C., 1978, pp. 39-47.
14. C. S. Chang, C. W. Adegokpe, and E. T. Selig. GEOTRACK Model for Railroad Track Performance. *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 106, No. GT11, 1980, pp. 1201-1218.
15. K. Terzaghi. *Theoretical Soil Mechanics*. John Wiley & Sons, Inc., New York, 1943.
16. G. G. Meyerhof. The Ultimate Bearing Capacity of Foundations. *Geotechnique*, Vol. 2, 1951, pp. 301-332.
17. G. G. Meyerhof. Influence of Roughness of Base and Groundwater Conditions on the Ultimate Bearing Capacity of Foundations. *Geotechnique*, Vol. 5, 1955, pp. 227-242.
18. G. G. Meyerhof. Some Recent Research on the Bearing Capacity of Foundations. *Canadian Geotechnical Journal*, Vol. 1, No. 1, 1963, pp. 16-26.
19. B. B. Broms. Effect of Degree of Saturation on Bearing Capacity of Flexible Pavements. Presented at 43rd Annual Meeting of the Committee on Flexible Pavement Design, Ithaca, New York, 1965.
20. A. M. Hanna and G. G. Meyerhof. Design Charts for Ultimate Bearing Capacity of Foundations on Sand Overlying Soft Clay. *Canadian Geotechnical Journal*, Vol. 17, 1980, pp. 300-303.
21. H. E. Stewart and E. T. Selig. Predicted and Measured Resilient Response of Track. *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 108, No. GT11, 1982, pp. 1423-1442.
22. L. M. Kraft and S. C. Helfrich. Bearing Capacity of Shallow Footing, Sand Over Clay. *Canadian Geotechnical Journal*, Vol. 20, 1983, pp. 182-185.
23. J. Brinch-Hansen. A Revised and Extended Formula for Bearing Capacity. Bulletin 28. Danish Geotechnical Institute, Copenhagen, Denmark, 1970, pp. 5-11.
24. A. S. Vesic. Analysis of Ultimate Loads of Shallow Foundations. *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 99, 1973, pp. 45-73.
25. P. J. Sattler and D. G. Fredlund. *GEOTRACK and GEOPLOT, STRESS, BEARING, and COMPARE User's Manuals (Subgrade Stress Analysis and Bearing Capacity)*. Document TP9523E. Submitted to the Transportation Development Center, Canadian National Railways, and Canadian Pacific Railways, 1988.

---

*Publication of this paper sponsored by Committee on Railroad Track Structure System Design.*