Crane Productivity and Ship Delay in Ports

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This paper studies the effect of crane operations on ship service at port terminals. It first proposes a simple, approximate approach to calculate the maximum berth throughput during periods of congestion. The key assumption is that the workload distribution (over time) for the ships at berth is the same as the workload distribution for the ship population as a whole. The validity of this assumption is tested with simple, exact models for a variety of scenarios involving different kinds of ships and crane operating strategies. The paper then examines the effect that two extreme crane operating strategies have on ship delay, when the traffic level does not exceed the maximum throughput. This is done for an idealized situation designed to highlight the impact of crane operations while admitting closed-form solutions. The average ship delay can vary considerably with the crane operating strategy.

A port's efficiency is often measured in terms of its throughput and typical ship turnaround time (i.e., a ship's time at berth plus any delay caused by the port). High turnaround times are not acceptable in the shipping industry because of the very large opportunity cost typically associated with ship delay. However, port construction, maintenance, and equipment are also very expensive. Thus it is important for ports to set an appropriate expenditure level, and to allocate their resources efficiently among their different functions. For example, they should decide carefully how berth length should be divided among the various traffic types, and how much cargo handling equipment should be allocated to each terminal. Although such decisions often depend on factors that cannot be quantified, rational solutions should be found with an understanding of how the ships' turnaround time and the port throughput depend on different resource allocation levels.

The port elements that influence ship turnaround most directly are berth space and crane availability. Although other elements have the potential for delaying operations (tugboat unavailability and land-side congestion, for example) they are not considered in this paper.

Even though queuing theory has been applied to ports [see, for example, work by Plumlee, Mettam, Jones and Blunden, Nicolau, Miller, Koenisberg and Meyers, Daskin and Walton, and Sabria (*1–8*), and other references in Sabria's dissertation], and to the berth system in particular, no models seem to recognize explicitly the interaction between berth availability and crane operating strategies. This may be because the requirements for onshore (un)loading equipment can vary considerably from ship to ship, and may also be subject to peculiar restrictions, which complicates matters. Work by Atkins (*9*) contains one of the best descriptions of the ship loading process for modern container ports.

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The goal of this paper is to develop an understanding of the impact that different crane scheduling strategies have in the long run on maximum throughput and ship delay. To achieve this goal, we will work with a representation of the world that, although highly idealized, preserves the phenomena of interest. The paper builds on previous work (*10,11*) that used the same idealized model to develop crane scheduling strategies.

The model in these references assumed that ships were divided into holds; that each hold had a certain amount of work that needed to be done (measured in time units of crane time); that certain holds could be handled without the need for a shore crane; and that shore cranes could be moved rapidly. The objective was to assign cranes to holds to reduce ship delays. Sometimes this meant that a large ship with little need for cranes would seize the cranes working on another ship that required more work.

For the most part all the ships were assumed to be already at berth, but a case in which ships had to queue for berth space was also discussed. For this purpose it was assumed that a ship departure always freed enough space for another ship and that ships were chosen from the queue in order of arrival. A justification for all of these modeling simplifications (which are also adopted here) can be found elsewhere (*10*).

This paper attempts to take these results one step further. It studies the system's steady-state performance as a function of the ship arrival pattern when the aforementioned crane operating rules are used. It presents simple expressions for maximum expected throughput as a function of the number of cranes and total berth length. It also provides ship delay formulas when ships have to queue for cranes but the berth space is never in short supply.

The next section gives approximate expressions for the average number of busy and idle cranes during periods of congestion. These expressions lead to berth throughput and crane productivity formulas. The approximation, which is proposed for reasonably efficient crane operations, is tested with exact expressions for a special case in which all the holds requiring a crane take the same amount of time to be handled. (This assumption, which still preserves the main phenomena we want to model, is also used in later sections.)

The following section applies the results from the previous one; it compares efficient and inefficient crane scheduling strategies and examines the trade-off between cranage cost and maximum productivity.

Next is a study of ship delay for a multipurpose terminal in which ships are either self-sufficient or require, at most, two cranes. It is assumed that berth space is never in short supply [this is reasonable from a port economics standpoint

(*8*)], but ships may not always get all the cranes they need immediately on arrival. The final section summarizes the results and suggests further work.

CRANE PRODUCTIVITY

Averaged over time, the number of busy port cranes is related to cargo throughput by the relationship:

$$\text{(cargo throughput)} = \text{(busy cranes)} \times \text{(crane capacity)} \quad (1)$$

where the crane capacity is the maximum number of cargo units that a fully used crane can handle per unit time.

It is thus important to be able to predict the number of busy cranes during periods of congestion. The result can indicate the maximum possible berth throughput.

A Simple Model

We assume that there is an infinite ship queue and that the berth can hold exactly S ships. The ith ship to enter the berth is assumed to have H_i holds requiring attention. The H_i are mutually independent, identically distributed random variables with cumulative distribution function, F_H.

At any given time, the number of busy port cranes equals the minimum of two values: the number of available cranes, C, and the number of active holds, A (holds still requiring attention at the time).

If the number of active holds present at a berth at a random time has the same cumulative distribution function (cdf) as the number of holds requiring attention for S ships randomly sampled from the queue, then berth throughput can be calculated simply. The accuracy of this assumption is tested in the next section. The resulting simple throughput expressions are derived next.

Because A is distributed like the sum of S independent, identically distributed random variables with cdf, $F_{H(h)}$ A is likely to be well approximated by a normal random variable and the expected number of busy cranes by the mean of the truncated normal variable, min $\{A, C\}$:

$$E\,\text{(busy cranes)} = C - \sigma\sqrt{S}\,\psi\,([C - Sm]/\sigma\sqrt{S}) \quad (2a)$$

and similarly

$$E\,\text{(idle cranes)} = \sigma\sqrt{S}\,\psi\,([C - Sm]/\sigma\sqrt{S}) \quad (2b)$$

In these equations, m and σ^2 are the mean and variance of H_i, and ψ (*) represents the integral of the standard normal cumulative distribution function. This function is given by $\varphi(*) + (*)\Phi(*)$, where $\Phi(*)$ and $\varphi(*)$ are the standard normal cdf and probability density function, respectively [see Clark (*12*) for a derivation]. The function $\psi(*)$ is positive, increasing, and convex; it approaches 0 as its argument approaches $-\infty$, and for large positive arguments (greater than 3) its value barely exceeds the argument. See Figure 1.

Equation 2b shows that the number of idle cranes depends on only two parameters: the "average crane surplus," $C - Sm$, and the "holds at berth variability," $\sigma\sqrt{S}$. Although the expected number of idle cranes always exceeds the average crane surplus, these two are close when there is little variability.

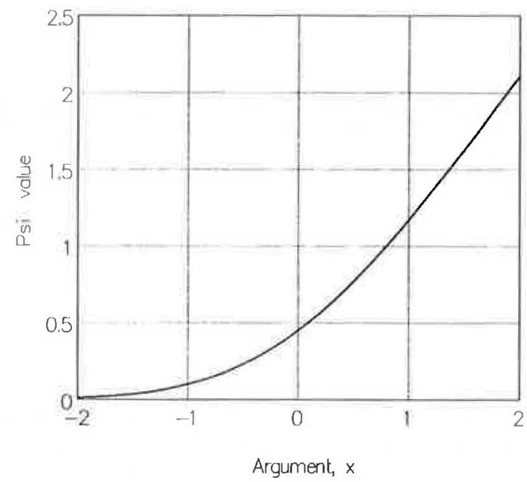

FIGURE 1 Graph of ψ (*).

Equation 2a can be used in conjunction with Equation 1 to calculate berth throughput.

An Assessment of Its Accuracy

Equations 2a and 2b are based on the assumption that the distribution of active holds per ship is the same at berth and in the queue.

Two factors that work in opposite directions (with an intensity that depends on the specific crane scheduling strategy) tend to disrupt this equality:

1. A ship's hold with little work may become inactive before the ship departs. If this happens often, it will tend to decrease the number of active holds at berth; and
2. Because, with an efficient strategy, ships with low workloads are given priority, the ships with most active holds will tend to be overrepresented at the berth. This tends to increase the number of active holds at berth.

The first factor should be most significant when the distribution of (active) hold workloads within a ship is very uneven. The second factor should be most significant when the workload changes drastically across ships.

The scheduling strategy discussed elsewhere (*10*) tends to reduce the impact of the first factor and increase the impact of the second. As mentioned in that reference, the strategy "tends to hoard at berth the holds that require work." In the remainder of this section we derive, for comparison purposes, exact expressions for two simple cases in which the first factor does not play a role and the crane allocation strategy proposed in earlier work (*10*) is used.

It is assumed that all the active holds take exactly the same amount of time (without loss of generality we take this time to be one unit) and that only one crane can work on a hold at a time. We start our observation with an empty system; thus, at time $t = 0$, the first S ships in the queue join the berth.

The preceding assumption, which is also used for the ship delay analysis, still preserves the main phenomena that we try to model; that is, because not all ships require the same number of cranes, the number of cranes needed by the ships at berth fluctuates. If at times there is a need for more cranes than the available number and at other times some cranes are idle, crane productivity is wasted. Our crane allocation rules are designed to restrain these fluctuations.

Before starting the analysis, strategy G needs to be described. For the simple cases studied in this paper (in which holds can be handled in exactly one time unit, etc.), strategy G reduces to the following:

Strategy G: Each time a new ship joins the berth reallocate all the cranes again; assign as many cranes as possible to the ship with fewest active holds; if some cranes are left, allocate as many as possible to the ship with second fewest holds; repeat this process until either no more cranes or no more ships are left.

The results of the analysis about to be presented indicate that the approximate and exact formulae are pretty close. Although the expressions should be tested further (with simulations geared to verify the importance of the first factor), the results suggest that Equations 2a and 2b may be good first-order approximations, useful for planning purposes.

Multipurpose Terminals

Two types of ships are considered in this subsection: type-0 ships that do not require the port's equipment ($H_i = 0$) and type-1 ships that require exactly one crane ($H_i = 1$). This situation could represent a multipurpose terminal. It is studied first because, with this traffic pattern, one does not require an involved crane scheduling algorithm. Allocating cranes to ships on a first berthed, first served, basis (which happens to be the result of strategy G) results in maximum productivity.

Type-0 ships spend exactly one time unit at berth, but type-1 ships may spend a little more time; they may have to wait for a crane if the berth has more type-1 ships than there are cranes. Thus Factor 2 applies. There will tend to be more active holds at berth than would be predicted from the queue, and Equations 2a and 2b should underpredict throughput.

Because all the holds take exactly one time unit to be handled, and because the system starts empty, ships and cranes move only at integer times ($t = 0, 1, 2, \ldots$). The number of active holds at berth can change only at these times. In fact, the whole system can be modeled exactly as a Markov chain embedded at integer times. The state is the number of (type-1) ships remaining at berth at the end of one period, but immediately before the next batch of ships joins the berth. It is thus possible to derive exact numerical results to compare them with the approximation.

Let p denote the fraction of type-1 ships. Then, the (i,j) element of the one-step transition probability matrix, M, m_{ij}, is:

$$m_{ij} = \Pr\{(C + j - i) \text{ type-1 ships join the berth}\}; \text{ if } j = 1, 2, \ldots, S - C$$
$$= \Pr\{(C - i) \text{ or less type-1 ships join the berth}\} \text{ if } j = 0.$$

For $j > 0$ the m_{ij} are the binomial probabilities:

$$m_{ij} = \binom{S-i}{C+j-i} p^{(C+j-i)}(1-p)^{(S-C-j)}$$

For $j = 0$, $m_{i0} = 1 - (m_{i1} + m_{i2} + \ldots + m_{i(S-C)})$

The steady-state probability (row) vector, $\mathbf{m}$, can be obtained by solving: $\mathbf{m} = \mathbf{mM}$, and ensuring that its elements, m_i, add up to 1.

The expected number of cranes in use, K, is

$$K = \sum_{i=0}^{S-C} m_i \left[\sum_{j=0}^{S-C-i} \min[j+i, C] \Pr\{j \text{ type-1 ships join the berth}\}\right]$$
$$= \sum_{i=0}^{S-C} m_i \left[\sum_{j=0}^{S-C-i} \min[j+i, C] \binom{S-i}{j} p^j(1-p)^{(S-i-j)}\right]$$

The expected number of cranes in use also gives the throughput of type-1 ships. Because the fraction of these ships is p and because ships join the berth on a first come, first served, basis, the total ship throughput, P, must be $P = K/p$.

Example: Assume that $S = 3$ and $C = 2$. In this case the calculations required by the preceding expressions are simple. We obtain the exact result:

$$P = 3 - [p^3(1 - p^2 + p^3)]$$

If the distribution of ships at berth is the same as in the queue, the number of busy cranes is the minimum of C and a binomial random variable with S trials and probability of success, p. For our case, the expectation of such a variable is $(3p - p^3)$, and

$$P = 3 - p^2$$

As expected, this expression underpredicts the exact one, but the maximum difference is only about 0.1 when $p \approx 0.6$. The error is much smaller when p is close to 0 or 1; it never exceeds 4 percent. See Figure 2.

The result derived from Equation 2a, which includes a normal approximation to the binomial, is quite close to this last expression (for $p = 2/3$, one obtains $P = 2.56$ with the last expression and $P = 2.51$ with Equation 2a). The normal approximation would be even better in a case with more berths and cranes, just when the binomial calculations become cumbersome.

In general, either approximation should be quite good if $p \ll C/S$, because then cranes are almost never in short supply and both ship types spend the same time at berth. The approximations should also be quite good when $p \gg C/S$, as then the exact and approximate formulas predict $K \approx C$. These observations are consistent with the example; the worst underprediction occurs when $p \approx C/S$.

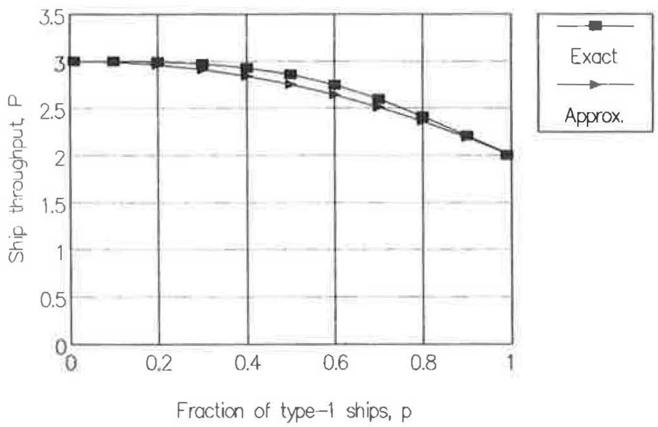

FIGURE 2 Comparison of exact and approximate throughput expressions for S = 3 and C = 2 at a multipurpose terminal.

Single-Purpose Terminals

This subsection explores the accuracy of our simple model in a more complex situation. It is assumed that all the ships require at least one crane, but that ships can have a varying number of active holds. Now the crane scheduling strategy can make a difference, and strategy G is used. (The implications of changing the strategy are examined in the Crane Usage Evaluation section.)

As in the preceding section, the system can be modeled as a Markov chain. Here the state is a vector composed of the numbers of ships with 1, 2, 3, . . . active holds that are still at berth immediately before the next batch of ships joins the berth. Although the state space is multidimensional, it is finite; numerical analysis is possible.

We present numerical results for a terminal with 4 cranes. In the first instance (Case A) we assume that the berth can hold 3 ships and that the ships request either 1 or 2 cranes each. For the second case (B) the berth can hold only 2 ships, but the ships can request either 1, 2, or 3 cranes.

Case A is characterized by a single parameter: the fraction, p, of ships that have 2 active holds. Only three possible states are possible because, at most, 1 ship can be left at berth, and this ship can only have either 1 or 2 active holds. This makes the search for the steady-state probability vector (and the associated measures of performance we seek) rather simple; the analysis is equally simple for an arbitrary number of ships and cranes. The process is similar to that outlined in the preceding section. Thus, only the results are given here.

The productivity in ships handled by the berth per unit time, P, is

$$P = 3 - p^2 [(3 + p)/(1 + p + 2p^2)] \quad (3)$$

Clearly, P cannot exceed 3.

In holds per unit time, the productivity is equal to the average number of busy cranes, K. Because the average number of holds per ship is $(1 + p)$, $K = P(1 + p)$. This reduces to

$$K = 4 - [(1 + p^2)(1 - p)^2/(1 + p + 2p^2)] \quad (4)$$

which cannot exceed C = 4. Note that, as expected, if all ships have 1 hold (p = 0), then P = 3 and K = 3; also as expected, if all ships have 2 holds (p = 1), P = 2 and K = 4. Figure 3 shows graphically how P and K vary with p.

We now test the accuracy of the assumption that states that the distribution of active holds at berth is equal to the distribution of active holds for S ships in the queue. We calculate K assuming that the 3 berthed ships have been randomly taken from the queue. Then, 4 cranes will be at work unless the 3 ships have exactly 1 hold each. This happens with probability $(1 - p)^3$, and thus K is approximately given by

$$K \approx 4 - (1 - p)^3 \quad (5)$$

The maximum difference between Expressions 4 and 5 occurs when p = 0.53, which results in K = 3.87 and 3.90, respectively. The discrepancy is less than 1 percent. As expected, Equation 5 yields larger values than does Equation 4. If one uses Equation 2a, which also includes a normal approximation, the result is not very different (3.87 instead of 3.90); just by chance, it nearly matches the exact value, which is also 3.87. In any case, it appears that our assumption (about the distribution of active holds at berth) does not lead to large inaccuracies for this example.

For Case B, the ship workload changes more from ship to ship, thus one expects the approximation to be less accurate. Two parameters now define the problem: p, the probability that a ship has 2 active holds, and q, the probability that the ship has 3 active holds. Of course, the fraction of ships with a single active hold is $(1 - p - q)$. The Markov analysis can still be used. In this case, too, only three possible states can arise: the berth either is empty or has 1 ship that can have either 1 or 2 active holds; no other possibilities exist.

The berth productivity (ships per unit time) is found to be:

$$P = 1 + 1/(1 + 2pq + q^2 + q^3) \quad (6)$$

This value remains between 4/3 and 2; if there are no ships with 3 holds (q = 0), then, as expected, P = 2. The crane usage, which coincides with the number of holds served per unit time is $K = P(1 + p + 2q)$, where the quantity in parentheses is the expected number of active holds per ship:

$$K = (1 + p + 2q)(1 + 1/(1 + 2pq + q^2 + q^3)) \quad (7)$$

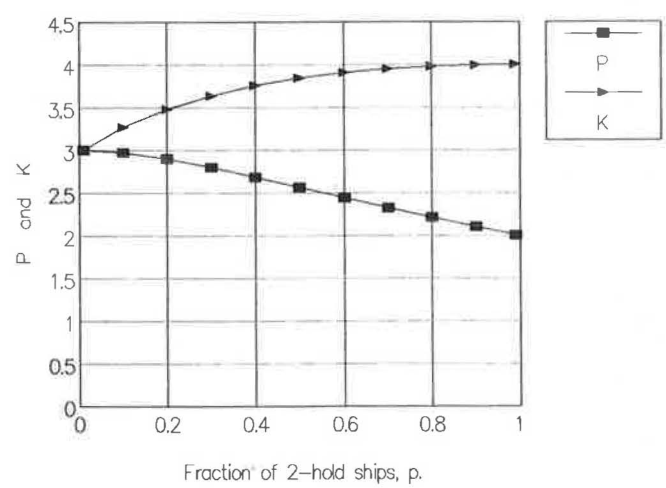

FIGURE 3 Exact expressions for berth throughput and crane usage at a single-purpose terminal with S = 3 and C = 4.

The approximation for K using the distribution of holds in the queue is:

$$K \approx 4 - 2(1 - q)(1 - p - q) \tag{8}$$

Note that if $(p + q) = 1$, Equation 8 yields $K \approx 4$, but consideration shows that it should be a little smaller: whenever 5 active holds are at berth, 1 ship with 1 active hold must remain for the next period. If the next ship requests only 2 cranes, 1 crane will have to be idle. Thus K cannot be 4 except when either $q = 1$ or $p = 1$. Then all the ships are identical, and 5 active holds can never be at berth; in that case the approximation is exact.

Equation 7 is consistent with these observations. The maximum difference between the exact and approximate expressions over all possible values of p and q occurs when $p = .72$ and $q = .28$. Then the exact value is 3.80, and the approximation is 4.00 (a 5.26 percent error). In most other instances the overprediction is less severe. The average (root mean square) error over all possible values of p and q is slightly less than 3 percent.

This error is not very large (given the rather large workload variability exhibited by this example), suggesting that Equations 2a and 2b may be reasonable predictors in actual situations. Still to be tested, however, is the extent to which Factor 1 counterbalances (and perhaps overcorrects) this error.

CRANE USAGE EVALUATION

The results from the section on crane productivity are now demonstrated. The section immediately following investigates the importance of an efficient crane allocation scheme and the subsequent section, the trade-off between cranage cost and maximum productivity.

Effect of a Bad Crane Allocation Method

In this subsection we explore the changes to productivity for the single-purpose terminal scenario of the preceding subsection, when a "bad" crane allocation method (strategy B) is used. This strategy, which is also described in an earlier work (*10*), is almost the exact opposite of strategy G. For the idealized scenarios in this paper, the strategy is easy to describe:

At every integer time ($t = 0, 1, 2 \ldots$), reallocate all the cranes (one at a time) to the ship with most active unattended holds.

As before, the system can be studied as a Markov chain, and the results are as follows:

$$P = 3 - p^2(3 + 2p + 2p^2)/(1 + p + 3p^2 + p^3 + p^4) \tag{9}$$

for case A, and

$$P = 1 + (1 - q^2)/(1 + 2pq + q^2) \tag{10}$$

for Case B.

Equations 3 and 9 are rather close. They differ the most in the range from $p = 0.4$ to 0.8, when the difference is on the order of 0.02 to 0.025. Thus for Case A, the specific crane allocation strategy used does not seem to matter much. The situations where the wrong crane choice influences productivity do not arise often enough. When $p = 0.5$, the cranes are idle 3.9 percent of the time with the good scheme but only 5.1 percent with the bad one.

For Case B we perform the same comparison when the average number of active holds per ship is 2. At one extreme, all the ships have 2 active holds ($p = 1$), and at the other extreme, half the ships have 1 hold and half, 3 holds ($p = 0$ and $q = 0.5$). When $p = 1$, both strategies are equal (clearly), and as one moves toward the other extreme the bad strategy deteriorates: 3.5 cranes are busy on average with the efficient strategy, but only 3.2 with the bad strategy. This is a 10 percent difference in productivity.

These comparisons illustrate the productivity increases that can be obtained with efficient operation. The increases are not enormous, but when ships are very different from one another, they can be significant. Even in these cases, however, the percentage changes in productivity are only a few percentage points larger than the errors in Equations 2a and 2b. This suggests that these expressions should be quite robust and applicable even if the scheduling strategy only vaguely resembles strategy G.

Although it may seem like a contradiction, increases in productivity comparable with the errors in Equation 2 should not be dismissed. A 5 percent increase in productivity would be highly desirable at a port, but a 5 percent error in our ability to predict it does not invalidate a preliminary planning tool (in fact, in the planning stages a 5 percent prediction error may be quite satisfactory).

The next subsection explores the trade-off between crane cost and productivity.

Optimum Number of Cranes

Clearly, there are some benefits associated with a high maximum productivity. If maximum productivity is increased, say, by purchasing more cranes, the terminal can attract more business and generate more revenue. Maximum productivity also increases with S (see Equation 2a). Thus, it is possible to use Equation 2a to determine the most cost-effective combination of berth capacity and number of cranes to achieve a certain productivity goal.

Equations 2a and 2b can also be used to determine the equipment needs for a given berth capacity. Let α denote the yearly marginal profit associated with one unit of productivity, and let us measure the productivity by the average number of busy cranes as given by Equation 2a. Let β denote the yearly cost associated with owning one crane. This cost does not include any operating costs, which should have been factored into α. Thus, the total yearly profit associated with owning C cranes is:

$$\text{Profit} = \alpha\{C - \sigma\sqrt{S}\,\psi([C - Sm]/\sigma\sqrt{S})\} - \beta C$$

This is a concave function of C, which will have a unique maximum at the point where the derivative vanishes: the root of the equation,

$$(1 - \beta/\alpha) = \Phi\{(C - Sm)/(\sigma\sqrt{S})\} \tag{11}$$

where $\Phi(*)$ stands for the standard normal cdf. Equation 11 has a solution if $\beta < \alpha$. The best number of cranes to have is the nearest positive integer to this solution. In reality one should never have $\beta > \alpha$ because this would mean that the profits obtained by continuous operation of a crane are not enough to offset its fixed cost: the terminal should not operate.

The calculations suggested in this subsection assume that the marginal profit associated with an extra productivity unit is constant. This is a coarse approximation that may be valid for long-term planning (when it is planned to use the terminal capacity nearly to its fullest), but not always. If, as is more common, to provide a good level of service to its users, the terminal is not used to its fullest, then the most significant benefit derived from the availability of more cranes is a reduction in ship delay; and ship delay is not linear with the number of cranes.

The next section derives ship delay expressions that can be used to address these questions.

SHIP DELAY

This section explores the relationship between ship delay and crane operations. As before, this is done by means of idealized models that can be solved analytically. It is assumed that ship arrivals to the terminal are stationary and random, and that while the terminal may not have enough cranes from time to time to serve all the ships at berth, the berth is long enough so that ship queuing is extremely rare. This should be the case at well-run ports and will help to separate the effects of crane operations on delay from those of berth availability.

The Model

Ships fall into two categories: type-0 ships that need no cranes and type-1 ships that need cranes. The service times of type-0 ships are arbitrary. Type-1 ships can be one- or two-hatched; that is, they may have either one or two active holds, which require exactly one time unit of a crane's attention.

Because the berth is almost never congested, it will be assumed that it never is; for all practical purposes, its length is infinity. This implies that the type-0 ships never interact with the type-1 ships and that the operations of both can be studied separately. Of course, to make sure that the infinite berth length assumption is reasonable, one will have to check a posteriori that the total number of type-0 and type-1 ships at berth is very unlikely to exceed the maximum possible number.

The two crane scheduling strategies already presented will be compared. Strategy G (good) gives priority to the ships with one active hold and strategy B (bad), to the ships with two holds.

For both ship types we seek the expectation and the variance of the number of ships at berth. The expectations give an indication of the cost of delay; and together with the variance they yield insight into the maximum number of ships that are likely to be present simultaneously at the berth.

Results

Let us focus our attention on the type-1 ships and imagine that all ships have exactly two holds; that is, one-hatched ships have another (empty) hold. Let Q denote the total number of holds at berth that are still active. This does not include any holds that have already been handled, even if the ship still is at berth. We define $Q = Q_1 + Q_2$, where Q_1 denotes the number of holds belonging to one-hatched ships and Q_2, to two-hatched ships.

The total number of type-1 ships, N_1, in the system can be obtained as a function of Q_1 and Q_2. This is because with strategy G all the active holds on two-hatched ships are spread across as few ships as possible. Thus,

$$N_1 = Q_1 + (Q_2/2)^+ \tag{12}$$

where the last term in this equation is rounded to the nearest integer, if Q_2 is odd. With strategy G, ships with one hold have priority. Thus, Q_1 can be visualized as the number of customers (holds) in a queuing system with C servers with deterministic (unit) service times. A simple model for Q_2, however, is not readily available (it would seem to require priority queues). To avoid this complication, we express N_1 as a function of Q_1 and Q. Because $Q = Q_1 + Q_2$, we can write:

$$N_1 = [(Q + Q_1)/2]^+ \tag{13a}$$

or approximately,

$$N_1 \approx (Q + Q_1)/2 \tag{13b}$$

This expression is more useful because the total number of holds can be modeled as a queuing system with C servers where the customers are the holds on all ships; some arrive in batches of two.

Queuing systems with many servers and a variety of arrival and service processes have been extensively studied. Here we use Newell's approximate formulas (*13*) because of their simplicity and generality. They apply to arrival processes that can be approximated by a diffusion process (e.g., with independent increments, compound Poisson).

A similar type of argument can be made for strategy B. Because now two-hatched ships have priority, Q_2 (and not Q_1) is easily predicted. Thus it is now advantageous to express Equation 12 as a function of Q and Q_2 as follows:

$$N_1 = [Q - (Q_2/2)]^+ \tag{14}$$

Newell's Queuing Expressions

For our deterministic service time queuing system (assuming that the customer arrival process follows a stationary process that can be approximated by a diffusion process), Newell's (*13*) approximate expressions simplify. Let λ denote the average customer arrival rate and σ^2, the variance of the number of arrivals in one time unit (this value equals λ for a Poisson process). These two parameters characterize the arrival process. Then the expected number of customers in the system (being and waiting to be served) is a function of λ, σ^2, and the following dimensionless constant, μ:

$$\mu = (C - \lambda)/\sigma \tag{15}$$

This constant represents how far the system is from being saturated. If it is negative, the system is oversaturated; a steady-state solution does not exist and the queue would grow steadily with time. If μ is close to zero but positive, the system has a steady state in which there usually is a queue; and if μ is greater than 2, queues arise only rarely. The probability that all the servers are busy is:

$$\Pr\{\text{busy}\} \approx \varphi(\mu)/\psi\,(\mu) \tag{16}$$

where $\psi(^*)$ is the function appearing in Equations 2a and 2b. The expected number of customers in the system is

$$E\{\text{no. customers}\} \approx \lambda + \sigma\{\mu\,\Phi(-\,\mu) + \varphi(\mu)/(2\,\mu\,\psi\,(\mu))\} \tag{17}$$

which for uncongested systems (μ >2) can be approximated by

$$E\{\text{no. customers}\} \approx \lambda + \sigma\,\varphi(\mu)$$

Note that as μ approaches infinity, the expected number of customers approaches λ. This is the result that is obtained for the infinite channel queue, and it is a lower bound to the actual number. Figure 4 displays the quantity in braces in Equation 17 and the probability that all servers are busy; both plotted against μ.

Expected Number of Ships and Expected Delay

To calculate $E(Q)$ and $E(Q_1)$ (or $E(Q_2)$) for strategy B, one needs to determine the mean and variance of the pertinent hold arrival process. Let a_1 and a_2 represent the arrival rates for one- and two-hatched ships, respectively, and σ_1 and σ_2 the corresponding variances per unit time. If ships are tramps (they do not follow a schedule), one would expect these variances to be close to the arrival rates. The arrival rates for holds (total, and on one- and two-hatched ships) are (a_1 + 2 a_2), a_1, and $2a_2$.

One can then use Equations 15 and 17 with these arrival rates and the corresponding variances. These are either (σ_1 + $4\sigma_2$), σ_1, or $4\sigma_2$. The coefficient 4 appears in these expressions because some holds arrive in batches of two.

Equations 15 and 16 can be used to calculate the probability that the system is busy, p_0, and the probability that the system is busy with all the cranes attending priority holds: p_1 for strategy G (where priority ships have only one hold) and p_2 for strategy B. Clearly, $p_0 > p_1, p_2$.

The expected number of ships at berth is given by the expectation of Equations 13a or 14. These are not linear functions of the Qs, but the equations need only to be rounded up when the system has an odd number of active holds belonging to two-hatched ships. For strategy G this can happen only when there is a queue, and then only about half the time. Thus the expectations of Equations 13a and 13b differ only by $p_{0/2}$; but Equation 13b is linear. Thus:

$$E(N_1) = (E(Q) + E(Q_1) + p_0)/2 \tag{18}$$

For strategy B, an odd number of active holds belonging to two hatched ships can arise only if the system has an odd number of cranes, and then only for about half the time when the system is saturated with these types of ships. Thus:

$$E(N_1) = (2E(Q) - E(Q_2) + p_2)/2 \text{ if } C \text{ is odd} \tag{19a}$$

$$= (2E(Q) - E(Q_2))/2 \text{ if } C \text{ is even} \tag{19b}$$

The average ship time in port is obtained by dividing these expressions by the average ship arrival rate:

$$E\{\text{time in port}\} = E(N_1)/(a_1 + a_2) \tag{20}$$

Example

To illustrate these expressions, assume that C = 4 and that ship arrivals are Poisson with a_1 = 1 and a_2 = 0.5. Then the total hold arrival rate is (1 + 2(0.5)) = 2, and the combined σ^2 is (1 + 4(0.5)) = 3. Thus, $\mu = 2/\sqrt{3}$, p_0 = 0.17, $E(Q)$ = 2.39, $E(Q_1)$ = 1.0, and $E(Q_2)$ = 1.07. The average number of ships with the good strategy is about 1.78 and with the bad strategy, 1.86. The average ship time in port is 1.19 time units for strategy G and 1.24 for strategy B. If cranes were never in short supply, these numbers would be 1. Thus, one can think of the excess (0.19 and 0.24 time units) as the delay caused by crane shortages; switching strategies can reduce this delay by about 25 percent (0.06 time units). If the delays are longer, choosing the best crane allocation strategy should be more important. With 3 cranes, for example, the average number of ships in the system is 2.25 with strategy G and 2.52 with strategy B. The corresponding times in port are 1.5 and 1.68 time units; the difference between the strategies still amounts to about 25 percent of the ship delay, but the difference is now larger in absolute value.

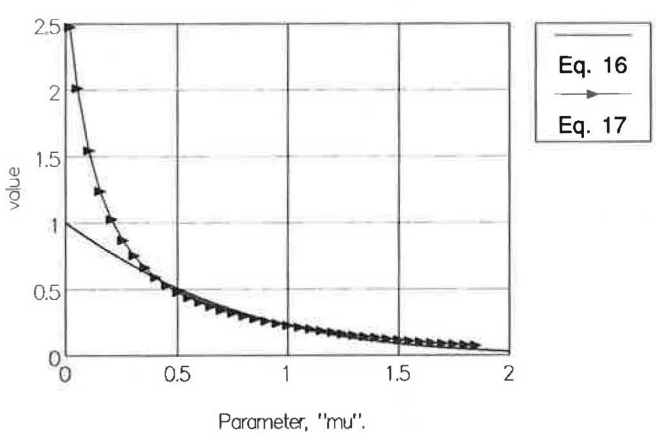

FIGURE 4 Graphs for quick evaluation of Equations 16 and 17.

Discussion

The results in the preceding section assumed that the berth is so long that ships never have to queue for berthing space and that the ship arrival process has independent increments. To check that ships do not have to queue for berthing space, one can calculate the mean and variance of the total number of ships at berth and verify that both are small enough. For type-0 ships, the mean and variance, $E(N_0)$ and var (N_0), can be obtained with the formulas for an infinite server system [see Newell (*13*)]. Earlier, formulas were given for the mean number of type-1 ships, $E(N_1)$, but not for its variance. If Q_1 and Q were independent (they should be positively correlated), Equation 13b would indicate that:

$$\mathrm{var}(N_1) = (\mathrm{var}(Q) + \mathrm{var}(Q_1))/4$$

where var(Q) and var(Q_1) are given by a formula, which is similar to Equation 17 but is not given here. If Q and Q_1 were perfectly correlated, the variance would instead be:

$$\mathrm{var}(N_1) = [(\mathrm{var}(Q))^{1/2} + (\mathrm{var}(Q_1))^{1/2}]^2/4$$

The actual value should be between these limits, which should then be added to var(N_0) to obtain the variance for the total number of ships. Although an exact value is not given here, the calculations may indicate whether the available berth space is likely to suffice; great accuracy is not always needed for this purpose. If some of the ships are liners, the assumption of an arrival process with independent increments does not hold. Some graphical simulations can be done. For example, if all the ships are liners, two cumulative plots of the number of cranes demanded by one- and two- hatched ships versus time (as per their schedules) can be constructed. These graphs will help determine when each hold gets served with algorithm G and the departure time of each ship. This yields the desired information. If only some of the ships are liners (and liners have priority), one can use the preceding process to determine how many free cranes there are on average after serving the liners. If this number does not fluctuate with time very much (the liner schedules could be fairly regular), one could use this average (instead of C) with the expressions in the earlier section to obtain a first estimate of tramp delay. Clearly, there are many situations where the queuing formulas presented in this section do not apply. Nonetheless, the results give an indication of the kind of delay savings that can be attained by efficient crane scheduling.

CONCLUSION

This paper represents an initial attempt at understanding crane operations at ports by means of simple analytical formulas. It provides some approximate expressions for the average number of busy cranes during congested periods (a measure directly related to the maximum terminal throughput) and for ship delay.

The maximum terminal throughput depends on several factors: the berth capacity (in ships), the number of cranes, the amount of work per hold and its variability within and across ships, and the crane operating strategy. The crane operating strategy influences throughput considerably less than the other factors. In all the cases examined, throughput does not change by more than about 10 percent when one switches from an inefficient to an efficient strategy. This indicates that detailed models of crane operations are not needed to obtain rough productivity estimates.

A simple formula, which is proposed for efficient crane operations, was tested against exact expressions for some special cases. The errors were on the order of just a few percent. Although further testing is needed, this suggests that such a formula may be useful for quick response economic and planning purposes, in instances where detailed simulations are not possible.

The paper also illustrates how the maximum productivity expressions can be used for evaluating the effectiveness of various terminal configurations. As an example, it calculates the optimum number of cranes when the berth capacity is fixed and the cost of additional cranes is counterbalanced by corresponding productivity increases.

The paper also studies the impact of crane scheduling on ship delay for a berth that has a finite number of cranes but is ample enough to hold all ships; ships arrive at random so some of them may have to wait for a crane if too many are already at berth. The paper examines idealized situations that can be modeled analytically, and yet are rich enough to be sensitive to the crane allocation strategy. For a given strategy, the expected delay depends on only three parameters: the number of cranes and the average and standard deviation of the number of arrivals in the time that it takes to serve one hold. For the examples studied, representing lightly congested conditions, the expected delay was reduced by about 25 percent when switching from an inefficient to an efficient crane scheduling strategy.

The results in this paper represent only an initial effort toward providing crane usage analytic models. It definitely would be desirable to validate the approximate productivity equations under a wider set of conditions, and to extend the queuing models to situations where berth space is not quite so plentiful.

ACKNOWLEDGMENT

This research was supported in part by a grant from American President Lines to the Institute of Transportation Studies, University of California, Berkeley.

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