Method for Determining Optimal Blading Frequency of Unpaved Roads

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Management systems for unpaved roads are often viewed as unwarranted because of the low levels of traffic normally found on these roads. However, unpaved roads in many developed and developing countries represent the larger portion of mileage in the network. Even at the low cost of maintenance per mile of unpaved roads, the total cost resulting from multiplying this value by the overall road mileage corresponds to a large financial outlay. Therefore, the efficient management of these roads is justified. Recognizing the need to optimize the blading and regraveling frequencies of unpaved roads, some agencies tried to develop methodologies for determining the appropriate maintenance strategies. The procedures vary from road classification-based maintenance to economic analyses of alternative maintenance frequencies. However, the general approaches used in solving the maintenance problem are unsatisfactory as they either require restrictive assumptions or do not give closed-form solutions. This paper presents a dynamic optimization approach for determining the optimum blading frequency for an unpaved road using the principles of optimal control. The model is based on a procedure developed for setting overlay frequency and thickness for paved roads. The optimization equations are formulated for unpaved roads and applied to hypothetical cases. A sensitivity analysis is performed to evaluate the parameters in the model. The study indicates this approach is appropriate for determining optimal blading strategies for unpaved roads. However, further research is required to develop suitable deterioration and user cost functions and to include the frequency of regraveling.

For low-volume unpaved roads, loose gravel or plain earth is the primary riding surface. These roads are designed to provide low-cost highways for accommodating low traffic volumes. They are found in many agricultural areas where access is needed for the transport of farm products. In many developing countries, unpaved roads constitute a significant portion of the total road mileage and play a major link in the overall economy. Some of the routes connecting major cities in these countries are unpaved, owing to the economic infeasibility of transforming these roads into hard-top, all-weather surfaces. Most third-world economies rely on farm-based industries, but several developed countries also have substantially extensive farming activities, giving rise to a large number of earth and gravel roads. The proportions of unpaved roads in representative developed countries in 1978 ranged from 5 to 63 percent, whereas for developing countries they are as high as 70 to 97 percent of the network (*1*).

Maintenance of unpaved roads involves blading using a motor grader or a modern tow-type blade to restore the shape and surface of the road to ensure drainage and enhance rideability. Regraveling is also performed for gravel-surfaced roads when the gravel thickness falls below a minimum value. The major difference between paved and unpaved roads with regard to maintenance is that the former, once built, will exist for a number of years with minimal or no maintenance whereas the latter deteriorate faster and require frequent maintenance.

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Compared with that of paved roads, very little attention has been given to the maintenance of unpaved roads. As significant portions of the road network are unpaved in both developed and developing countries, even low-level maintenance consumes valuable resources. The development of management systems for unpaved roads aims at the efficient allocation of these resources. Existing systems either use unrealistic simplifying assumptions or do not give closed-form solutions.

This paper develops an analytical method to determine optimal blading frequencies for unpaved roads. The dynamic optimization technique optimal control is used. This procedure was applied by Tsunokawa (*2*) in solving for the optimum frequency and thickness of overlay in the rehabilitation of highway pavements. Tsunokawa's formulation, however, assumed that the performance of a pavement can be expressed in terms of a single measure, which is roughness, and that there is only one maintenance activity to correct for roughness at various levels. Deterioration of the pavement is manifested in many ways and is corrected or remedied by maintenance activities that vary with the extent and type of deterioration. For an unpaved road, roughness is the primary indicator of condition, and routine maintenance is limited to blading. Using performance functions derived from previous studies, a dynamic optimization model is formulated using roughness as the state variable and blading frequency as the decision or control variable.

The following section presents measures of condition for unpaved roads and reviews studies of performance and existing management systems. Then, the optimal control model for unpaved roads is developed and a case study, including sensitivity analysis, is presented. This research demonstrates the applicability of optimal control technique for determining blading frequencies and identifies areas for further research to determine its feasibility.

BACKGROUND

Previous research on the maintenance of unpaved roads has focused on measures of condition or deterioration, performance, and maintenance systems (*3*). Measures of condition and the performance of unpaved roads are fundamental elements of any maintenance management system as they are used to quantify the impacts of maintenance. The relationships between condition and impacts have been developed in many of the existing maintenance management systems. A comprehensive review is presented in Alfelor (*4*). These relationships are briefly reviewed in this section as they are used in the determination of optimal blading frequencies. Some representative approaches to maintenance management are also described to illustrate the limitations of existing approaches.

Measures of Condition

Deterioration of unpaved roads is manifested and quantified in terms of the following measures of condition:

1. Surface roughness,
2. Gravel loss of the wearing surface,
3. Rut depth, and
4. Depth of loose surface materials.

Roughness is the primary component of serviceability of the road (*5*), and the way it is perceived by the road user is very important. In terms of profile, roughness can be defined as the summation of variations in the surface profile. For a gravel-surfaced road, traffic and environment act together to cause reduction in gravel thickness. This change in gravel height measured over a period of time is called gravel loss. Excessive gravel loss results in earth riding surfaces that become impassable during the rainy season. Ruts decrease the serviceability of the road because they cause vehicle displacements. The operating speed of the vehicle is substantially reduced because the vibration increases with speed. Loose material on the road leads to loss of traction and was found to increase fuel consumption for a wide spectrum of vehicles in the Kenya study (*6*). The present study focuses only on roughness because for unpaved roads, all other measures of condition contribute to surface roughness.

Roughness measures are classified as either profile numeric or summary numeric. For profile numerics, the longitudinal elevation profile of the road is measured and then analyzed to obtain one or more roughness indices. High-speed and manual profilometers are in use. The most popular measures of roughness using statistics from profiles are the root-mean square deviation (RMSD) and the Quarter Car Index (QI). The former was developed by the British Transport and Road Research Laboratory (TRRL) using an instrument for statically measuring profiles called the TRRL beam (*7*). The Quarter Car Index, on the other hand, was developed for the Brazil road cost study and was originally measured by the General Motors Surface Dynamics Profilometer. The research team in Brazil later adopted a simplified method of obtaining the index using rod and level.

Summary numerics are measured using instruments known as Response-Type Road Roughness Measuring Systems (RTRRMS). For these systems, a vehicle is instrumented with a road meter that produces a roughness reading as a result of the vehicle motions that occur while traversing the road. RTRRMS provide means to acquire roughness data using relatively low-cost equipment. The main disadvantage of these systems is that the roughness measure is intimately tied to the vehicle response, which varies among vehicles and likewise with time, vehicle condition, and weather. The quarter car model is a response-type system that produces the quarter car index and is used as a standard measure. The most popular RTRRMS in the United States is the Mays Road Meter because it is simple and cheap.

The condition measures just described are used in studies of the performance of unpaved roads and in the development of management systems.

Studies of Performance

The majority of research on unpaved roads was performed as part of a larger study that included paved roads. Hence, the focus of the study was not necessarily on unpaved roads themselves. These major studies were conducted in developing countries with the objective of establishing rational, quantitative bases for highway decision-making in those countries. The World Bank realized that situations in developing countries (i.e., economic, labor, and technology) are different from those of developed countries where pavement management systems exist, and concluded that those systems are not appropriate for use in entirely different environments. For this reason the World Bank initiated collaborative research with institutions in several countries, and took a share in funding this research. The original research was conducted in Kenya and included both paved and unpaved roads. This was followed by studies in the Caribbean, Brazil, India, and Bolivia, among others. The result is a large data base and empirical models that can be used for economic evaluation of unpaved roads. Included in the studies of performance are deterioration and vehicle operating costs as well as impacts of maintenance on condition. The result of this study was used in some of the existing management systems described next.

Existing Approaches to Maintenance Management

Maintenance management systems provide data for planning as they determine maintenance needs and the cost of executing a desired level of maintenance. Two types of management systems are currently used for unpaved roads—namely, the road-classification based and mathematical optimization techniques. Road classification is a simple way to assign maintenance to a road. The procedure is to divide the roads into different classes based on characteristics such as traffic volume. For each class a level of maintenance is defined. One example is the Ontario Road Classification System (*8*). In this system, roads are divided into three classes based on four quality-of-service characteristics: (a) average daily traffic, (b) visibility, (c) ease of passage, and (d) all-season travel. The main criterion used is the average daily traffic. The overlaps in the classification are taken care of by the other characteristics. The purpose of the classification is to establish a basis for distributing maintenance funds. A formula was developed that relates maintenance costs to each class in a linear manner. Given the ratios of maintenance costs among the classes, the portion of budget to be allocated to each class is computed. This system does not attempt to come up with optimal maintenance strategies, as it is designed only for allocating maintenance funds. It is also not clear how the other criteria (i.e., visibility and ease of passage) can be quantified as they are very subjective. The assumption on linearity of cost with average daily traffic is very unrealistic.

The more elaborate procedures applied in setting maintenance frequencies for unpaved roads consist of analysis of quantitative relationships describing the road's performance and determination of the most economic strategy using a certain objective function. Kerali and Snaith (*9*) proposed a simple analytical model that aims at minimizing the total costs involved after a road is constructed. The performance relationships are taken from the Kenya Study that predicts distresses on unpaved roads as functions of cumulative traffic and surfacing materials (*6*). The same study determined that roughness was the surface condition measure by which road user costs can be predicted. Assuming that the geometric and traffic characteristics of the road remain fixed, Kerali concluded that the vehicle operating cost incurred on a given road per vehicle-kilometer will change only if the road roughness changes. The relationship between unit vehicle operating cost and cumulative number of vehicles (and roughness) is graphed for a particular type of vehicle and surfacing material and a maintenance activity assumed to be repeated several times after a constant traffic interval. Assuming zero traffic growth, the cumulative increase in postconstruction cost (VOC + maintenance cost) is drawn against the cumulative traffic. Cost of maintenance is a fixed vertical line every time maintenance is performed. A Total Cost Line (TCL) connects the total cost coordinates for each maintenance cycle. The slope of the TCL will depend on the shape of the excess VOC curve and on the unit cost and interval of maintenance activity. The optimum maintenance interval is given by the TCL with the least gradient. The major flaw in the analysis is the assumption that roughness is brought back to the constructed value everytime maintenance takes place. Maintenance cost was also assumed fixed no matter how rough the road is before blading.

A model called Maintenance and Design System (MDS) that evaluates alternative regraveling and blading strategies for unpaved roads was developed by Visser (*1*) using the performance relationships estimated in the Brazil study but calibrated for South African conditions. The criterion used in the evaluation was total transport costs, including road maintenance and road user costs. The model generates blading alternatives expressed in number of bladings per year. Annual average roughness is computed by integrating the roughness-time relation for every grade/curvature combination and obtaining the weighted average over the road link using as weights the proportion of the road link in each grade/curvature combination. This annual average roughness is used in the user cost computations. The total costs of maintenance and vehicle operation are computed for each maintenance strategy. The program terminates by ordering the strategies in terms of increasing discounted total costs. The MDS model is one of the most comprehensive ever applied to unpaved roads. The main disadvantage of using the model is that it has no closed-form solution: the user has to define several alternatives, simulate each alternative, and pick the one with the least cost that is not guaranteed to be the optimal solution.

From the results of the Brazil study, the World Bank developed a comprehensive model for evaluating investments on highway design, construction, and maintenance in developing countries. The system is called Highway Design and Maintenance Standards Model (HDM) (*10*). It performs financial and economic analyses of user-defined alternatives for both paved and unpaved roads. In the present working model (HDM III), sets of design, construction, and maintenance options are input as alternatives and analyzed by calculating their life-cycle costs. For unpaved roads, maintenance options are entered as number of bladings per year (blading frequency). A steady-state roughness cycle is assumed that represents an equilibrium condition given a specific blading frequency. The analysis period used is equal to one regraveling cycle. The HDM model is designed to evaluate new construction instead of exclusively the costs involved after the road is constructed (which include maintenance and road user costs). Management systems are generally applied to already constructed highways, and HDM is not the appropriate tool for this purpose.

In general, the existing maintenance management systems applied to unpaved roads could be improved by applying a technique that generates the optimal blading frequency as a function of the road characteristics and their actual relationships with vehicle operating costs and maintenance costs. An ideal system is that which does not require the user to define alternatives, which is the more rigorous solution approach. In view of this, the optimal control technique is worth exploring.

OPTIMAL CONTROL MODEL

The maintenance of unpaved roads can be characterized as a dynamic system where both condition and performance change over time. For unpaved roads, dynamic optimization techniques have the advantage of giving a closed-form solution to the highway maintenance problem compared with a simulation model like the MDS, described previously. The interaction among maintenance, deterioration, and performance can also be modeled more realistically in the dynamic optimization framework, avoiding such restrictive assumptions as constant roughness after blading or constant maintenance cost, which were assumed in the graphical model.

A review of highway literature indicates extensive studies dealing with dynamic optimization of routine maintenance and rehabilitation of paved roads. Probabilistic dynamic programming was used by Carnahan and colleagues (*11*) in determining optimal maintenance decisions for a pavement system. Balta (*12*) formulated a dynamic control model using the principles of optimal control to compute the optimum time for rehabilitation of either flexible or concrete pavements. However, the jump in the performance function resulting from application of maintenance is difficult to model in the dynamic control framework because of the discontinuity over time of the state and control variables. This difficulty constrained Balta to consider only single overlay in his formulations. In a later study, Tsunokawa (*2*) proposed a procedure for approximating the discontinuous performance function by a continuous curve, and solved the maintenance problem using optimal control. This procedure is applied to determine the optimal blading frequency for unpaved roads.

In optimal control problems, variables are divided into two classes: state variables and control variables (*13*). The simplest form of the control problem is to choose the continuous control function $u(t)$, $t_0 \leq t \leq t_1$, to solve

$$\max \pi = \int_{t_0}^{t_1} f[t, x(t), u(t)]\, dt \tag{1}$$

subject to $x'(t) = g[t, x(t), u(t)]$

$$t_0, t_1, x(t_0) = x_0 \text{ fixed}; x(t_1) \text{ free} \qquad (2)$$

The functions f and g should be continuously differentiable. The control function $u(t)$ is required to be piecewise continuous with time and affects both the performance function π through its own value and the change in the state variable $x(t)$. Equation 2 is called the state equation. Solution of this type of optimization problem involves forming the *Hamiltonian* function (H) as follows:

$$H[t, x(t), u(t), \Omega(t)] = f(t, x, u) + \Omega^* g(t, x, u) \qquad (3)$$

where $\Omega(t) = dH/dg$ is called adjoint, auxiliary, or co-state variable. The Hamiltonian function is similar to the Lagrangian equation used in solving a nonlinear programming problem, with Ω as the Lagrangian multiplier or shadow price. This variable represents the marginal contribution of the change in the state variable to the performance or objective function. To determine the optimal control variable $u^*(t)$, the derivative of the Hamiltonian function with respect to u, dH/du, is equated to zero. The formulation presented in Equations 1 and 2 is equivalent to a formulation for determining the optimal maintenance strategy where the function f is the discounted user and agency costs, $x(t)$ is the condition of the road, and $u(t)$ is the maintenance strategy.

Roughness Trend Curve

The extension of the general control problem to highway maintenance poses some problems because of the discontinuity in state and control variables. Figure 1 shows the state variable (roughness) as a function of time under a maintenance strategy (blading). This problem may be overcome by the use of the concept of *roughness trend curve*, which is a continuous approximation to the roughness sawtooth curve shown in Figure 1. The times t_1, t_2, and t_3 represent the blading times. The reduction G in the roughness after the first blading is a function of the roughness before blading. To solve the problem using optimal control, the sawtooth roughness curve in Figure 1 is first approximated by a continuous *average roughness curve* that passes through all the midpoints of the spikes (line AB in Figure 1). Note that the time is in days, the reason being that deterioration of unpaved roads is relatively rapid. The difference between roughnesses (ΔAB) of the two points A and B on the midpoints of the spikes is equal to

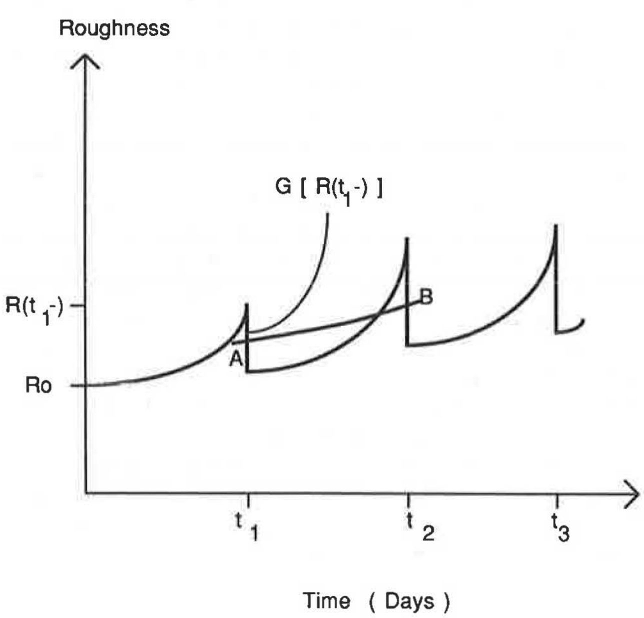

FIGURE 1 Sawtooth roughness trajectory curve.

$$\Delta AB = \int \dot{R}(t)\,dt - 1/2 * [\bar{G}(R(t_{n+})) + G(R(t_{n-1^-}))] \qquad (4)$$

where

$\dot{R}(t)$ = deterioration rate of the road = $f(R(t))$,
$G(R(t_{n-1^-}))$ = reduction in roughness due to blading at roughness $R(t_{n-1^-})$, and
$\bar{G}(R(t_{n+}))$ = inverse of G obtained from rewriting $x = y - G(y)$ as $y = x + \bar{G}(x)$; equal to the reduction in roughness needed to bring roughness to condition $R(t_{n+})$.

The concept of maintenance application rate $h(t)$ is introduced such that

$$\int_{t_{n-1}}^{t_n} h(t)dt = 1 \qquad (5)$$

This is simply a uniform rate of blading that has the equivalent of the impact on the average roughness in the period t_{n-1} to t_n as the average impact of blading at times t_{n-1} and t_n.

Using this relationship, the approximation of the average roughness curve by the roughness trend curve is derived as follows:

$$\int h(t) * \Delta AB\,dt \cong \int \dot{R}(t)dt - \int h(t)/2 * [\bar{G}(R(t)) + G(R(t))]dt$$

$$\cong \int [\dot{R}(t) - h(t) * K(R)]dt \qquad (6)$$

where

$R(t)$ = average roughness at time t, and
$K(R)$ = blading impact function, which = $[\bar{G}(R) + G(R)]/2$. (7)

The slope of the average roughness curve is given by

$$h(t) * \Delta AB \cong \dot{R}(t) - h(t) * K(R)$$

$$\cong f[R(t)] - h(t) * K(R) \qquad (8)$$

For a small interval dt, Tsunokawa (2) shows that the slope of the average roughness curve is approximated by the slope of a roughness trend curve (S) equal to

$$\dot{S} = dS/dt \cong f[S(t)] - h(t) * K(S(t)) \qquad (9)$$

Problem Formulation

The rate of change in roughness for the roughness trend curve is substituted into the Hamiltonian function for a general objective function of the form

$$\min H = C(s(t)) + h(t) * M(S(t)) + z(t) * [f(S(t)) - h(t) * K(S(t))] \quad \text{subject to } h_1 \leq h \leq h_2 \qquad (10)$$

where

$C(S(t))$ = user cost function,
$h(t)$ = blading application rate,
$M(S(t))$ = agency maintenance cost function,
$z(t)$ = current value adjoint variable,
$f(S(t)) - h(t) * K(S(t)) = \dot{S} = g(S,t,h)$, and
h_1, h_2 = minimum and maximum blading rates, respectively.

The necessary conditions for this minimization problem are

$$h = \begin{Bmatrix} h_1 \\ h \\ h_2 \end{Bmatrix} \text{ if } H_h = M(S) - z * K(S) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \qquad \begin{matrix} (11) \\ (12) \\ (13) \end{matrix}$$

Equation 11 means that if the change in total cost with respect to the application rate $h(t)$ is greater than zero, then it is best to blade at the lowest frequency (h_1). Conversely, if the marginal value of the total cost decreases with the application rate (Equation 13), blading should be done as often as possible. The solutions defined by these two equations are called bang-bang controls. When the term H_h is equal to zero (Equation 12), h assumes values between h_1 and h_2 and is called a singular control solution. In practice the constraints h_1 and h_2 may be determined by resource constraints or manpower equipment, capital resource use, and minimum acceptable comfort levels.

The road deterioration equation used in the analysis is an approximation to the relationship derived from the Brazil study (*14*) and is given by

$$R = R_0 * \exp T(0.0034 + 1.3e-5 * Q) \qquad (14)$$

where R is the roughness at time T, R_0 is the initial roughness, and Q is the average daily traffic in passenger car units (pcu). Differentiating Equation 14 with respect to T yields

$$dR/dt = R_0 * \exp K_1 * T * K_1 = R * K_1 \qquad (15)$$

where K_1 is a constant term depending on the assumed value of Q. An equation predicting roughness after blading was estimated as follows:

$$RA = RB^{0.63} * \exp K_2 \qquad (16)$$

where RB and RA are the roughnesses before and after blading, respectively, and K_2 is a constant term defined by the input variables (i.e., width, plasticity index, traffic, surfacing material, etc.). The roughness values are in counts/kilometer.

Letting $K_3 = \exp K_2$, the improvement in road condition or decrease in roughness G as a result of blading is just equal to

$$G = RB - RA = RB - K_3 * RB^{0.63} \qquad (17)$$

For the continuous approximation, RB is equal to the roughness trend curve value S. The blading impact function is defined by Equation 7. The derivative of this function with respect to roughness is given by

$$K_s = \frac{1}{2} * (2 * G_s - G_s^2)/(1 - G_s) \qquad (18)$$

Differentiating Equation 16 with respect to S, substituting this to Equation 18 for K_s and finally integrating with respect to S results in the blading impact function

$$K = 0.58 * S^{1.37}/K_3 - 0.5 * K_3 * S^{0.63} \qquad (19)$$

Equations 15 and 19 can be used to define the state equation $\dot{S}$ that is required to solve the Hamiltonian function. This equation is written as

$$\dot{S} = K_1 * S - h(t) * (0.58 * S^{1.37}/K_3 - 0.5 * K_3 * S^{0.63}) \qquad (20)$$

For simplicity in calculation, the only component included in the vehicle operating cost function is the cost of fuel consumption. The analysis can be generalized by including other cost components. The expressions for vehicle speed and fuel consumption for a passenger car are taken from the Caribbean study (*15*). Cost of maintenance is influenced by the productivity of the motor grader, which is measured in terms of the number of kilometer-passes that a motor grader can blade for a given day depending on the roughness of the road to be bladed. An exponential approximation to the relationship derived from studies in South Africa (*1*) is used, which is given by

$$N(RB) = 60/\exp(0.009 * RB) \qquad (21)$$

where $N(RB)$ is number of kilometer passes/day and RB is the roughness before blading in counts/kilometer. For a road length of L kilometers and a daily cost of grader equal to CG, the maintenance cost is given by

$$M(RB) = L * CG * \exp(0.009 * RB)/60 \qquad (22)$$

This is the equation used in the optimization model, again substituting S to RB for the roughness trend curve. Given the expressions for vehicle operating costs, maintenance costs, and the differential equation representing the change in roughness trend curve with time, the general optimal control problem for unpaved roads is formulated using the Hamiltonian equation.

An upper limit is set for the frequency with which maintenance is applied. It is assumed that the most frequent interval of blading is every 25 days ($h_2 = 0.04$) or approximately once every month. To avoid the computational problems that arise when very low values of h are considered (*2*), a lower bound (h_1) for the application rate is defined, which is 0.005 or once every 200 days. Again, these bounds may be set to reflect resource constraints, resource use, and minimum comfort levels.

In summary, the optimal control model for unpaved roads has been developed using the deterioration and maintenance equations estimated in previous road studies. To be able to generate the components of the objective function (vehicle operating costs and maintenance costs) that will be minimized in the optimization problem, prediction equations for fuel consumption and number of days required to blade a road of certain roughness have been defined. The solution to the optimization problem using these equations is not expected to generate results that correspond to actual field experience because of the nature of equations used. For instance, fuel consumption is the only component of vehicle operating cost included. This cost function monotonically increases with speed. Because speed is inversely related to roughness, the cost function used decreases with roughness. This is not the case when other costs are included, such as vehicle depreciation, because this cost component increases with roughness. On the other hand, the cost of blading increases with the roughness of the road. The cost of maintenance then increases with the roughness. The functional equation used for maintenance cost is negative exponential, which simply means that extremely high blading costs are incurred at high roughnesses. This will offset the alternative to keep the road very rough and fuel consumption at its minimum, as the road needs to be bladed at least every year and the cost for blading an extremely rough road is extremely high. The following case study is intended to illustrate the application of the model and its sensitivity to changes in parameters based on the cost components used. The results should not be compared with real field solutions but are intended to demonstrate the applicability of the solution method.

CASE STUDY

Two *hypothetical* cases with different levels of traffic are used to test the preceding models. The volumes are 30 pcus/day and 250 pcus/day, respectively. Table 1 shows the parameters assumed for both cases. These parameters were assumed constant in the solution.

TABLE 1 VARIABLES USED IN CASE STUDY

Cost of Grading	$ 200 / day
Rise	5 meters/km.
Fall	5 meters/km.
Interest Rate	8 % per annum
Fuel Cost	28 ¢ / liter
Curvature	5^{o} / km.
Type of Surface	lateritic gravel
Plasticity Index	5 %
Length of Road	10 kms.
% Passing 0.075 mm Sieve	15 %
Width of Road	10 meters

Results

The analysis begins by plotting the two curves $H_h = 0$ and $\dot{H}_h = 0$ on the *S-z* plane, as shown in Figures 2 and 3. The intersection of the two curves represents the singular control solution that also satisfies the steady-state conditions ($\dot{S} = 0$ and $\dot{z} = 0$). At steady states, S and z both attain time-invariant values corresponding to the coordinates of the intersection of the two curves defined by the following parameters:

Case 1	Case 2
$Q = 30$ pcus/days	$Q = 250$ pcus/day
$S = 150.5$ QI	$S = 266$ QI
$z = 1.26$	$z = 1.52$
$h = 0.0056$ (every 180 days)	$h = 0.0074$ (every 136 days)

The singular controls found in the two cases are between the minimum and maximum frequencies set for the problem, hence there are no bounded control solutions for this problem.

Because the initial roughness of the road differs from the steady-state roughness, a roughness trajectory curve (*2*) is constructed for each case. Assuming a range of roughness values within which the initial roughness is assumed to fall, it is determined whether all values in this range actually converge to the steady-state solution. The curves defined by such roughness values are called stable branches. Stable branches are derived by integrating the canonical equations $\dot{S} = 0$ and $\dot{z} = 0$ with respect to t and solving for the appropriate terms in the expressions $S(t)$ and $z(t)$ such that when both terms are differentiated, both S and z are time invariant. However, the expressions for S and z in the unpaved roads problems do not allow such calculations to be made. Hence, the stable branches are determined by calculating S and z at different points on the *S-z* plane and drawing the curves that converge to the steady-state solutions. Figures 4 and 5 show the minimum roughnesses (70 for Case 1 and 230 for Case 2) that exist for the stable branches. A maximum roughness value of 300 for the roughness trend curve was arbitrarily assumed. These extreme values are the ranges to be used in the analysis.

The roughness trend curves are drawn assuming an initial roughness within the ranges defined. Such curves are shown on Figures 6 and 7 for Case 1. Equation 5 is used to convert the roughness trend curve to the true sawtooth curve that is also shown on Figures 6 and 7. The discrete blading times (t_n) are determined from the sawtooth curve. For the first blading, the roughness trend curve passes through the midpoint of the first spike.

To check if the given solutions are the true minima, the total costs of the steady-state solutions are compared with the

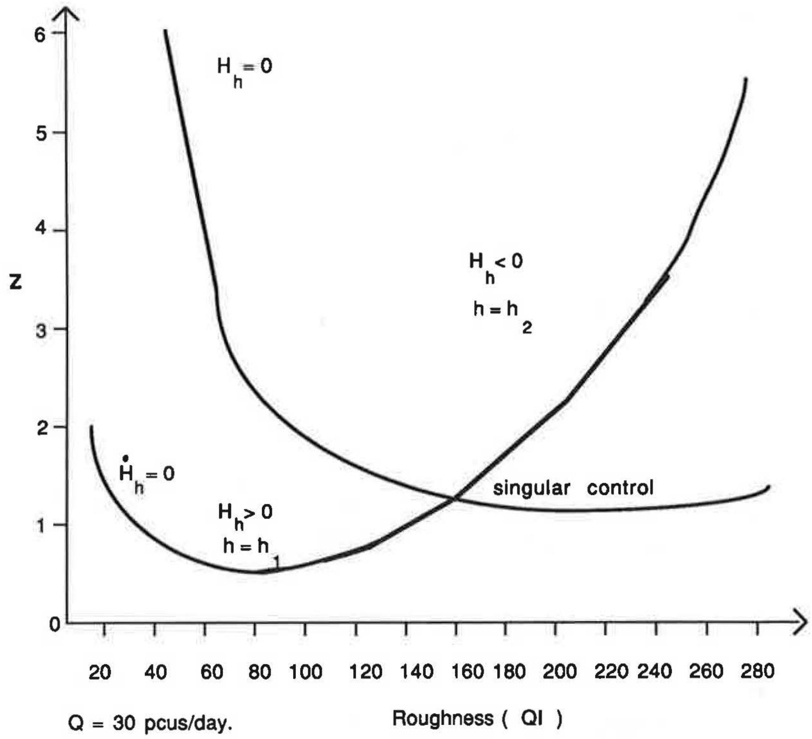

FIGURE 2 Graph of $H_h = 0$ and $\dot{H}_h = 0$ for Case 1.

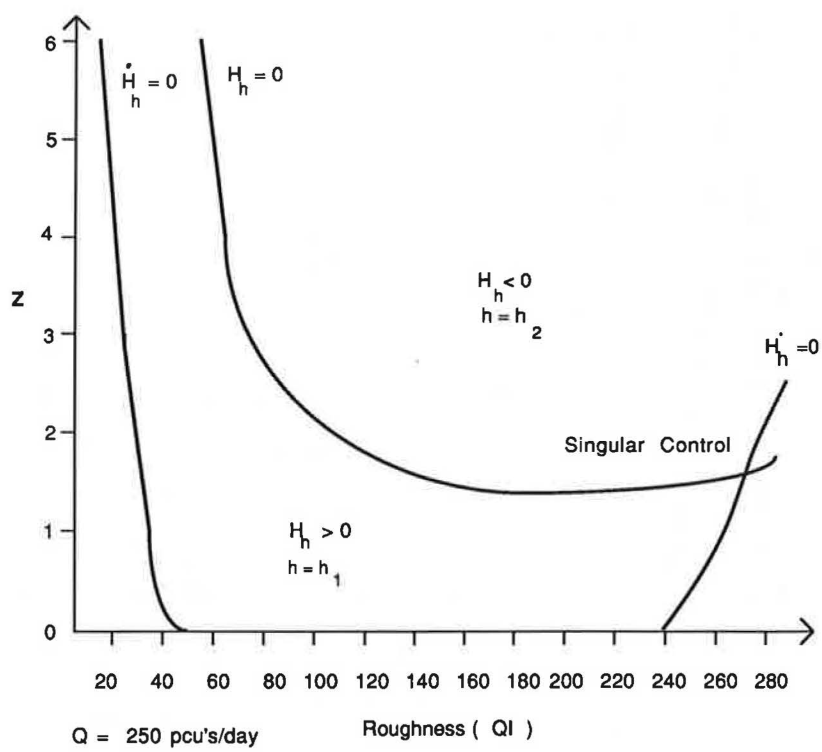

FIGURE 3 Graph of $H_h = 0$ and $H_h = 0$ for Case 2.

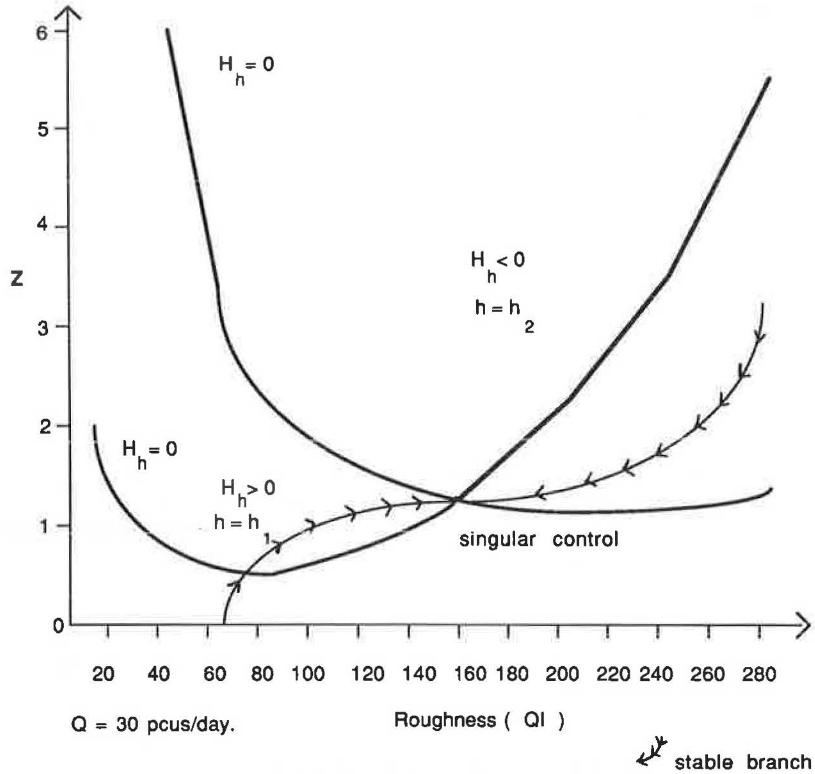

FIGURE 4 Stable branches for Case 1.

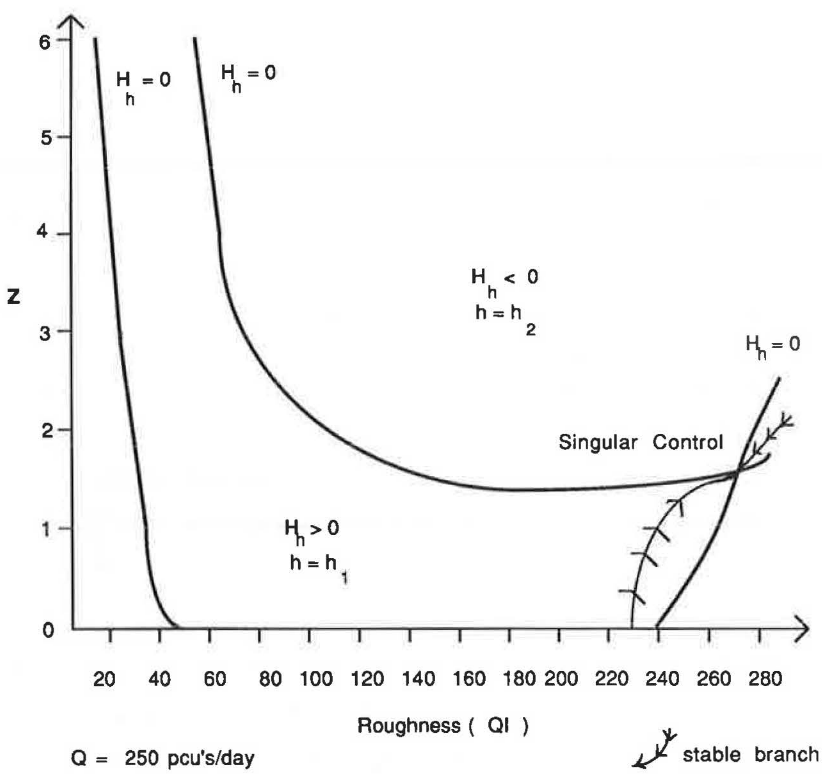

FIGURE 5 Stable branches for Case 2.

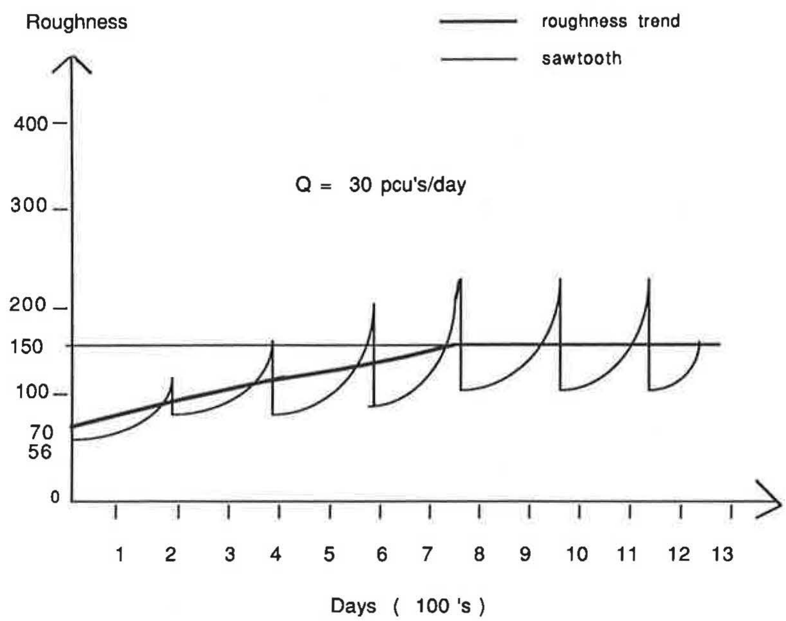

FIGURE 6 Roughness trend curve and sawtooth curve for Case 1 with initial roughness = 70 QI.

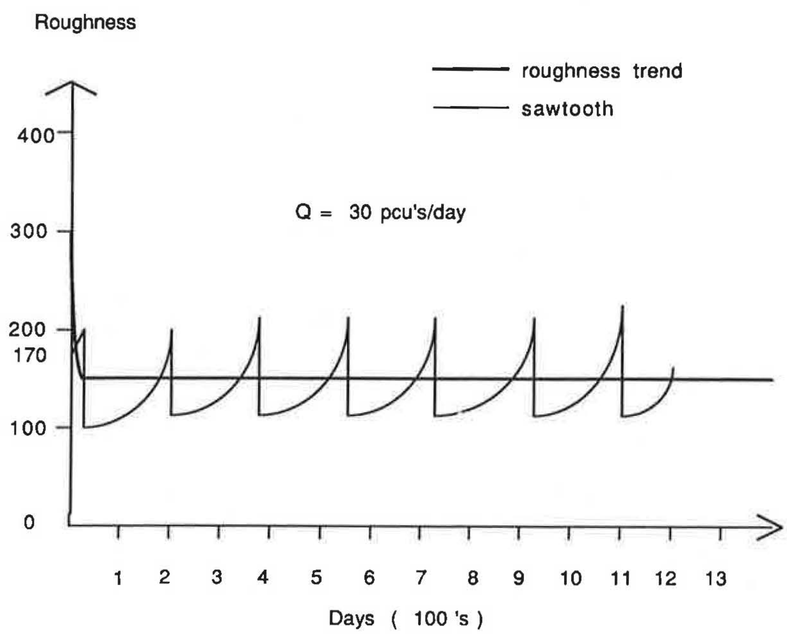

FIGURE 7 Roughness trend curve and sawtooth curve for Case 1 with initial roughness = 300 QI.

costs associated with other blading frequencies. The results are shown on Table 2. The values shown are not significantly different from each other. However, note that the case study is only for a 10-km road. If the same analysis is made for the entire unpaved road network, say, in Bolivia, which has a total length of 36,155 km, the amount that represents the difference in blading interval of 30 days (every 150 days instead of 180 days) for Case 1a is equal to $6.25 million for the entire planning horizon or $0.5 million annually using an interest rate of 8 percent. If traffic were to increase to 250 pcus/day as in Case 2, the amount corresponding to a difference in blading interval of 36 days (100 instead of 136) is equal to $17.8 million for an infinite planning horizon, or $1.42 million annually. For a developing country like Bolivia, these amounts represent significant improvement in the overall highway economy. The impacts are emphasized by comparing the total costs for the optimal solutions in both cases when applied to the total road network in Bolivia. Table 3 gives us some idea of the order of costs associated with a system of unpaved roads in a developing country like Bolivia.

TABLE 2 TOTAL COSTS IN DOLLARS FOR OPTIMAL AND NONOPTIMAL SOLUTIONS

	FREQUENCY		
	optimum		
Case 1a	every 180 days	every 150 days	every 200 days
Ro = 56 QI	37,234	38,963	43,105
Case 2a	every 136 days	every 100 days	every 150 days
Ro = 132 QI	287,496	292,426	298,782

TABLE 3 TOTAL COSTS (OPTIMAL) WHEN EITHER SCENARIO APPLIES IN BOLIVIA

	for a 10 km. road	for the whole network
Case 1a	$ 37,237	$ 134.6 B (entire horizon)
		$ 10.8 M (annually)
Case 2a	$ 287,496	$ 1039 B (entire horizon)
		$ 83.2 M (annually)

Sensitivity Analysis

The objective of this exercise is to see how the solution shifts from the optimal solution of the base problems when other variables are changed and determine whether the shifts make sense intuitively on the basis of the relationships between maintenance and vehicle operating costs used in the formulation. It is the direction of the changes that are important, not the absolute change. For example, if the traffic level is higher then it is expected, that blading will be required more frequently. Table 4 shows the results of these tests. The optimum frequency of blading is indeed a function of the different variables, notably the volume of traffic and the cost of fuel. For the base case with an average daily traffic of 30 pcus, it can be observed that increasing the fuel cost results in less frequent optimum blading (therefore higher steady-state roughness) because fuel consumption increases with speed; therefore, it is better to keep the road rough to reduce the vehicle speeds. On the other hand, if the daily cost of grading is doubled, the optimal solution is to blade more frequently so that the steady-state roughness declines. Because the maintenance cost increases with roughness before grading, it is more economical to keep the average roughness low, reducing the cost per blading and the user costs. Interest rate is found to be insignificant in the optimal solution.

Increasing the average daily traffic to 250 pcus raises both the frequency of maintenance and the steady-state roughness. This makes sense intuitively because doing more frequent maintenance in this case does not stop the average roughness from attaining a high value as a result of the traffic. When the fuel cost is increased to $1 per liter and the traffic remains the same, no optimum is found. However, decreasing the price of gasoline to $0.1 per liter shifts the optimum blading frequency to a higher value. With lower fuel prices, the vehicle speeds may be increased without considerably increasing the vehicle operating costs. The optimum solution is to blade the road more frequently so that the average roughness (and therefore the cost of maintenance) can be lowered. Further increase in the traffic volume (900 pcus daily) results in no optimal solution because of the exponential form of the deterioration function that predicts extremely high roughness for this volume.

The preceding sensitivity analysis is made to test how the model responds to changes in the different parameters and whether the results of applying it to different scenarios correspond with the expected results on the basis of the equations used in the formulation. The results that will be generated by using a different set of deterioration and cost functions will be different.

Computer Implementation

The amount of computation involved in solving the optimal control problem requires the use of a computer. For this reason, a program was written that gives the values of the roughness (S) and the adjoint variable (z) that were plotted on the S-z graphs. The steady-state solutions (singular and bounded controls) are also calculated by the program. In determining stable branches, however, manual computation was made, although this could have been done with the computer as well. Finally, the path traced by the roughness trend curve from the initial roughness was computed using a numerical approach. Cost computations were done using numerical integration.

SUMMARY AND RECOMMENDATIONS

The application of dynamic optimization in setting optimal blading frequencies for unpaved roads is shown to be useful in the maintenance of unpaved roads. A review of the existing systems used to manage unpaved roads indicates that the classification-based types of maintenance systems are popular in developed countries. Some mathematical optimization and simulation techniques have been applied. These systems, however, either suffer from very restrictive assumptions to solve the problem or do not give closed-form solutions. The dynamic optimization approach solves both problems. It has the advantage of being more realistic than the graphical anal-

TABLE 4 SENSITIVITY ANALYSIS

PARAMETERS	CASES										
	1*	2	3	4	5	6	7	8	9	10	11
No. of PCUS.	30	250			900		250	250	250	250	250
Fuel Cost ($/liter)	0.28					1	1	1	1	0.1	
Cost of Grading ($/day)	200		400								400
Length (km.)	10										
% passing 0.0075 mm sieve	15										
Plasticity Index	5										
Interest Rate (%)	8			20					20		
Width of Road (m)	10										
Surfacing T1 T2	0 0										
Rise (m/km)	5										
Fall (m/km)	5										
Blading Season	wet										
Curvature (deg/km)	5										
No. of Bladings /year	2	2.7	2.4	1.97	—	1.8	—	—	—	3.4	3.1
Steady State Roughness	150	266	124	158		175				182	208
Adjoint Variable (z)	1.26	1.52	2.79	1.24		-1.1				1.32	2.68

— no solution

* value for block with no entry same as that for Case 1

ysis. Likewise, the user does not have to define possible maintenance strategies because the solution is explicitly defined by the problem. The optimization technique chosen is optimal control, because the condition (roughness) curve is not constant between blading and this is very difficult to model in the dynamic programming framework. However, the discrete jumps in the condition function due to application of maintenance make the optimal control solution infeasible. This was overcome by approximating the sawtooth condition curve by a continuous curve, following Tsunokawa's approximation.

The results of this research must be interpreted carefully as the prediction equations and performance functions used in the analysis with which the optimization formulation was made are just approximations to the existing models and are in functional forms that may not represent the true relationship between deterioration, maintenance, and vehicle operating cost. The existing models have very poor explanatory power and functional forms not suitable for the optimization formulation. Hence, the research is basically an exploratory analysis of the solution to the optimization problem. Further research is needed on (a) developing equations predicting highway deterioration, vehicle operating costs, and maintenance costs with functional forms suitable for optimization and estimated from actual data; (b) solving the more general case of varying traffic volume that was assumed constant in the model; (c) incorporating other components of vehicle operating costs, such as lubrication costs, tire wear, and parts consumption; and (d) including regraveling in the maintenance decision problem.

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