Dual Equilibrium Model of Urban Commercial Activity and Travel

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A retail activity allocation/shopping trip assignment model is developed, in which the zonal retail price as well as the travel times are in equilibrium. The model is based on two main assumptions. First, in each zone, retailers maximize their profits from retail sales. Second, prospective shoppers in each residential zone select their shopping destination so as to minimize the cost of shopping and the cost of traveling. Use of the model to simulate the behavior of the commercial activity and travel system is illustrated.

In a previous Transportation Research Record paper (*1*), the author developed a model for retail activity allocation and travel that assumed there is only one link/itinerary from a given place of residence to a given place of shopping. This simplification, although assuming the transportation network was congestible, obviated the need to assign the shopping trips to the network. In other words, the model did not allow for predicting the routes that shoppers would follow to their chosen shopping areas.

This may be of obvious interest, not only to transportation planners, but also to retailers. The purpose of this paper is thus to extend the previous formulation to include a full transportation network. Specifically, there will now be several possible routes between any origin/destination pair, so that a complete description of the retail activity and travel system may be possible.

EQUILIBRIUM FORMULATIONS OF ACTIVITY ALLOCATION AND TRAVEL

The starting point is the standard network equilibrium model of urban location and travel (*2*). Specifically, given the numbers of trips O_i made for the purpose of some activity that originates in a given origin zone i, the link congestion functions $g_a(.)$, and a measure of attractiveness R_j for each of the potential locations j for the conduct of the activity, the number of interzonal trips Y_{ij} and the link volumes x_a may be determined as the solution of the standard combined distribution/assignment problem (*3*):

$$\underset{(Y_{ij},\, x_a)}{\text{Min O}} = \sum_a \int_0^{x_a} g_a(x)dx - \sum_j \sum_i R_j Y_{ij} \tag{1}$$

such that

$$\sum_j Y_{ij} = O_i \qquad j = 1,2, \ldots, n \tag{2}$$

$$\sum_r F^r_{ij} = Y_{ij} \qquad i,j = 1,2, \ldots, n \tag{3}$$

$$X_a = \sum_i \sum_j \sum_r F^r_{ij} \delta^{i,j}_{a,r} \qquad i,j = 1,2, \ldots, n \tag{4}$$

where F^r_{ij} is the flow on route r connecting i and j, and δ^{ij}_{ar} is equal to 1 if link a is part of route r between i and j, and zero otherwise.

From the Y_{ij}'s, the numbers of trip ends, Y_j, which measure the activity level in the zone, may then be determined.

In this formulation, however, the attractiveness of a given zone as a place to conduct the activity—shopping—is assumed given. This may not always be realistic because the attractiveness may be a function of the activity level in the zone. For instance, congestion at the destination zone (e.g., due to limited parking) may be a factor in the choice of destination, in the same manner that congestion on the network's links is assumed to be a factor in the choice of route. In general, from an economic standpoint, the cost, or price of the activity, will depend on the level of activity, in the same manner as the price of a commodity depends on the demand for it.

Thus, the destination attractiveness terms R_j should be endogenous variables, as are the link travel costs. This necessitates the incorporation of two additional relationships in the model above. The first is a "destination cost function" specifying the value of R_j as a function of Y_j to play a role similar to the "link cost function." The second is the statement of an equilibrium principle for activity levels, in addition to the "user equilibrium" principle for link volumes.

This paper presents the development of a retail activity allocation/shopping trip assignment model based on this approach.

FORMULATING THE MODEL

Because the purpose of travel is shopping, and in keeping with the previous version of the model (*1*), we may equate the destination's attractiveness R_j to the unit retail price of a basket of commodities. The demand and supply functions for the activity in each zone must then be specified. The demand function most often used in activity allocation models, and

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which in fact was used in the previous version of the model (*1*), is the logit function

$$Y_{ij} = O_i \frac{\exp(\beta M_{ij})}{\sum_j \exp(\beta M_{ij})} \quad \text{for} \quad i,j = 1,2, \ldots ,n \tag{5}$$

The assumption underlying the choice of this function is that residents choose their destination so as to minimize the (perceived) total cost (dis)utility M_{ij} of traveling from i to j and shopping in j.

The supply function will be determined in the standard fashion from maximizing the supplier's (retailer) revenues. The cost C_j of operating shopping facilities is assumed to be a power function of the level of activity (sales)

$$C_j = kY_j^{\omega}; \qquad \text{for} \qquad j = 1, 2, \ldots , n \tag{6}$$

k has the dimension of a unit cost per sale. The value of parameter ω translates the magnitude of (dis)economies of scale in store operation. Because the revenues are equal to Y_jR_j, the level of supply Y_j is such that it maximizes the profit

$$p_j = Y_jR_j - kY_j^{\omega}$$

thus:

$$\frac{\partial P_j}{\partial Y_j} = R_j - \omega k Y_j^{\omega - 1} = 0 \rightarrow R_j = \omega k Y_j^{\omega - 1} \tag{7}$$

[It may be of interest to note that in the previous version of the model, zonal price was equal to

$$R_j = KY_j^{\omega - 1}$$

This translated the assumption that in each zone/store, revenues were in balance with operational costs. By rewriting R_j as

$$R_j = (K/\omega)\omega Y_j^{\omega - 1} = k'\omega Y_j^{\omega - 1}$$

it may be seen that these two assumptions are equivalent when the cost function is a power function of the activity level.]

In keeping with Equations 1–4, a non-linear program may be developed, the solution to which provides a retail activity allocation and trip assignment model conforming to the principles described in the introduction, as well as to the functional requirements (Equation 5 and Equation 7).

The new program's objective is:

$$\underset{(Y_{ij}, X_a)}{\text{Min O}} = \tau \sum_a \int_0^{X_a} g_a(x)dx + \frac{1}{\beta} \sum_i \sum_j Y_{ij}(\log Y_{ij} - 1) + \sum_j \int_0^{\Sigma_i Y_{ij}} z(x)dx \tag{8}$$

in which the functions $z(.)$ and $g_a(.)$ are respectively defined as

$$Z(x) = k\omega x^{\omega - 1} \tag{9a}$$

and

$$g_a(x) = C_a^0 [1 + 0.15(x_a/K_a)^4]$$
$$a = 1, 2, \ldots , A \tag{9b}$$

$g(.)$ is the standard "B.P.R." link cost function, where K_a is the link's "practical capacity" and C_a^0 is the "free flow travel time" (*4*).

The constraints are the same as in the program above, i.e. Equations 2–4, and all variables are again non-negative. The signs of the parameters are $\beta \geq 0$, $\omega > 0$, $k > 0$, and $\tau \geq 0$. The meaning of the parameters β and τ will become apparent during the ensuing derivations.

It may be shown that the solution to program Equations 8, 9, 2–4 has the following characteristics. First, the interzonal flows are equal to

$$Y_{ij} = \frac{O_i e^{-\beta(R_j + \tau \dot{t}_{ij})}}{\sum_j e^{-\beta(R_j + \tau \dot{t}_{ij})}} \tag{10}$$

Thus, shopping trips originating in zone i are distributed to shopping zones according to a logit function as was required in Equation 5. Its argument is the sum of the minimum travel cost from i to j plus the zonal unit price of goods R_j in the zone. Thus Equation 10 states that shoppers choose their shopping zone so as to minimize the perceived total cost of shopping and traveling. Furthermore, since R_j is given by Equation 7, the assumption that retailers maximize their revenues is represented as well. (The role of parameter β is to set the spatial dispersion of commercial activity across zones. A value of zero implies that cost is not a factor of destination choice, since the resulting distribution of shopping demands from a given origin is uniform. Conversely, an infinite value results in an "all-or-nothing" distribution in which the trip ends are concentrated in a single zone, that with the lowest cost from the given origin.) In addition, the solution to the program above is such that the loadings on the network's links conform to a "user equilibrium," that is, reflect the minimization of travel time by shoppers. (Trip volumes for purposes other than shopping are included, but must be given externally to the model.)

Finally, it may also be shown (*5*) that such a solution to the problem above will always exist and always be unique for all values of the parameters.

SIMULATING THE RETAIL ACTIVITY AND TRAVEL SYSTEM

To illustrate its uses for policy analysis and decision-making purposes, the model developed above was applied to the simulation of a hypothetical urban retail activity and travel system, under various circumstances. A prototype configuration of nine zones and 25 links was used, as represented in Figure 1.

Zone numbers are represented in Figure 1 in shadowed script, and link numbers in italics, including intra-zonal links. Bold numbers represent the link capacities, while plain numbers represent the "free flow" travel times C_0. Each (repre-

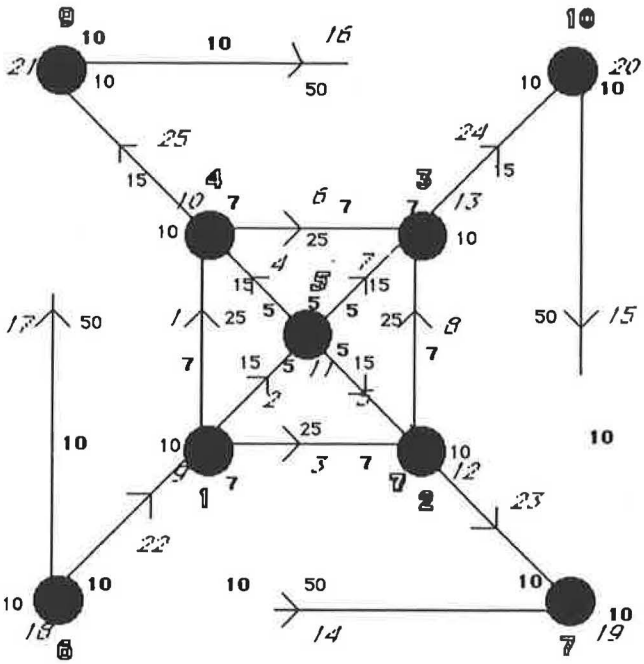

FIGURE 1 Simple hypothetical spatial system.

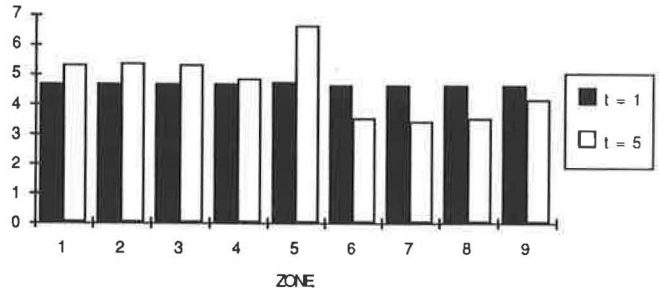

FIGURE 2 Shopping trip ends.

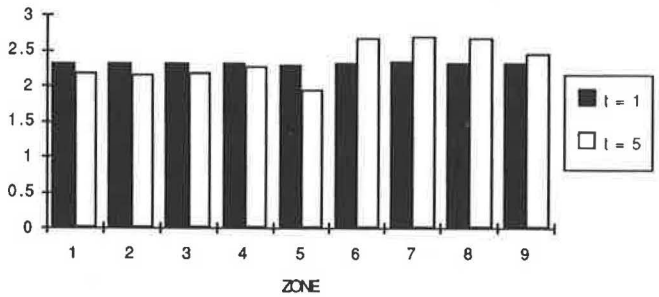

FIGURE 3 Retail prices.

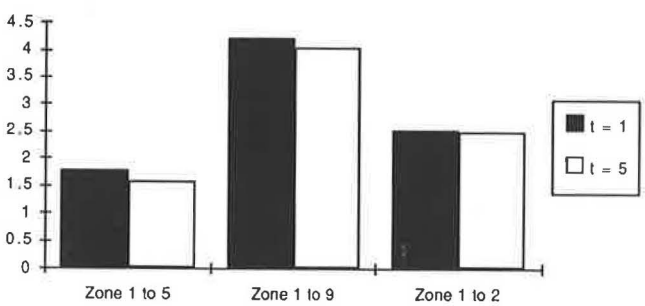

FIGURE 4 Interzonal travel time.

sented) directional link has an unrepresented counterpart in the opposite direction, with the same characteristics. Its number is equal to the link's number plus 25. The number of paths between two given zones is either one or three, depending on whether there is a direct, one-link connection or not. The total number of routes between all zones is 112. Also, the residential population levels were assumed to be highest in the central zone with the intermediate ring zones each at 60 percent of the central level, and the outer ring zones each at 20 percent.

The solution of program (Equations 8, 9, 2–4) provides the values of the interzonal flows Y_{ij}, the link flows x_a, the path flows F^r_{ij}, the zonal commodity prices R_j, the zonal levels of shopping activity Y_j, and the minimum cost routes from a given residential zone i to a given shopping zone j

In order to illustrate the model's application to the assessment of the sensitivity of the retail activity and travel system to changes in the prevailing conditions, the value of parameter τ, measuring the importance of travel time relative to that of the unit retail price, was varied, as represented as follows:

1. Travel time's importance is equal to that of the retail price ($t = 1$).
2. Travel time's importance is five times higher than that of the retail price ($t = 5$).

Parameter ω was set at 0.8, representing medium economies of scale. The value of β was set at 0.2, resulting in a ratio between lowest and highest activity levels of about 7. (The commodity prices and travel costs were between 1 and 10.) The value of k was set at 0.1, resulting in comparable magnitudes for the average (i.e., typical) unit retail price and the average interzonal travel time.

The results, represented in Figures 2–4, illustrate the complex interactions between the various system variables.

For instance, it may be seen that when the importance of travel time is equal to that of the unit cost of goods, the distribution of shopping trip ends (e.g., retail activity levels) is nearly uniform. (This does not imply that travel cost has no influence on the choice of shopping area, as the distribution of population is not uniform.) However, when travel time becomes much more (i.e., five times) important than shopping costs (corresponding, for example, to a change in shopping for high cost items to low cost items), the outlying zones (Numbers 6–9), those the farthest away from the center of gravity of the residential distribution, experience a decrease of about 30 percent in their level of retail activity. On the other hand, the level of activity in the central zone, Number 5, goes up by about 40 percent, while the middle ring zones (Numbers 1–4) see theirs increase by about 15 percent.

Concurrently, the effects on the levels of zonal retail prices reflect the inverse relationship between activity and prices. That is, as may be seen in Figure 3, prices in the outlying zones go up by about 15 percent, while the price in the central

zone goes down by about 15 percent, and about 10 percent in the middle ring zones.

Finally, Figure 4 illustrates the effects of the change above on the travel time from Zone 1 to Zones 5, 9, and 2, respectively. It may be seen that the changes observed above on the zonal distributions of retail activity (trip ends) and the corresponding zonal distribution of unit retail prices (zonal destination attractivity) are translated into changes on the link travel times through network equilibrium loading and consequently on the interzonal travel times.

This brief application of the model assessing the impacts of changes in the prevailing conditions on the state of the retail activity system, is but an example of the large variety of other scenarios that may be similarly analysed through changes in the various parameters of the model.

SUMMARY AND FUTURE EXTENSIONS

The model of retail activity and travel presented above possesses the following features. First, retailers set the levels of sales of retail goods in each zone such that they maximize their profits. The resulting levels of zonal retail activity supply are in equilibrium with the zonal demand for shopping. This is the first equilibrium. Second, shoppers in each zone of residence choose a shopping zone, together with an itinerary to it, such that they minimize their traveling and shopping costs. The resulting congestion-related link travel costs are such that they correspond to the travel costs that give rise to the choices of shopping zone as demonstrated above.

From a practical standpoint, the model's inputs are relatively few. They include the spending budgets in each of the residential zones, the capacities and free-flow travel times of each of the links, and the cost functions for each of the retail facilities. The model's output includes the interzonal shopping flows, the volumes of shopping trips on each of the links and each of the routes on the network, the zonal retail prices R_j, the zonal levels of retail sales, and the minimum cost routes from a given residential zone i to a given shopping zone j. Many derived zonal measures of system performance may be evaluated from these variables, including the revenues to retailers, and spending by residents.

The model, however, does not represent the interactions between a retailer and his suppliers. This is important because such interactions have implications for land use, specifically, the allocation of wholesale activity. They also have implications for transportation, specifically, the assignment of goods movements/truck trips to the same network. The fact that wholesale activity will now compete with retail activity for the use of land, and delivery trucks will compete with shoppers' cars for the use of the transportation network will change the configuration of the activity and travel system. An extension of the present model describing such interactions is being developed and will be presented in the future.

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