Scheduling Transit Extraboard Personnel

ISAM KAYSI AND NIGEL H. M. WILSON

Because operators are sometimes absent and daily workloads are often uncertain, transit agencies employ more operators than required by the timetable to ensure reliable service. These extra operators are usually referred to as extraboard or cover operators because they are used to cover the assignments of absent operators and to provide required, but unscheduled, work. Operators who do not have specific work assignments are told to report for work at specific times of the day to cover work that may be open at those times. A methodology is proposed to deal with the problem of assigning report times to extraboard personnel. The proposed methodology is sensitive to the variability of unanticipated requirements, work rules applying to extraboard personnel, reliability objectives, and availability of regular operators to work overtime in case unanticipated requirements cannot be covered off the extraboard. The methodology is applied to a large bus garage at the Massachusetts Bay Transportation Authority to test the quality of the resulting solution under different work rules. This case study demonstrates the potential of the methodology to produce significant improvements over current practice and to serve as a valuable policy analysis tool.

In order to provide reliable service despite operators' being absent from work and to accommodate uncertainty about the amount of work actually required on a given day, transit agencies employ more operators than required by the timetable. These extra operators are usually referred to as extraboard or cover operators because they are used to cover the assignments of absent operators and to provide required, but unscheduled, work. Although most extraboard operators are directly assigned to fill in for scheduled operators whose absences are known in advance, the remainder are assigned report times at which they must be available to cover work that may be open at that time. A method was developed to assign report times for these extraboard operators; the value of the method is demonstrated through a case study of a single large bus garage at the MBTA.

In the transit industry, the issue of operator workforce planning has been receiving increased attention in the past decade, primarily because of the prospects of cost savings through improved operator management methods. Although some of these efforts have focused on the staffing levels required, and hence on the size of the extraboard, little has been reported in terms of analytical methods for assigning report times to extraboard operators who have no specific, known-in-advance work assignments.

MacDorman and MacDorman (*1*) presented the first effort at analytically determining the extraboard size by identifying the major cost factors influencing it. MacDorman (*2*) more directly addressed issues relating to stand-by, or report, operators. MacDorman (*2*) discussed the real-time assignment of stand-by operators to fill open work that was not anticipated. He concluded that fitting manpower levels to the dynamics of open work was of utmost importance. The study also categorized open work, evaluated stand-by operator distribution strategies, and emphasized the importance of considering the complete range and variation of manpower demand.

Booz-Allen & Hamilton (*3*) targeted operator availability management and presented a number of related case studies. However, the issue of assigning report extraboard operators was only mentioned briefly in such general statements as "daily dispatching is responsible for report crew assignment" and "early report operators are split if they receive no assignment." For most, if not all, transit agencies, report operator assignment is based on agency experience without reference to analytical tools.

Koutsopoulos (*4*) and Koutsopoulos and Wilson (*5*) presented a general framework for addressing workforce planning issues at three levels: strategic, tactical, and operational. At the operational level, which is central, the available extraboard personnel are assigned specific times for report duty. These two works form the basis for the methodology presented here.

PROBLEM DESCRIPTION

One of the key tasks of the operator workforce planning process is the management of available operators. In the context of the extraboard, this task translates into the assignment of operators to cover open work, which consists of the following three elements:

1. Covering absences—substitution for absent regular operators;
2. Covering extra work—the operation of extra service for unexpected events and the relaying and shifting of vehicles for in-service breakdowns or major delays; and
3. Operating trippers—working known-in-advance short pieces of work that are not built into scheduled runs.

Extraboard operators may also be called on to provide optional extra service when surplus manpower is available. Because such extra service is not required, but is simply offered when the personnel and vehicles are available at low marginal cost, it should not be considered in sizing the extraboard.

One way to look at extraboard tasks is by their predictability. Some requirements may be known well in advance (trippers, for example); others may be known only a day or so in advance; whereas still others may be completely unanticipated (due to sickness, accidents, and breakdowns). Because of this variation in the predictability of open work, extraboard operators are typically assigned work in the following two-step sequence:

Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

1. Runs are built to cover known-in-advance requirements.
2. The remaining unassigned extraboard operators are assigned report times to cover work that may become open during the day.

The second step, which is the problem to be addressed, involves selecting report times for a given number of extraboard operators so that operator availability best matches expected needs throughout the day. This broad objective can be translated into more precise objectives that are expressed in terms of uncovered open work, unassigned cover, and reliability.

Clearly, the determination of report times should be sensitive to the probable unanticipated requirements (whether caused by absence or extra service) by hour of day, the probable availability of regular employees to work overtime, the number of available (i.e., unassigned) extraboard employees, and the work rules. Consideration of all these issues complicates the problem.

Because the problem of sizing the extraboard is not addressed here, any considerations relating to the differential costs of using part-time, full-time, or overtime personnel to fill extraboard requirements are irrelevant. The usage of a given number of extraboard operators is maximized (or their unproductive time minimized) by assigning them to best match anticipated requirements. Clearly, results obtained from this research are valuable for the subsequent task of extraboard sizing.

PROBLEM FORMULATION

To formulate the report time assignment problem, two components must be considered that describe the state of the system at time of day t, namely, the number of available extraboard operators and the number of open pieces of work (or runs). The number of extraboard operators available at time t is the sum of those operators who are assigned to report at time t and those operators who reported earlier but are still available because of lack of open work before t. Similarly, the number of open runs is the sum of runs that become open at time t and earlier open runs that are not yet covered. It is evident that the state of the system at time t depends on the history of, and interactions between, the two variables. These factors complicate an exact formulation of the problem; therefore, a simplified formulation was developed.

The simplified formulation is based on defining two time-of-day profiles, which are shown in Figure 1. The operator availability profile, denoted by $x_{(t)}$, is a function of the assigned report times and the work rules and gives the total number

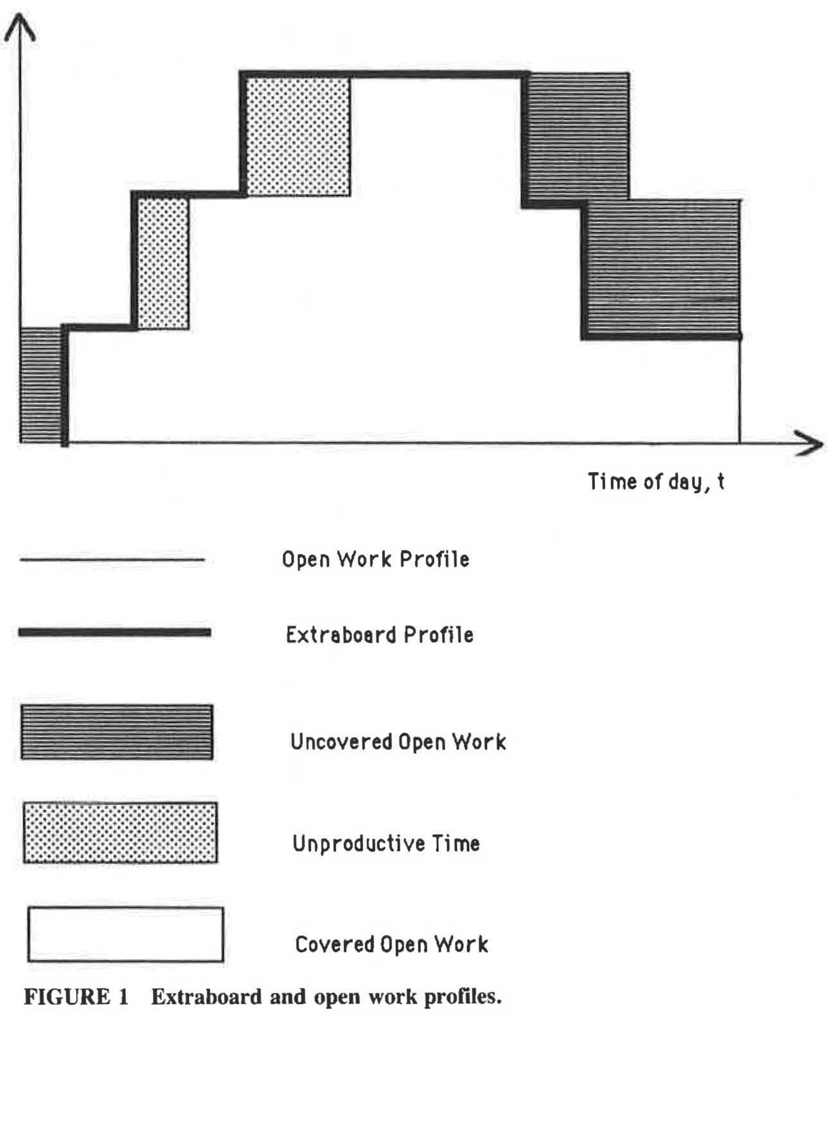

FIGURE 1 Extraboard and open work profiles.

of extraboard operators available at time t. The open work profile, denoted by $\varepsilon_{(t)}$, is the number of open runs that exist at time t. The variable $x_{(t)}$ is a deterministic decision variable, whereas $\varepsilon_{(t)}$ is a random variable in the sense that $\varepsilon_{(t)}$ is known only probabilistically at the time $x_{(t)}$ must be determined.

Using these profiles, expressions can be derived for expected uncovered open work, unproductive time, and system reliability (usually measured by missed trips as a percentage of all scheduled trips). Uncovered open work (UOW) and unproductive time (UT) are also shown in Figure 1 and can be formulated as functions of time t as follows:

$$\mathrm{UOW}_{(t)} = \max\,[0, \varepsilon_{(t)} - x_{(t)}] \qquad (1)$$

$$\mathrm{UT}_{(t)} = \max\,[0, x_{(t)} - \varepsilon_{(t)}] \qquad (2)$$

As far as system reliability is concerned, any UOW will be split between missed service and overtime in a manner that depends on the availability of regular operators for overtime work. The likelihood that UOW will result in missed service will depend on the time of day at which it occurs; at certain times of day, operators are more likely to be available for overtime work than at other times.

In the following sections, time of day is treated as a discrete rather than a continuous variable. In other words, the day is divided into subperiods, each of which is treated as homogeneous. This simplification of the problem does not entail real sacrifice in the accuracy of the results but makes the solution algorithm computationally more tractable.

Figure 2 shows a model that is used to represent the relationship between UOW and missed service after dividing the operating day into discrete time periods. The model is period-specific and is based on two parameters, $\mathrm{xinter}_{(i)}$ and $\mathrm{slope}_{(i)}$, for period i. If UOW is less than $\mathrm{xinter}_{(i)}$, then all UOW can be worked as overtime. Thus, $\mathrm{xinter}_{(i)}$ represents a lower bound on the overtime hours available during period i. If UOW is greater than $\mathrm{xinter}_{(i)}$, then the surplus will be split between missed service and overtime with the fraction resulting in missed service equal to $\mathrm{slope}_{(i)}$. $\mathrm{Slope}_{(i)}$ can also be viewed as the probability that UOW in excess of $\mathrm{xinter}_{(i)}$ will be translated into missed service. The period-specific parameters $\mathrm{xinter}_{(i)}$ and $\mathrm{slope}_{(i)}$ would reflect the likely availability of operators for overtime work at different times of the day. For example, it is quite likely that no operators will be available for overtime work during the early morning; consequently, $\mathrm{xinter}_{(i)}$ may be set to 0.0 and $\mathrm{slope}_{(i)}$ to 1.0 for these periods. For other periods of the day, however, $\mathrm{slope}_{(i)}$ would depend on the exact time at which open work occurs and the availability of operators willing to work overtime at that time. Consequently, $\mathrm{slope}_{(i)}$ may be less than 1.0.

The proposed model allows the expected missed service hours to be predicted for each period based on the UOW for that period. By summing these missed service hours over all periods of the day, the total expected missed service for that day can be obtained. Moreover, if this measure is used as the objective function in the proposed problem formulation, then a third objective is available, namely, minimization of missed service. Therefore, system reliability can be treated directly as an objective within the proposed methodology, although this requires that both UOW and a basis for splitting it between missed service and overtime (as shown in Figure 2) be available for each period of the day. Alternatively, reliability could be treated as a constraint on the solution rather than as another objective.

To model the possible objectives, expressions are required for UOW (which is closely related to the reliability objective) and UT. It can readily be shown, however, that the objectives of minimizing UOW and minimizing UT are equivalent (*6*). Furthermore, the reliability objective is also a linear transformation of UOW, and any mixed objective related to reliability and overall efficiency can be expressed by appropriate weightings of period-level UOW.

Consequently, the minimization of expected UOW is the central objective adopted in the analysis with the solution subject to work rule constraints. The objective function and the work rule constraints are developed in the following two sections.

OBJECTIVE FUNCTION FORMULATION

By dividing the day into N time periods, the expected UOW can be represented as

$$z = E\left(\sum_{i=1}^{N} \mathrm{UOW}_{(i)} d_{(i)}\right) = \sum_{i=1}^{N} d_{(i)} E(\mathrm{UOW}_{(i)}) \qquad (3)$$

Here, $d_{(i)}$ is the length of period i, E stands for the expected value functional, and both $\varepsilon_{(i)}$ and $x_{(i)}$ are assumed constant within period i.

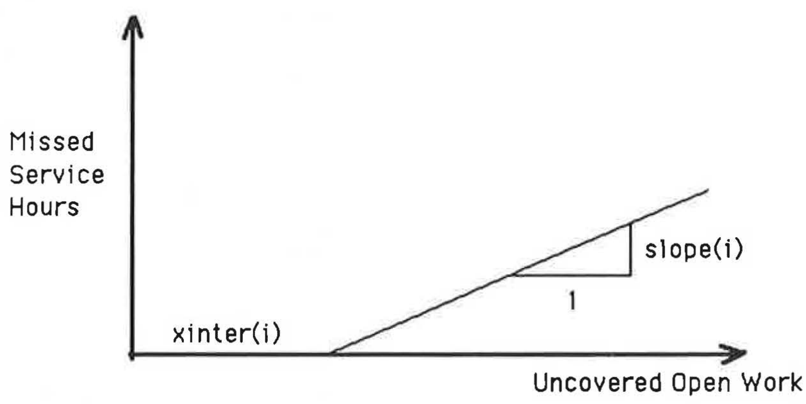

FIGURE 2 Missed service relationship.

Now, $UOW_{(i)}$ is a function of $\varepsilon_{(i)}$ and $x_{(i)}$ in each period as follows:

$$UOW_{(i)} = \begin{cases} 0 & \text{when } x_{(i)} \geq \varepsilon_{(i)} \\ \varepsilon_{(i)} - x_{(i)} & \text{when } x_{(i)} < \varepsilon_{(i)} \end{cases} \quad (4)$$

Consequently, the expected value of $UOW_{(i)}$ becomes the following (dropping the subscript i for the moment):

$$E(UOW) = \sum_{x}^{\infty} (\varepsilon - x)P(\varepsilon) = \sum_{x}^{\infty} \varepsilon P(\varepsilon) - x \sum_{x}^{\infty} P(\varepsilon)$$
$$= E(\varepsilon) - \sum_{0}^{x} \varepsilon P(\varepsilon) - x \sum_{x}^{\infty} P(\varepsilon) \quad (5)$$

The next critical step is defining the function $P(\varepsilon_{(i)})$, which describes the open run probability density function for each time period i. Open run probabilities in successive periods may not be independent; a run that is open in period i will most likely also be open in period $i + 1$. However, this does not affect the formulation. To define $P(\varepsilon_{(i)})$ is a matter of selecting the discrete probability density function that best describes the occurrence of open work.

Any run scheduled during period i has a probability $p_{(i)}$ of being open and $(1 - p_{(i)})$ of being filled as scheduled. Such an outcome associated with any scheduled run is conceptually equivalent to the outcome of a Bernoulli trial. Moreover, period i has $n_{(i)}$ scheduled runs, each with the same probability of being open. These runs constitute a Bernoulli process, which is a series of independent Bernoulli trials. The probability of exactly $\varepsilon_{(i)}$ runs being open out of a total of $n_{(i)}$ independent scheduled runs in period i is given by the following binomial distribution:

$$P(\varepsilon_{(i)}) = \binom{n_{(i)}}{\varepsilon_{(i)}} p_{(i)}^{\varepsilon(i)} (1 - p_{(i)})^{n(i)-\varepsilon(i)}$$
$$\varepsilon_{(i)} = 0, 1, \ldots, n_{(i)} \quad (6)$$

where

$\varepsilon_{(i)}$ = number of open runs in period i;
$p_{(i)}$ = probability of any run being open in period i; and
$n_{(i)}$ = total number of runs in period i, as given by the scheduled operator profile.

The objective function for UOW minimization can then be rewritten as follows:

$$z = \sum_{i=1}^{N} d_{(i)} \left\{ E(\varepsilon_{(i)}) - \sum_{\varepsilon(i)=0}^{x(i)} \varepsilon_{(i)} P(\varepsilon_{(i)}) - x_{(i)} \left[1 - \sum_{\varepsilon(i)=0}^{x(i)} P(\varepsilon_{(i)}) \right] \right\} \quad (7)$$

with the probability of a particular number of open runs given by the binomial distribution of Equation 6.

WORK RULES

The work rules in effect for extraboard personnel both constrain the feasible solutions and place financial penalties on specific types of solutions. Therefore, it is essential to model those constraints accurately. Extraboard operators may be able to perform either continuous or split assignments on the basis of the work rules; consequently, both types of assignments are considered in this formulation. The formulation allows part-time employees to be assigned to the extraboard, which is the more complex case.

In the simpler case of only continuous assignments' being permitted, two sets of constraints are required. First, the total number of extraboard operators available in period i is simply the sum of the full-time and part-time extraboard operator profiles in period i. These profiles are determined by the number of operators who reported at some earlier period k but are still on duty during period i. Second, the sum of extraboard full-time operators (FTOs) and part-time operators (PTOs) reporting at each time period should equal the total FTOs and PTOs to be assigned report times:

$$x_{(i)} = xf_{(i)} + xp_{(i)} = \sum_{k \varepsilon If(i)} yf(k) + \sum_{k \varepsilon Ip(i)} yp_{(k)} \quad (8)$$

$$\sum_{k=1}^{N} yf(k) = Nf; \qquad \sum_{k=1}^{N} yp(k) = Np \quad (9)$$

where

$x_{(i)}$ = total number of extraboard FTOs and PTOs available in period i;
$xf_{(i)}$ = full-time extraboard operator profile in period i, representing the number of FTOs who reported at or before period i but are still on duty, according to the work rules;
$xp_{(i)}$ = extraboard PTO profile in period i;
$yf_{(i)}$ = extraboard FTOs reporting in period i;
$yp_{(i)}$ = extraboard PTOs reporting in period i;
$If_{(i)}$ = set of report times t for which an extraboard FTO who reports at time t is still available at time i;
$Ip_{(i)}$ = set of report times t for which an extraboard PTO who reports at time t is still available at time i;
Nf = number of extraboard FTOs to be assigned report times; and
Np = number of extraboard PTOs to be assigned report times.

In some transit authorities, work rules permit management to make split shift extraboard assignments, which consist of two piece assignments with an unpaid break in between. This flexibility provides a greater potential to cover both peak periods with a single cover operator. Alternatively, if an operator who is assigned an early report time is not used, that operator might be released and asked to report later in the day. In this case, within a defined period following the operator's first report, the garage manager has the option to excuse the operator and assign a later report time if the operator has not yet been assigned work. The latest time at which a new report time can be assigned is known as the "decision point." Moreover, there is a spread premium when the total time from the time of first report to the end of the second piece exceeds a certain amount, typically 10½ or 11 hr.

With split assignments resulting from these rules, the operator profile is not fully determined by the first report time, unlike continuous assignments. In fact, the operator profile is not deterministic but depends on whether or not each report operator is assigned a run between the time of first report and the decision point. This assignment depends on the occurrence of an open run during this time span, which in turn depends on the number of bus pullouts and reliefs and the probability of an individual piece's becoming open during each period. As a result, the operator profile is stochastic based on the probability of each report operator being excused at the decision point and assigned a later report time or being assigned work before the decision point.

Although there are many possible work rules, it is assumed that a report operator who is assigned work before the decision point will work a continuous assignment. Moreover, the output of the proposed model includes a first and a second report time for each report operator, with the second report time's going into effect only if the operator is excused at the decision point. Finally, the first and second report times for each operator are assigned in such a way that the total spread is restricted so that no spread premiums are incurred.

Each report operator j will work either a straight shift, determined by the first report (with a probability $pc_{(j)}$ of being assigned a run before the decision point), or a split shift, determined by the first report time, the decision point, and the second report time (with a probability $1 - pc_{(j)}$). Clearly, $pc_{(j)}$ will depend on the first report time and on the report times for other extraboard operators. That is, $pc_{(j)}$ will be a function of the number of bus pullouts and reliefs, the probability of any of these pullouts being open, and the availability of other report operators for the time span extending from the first report time to the decision point.

Consequently, the previous constraint set used for continuous assignments must be modified in the case of split assignments with the following redefinitions of $xf_{(i)}$ and $xp_{(i)}$:

$$xf_{(i)} = \sum_{j=1}^{Nf} pc_{(j)} \cdot rcf_{(ij)} + \sum_{j=1}^{Nf} (1 - pc_{(j)}) \cdot rsf_{(ij)} \qquad (10)$$

$$xp_{(i)} = \sum_{l=1}^{Np} pc_{(l)} \cdot rcp_{(il)} + \sum_{l=1}^{Np} (1 - pc_{(l)}) \cdot rsp_{(il)} \qquad (11)$$

where

$xf_{(i)}$ = extraboard FTO profile in period i;
$xp_{(i)}$ = extraboard PTO profile in period i;
$rcf_{(ij)}$ = 1 if continuous shift of FTO j includes period i, 0 otherwise;
$rsf_{(ij)}$ = 1 if split shift of FTO j includes period i, 0 otherwise;
$rcp_{(il)}$ = 1 if continuous shift of PTO l includes period i, 0 otherwise;
$rsp_{(il)}$ = 1 if split shift of PTO l includes period i, 0 otherwise; and
$pc_{(j)}$ = probability that report operator j will be assigned work before the decision point.

SOLUTION ALGORITHM

For even a small number of extraboard operators, many combinations of report times are possible. Because it would be computationally prohibitive to evaluate the expected overtime (or unproductive time) associated with all assignment combinations, another solution algorithm was adopted.

The problem formulation requires that the decision variables (i.e., the number of operators reporting in each time period) be integers. However, because most algorithms to determine optimal integer solutions are computationally inefficient, a greedy heuristic procedure was used to solve the problem. It involved the incremental allocation of the available extraboard operators, with each operator assumed to report at the beginning of a period. At each iteration, the appropriate report time is determined on the basis of maximizing the expected marginal reduction of UOW for the operator being assigned at that iteration, while keeping the previous assignments fixed. This process is repeated until all report operators have been assigned report times. The measure of marginal reduction of UOW for each operator is the total reduction in UOW by assigning i to time period t. By adopting this measure, the algorithm assigns all FTOs first and then assigns any PTOs.

Lower bounds on the optimal solution are particularly useful when a heuristic is being proposed because they can be used to determine an upper bound on the difference between the heuristic solution and the optimal solution. In this case, there are two interesting lower bounds on the solution. One lower bound is provided by the solution to the problem with the integrality constraint relaxed. Another lower bound results from relaxing the work rule constraints so that operator duties are not restricted to shifts of fixed length and may be as short as one period. In effect, this assumes that a total supply of report hours is available equal to the total hours to be worked by report operators. Consequently, the incremental allocation in the solution methodology is based on the available extraboard operator periods (e.g., quarter hours) instead of operator shifts. The result is an "ideal" profile, which can also be a lower bound in estimating the savings that may result from relaxed work rules.

CASE STUDY

The methodology was applied to a large bus garage in the Massachusetts Bay Transportation Authority (MBTA), making use of existing MBTA extraboard work rules and other labor contract provisions. At MBTA, both FTOs on 8-hr shifts and PTOs on 6-hr shifts are assigned to the extraboard, but only straight (continuous) shifts can be worked by all operators. A virtually unique feature of MBTA is that, by law, management has the right to use an unrestricted number of PTOs. In addition, extraboard FTOs and PTOs can cover any open work, regardless of whether it is due to the unavailability of an FTO or a PTO. Another MBTA characteristic is that trippers are not permitted, which eliminates one type of task that extraboard operators are usually required to perform.

The spring 1985 schedule was chosen for the primary application of the methodology because extensive data were available for that period. The weekday profile of regular operators (FTOs and PTOs) by time of day shows the heavy peaking that is characteristic of many large U.S. transit authorities (see Figure 3). The 22-hr operating day was divided into 15-min periods for this analysis. Data describing unexpected absence patterns by time of day for each day of the week

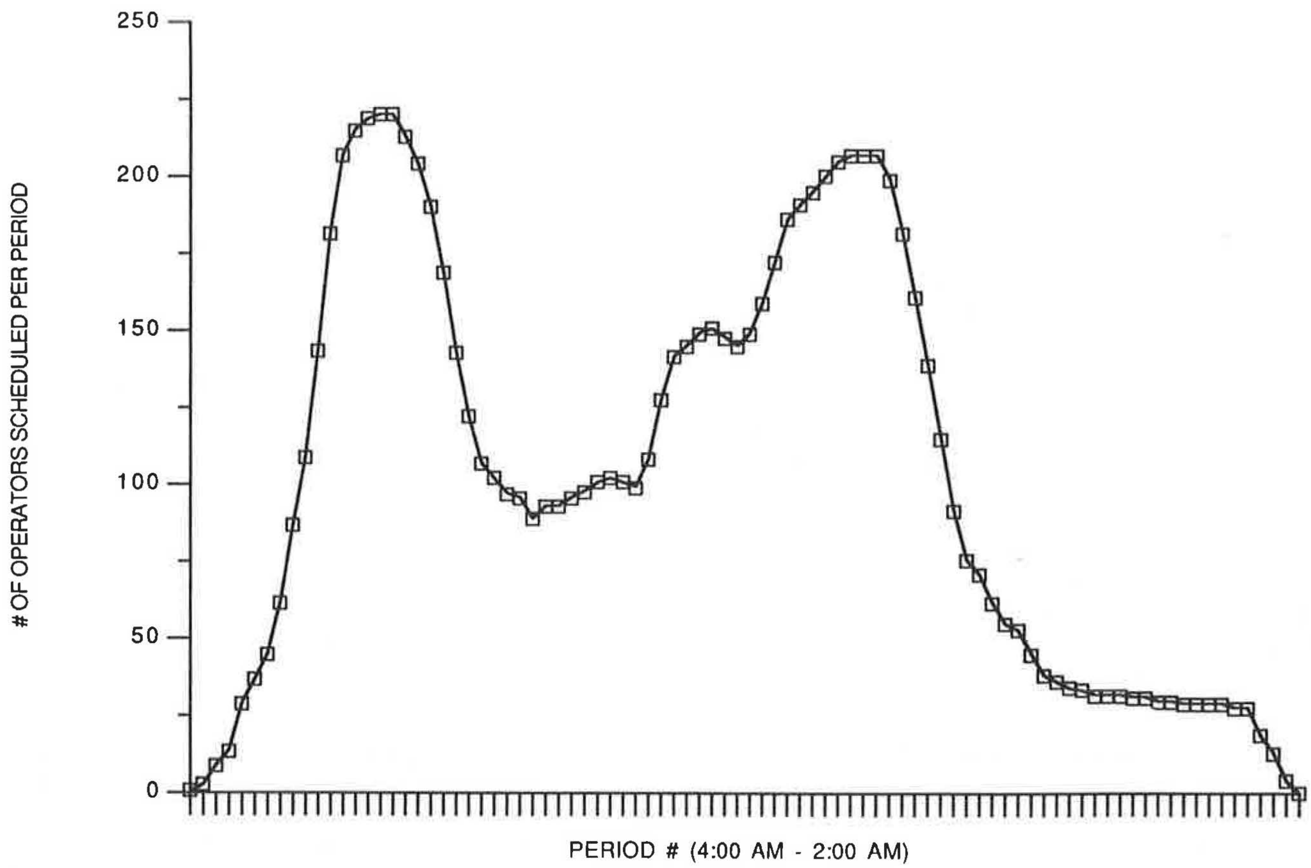

FIGURE 3 Profile of scheduled runs.

were used to determine the probability density functions describing unanticipated requirements by time of day. The number of unexpected absences during each time period was divided by the number of scheduled operators during the same period. This ratio yields the probability of a run being open during that period. By estimating this probability for all periods of the day and smoothing the resulting profile, the open run probability profile was developed for each day of the week. Figure 4 shows the smoothed open run probability profile for a typical Monday.

Unless otherwise stated, the results in this section apply to 10 report operators, 6 full-time and 4 part-time, which was a typical (though not necessarily optimal) weekday level of cover for the selected garage.

Reliability

As described earlier, UOW that occurs in different periods of the day can be split into missed service (MS) and overtime (OT), according to the model shown in Figure 2. This function can be used either to estimate the MS that will occur under the different objectives or to establish minimization of MS itself as an objective. Open work occurring in the early morning (i.e., before 9:00 a.m.) not covered by extraboard operators is unlikely to be filled due to the lack of available operators and their unwillingness to work overtime at that time of the day. Consequently, morning UOW is more likely to result in MS, which will affect service reliability. One way of dealing with this problem is to introduce the morning UOW measure directly into the objective function. Within the framework of the proposed methodology, this can be accomplished by placing a weight, W, in the objective function on UOW occurring before 9:00 a.m. Another way is to define MS minimization as the objective.

Both approaches to reliability were tested for three weekdays (Monday, Tuesday, and Friday), which have different absence patterns. Table 1 presents UOW, morning UOW, MS, OT, and distribution of report hours by time of day resulting from the objectives of minimizing UOW (with W = 1, 2, and 4) and minimizing MS in the proposed methodology. The parameters relating to the split of UOW between MS and OT at different periods of the day were estimated by dividing the operating day into two periods (before and after 9:00 a.m.) and setting parameters in such a way that the overall daily level of MS was the same as that actually experienced at the garage. In the early a.m. period, it was assumed that all UOW would result in missed trips. This procedure was followed because no data relating to the actual split of UOW by time of day were available. However, when data are available, these parameters should be related to the actual split between MS and OT observed during different periods of the day.

As presented in Table 1, $W = 2$ seems to offer a reasonable balance between reductions in morning UOW and increases in total UOW and, in fact, closely approximates the results obtained from minimizing MS directly. Between $W = 1$ and $W = 2$, the increase in total UOW is small, whereas the

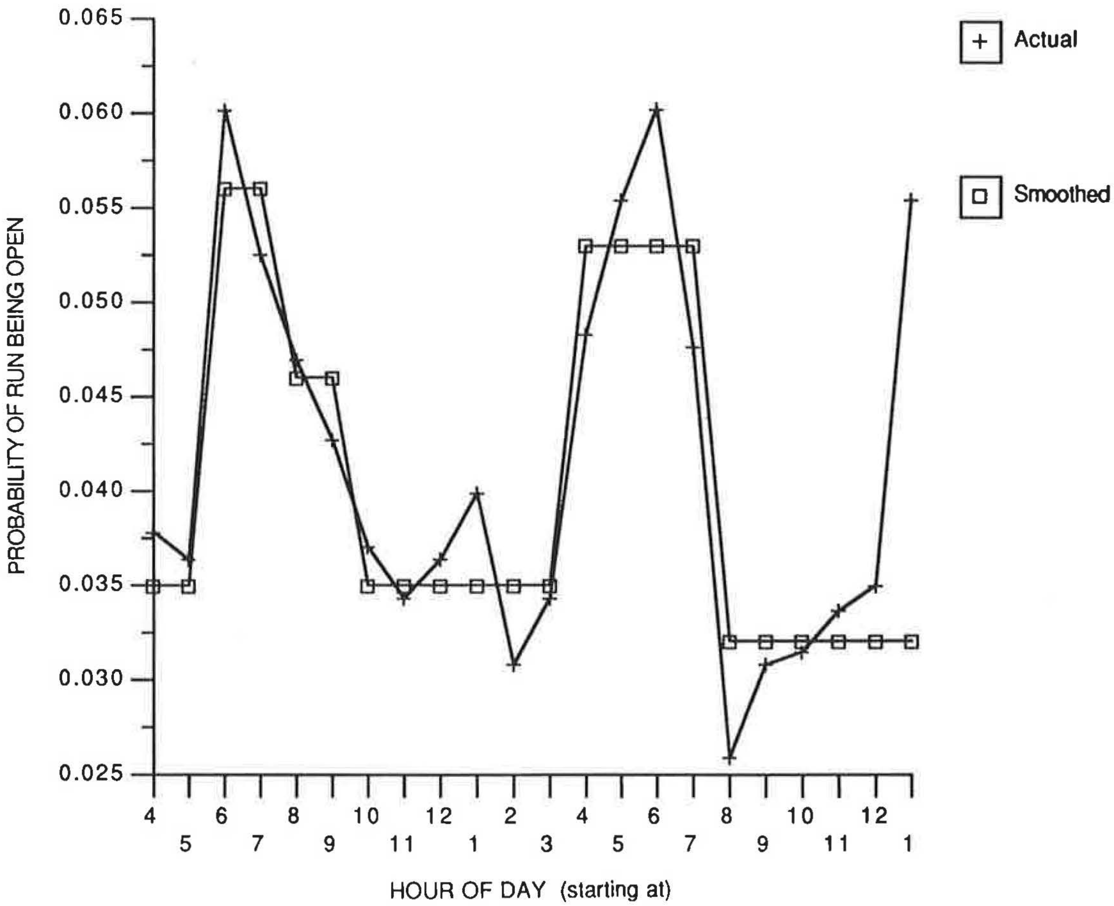

FIGURE 4 Open-run probability profile: Monday.

TABLE 1 SERVICE RELIABILITY

		UOW	Morn UOW	MS	OT	Distribution
Mon	W=1	40.1	14.5	24.1	16.0	28/18/26
	W=2	43.9	9.5	23.7	20.2	39/17/16
	W=4	55.7	4.0	27.2	28.5	55/17/0
	min MS	44.1	9.4	23.7	20.4	40/16/16
Tue	W=1	15.8	3.4	6.7	9.1	27/20/25
	W=2	15.9	3.5	6.8	9.1	27/19/26
	W=4	17.6	2.1	7.0	10.7	34/18/20
	min MS	16.1	3.3	6.7	9.4	28/18/26
Fri	W=1	33.8	9.1	18.2	15.7	21/20/31
	W=2	35.1	6.3	17.5	17.6	27/19/26
	W=4	42.5	2.9	19.9	22.6	38/19/15
	Min MS	35.1	6.3	17.4	17.7	27/19/26

The notation (a/b/c) refers to:

a= hours of morning report time (before 11 AM)

b= hours of mid-day report time (11 AM - 3 PM)

c= hours of PM report time (after 3 PM)

reduction in morning UOW is significant. Between $W = 2$ and $W = 4$, the increase in total UOW is large relative to the reduction in morning UOW. As expected, the distribution of report hours is significantly affected by the value of W, with more reports being shifted into the morning as W increases.

Figure 5 contrasts the occurrence of MS over the course of a Monday between the two cases of minimizing UOW with $W = 1$ and minimizing MS. It is apparent that significantly more MS occurs during the morning peak in the case of $W = 1$ than under the MS minimization objective. However, this situation is reversed during the afternoon. As far as OT is concerned, the $W = 1$ case results in less OT in almost all periods of the day. No OT is required in the early a.m. hours because, according to the UOW split model, all UOW that occurs before 9:00 a.m. is translated into MS. These observations correlate directly with the fact that the MS minimization objective (or the $W = 2$ case) assigns more report operators in the morning hours.

Obviously, the proper W for MBTA will depend on the value MBTA places on avoiding missed trips and its ability to get drivers to work overtime at different times of the day. For this paper, $W = 2$ appears to be a suitable weight for addressing constraints on available OT operators.

Data Issues

The availability of accurate and detailed data may have a significant effect on the outcome of the analysis. Data describing patterns of unanticipated work requirements by day of week and by time of day are critical inputs to the proposed methodology for setting report times. On a day-of-week level, unexpected absence hours, and consequently the probabilities of open runs, varied for the MBTA garage during the spring 1985 schedule. This suggests that different numbers of report personnel are appropriate for different days of the week. Moreover, different report times are likely to be appropriate for each day of the week based on the patterns of unexpected absences for each day. For example, although Monday and Friday have similar overall levels of unexpected absence, the patterns of absence over the course of each day are somewhat different. Because Monday has more morning absences and Friday more afternoon absences, more early a.m. reports should be provided for Mondays than for Fridays.

Analysis that is sensitive to this level of detail in the unexpected absence patterns requires a more extensive data base but is almost guaranteed to produce a better solution in terms of matching resources to needs. This raises the issue of the

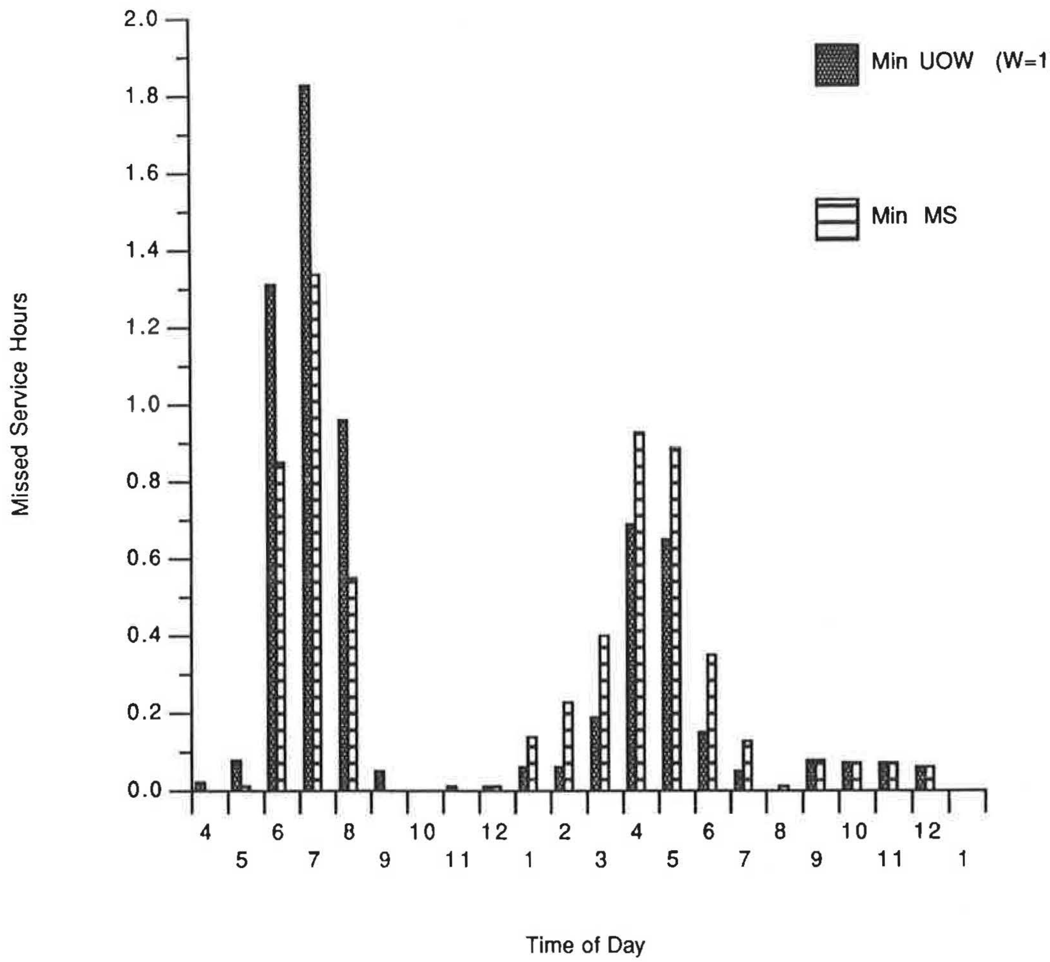

FIGURE 5 Missed service by time of day.

tradeoff between the overhead cost of maintaining and updating a more extensive data base and the cost of the inefficiency introduced into report time setting by using more limited data.

To determine the appropriate level of data, three scenarios were analyzed that feature various assumptions concerning the availability of data on unexpected absences:

1. *Day- and Hour-Specific (DHS).* The open run profiles were set up to differentiate between day of week and time of day assuming full information. This was the basis for comparing the scenarios' performance.
2. *Hour-Specific (HS).* A single open run profile was used for all days of the week that reflected the average patterns of unexpected requirements over the course of a typical day.
3. *Flat (FLAT).* An overall flat rate of unexpected absence was assumed and used to define a single open run profile for all days of the week.

To test the impact of the different levels of data availability, the proposed methodology was used to assign report times under each scenario. Ten report operators (six full-time and four part-time) were assigned report times for a Monday, Tuesday, and Friday for each of the three data scenarios. Obviously, actual levels of report operators to be assigned report times will normally be different for each day of the week based on expected requirements for that day, but such a distinction is not the purpose of this paper. As previously mentioned, a number of planning models exist for optimal sizing of the extraboard and for the daily allocation of extraboard manpower.

Surprisingly, there were no major differences in expected UOW for the three data scenarios (see Table 2), even though the three profiles of open run probability by time of day are somewhat different. The primary differences occur in the distribution of report hours over the day, as evidenced by the (a/b/c) notation. As expected, both the HS and FLAT scenarios are insensitive to differences among days of the week. For example, whereas in Table 2 the DHS solution produces a 39/17/16 distribution for Monday and a 27/18/27 distribution for Friday, the HS and FLAT scenarios produce 27/18/27 and 33/18/21, respectively, for both days. This difference affects the expected morning UOW, as depicted in the same table. For example, the DHS solution yields 9.45 morning UOW hours for Monday, whereas the HS and FLAT solutions yield 14.93 and 12.00 hr, respectively. Therefore, given an equal number of report hours on Monday and Friday, the DHS solution assigns more morning reports on Monday than on Friday as a result of the higher absence levels on Monday mornings.

Figure 6 presents the expected number of open runs on a typical Monday by time of day for each of the three scenarios. This was expected to be a major determinant of the report time assignments. While there are differences among the scenarios in terms of the mean number of open runs, they have the same overall peaking pattern during the day, and it is likely that this pattern, rather than the exact values occurring at the peaks, is the major determinant of report times. In fact, the optimal availability of report operators in the morning hours as determined by the proposed methodology is not much different for the three scenarios, even on Monday.

The similarity of results for the three scenarios on the three days suggests that overall extraboard effectiveness is not significantly increased by additional information although reliability at specific times of day may be affected. However, this conclusion is limited to the case of straight shift assignments and is not expected to be valid in the case of split assignments. For split assignments, the peaks can be covered more efficiently and the peak values, in addition to peaking patterns, are expected to be of significance in report time determination.

TABLE 2 EFFECT OF LEVEL OF DATA AVAILABLE

		DHS	HS	FLAT
Mon	Exp. morn UOW	9.5	14.9	12.0
	Exp. UOW	43.9	40.0	41.1
	Exp. wted UOW	53.3	55.0	53.1
	Distribution	39/17/16	27/18/27	33/18/21
Tue	Exp. morn UOW	3.5	3.4	2.1
	Exp. UOW	15.9	15.8	17.4
	Exp. wted UOW	19.4	19.2	19.4
	Distribution	27/19/26	27/18/27	33/18/21
Fri	Exp. morn UOW	6.3	6.4	4.4
	Exp. UOW	35.1	34.9	37.5
	Exp. wted UOW	41.4	41.3	41.9
	Distribution	27/18/27	27/18/27	33/18/21

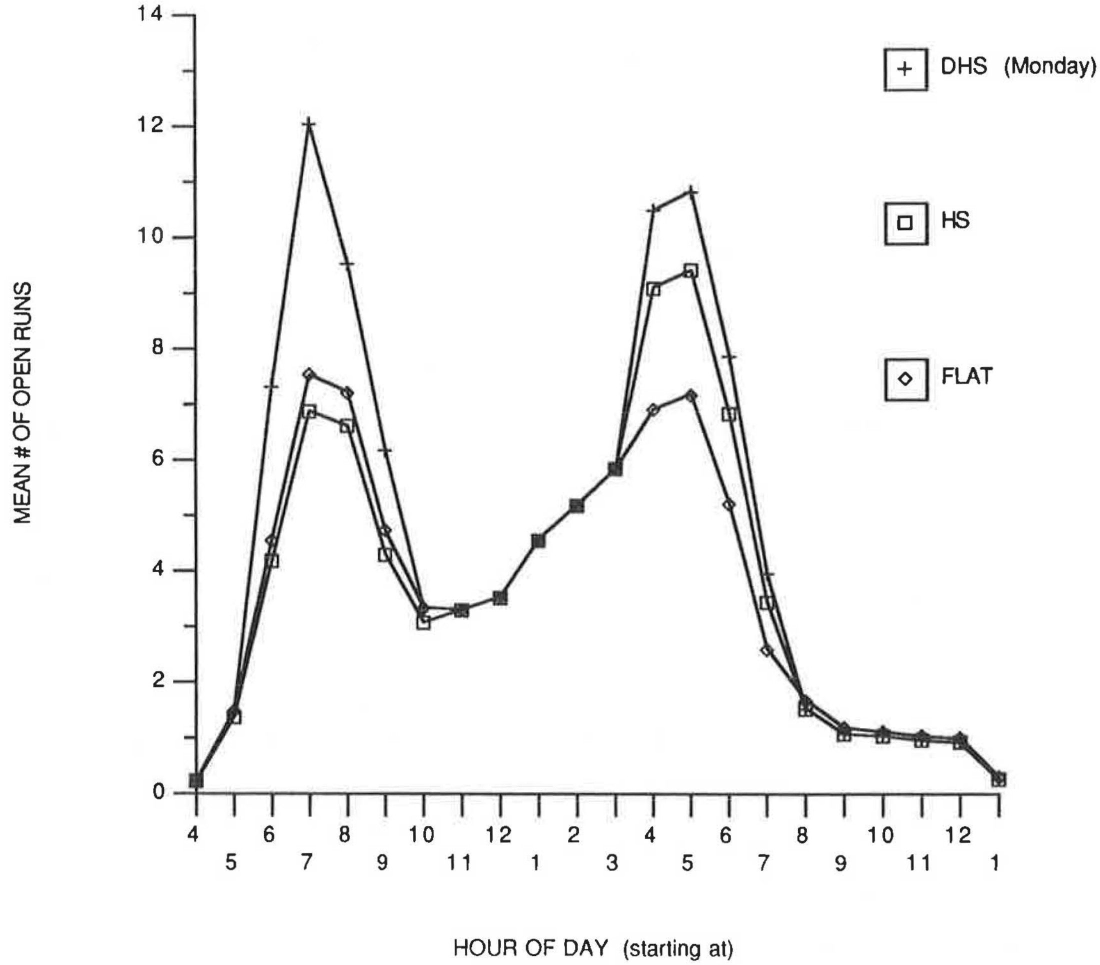

FIGURE 6 Open-run profile.

Lower Bound Results

To evaluate the potential savings from a relaxation of work rules and to assess the effectiveness of the heuristic, a lower bound (LB) was obtained by assigning operator shifts that are not restricted in length. Under this strategy, the allocation of available report time to the different periods of the day yields an ideal profile of report operator assignment. As shown in Table 3, the LB values are from 8.5 to 16.4 percent lower than the model values, after the full 72 report hours are assigned. Because the LB value corresponds to the ideal case of no work rule constraints, it indicates both how far the model results are from the ideal results and how much may be gained from a complete relaxation of the work rules.

Obviously, relaxation of any one work rule would not come close to the "no work rules" scenario; hence, the savings from work rule relaxation will achieve only a fraction of the savings indicated by the LB values. Thus, this lower bound is not very tight and serves only to indicate the general proximity of the model results to the ideal results. This bound is the second lower bound referred to earlier in this paper. It was felt that these results were sufficient to indicate that this method produces close to optimal results and that the alternative lower bound introduced earlier was not necessary in this case.

Evaluating Current Practice

At MBTA, report time setting relies heavily on experience and judgment and little on formal analysis. For this reason, it was expected that introducing analytic tools into the process of setting report times might produce tangible benefits. To evaluate the possible improvements, an experiment was conducted using data from the same MBTA garage during the summer 1987 schedule but relying on unexpected absence patterns developed for spring 1985. The report times actually used at the garage were assessed to estimate the expected UOW if these report times were utilized with the anticipated absence patterns in effect. The expected UOW value was then compared with the values obtained from using the report times produced by the heuristic algorithm.

As shown in Table 4, the methodology, as applied to five consecutive Mondays, consistently outperformed the manual approach used at MBTA. As might be expected, the number of operators actually available for report assignments varies considerably across these days, and so, of course, does the amount of UOW. The expected total UOW or morning UOW levels resulting from the methodology, both for $W = 1$ and $W = 2$, are significantly lower than those expected to occur under the report times actually used. In fact, the $W = 2$ case

TABLE 3 LOWER BOUND RESULTS (HOURS OF UNCOVERED OPEN WORK)

Report	Mon.		Tues.		Fri.	
Hours	Model	LB	Model	LB	Model	LB
0	101.7	101.7	70.4	70.4	96.9	96.9
8	93.7	93.7	62.5	62.4	89.0	88.9
16	86.0	85.7	55.0	54.6	81.2	81.0
24	78.5	77.7	47.9	46.9	73.5	73.0
32	71.4	69.8	41.4	39.7	66.2	65.2
40	64.7	62.1	35.2	33.0	59.2	57.6
48	58.5	54.6	29.8	27.0	52.6	50.2
54	53.3	49.2	25.5	22.9	47.5	45.0
60	48.7	44.0	21.9	19.3	42.5	40.0
66	44.1	39.0	18.5	16.1	37.9	35.3
72	40.1	34.5	15.8	13.2	33.8	31.0
% Diff		14.1		16.4		8.5

TABLE 4 EVALUATING CURRENT PRACTICE

				DHS	
Day	Report Operators (FT-PT)		Act. Rep.	W=1	W=2
Mon 6/29	11-7	morn UOW	6.6	7.2	3.9
		UOW	29.6	19.6	21.9
Mon 7/6	3-0	morn UOW	25.6	26.6	19.9
		UOW	92.5	91.2	92.4
Mon 7/13	6-6	morn UOW	10.8	14.4	9.2
		UOW	53.2	41.6	45.1
Mon 7/20	8-12	morn UOW	4.6	5.4	3.9
		UOW	35.5	16.3	18.1
Mon 7/27	10-5	morn UOW	1.7	9.1	5.4
		UOW	51.4	28.1	31.2

presents a reduction in either morning UOW or total UOW ranging between 15 and 49 percent for each of the Mondays analyzed. Table 5 presents the actual and recommended report times for two Mondays. These results should be viewed with caution because the assumed absence patterns relate to the spring 1985 schedule. It was not possible to base the evaluation on actual summer 1987 absences.

Value of Split Shifts

A final issue to be evaluated in the context of the MBTA case study is the improved efficiency in covering open work that might result from permitting split assignments. Currently, the MBTA work rules only permit continuous assignments for report operators. The following analysis is based on the relax-

TABLE 5 ACTUAL VERSUS RECOMMENDED REPORT TIMES

Mon 7/13		Mon 7/27	
Actual	Recom.	Actual	Recom.
	4.45	4.30	4.30
5.00	5.00	4.30	
	5.30		4.45
	5.45	5.00	
6.00	6.00	5.00	5.00
6.00	6.00	5.00	
	6.00	5.30	5.30
7.00			5.45
7.00		6.00	
7.00		6.00	6.00
7.00		6.00	6.00
8.00		6.00	6.00
8.00		6.00	
	13.30		6.15
	14.00	6.30	
	14.00	8.00	
	14.00	12.00	
	15.30	13.00	
15.45			13.30
18.15			14.00
20.00			14.00
			14.00
			14.45
			15.30

ation of this restriction and on assigning split shifts to report operators according to two rules:

1. A report operator who is not assigned work during the first 2 hr 15 min will be excused and assigned a later report time (defining the decision point).
2. The maximum time allowed from the first report time to the end of the second piece is not to exceed 10 hr 15 min (defining the spread).

This analysis was run for a typical Monday for the same MBTA garage using scheduled run data for fall '87 and assuming 10 full-time and 8 part-time report operators. Table 6 presents values for the slight reductions in UOW and MS achievable by making split assignments instead of continuous assignments. Figure 7 shows the reduction in MS with the incremental allocation of each operator. The slight improvements result from the flexibility afforded by split shifts to match the peaked nature of demand over the course of the day.

TABLE 6 VALUE OF SPLIT SHIFTS TO MBTA

	Oper.	UOW		MS	
		cont.	split	cont.	split
FTO	1	82.2	82.2	47.3	47.3
	2	74.8	74.8	42.1	42.1
	3	68.0	67.5	37.3	37.3
	4	62.0	60.6	33.1	32.8
	5	54.3	54.0	29.3	28.6
	6	49.9	49.1	25.8	24.8
	7	42.9	41.8	22.3	21.1
	8	36.6	36.2	19.2	17.8
	9	32.8	30.0	16.7	14.8
	10	27.3	24.8	14.1	12.1
PTO	11	24.9	21.9	12.0	9.7
	12	20.7	18.1	10.0	8.1
	13	17.3	15.5	8.3	6.3
	14	15.5	12.4	6.8	4.8
	15	12.9	10.6	5.6	3.6
	16	10.3	8.4	4.6	2.8
	17	9.0	6.9	3.5	2.1
	18	7.1	5.2	2.8	1.6
	% Red.		26.8		42.8

CONCLUSIONS

A formulation and solution method were presented for the problem of scheduling a fixed number of operators for report duties at a transit authority. The proposed method incorporates all the important inputs to the problem: the variability of unanticipated work requirements, work rules relating to extraboard personnel and affecting their availability, reliability objectives and constraints, and the availability of regular operators for overtime work. A heuristic algorithm was presented that solves the problem efficiently and is based on the assignment of each operator to maximize the incremental contribution to the objective. The quality of the resulting

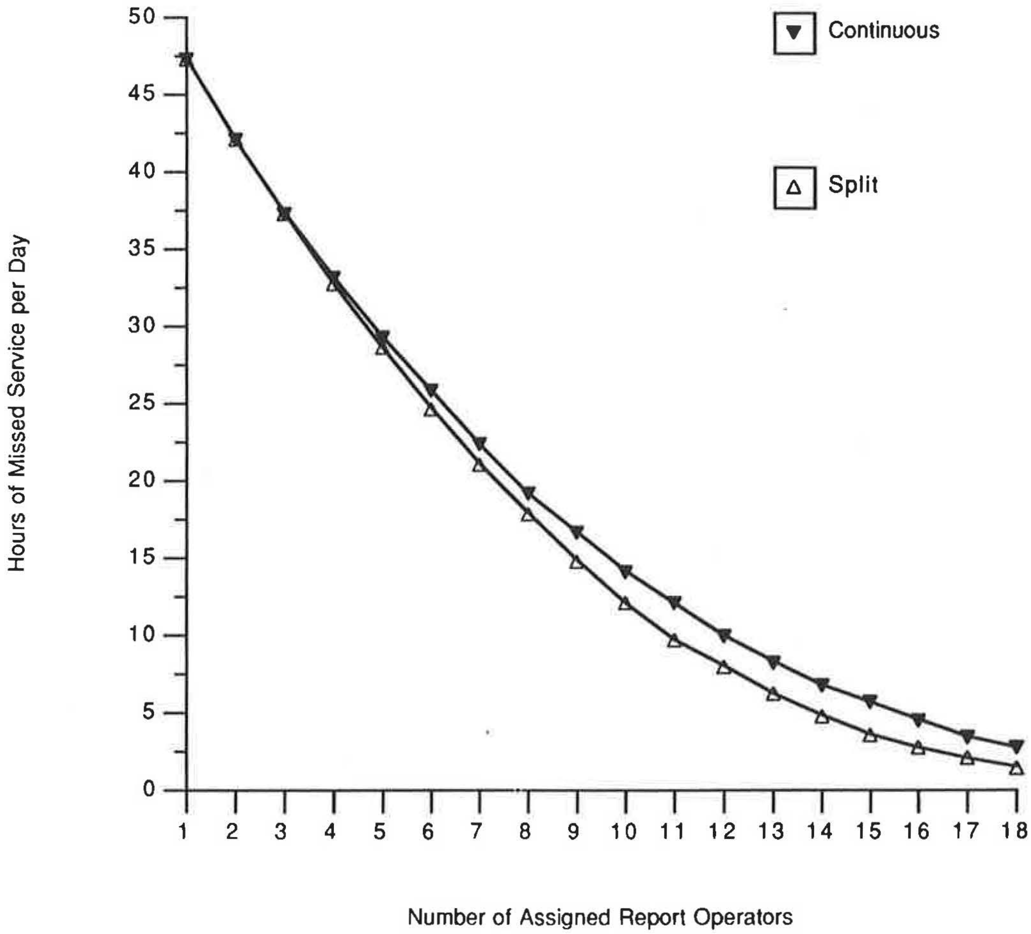

FIGURE 7 Effect of split shifts on missed service.

solution was found to be good when compared with a lower bound to the solution that was also developed within the framework of this study.

The proposed methodology was applied to a case study involving MBTA. The methodology was applied with the objective of minimizing total uncovered open work while meeting system reliability objectives by placing a weight ($W = 2$) in the objective function relating to minimizing uncovered open work on any such work occurring before 9:00 a.m. This produced results that were similar to the case of minimizing missed service for the whole operating day. The case study also indicated that the proposed methodology can be applied based on minimal data requirements (such as a flat rate of unexpected absences) with results offering tangible improvements over current report time setting practices. Other results indicated that small additional benefits can be achieved by having the freedom to assign split shifts to report operators and by having part-time operators work at least some of the report duties. Finally, it was clear from the case study that report time setting based on an analysis of the major inputs to the scheduling problem can offer significant improvements over report times that are based only on experience and common sense. This was evidenced by comparing uncovered open work resulting from report times actually used at the MBTA garage with uncovered open work expected from the implementation of the report times produced by the proposed methodology.

ACKNOWLEDGMENTS

The research described in this paper was funded in part by the Urban Mass Transportation Administration and the Massachusetts Bay Transportation Authority. Particular thanks are due to Brian McCollom, the UMTA project monitor for this research, for his many valuable suggestions.

REFERENCES

1. L. C. MacDorman and J. C. MacDorman. The Transit Extraboard: Some Opportunities for Cost Savings. Presented at the Annual Meeting of the American Public Transit Association, 1982.
2. L. C. MacDorman. *Extraboard Management: Procedures and Tools*. NCTRP Synthesis of Transit Practice 5. TRB, National Research Council, Washington, D.C., June 1985.
3. Booz-Allen & Hamilton, Inc. *Operator Availability Management Methods*. Technical Assistance Program. UMTA, U.S. Department of Transportation, 1984.
4. H. N. Koutsopoulos. *A Methodology for Transit Operator Workforce Planning*. Ph.D. thesis. Massachusetts Institute of Technology, Cambridge, 1986.
5. H. N. Koutsopoulos and N. H. M. Wilson. Operator Workforce Planning in the Transit Industry. *Transportation Research*, Vol. 21A, No. 2, 1987, pp. 127–138.
6. I. Kaysi. *Extraboard Personnel Scheduling in the Transit Industry*. M.S. thesis. Massachusetts Institute of Technology, Cambridge, 1988.

Publication of this paper sponsored by Committee on Transit Management and Performance.