Stochastic Optimization Subsystem of a Network-Level Bridge Management System

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The prediction and stochastic optimization modules are two of seven modules that make up a stochastic network-level bridge management system which is under development. An overview of major portions of the bridge management system is provided. The prediction model of structural degradation generates initial estimates of transition probabilities (tp values). A tp value is defined as the probability that a bridge segment will move from one condition state to another within 1 year given the maintenance scope assigned to it. The tp values are updated with new survey data using a Bayesian updating procedure. Methods are developed to account for the fact that structural surveys may be performed on a multiyear basis, while yearly tp values are needed for the optimization models. The optimization module, which minimizes cost subject to top management's performance objectives, is a Markovian-based linear program that stratifies the bridge network to improve degradation predictions. Rather than using single ratings for a major bridge element (e.g., bridge deck), the program optimizes on a bridge segment level to maximize the use of structural condition information. The condition state of a segment can include selected functional deficiencies as well as structural condition ratings.

A network-level bridge management system (BMS) based on the Markovian decision process is under development. Seven modules make up this BMS. The condition module uses surveyed condition-rating data to derive condition states that characterize the overall condition of each bridge segment. The use of this module to derive engineering-based maintenance solutions has been described by Harper et al. in a companion paper in this record.

The maintenance and repair (M&R) scopes module contains 40 possible levels of M&R intensity (under the categories of repair, replacement, rehabilitation, and routine maintenance) for the condition states. Each M&R scope has a defined effect on each condition level, so that improvements resulting from the application of these scopes can be modeled. The M&R scopes provide input to the prediction, cost, optimization, packaging, and comparator modules.

The prediction module estimates transition probabilities (tp) and uses Bayesian techniques to update them to predict the probability that a given segment will move from one condition state to another over time. The prediction module covers all M&R scopes, so that long-term segment changes can be predicted.

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The cost module uses historical cost data, condition states, M&R scopes, and other inputs to estimate unit costs of the M&R scopes. This module includes a parametric equation that can be used to aid in the generation of user costs.

The optimization module consists of three network optimization solution models based on a Markovian decision model using linear programming techniques. A separate linear program is solved for each stratum. Bridges are stratified according to factors such as bridge type, climate, and functional class. The first year's solution (of the multiyear model) provides the network-level guidance used in the subsequent modules.

The packaging module packages the first year of the optimized network solutions into individual work projects in which the generalized M&R scopes are made specific. In the project-level analyses by the packager, maintenance costs identified by the optimizer will be more accurately assessed.

The comparator module performs a quality control role on the performance and implementation of the BMS and provides necessary comparisons of the cost and predictive capabilities of the models with actual experience when the BMS solutions are implemented.

BASIC UNIT OF MANAGEMENT

Network bridge optimization can be approached in two ways in terms of the basic unit being modeled. Either the bridge or a subset thereof can be the fundamental unit for the optimization model. This BMS can work on bridge segments as this subset. A segment is defined as one superstructure span with a unit of substructure (either a pier or an abutment).

The difficulty in using the bridge as the unit of optimization is that many M&R activities will apply only to a given segment, and not to the entire structure. Although tasks for a particular bridge can be determined given the survey information, it would be extremely difficult to predict future bridge maintenance needs without subdividing the structure into smaller units. Also, better cost estimations are possible when segments are the basic unit.

Each bridge segment is categorized by its condition state and the strata to which it is assigned. Core condition states are developed on the basis of the structural condition ratings. The core condition state assigns a condition level of good, fair, poor, or critical to the major bridge elements, the deck, superstructure, and substructure. This level is based on the detailed ratings of the individual components that comprise each element. Supplementing the core condition state parameters are user-defined parameters, such as element age, or various functional deficiency parameters, such as insufficient deck width.

Strata have been created in this BMS for two reasons. The first is to develop groups of bridges that exhibit similar degradation patterns. The second is to form groups that have approximately the same costs for the various M&R scopes. Subject to the desired performance goals in the optimization module, the interaction of cost and transition probabilities determines the optimal policy. User-defined stratifying variables may include such items as bridge type, climate, and functional class.

PREDICTION MODULE

The prediction module is based on a Bayesian updating of tp values. These tp values are estimates of the probability of a segment moving from one condition state to another for the various M&R scopes. In the following paragraphs, the development of initial tp values is followed by a brief description of a direct Bayesian updating of the tp matrices that are generated for later input to the Markovian-based optimization models.

A Bayesian updating of the tp values necessitates assigning a prior probability distribution. This approach uses the multivariate Dirichlet distribution (*1*). Given the current condition state, the tp values for moving from that state to all possible states in the next year must add up to 1. The survey data updating the tp values is multinomial. The Dirichlet distribution is a conjugate prior for the multinomial and may be considered a multivariate generalization of the beta distribution, as the multinomial is a generalization of the binomial distribution. The Dirichlet distribution simultaneously updates each individual tp value for a given initial state and ensures that the resulting sum is 1. A separate Dirichlet distribution is used for each row of a given tp matrix.

Each prior tp estimate can be treated as coming from a beta distribution when a Dirichlet multivariate prior distribution is assumed for a given set of tp values. The beta prior distributions can be handled individually, and the probabilistic aspects of the posterior tp values are preserved (the sum of tp values for any row in the tp matrix equals 1).

Given a Dirichlet prior distribution and multinomial observed data, the resulting posterior distribution is a Dirichlet distribution. The tp values needed may be easily determined from the Bayesian updated matrix once all the new data have been used in the updating procedure (*2*).

INITIAL DEVELOPMENT OF TRANSITION PROBABILITIES

To develop initial tp values for each stratum is necessary. These tp values will provide the first prior distributions for the initial Bayesian updating. After implementation of the BMS, the Bayesian updating will result in self-adjustment of the tp values to the specific conditions for each stratum. However, during the first years of its operation the initial tp values will still influence the operation of the Markovian decision-based optimization models.

Expert opinion has been used to estimate the remaining useful life (RUL) of the deck, superstructure, and substructure for different bridge types on the basis of current condition ratings. This information is used to develop the initial tp values, which are updated annually with the actual survey data using Bayesian statistical methods. The initial tp matrix is made specific for each stratum. If sufficient historical data exist, they should be used to develop the initial tp values.

The following example generates the initial tp values for a bridge deck changing condition levels from good to the possible condition levels good, fair, poor, or critical in 1 year under routine maintenance. The same procedure will be used to develop the tp values for both superstructure and substructure. These tp values include adjustment for the dependence of the elements. The initial core tp matrix (based on element condition ratings) results from multiplication of the associated individual-element tp value.

Using the results of the analyzed expert opinion, the estimated RUL for an average deck in good condition results in a good RUL_{Deck} of 30 years (*2*). Similarly, the expected RUL for a top-of-the-range fair deck, (Top of fair RUL_{Deck}) equals 22 years. The difference between the expected RUL for a typical good deck and the top of the fair deck level results in an expected difference [Delta(RUL)] of 8 years.

A structural dependency table is then used to adjust the Delta(RUL), if necessary, to account for the condition of the structural elements. This procedure results in an AdjDelta(RUL). Assuming the other elements are in good condition, this adjustment results in an AdjDelta(RUL) that is still 8 years. The resulting tp value for a deck going from good to fair, when the other elements are in good condition, may be estimated as ⅛, which equals 0.125. The general formula for converting an AdjDelta(RUL) to a tp value is as follows:

$$tp = [\text{AdjDelta(RUL)}]^{-1} \quad (1)$$

This formulation results in the correct expected transition times from one level to another. In the development of the initial tp matrix, an additional assumption is made that under normal conditions, a structural element will not degrade more than one level in a 1-year time period. Thus, the tp values for a deck transitioning from good to each of the four levels (when the other elements are in good condition) are calculated as follows:

To Condition Level	*tp Value*
Good	1.0 − 0.125 = 0.875
Fair	1.0/(30 − 22) = 0.125
Poor	0.0
Critical	0.0

Using the same approach, the other deck tp values may be calculated. In a similar manner, the tp values will be calculated for superstructure and substructure. These tp values are combined to give the initial joint tp values for the core condition states under routine maintenance. From this tp matrix, all other M&R scope tp matrices are generated.

BAYESIAN UPDATING USING MULTIYEAR SURVEYS

The initial tp values are 1-year probabilities. Surveys performed in a given year may only cover part of the structures each year. The resulting data then reflect multiyear tp values instead of the needed 1-year tp values. Thus, the survey will represent different periods of time ($k = 1, 2, \ldots,$ years) since the last survey on various structures. In this section, the methodology is introduced for using the multiyear data to generate 1-year tp values. If all bridges are surveyed each year, these steps are not necessary.

After each year's survey, the prior tp values can easily be converted from 1-year tp value estimates to k-year (where k is a positive integer) tp value estimates by multiplying the tp matrix by itself as follows:

$$\mathbf{T}^{(k)} = \mathbf{T}^k \tag{2}$$

where

$\mathbf{T}^{(k)}$ = k-year tp matrix,
$\mathbf{T}$ = 1-year tp matrix, and
$\mathbf{T}^k$ = $\mathbf{T}$ multiplied by itself k times.

Using the matrix $\mathbf{T}^{(k)}$, the Bayesian updating algorithm presented in *B&SMS Conceptual Framework* (*2*) may be applied using Year k survey data (k years since last survey). Thus $\mathbf{T}^{(k)}$ represents the prior tp matrix that will be updated with the Year k survey data. A prior tp matrix is needed for each Year k that the current survey represents. For each Year k, there is a Bayesian updating step. Year $k = 1$ survey data will be used first, then Year $k = 2$ survey data, and so forth until all the survey data have been used in the tp updating. For example, if current survey data are available for bridges that were last surveyed 1, 2, 3, and 5 years ago, the mathematical process will generate the needed prior tp matrix for each of the 4 years represented in the survey.

The resulting updated tp values are k-year tp values. These tp values then need to be converted to $(k + 1)$-year tp values so that Year $(k + 1)$ survey data can be used for updating purposes. After all the survey data have been used, the final conversion to 1-year tp values is used for the Markovian decision linear program process. In the following, $\mathbf{B}_k$ represents the posterior k-year tp matrix (resulting matrix after the Bayesian updating using Year k data), and $\mathbf{A}_k$ represents the posterior 1-year tp matrix after updating with Year k data. The mathematics for this is as follows:

$$\mathbf{B}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{U}_k^{-1} \tag{3}$$

where

$\mathbf{B}_k$ = k-year updated (posterior) tp matrix,
$\mathbf{U}_k$ = eigenvector matrix of $\mathbf{B}_k$, and
$\mathbf{D}_k$ = diagonal matrix with eigenvalues on diagonal.

Then,

$$\mathbf{A}_k = \mathbf{U}_k\mathbf{D}_k^{1/k}\mathbf{U}_k^{-1} \tag{4}$$

where

$\mathbf{A}_k$ = annual (or 1-year) updated tp matrix after updating with Year k data,
$\mathbf{U}_k$ = eigenvector matrix of $\mathbf{B}_k$, and
$\mathbf{D}_k^{1/k}$ = diagonal matrix $\mathbf{D}_k$ replaced with kth roots of its diagonals.

It is easily seen that

$$(\mathbf{A}_k)^k = (\mathbf{U}_k\mathbf{D}_k^{1/k}\mathbf{U}_k^{-1})^k = \mathbf{U}_k\mathbf{D}_k\mathbf{U}_k^{-1} = \mathbf{B}_k \tag{5}$$

After all the survey data have been used, the desired Bayesian, updated, 1-year tp values are in the final tp matrix $\mathbf{A}_F$. There is a separate matrix $\mathbf{A}_F$ for each stratum. This posterior matrix $\mathbf{A}_F$ becomes the prior tp matrix $\mathbf{T}$ in the subsequent year. This approach using eigenvalues also eliminates the need to multiply the matrix $\mathbf{T}$ by itself k times to generate the matrix $\mathbf{T}^{(k)}$. Similarly, the steps can be reduced by moving directly from a posterior k-year tp matrix to a prior $(k + 1)$-year tp matrix without having to create an intermediate posterior 1-year tp matrix after each Year k Bayesian updating.

As an illustration of this procedure, assume that the recent survey provides data for bridges that were last surveyed 1, 2, 3, and 5 years ago. The prior matrix $\mathbf{T}$ will be updated using the methodology in *B&SMS Conceptual Framework* (*2*) with the Year $k = 1$ data, resulting in the posterior tp matrix $\mathbf{B}_1$. Obviously, $\mathbf{A}_1 = \mathbf{B}_1$. Following the mathematics illustrated with matrix $\mathbf{T}$, the matrix $\mathbf{A}_1$ becomes the prior 1-year tp matrix for updating with Year $k = 2$ data. Therefore the prior 2-year tp matrix is $\mathbf{A}_1^2 = \mathbf{A}_1 * \mathbf{A}_1$ (or could be generated using the eigenvalue approach). This tp matrix is updated with the Year $k = 2$ data, resulting in the posterior 2-year tp matrix $\mathbf{B}_2$. From this matrix, $\mathbf{A}_2$ is obtained, representing the posterior 1-year tp matrix after updating with Year $k = 2$ data. This in turn becomes the prior 1-year tp matrix for updating with Year $k = 3$ data. As a result, $\mathbf{A}_2^3$ is the needed prior 3-year tp matrix. (Another way to obtain $\mathbf{A}_2^3$ is to bypass generating the 1-year posterior tp matrix and go directly from the Year 2 posterior tp matrix $\mathbf{B}_2$ to the desired prior tp matrix, $\mathbf{A}_2^3$. Because $\mathbf{B}_2 = \mathbf{U}_2\mathbf{D}_2\mathbf{U}_2^{-1}$, $\mathbf{A}_2^3 = \mathbf{U}_2\mathbf{D}_2^{3/2}\mathbf{U}_2^{-1}$.) $\mathbf{A}_2^3$ is updated, resulting in the posterior $\mathbf{B}_3$ (3-year posterior tp matrix). After Year $k = 3$ data have been used, $\mathbf{A}_3$ is the 1-year posterior tp matrix and is then the prior 1-year tp matrix for subsequent updating with the Year $k = 5$ data. $\mathbf{A}_3^5$ is the prior 5-year tp matrix that is updated to give $\mathbf{B}_5$, the posterior 5-year tp matrix. From this matrix, $\mathbf{A}_5$ is the desired 1-year tp matrix $\mathbf{A}_F$ that provides the tp matrix needed for the Markov-based linear programs. Of course, $\mathbf{A}_5$ will be the prior 1-year tp matrix $\mathbf{T}$ in the following year.

BMS OPTIMIZATION MODULE

The three optimization models are the long-term (steady state) goal-setting model, the multiyear (short-term) planning model, and the financial exigency planning model. The long-term model is used to establish the steady state goals that provide targets for the multiyear and financial exigency models. The multiyear model addresses the year-by-year maintenance needs for the planning horizon. Both the long-term and multiyear models solve a separate linear program for each stratum. The financial exigency model imposes a network-wide budget constraint across all strata if the budget is insufficient to satisfy the sum of the multiyear models of the individual strata.

The optimization models can be used to develop a set of maintenance plans for a bridge system over the desired planning horizon. Various inputs are required to run each of the models. Management input, cost parameters, tp values, and condition survey data are necessary. This process is iterative. If satisfactory results are not obtained, looping backwards may be necessary. For example, if satisfactory budget estimates cannot be obtained from the short-term model, new performance objectives may need to be set. This would require running both the steady state and short-term models again.

The following planning steps are necessary:

Step 1. Survey the bridge system. The survey results are used to update the present estimates of tp values. They are also used to compute proportions of the bridges in each condition state to be used in the multiyear planning model and the comparator.

Step 2. Determine realistic long-term performance goals by solving the long-term model until an acceptable level of annual expenditures is achieved. This iteration may involve lowering performance objectives to obtain a satisfactory budget level. The final result becomes a goal to be reached in the final year of the planning horizon for the multiyear optimization.

Step 3. Determine performance objectives to be achieved for each year of the planning horizon. The present surveyed condition states describe the present performance level, and the long-term model solution indicates the performance objectives for the final year of the planning horizon.

Step 4. Solve the multiyear model to determine the optimal maintenance policy for each year in the planning horizon and to develop the expected expenditures. If budgeting requirements are too high between the first and last years of the planning horizon, the performance objectives can be revised.

Step 5. If the multiyear solution is satisfactory, the first-year maintenance policy is packaged into actual projects using the packager module. If the result is not satisfactory, Step 6 is required.

Step 6. If the multiyear solution does not provide satisfactory results because of inadequate fund availability for the first year, the financial exigency model is solved. This solution indicates the optimal first-year maintenance policy that stays within the first-year budget while computing the additional expenditures needed to successfully achieve the performance objectives for the remaining years in the planning horizon. Management must then decide whether this additional cost is excessive. If not, the solution may be considered as the plan for the entire planning horizon, and Step 5 is executed. If the additional cost is considered excessive, two options are (a) to request supplemental funds for the first year to relax the budget constraint, or (b) to reduce performance objectives (and subsequent costs) for succeeding years. As with the multiyear model, the financial exigency model may be used iteratively to revise performance objectives or to justify supplemental budget requests. Once an acceptable solution is found, Step 5 is performed. The final step is to implement the packaged projects.

The linear programs for each of the three network models minimize cost subject to meeting the desired performance goals of top management. The Markovian-based linear programs optimize the M&R scopes for bridge segments instead of the entire bridge, because this allows a more in-depth use of the available information. The system accommodates mandatory projects (in which specific actions are mandated for a given bridge) that have been determined on the basis of engineering or policy decisions. The mathematical structures of the linear programs are provided in the following section. More detailed explanation of all three optimization models are given in *B&SMS Conceptual Framework* (*2*). Modifying these models to include additional performance constraints is not difficult.

LONG-TERM OPTIMIZATION MODEL

The parameters of the long-term (or steady-state) model are defined as follows:

Input Parameters

I = index set $(1, 2, \ldots, n)$ of condition states;
D = index set $(i_1, i_2, \ldots, i_g)$ of desirable condition states;
U = index set $(i_1, i_2, \ldots, i_b)$ of undesirable condition states;
S = index set $(1, 2, \ldots, m)$ of bridge strata;
M_i = index set $(a_1, a_2, \ldots, a_{m_i})$ of feasible maintenance scopes a for bridge segments in condition state i;
$C_{ia}(s)$ = average cost of maintenance scope a applied to one bridge segment in stratum s and condition state i;
$P_{iaj}(s)$ = the probability that a segment in stratum s and condition state i that has scope a applied to it will transition into condition state j in 1 year;
$\bar{p}(s)$ = maximum proportion of segments in stratum s that is allowed in an undesirable condition state;
$\underline{p}(s)$ = minimum proportion of segments in stratum s that should be in a desirable condition state; and
$N(s)$ = number of segments in stratum s.

Output Parameters (Decision Variables)

$w_{ia}(s)$ = proportion of the segments in stratum s that are in condition state i and should receive maintenance scope a; and
$C(s)$ = expected maintenance cost per segment in stratum s.

The long-term optimization model for stratum s requires minimizing the expression

$$C(s) = \sum_{i \in I} \sum_{a \in M_i} w_{ia}(s) C_{ia}(s) \tag{6}$$

subject to

$$w_{ia}(s) \geq 0 \qquad \text{for all } a \text{ in } M_i \text{ and } i \text{ in } I \tag{7}$$

$$\sum_{i \in I} \sum_{a \in M_i} w_{ia}(s) = 1 \tag{8}$$

$$\sum_{a \in M_j} w_{ja}(s) - \sum_{i \in I} \sum_{a \in M_i} w_{ia}(s) P_{iaj}(s) = 0 \qquad \text{for all } j \text{ in } I \tag{9}$$

$$\sum_{i \in D} \sum_{a \in M_i} w_{ia}(s) \geq \underline{p}(s) \tag{10}$$

$$\sum_{i \in U} \sum_{a \in M_i} w_{ia}(s) \leq \bar{p}(s) \tag{11}$$

Minimizing the objective function of Equation 6 minimizes the average cost per segment in stratum s. To get the total expected long-term cost for the stratum, the solution $C(s)$ must be multiplied by $N(s)$, the number of segments in the stratum. Constraints 7 and 8 ensure that solutions satisfy the probability axioms. The $w_{ia}(s)$ functions may be thought of as the elements of a discrete joint probability distribution. Constraint 7 ensures the nonnegativity of each individual element in this joint probability distribution, and Constraint 8 forces its sum over the possible sample space to equal 1. Constraint 9 provides the steady state equations for a Markovian process (forcing the proportion of the network in condition state i to remain fixed, i.e., at a steady state). Constraints 10 and 11 enforce the lower bound on the proportion of segments in desirable condition states and the upper bound on the proportion in undesirable states, respectively. Additional constraints to satisfy particular functional deficiencies can be easily added as desired.

MULTIYEAR OPTIMIZATION MODEL

The notation for the multiyear model is defined as follows:

Input Parameters

- I = index set $(1, 2, \ldots, n)$ of condition states;
- D = index set $(i_1, i_2, \ldots, i_g)$ of desirable condition states;
- U = index set $(i_1, i_2, \ldots, i_b)$ of undesirable condition states;
- S = index set $(1, 2, \ldots, m)$ of bridge strata;
- M_i = index set $(a_1, a_2, \ldots, a_{m_i})$ of feasible maintenance scopes a for bridge segments in condition state i;
- $C_{ia}(s)$ = average cost of applying maintenance scope a to one bridge segment in stratum s and condition state i;
- $P_{iaj}(s)$ = probability that a segment in stratum s and condition state i that has scope a applied to it will change into condition state j in 1 year;
- $\bar{p}^t(s)$ = maximum proportion of segments in stratum s allowed in an undesirable state in year t;
- $\underline{p}^t(s)$ = minimum proportion of segments in stratum s that should be in a desirable state in year t;
- $\hat{w}_{ia}(s)$ = lower bound on the proportion of segments in stratum s that is in condition state i and will receive maintenance scope a in Year 1, for mandatory projects;
- $q_i(s)$ = proportion of the segments in stratum s in condition state i at the beginning of Year 1;
- ϕ = parameter for uniformly relaxing minimum desirable condition state standards in Year 2;
- f = parameter for uniformly relaxing maximum undesirable condition state standards in Year 2;
- g, h = tolerances;
- $w^*_{ia}(s)$ = optimal values for the steady state (long-term) problem;
- r = discount rate for computing net present value; and
- $C^*(s)$ = optimal cost per segment in stratum s from the steady state (long-term) model.

Output Parameters (Decision Variable)

- $w^t_{ia}(s)$ = proportion of the segments in stratum s that are in condition state i and should receive maintenance scope a in year t,
- $\hat{C}(s)$ = expected net present value of cost per segment in stratum s of a maintenance policy, and
- $E^t(s)$ = expected expenditures in year t in stratum s.

The finalized multiyear (short-term) optimization model for stratum s requires minimizing the expression

$$\hat{C}(s) = \sum_{t=1}^{T} \sum_{i \in I} \sum_{a \in M_i} (1 + r)^{1-t}\, w^t_{ia}(s) C_{ia}(s) \tag{12}$$

subject to

$$w^t_{ia}(s) \geq \hat{w}^t_{ia}(s) \quad \text{for all } i \text{ in } I \text{ and } a \text{ in } M_i, \text{ for } t \text{ with known mandatory projects} \tag{13}$$

$$w^t_{ia}(s) \geq 0 \quad \text{for all } i \text{ in } I,\ a \text{ in } M_i, \text{ and } 2 \leq t \leq T \tag{14}$$

$$\sum_{i \in I} \sum_{a \in M_i} w^t_{ia}(s) = 1 \quad 1 \leq t \leq T \tag{15}$$

$$\sum_{a \in M_i} w^1_{ia}(s) = q_i(s) \quad \text{for all } i \text{ in } I \tag{16}$$

$$\sum_{a \in M_i} w^t_{ja}(s) - \sum_{i \in I} \sum_{a \in M_i} w^{t-1}_{ia}(s) P_{iaj}(s) = 0 \text{ for all } j \text{ in } I \text{ and } 2 \leq t \leq T \tag{17}$$

$$\sum_{i \in D} \sum_{a \in M_i} w^2_{ia}(s) \geq \underline{p}^2(s)\phi \tag{18}$$

$$\sum_{i \in D} \sum_{a \in M_i} w^t_{ia}(s) \geq \underline{p}^t(s) \quad \text{for } 3 \leq t < T \tag{19}$$

$$\sum_{i \in U} \sum_{a \in M_i} w^2_{ia}(s) \leq \bar{p}^2(s) f \tag{20}$$

$$\sum_{i \in U} \sum_{a \in M_i} w^t_{ia}(s) \leq \bar{p}^t(s) \quad \text{for } 3 \leq t < T \tag{21}$$

$$\sum_{a \in M_i} w^T_{ia}(s) \geq \sum_{a \in M_i} (1 - g) w^*_{ia}(s) \quad \text{for all } i \text{ in } I \tag{22}$$

$$\sum_{a \in M_i} w^T_{ia}(s) \leq \sum_{a \in M_i} (1 + g) w^*_{ia}(s) \quad \text{for all } i \text{ in } I \tag{23}$$

$$\sum_{i \in I} \sum_{a \in M_i} w^T_{ia}(s) C_{ia}(s) \leq (1 + h) C^*(s) \tag{24}$$

Minimizing the objective function of Equation 12 minimizes the average present cost per segment of maintenance over the time horizon of interest. To get the necessary (least) budget, $E^t(s)$, for stratum s for Year t, the following calculation is necessary:

$$E^t(s) = N(s) \sum_{i \in I} \sum_{a \in M_i} w^t_{ia}(s) C_{ia}(s)$$

Constraint 13 accommodates the mandatory projects for Year 1 and beyond, if planned. Constraints 14 and 15 are based on probability and are needed to satisfy the fact that $w_{ia}^{t}(s)$ constitutes a discrete joint probability distribution. Constraint 16 ensures that the optimal scopes associated with each condition state are assigned to the correct percentage of the network, a boundary condition that sets $q_i(s)$, the proportion of the network in each condition state in the first year, on the basis of survey results. Constraint 17 is a condition state balance equation from Year $t - 1$ to Year t based on the use of the transition probabilities $P_{iaj}(s)$.

Constraints 18 through 21 force the optimization to meet the performance objectives established by top management. Constraints 18 and 20 also allow a possible relaxation of the second-year performance objectives if desired, for instance, budget is insufficient. Constraints 22 and 23 allow a relaxation, if desired, in meeting the optimal steady state proportions. Constraint 24 allows a similar flexibility in meeting the optimal steady state average cost per segment in the last year (T) of the multiyear planning horizon. As with the long-term model, constraints can easily be added or modified to satisfy the goals of the organization.

FINANCIAL EXIGENCY MODEL

The multiyear model formulated in Equations 12 through 24 is actually a series of identical (in mathematical structure) and independent models, one for each stratum. The financial exigency model ties all the strata models together with a common budget constraint and has a combined objective function. Constraints 13 through 24 do not change in the financial exigency model. This combined model is too large to solve directly by the simplex method used in commercial linear programming packages, but it may be solved using Lagrangean methods. The model requires minimizing the expression

$$\sum_{s \in S} N(s) \sum_{i \in I} \sum_{a \in M_i} \sum_{t=2}^{T} (1 + r)^{1-t} \, w_{ia}^{t}(s) C_{ia}(s)$$

where S is the set of all strata indices, subject to Constraints 13 through 24 for all s in S and the condition

$$\sum_{s \in S} N(s) \sum_{i \in I} \sum_{a \in M_i} w_{ia}^{1}(s) C_{ia}(s) \leq B \qquad (25)$$

where B is the available budget for the first year.

This model seeks to minimize the present worth of the expected cost in Years 2 through T of the maintenance policies for Years 1 through T. Constraint 25 prevents expenditures in Year 1 from exceeding the budget. This constraint combines the problems of different strata into a single problem and destroys their independence. On the basis of work by Everett (*3*), the following modified version of the financial exigency model can be solved by selecting values for the Lagrange multiplier α (*2,4*) that minimize the expression

$$\sum_{s \in S} N(s) \sum_{i \in I} \sum_{a \in M_i} \left[\sum_{t=2}^{T} (1 + r)^{1-t} w_{ia}^{t}(s) C_{ia}(s) \right] + \alpha w_{ia}^{1}(s) C_{ia}(s)$$

subject to Constraints 12 through 24 for all s in S.

Because this version has no budget constraint, it may be separated into the independent strata problems. Once separated, it is exactly like the original multiyear models, Equations 12 through 24, except that the coefficient of the first-year expenditures is α. Different values of α will yield solutions that expend different amounts in Year 1.

Everett's (*3*) results applied to this problem indicate that the amount expended in Year 1 is a monotonic, nonincreasing function of α. Therefore, if, for a given α, the solution prescribes a policy that expends too much money in the first year, a new solution can be obtained for a larger value of α that will expend a smaller amount in Year 1. Everett also proves that if a given value of α produces a solution in which the total of all first-year expenditures among the strata is equal to the first-year budget (B), then such a solution is a globally optimal solution to the original financial exigency model. The results of the financial exigency model are a step function for different values of α. Thus the optimal solution is for the value of α that either results in a sum of B or is as close as it can get to B. An efficient searching procedure on α is used to find the requisite solution for the financial exigency model.

PACKAGING MODULE

The results of the optimization module are the proportions of the segments in a given stratum that should receive a particular M&R scope. The packaging module converts these figures to detailed bridge-by-bridge maintenance actions for the entire network for the first year of the planning horizon. Four major processes are involved: translation, specification, ranking, and aggregation. The packager focuses on a project-level analysis and uses the detailed bridge-by-bridge survey information.

The translation step converts the optimization output to specific M&R scopes for each segment in the network. The specification step uses the in-depth detailed survey information to further refine the M&R scopes into detailed activities for the entire bridge. The ranking step results in an ordered list of bridges that will guide the scheduling of the needed bridge work in the first year of the planning horizon. The aggregation step (if needed) consolidates all the bridge maintenance projects for a given district, geographical area, or other desired subset of the network.

COMPARATOR MODULE

The role of the comparator module is to perform a quality control check on both the BMS and its implementation. The comparator module is a means of evaluating and monitoring the performance of the BMS against established practices and engineering judgment.

The question of how well the BMS is performing and being implemented is addressed through items such as the following:

- Differences between planned and actual M&R activities,
- Differences between planned and actual costs, and
- Differences between planned and actual proportion of segments in desirable condition.

The comparator module also determines the cause for these deviations. Many additional questions are addressed in the comparator module, which provides the feedback necessary both to improve the actual BMS mathematical models and to ensure that its results are being properly implemented.

SUMMARY

The BMS is a modular network stochastic optimization model that also addresses project-specific needs. It provides automatic updating of the degradation models (transition probabilities) using Bayesian statistical procedures. If insufficient historical data are available, the system provides a methodology to generate initial degradation models using expert opinion. In the near future, organizations will find the advantages of such BMS network models as they have with pavement management systems.

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