Resource-Constrained Capital Budgeting Model for Bridge Maintenance, Rehabilitation, and Replacement

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A mathematical formulation is presented for optimal allocation of a granted budget among a system of bridges that are under the jurisdiction of a transportation agency. The formulation is based on a 0–1 integer-linear programming algorithm with multiple-choice constraints. Three improvement alternatives are assumed possible for a bridge at any point in time—replacement, rehabilitation, and major maintenance. Provision is also made for routine preventive maintenance. The optimal alternatives are selected on the basis of the criterion of maximizing reductions in equivalent uniform annual costs to the ultimate owner, the user-taxpayer.

A mathematical formulation is presented for optimal allocation of a granted budget among a system of bridges that are under the jurisdiction of an agency. The formulation is based on a 0–1 integer linear programming algorithm with multiple-choice constraints. The formulation is a part of Optimum Bridge Budget Forecasting and Allocation Module (OPBRIDGE), a computerized decision support system that was developed for the North Carolina Department of Transportation (NCDOT) for managing its 14,100-bridge system (*1*).

ANNUAL COST OF AN EXISTING BRIDGE

The annual cost of an existing Bridge i at the beginning of Year t, AMUC(i,t), consists of two types of costs: (a) annual user cost AURC(i,t), and (b) annual routine maintenance cost ARMC(i,t). These costs can be estimated by the methods developed by Chen and Johnston (*2*). Annual bridge user costs are caused by deficiencies related to narrow width, low vertical clearance, poor alignment, and low load capacity. Bridges having narrow width, low vertical clearance, or poor alignment have a higher accident-inducing probability. Bridges with low vertical clearance or low load capacity cause various proportions of vehicles to be detoured. As the volume of traffic increases, the number of accidents and detours also increases. Thus, the annual user cost increases over time because of continuous increase in average daily traffic, ADT(i,t), and continuous decline in bridge load capacity.

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The sum of the annual routine maintenance costs of the bridge components (e.g., deck, superstructure, and substructure) constitutes the total annual routine maintenance cost needed by a particular bridge. Annual routine maintenance cost also increases over time because of deterioration of element conditions. The resulting annual cost of an existing bridge because of user costs and routine maintenance costs increases with time as shown in Figure 1.

COST PARAMETERS FOR IMPROVEMENT ALTERNATIVES

Three types of improvement alternatives are usually available for a bridge: replacement, rehabilitation, and major maintenance. This section describes methods available to estimate their initial costs, IC(i,j,t), and their equivalent uniform annual costs, EUAC(i,j,t):

IC(i,j,t) = initial cost of Improvement Alternative j for Bridge i at the beginning of Year t; and
EUAC(i,j,t) = equivalent uniform annual cost of improvement alternative j for Bridge i at the beginning of Year t.

Following the methods proposed in the literature (*2–4*), cost profiles of the different alternatives can be developed.

New Bridge Alternative

The first alternative is to replace the existing bridge with a new one having new condition ratings and desirable user levels of service. Conditions of various elements of a new bridge gradually deteriorate with age causing maintenance needs to increase over time. A major rehabilitation is assumed when one of the condition ratings drops below the minimum allowable condition rating. The rehabilitation alternative improves condition ratings to the highest rehabilitation condition ratings, and might improve load capacity, vertical clearance, and width of the bridge. As a result of rehabilitation, the service life of the bridge is extended for a few more years during which routine maintenance is needed.

NCDOT uses the following equations to estimate the initial cost, IC(i,NB,t), of a new bridge alternative for any existing Bridge i at the beginning of Year t (*2*):

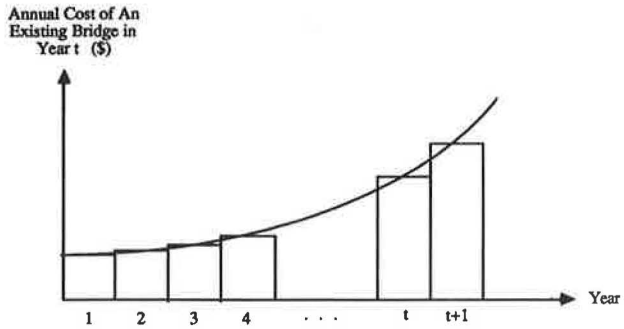

FIGURE 1 Annual cost of an existing bridge estimated discretely at the beginning of every year in the analysis horizon.

$$\text{BASCOS}(i,t) = \text{NBLEN}(i,t) * \text{NBWID}(i,t) * \text{UCDK}(t) \quad (1)$$

$$\text{IC}(i,\text{NB},t) = \text{BASCOS}(i,t) * (1 + \text{DESFEE}/100) + \text{FIXCOS}(t) \quad (2)$$

where

BASCOS(i,t) = basic construction cost of the new bridge that will replace existing Bridge i at the beginning of Year t;
NBLEN(i,t) = length (ft) of the new bridge to replace existing Bridge i at the beginning of Year t;
NBWID(i,t) = width (ft) of the new bridge to replace existing Bridge i at the beginning of Year t;
UCDK(t) = unit cost per deck area ($/ft^2) of constructing a new bridge at the beginning of Year t;
DESFEE = estimated design fee percentage; and
FIXCOS(t) = fixed cost associated with new bridge construction at the beginning of Year t.

The cost profile for one replacement cycle of a new bridge is shown in Figure 2. The replacement cycle cost, RCC(i,NB,t), of a new bridge alternative for bridge i at the beginning of Year t can be expressed as

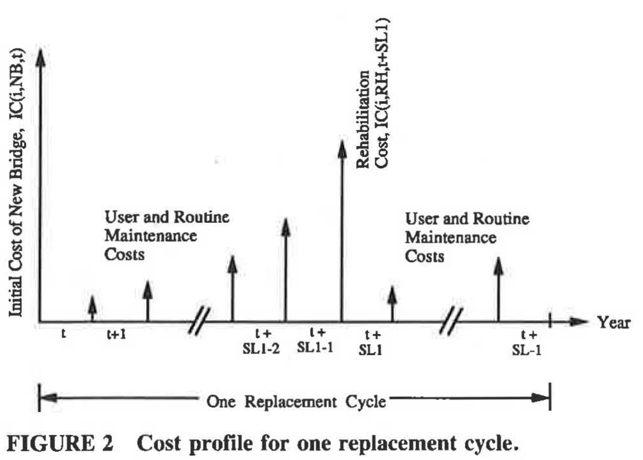

FIGURE 2 Cost profile for one replacement cycle.

$$\text{RCC}(i,\text{NB},t) = \text{IC}(i,\text{NB},t) + \sum_{tt=t}^{t+\text{SL}-1} [\text{AMUC}(i,tt)] * (\text{P/F},\text{RRRR},tt-t) + \text{IC}(i,\text{RH},t+\text{SL1}) * (\text{P/F},\text{RRRR},\text{SL1}) \quad (3)$$

where

AMUC(i,tt) = annual routine maintenance and user costs of Bridge i at the beginning of Year tt;
IC$(i,\text{RH},t + \text{SL1} + 1)$ = initial cost of rehabilitation alternative for Bridge i at the beginning of year $(t + \text{SL1} + 1)$;
(P/F,RRRR,$tt-t$) = single-payment present-value factor;
RRRR = real required rate of return;
SL1 = expected service life from new construction to rehabilitation; and
SL = expected service life of the bridge.

The initial cost, IC(i,RH,t), of the rehabilitation alternative for Bridge i at the beginning of any Year t can be computed as

$$\text{IC}(i,\text{RH},t) = \sum_{k=1}^{N} \text{RHC}(i,k,t) \quad (4)$$

where RHC(i,k,t) is the rehabilitation cost for Element Type k of Bridge i at the beginning of Year t and N is the total number of bridge components that may need rehabilitation (deck, superstructure, and substructure).

The equivalent uniform annual cost, EUAC(i,NB,t), of the new bridge alternative for Bridge i, constructed at the beginning of Year t, over its service life can be estimated as

$$\text{EUAC}(i,\text{NB},t) = \text{RCC}(i,\text{NB},t) * (\text{A/P},\text{RRRR},\text{SL}) \quad (5)$$

where (A/P,RRRR,SL) is the capital recovery factor.

Because bridge service is assumed to be always required, the replacement cycle cost, RCC(i,NB,t), would be repeated at SL intervals. The cost profile for repeated replacement cycles in perpetuity (i.e., forever) is shown in Figure 3. Thus,

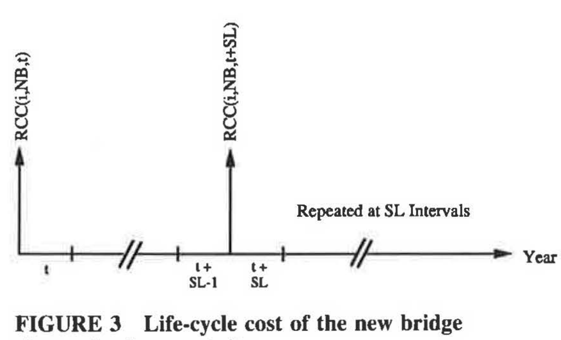

FIGURE 3 Life-cycle cost of the new bridge alternative in perpetuity.

the life cycle cost, LCC(i,NB,t), of the new bridge alternative in perpetuity for Bridge i at the beginning of Year t can be expressed as

$$\text{LCC}(i,\text{NB},t) = \frac{\text{RCC}(i,\text{NB},t)}{1 - (1 + \text{RRRR})^{-\text{SL}}} \quad (6)$$

Rehabilitation Alternative

The second improvement alternative may be to rehabilitate the bridge. Rehabilitation may extend the life of the bridge by several years. Rehabilitation upgrades all bridge element conditions to a desirable rehabilitation condition rating. Thus, the extended service life E is estimated as the number of years until one of the condition ratings drops below the minimum allowable condition rating. At the end of the extended service life E, a new bridge is constructed to replace the rehabilitated bridge.

The life cycle cost, LCC(i,RH,t), of a rehabilitation alternative in perpetuity for Bridge i at the beginning of Year t (Figure 4), can be computed as

$$\text{LCC}(i,\text{RH},t) = \text{IC}(i,\text{RH},t) + \sum_{tt=t}^{t+E-1} \text{AMUC}(i,\text{tt}) * (\text{P/F},\text{RRRR},tt-t) + \text{LCC}(i,\text{NB},t+E) * (\text{P/F},\text{RRRR},E) \quad (7)$$

Therefore, the equivalent uniform annual cost, EUAC(i,RH,t), of a rehabilitation alternative for Bridge i in perpetuity estimated at the beginning of Year t, is

$$\text{EUAC}(i,\text{RH},t) = \text{LCC}(i,\text{RH},t) * \text{RRRR} \quad (8)$$

Major Maintenance Alternative

The third improvement alternative is major maintenance, which has also been termed an "interim rehabilitation" by Chen and Johnston (*2*). The intent is to improve the element in poor condition to a higher condition level compatible with the good elements as shown in Figure 5. All major maintenance cost parameters are estimated from the rehabilitation tables. However, its funding often comes from the maintenance budget.

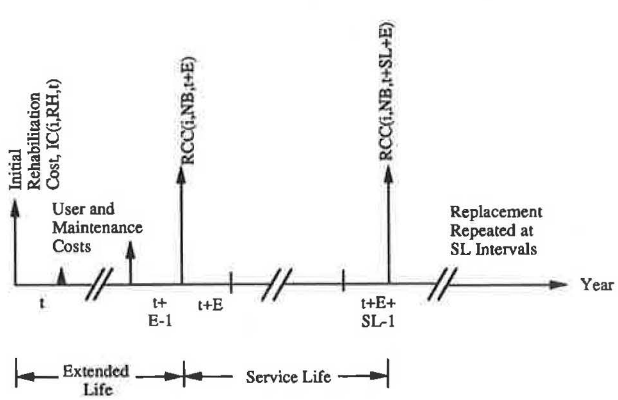

FIGURE 4 Life-cycle cost of the rehabilitation alternative in perpetuity.

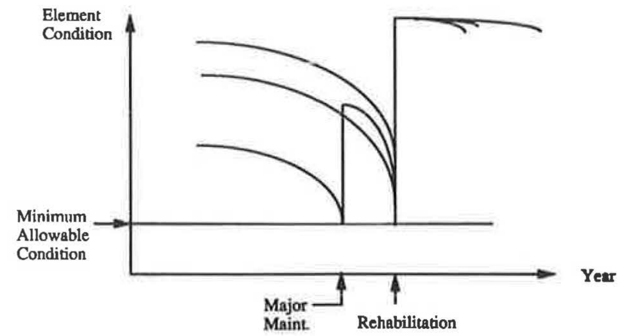

FIGURE 5 Major maintenance evaluation.

Following the approach of Chen and Johnston (*2*), one of two mutually exclusive cases is assumed:

- Case 1—If only one condition rating is less than 6, and the difference between the average of the two higher condition ratings and the lowest condition rating is greater than or equal to 2 points, then rehabilitate the bridge element with the lowest condition rating to the average of the other two higher ratings; or
- Case 2—If only one condition rating is greater than or equal to 6, and the difference of this highest condition and the lowest condition is greater than or equal to 2 points, then rehabilitate the bridge elements with the lowest two condition ratings to the single highest condition rating.

The extended service life produced by a major maintenance action, e, is assumed to be followed by a rehabilitation. Therefore, the equivalent uniform annual cost, EUAC(i,MN2,t), of a major maintenance alternative (MN2) for Bridge i at the beginning of Year t can be computed as

$$\text{EUAC}(i,\text{MN2},t) = \Big[\text{IC}(i,\text{MN2},t) + \sum_{tt=t}^{t+e-1} \text{AMUC}(i,tt) * (\text{P/F},\text{RRRR},tt-t) + \text{LCC}(i,\text{RH},t+e) * (\text{P/F},\text{RRRR},e)\Big] * \text{RRRR} \quad (9)$$

REDUCTION IN EQUIVALENT UNIFORM ANNUAL COST

At the beginning of every year in the analysis horizon, the following question needs to be answered for every bridge: "Will the bridge be routinely maintained and retained in service for one more year, or will it be replaced, rehabilitated, or maintained with a major-maintenance alternative?"

At the bridge level and under unlimited budget assumption, the annual cost of the existing bridge, AMUC(i,t), is compared with the equivalent uniform annual cost, EUAC(i,j,t), of the three Improvement Alternatives j: major maintenance, rehabilitation, and replacement. The alternative with the minimum annual cost is selected (Figure 6).

The procedure suggested by Blank and Tarquine (*5*) requires that the equivalent uniform annual cost of routine maintenance over the remaining life of the existing bridge be

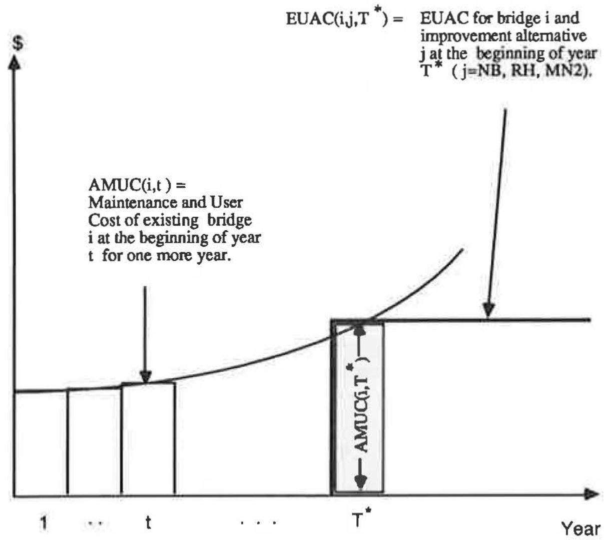

FIGURE 6 Comparison of AMUC(i,t) and EUAC(i,j,t) at the beginning of every year in the analysis horizon to determine optimal time for improving existing bridge.

computed and compared with EUAC(i,j,t) of other improvement alternatives, if AMUC(i,t) is greater than EUAC(i,j,t). However, this procedure is not necessary in this problem because it is always true that after T^*, AMUC(i,t) is smaller than the equivalent uniform annual cost of the routine maintenance alternative over the remaining life of the existing bridge. This could be concluded from the continuous increase in the annual cost of the existing bridge, as shown in Figure 1.

Minimizing EUAC(i,j,t) or AMUC(i,t) does not necessarily produce the optimal solution at the system level and under budgetary constraints, as can be concluded from the following example.

EXAMPLE

At the beginning of Year t, assume that two bridges have two alternatives, routine maintenance or replacement with a new bridge (NB), with the following values:

Bridge	*AURC(i,t)* (*$*)	*ARMC(i,t)* (*$*)	*AMUC(i,t)* (*$*)	*EUAC(i,NB,t)* (*$*)
1	30,000	1,000	31,000	15,000
2	21,000	500	21,500	15,000

Assume that the cost of replacing a bridge is the same for each. Further, assume that there is enough budget to replace only one bridge. Both bridges have a replacement alternative with the same minimum EUAC(i,NB,t), but only one of the two bridges can be replaced. To solve the problem, the analysis process computes reductions (savings) in annual costs as follows:

$$\mathrm{REUAC}(i,j,t) = \mathrm{AMUC}(i,t) - \mathrm{EUAC}(i,j,t) \tag{10}$$

where REUAC(i,j,t) is the reduction in EUAC for Bridge i produced by Improvement Alternative j at the beginning of Year t, and then maximizes the total amount of these reductions in annual cost under various constraints such as funding available for bridge improvements. Thus, in the example for Bridge 1,

$$\mathrm{REUAC}(1,\mathrm{NB},t) = \$31{,}000 - \$15{,}000 = \$16{,}000$$

and for Bridge 2,

$$\mathrm{REUAC}(2,\mathrm{NB},t) = \$21{,}500 - \$15{,}000 = \$6{,}500$$

Bridge 1 should be selected because the reduction in user and agency costs will be greater than that for Bridge 2 for the same equivalent annual investment.

If REUAC(i,j,t) is negative in Equation 10, the routine maintenance alternative for Bridge i at the beginning of Year t is the optimum action (Figure 6). However, if the bridge is deficient with respect to the user level-of-service goals (*6*), a Major Improvement j for Bridge i at the beginning of Year t with a negative REUAC(i,j,t) can be selected if an immediate improvement for deficient bridges is requested by the decision maker and the budget is enough to allow for such a selection.

It is important to understand the reasons for including AMUC(i,t) in Equation 10. AMUC(i,t) includes both current annual user and routine maintenance costs. Current annual user cost is included because users are the ultimate owners of the bridges. Current annual routine (preventive) maintenance cost is included because routine maintenance protects the bridge system against accelerated deterioration. Routine maintenance is generally recommended by modern maintenance systems for many types of facilities and plants. More importantly, all cost and deterioration tables were estimated on the basis of the assumption that routine maintenance is provided for all bridges.

The traffic parameters, cost factors, and deterioration relationships developed by Chen and Johnston (*2*) are used to estimate the initial and equivalent uniform annual costs (*1*).

MATHEMATICAL FORMULATION

The optimization problem, formulated at the beginning of every year in the analysis horizon, is a 0–1 integer linear programming algorithm with multiple-choice constraints, also called generalized upper bound (GUB) constraints. For every Year t in the analysis horizon, H, the yearly budgets are optimally allocated by maximizing the overall reductions in equivalent uniform annual costs.

$$\text{Maximize} \sum_{i=1}^{Nb} \sum_{j=1}^{\mathrm{NALT}(i,t)} \mathrm{REUAC}(i,j,t)\, X(i,j,t) \tag{11}$$

subject to the following constraints:

1. Total budget constraint:

$$\sum_{i=1}^{Nb} \sum_{j=1}^{\mathrm{NALT}(i,t)} \mathrm{IC}(i,j,t)\, X(i,j,t) \leq \mathrm{B}(t,\mathrm{TOTAL}) \tag{12}$$

2. Maintenance budget constraint:

$$\sum_{i=1}^{Nb} \mathrm{IC}(i,\mathrm{MN2},t) X(i,\mathrm{MN2},t) \leq \mathrm{B}(t,\mathrm{MN}) \tag{13}$$

3. Rehabilitation budget constraint:

$$\sum_{i=1}^{N_b} IC(i,RH,t)X(i,RH,t) \leq B(t,RH) \quad (14)$$

4. New bridge budget constraint:

$$\sum_{i=1}^{N_b} IC(i,NB,t)X(i,NB,t) \leq B(t,NB) \quad (15)$$

5. User level-of-service goals constraint:

$$LOS(i,g,t) + \sum_{j=1}^{NALT(i,t)} D(i,j,g,t)X(i,j,t) \geq MINREQ(1)$$

for $i = 1, 2, \ldots, N_b$ and

$$g = 1, 2, \ldots, N_g \quad (16)$$

6. Minimum allowable condition rating constraint:

$$CR(i,c,t) + \sum_{j=1}^{NALT(i,t)} G(i,j,c,t)X(i,j,t) \geq MINREQ(2)$$

for $i = 1, 2, \ldots, N_b$ and

$$c = 1, 2, \ldots, N_c \quad (17)$$

7. Multiple-choice decision variable constraint:

$$\sum_{j=1}^{NALT(i,t)} X(i,j,t) \leq 1 \qquad \text{for } i = 1, 2, \ldots, N_b \quad (18)$$

8. Decision variable constraint:

$$X(i,j,t) = 0,1 \qquad \text{for } i = 1, 2, \ldots, N_b \text{ and } j = 1, 2, \ldots NALT(i,t) \quad (19)$$

where

N_b = number of bridges in the system;
$NALT(i,t)$ = number of improvement alternatives for Bridge i in Year t, or number of improvement alternatives in the ith GUB constraint for Year t, normally three, the new bridge (NB) alternative, major maintenance alternative (MN2), and rehabilitation (RH) alternative;
$REUAC(i,j,t)$ = reduction in equivalent uniform annual cost for Improvement Alternative j, Bridge i, and Year t, computed by Equation 10;
$X(i,j,t)$ = decision variable for Bridge i, Alternative j, and Year t. It is 1 if the alternative is selected and 0 otherwise;
$IC(i,j,t)$ = initial cost for Alternative j, Bridge i, and Year t;
$B(t,TOTAL)$ = total budget for Year t;
$B(t,MN)$ = budget for maintenance activities in Year t;
$B(t,RH)$ = budget for rehabilitation activities in Year t;
$B(t,NB)$ = budget for new bridge activities in Year t;
$LOS(i,g,t)$ = level of service of Bridge i with respect to Goal g at the beginning of Year t;
$D(i,j,g,t)$ = gain in level of service of Bridge i with respect to Goal g if Alternative j is selected for implementation during Year t;
MINREQ(1) = user level-of-service goal selected as a part of the minimum performance requirements to be either (a) acceptable or (b) desirable;
N_g = number of user level-of-service bridge attributes measured on the scale of MINREQ(1), normally four, consisting of load capacity, clear deck width, vertical roadway underclearance, and vertical roadway overclearance;
$CR(i,c,t)$ = condition rating of Component c of Bridge i at the beginning of Year t;
$G(i,j,c,t)$ = gain in condition rating of Component c of Bridge i if Alternative j is selected for implementation during Year t;
MINREQ(2) = minimum allowable condition rating; and
N_c = number of major bridge components, normally three, consisting of deck, superstructure, and substructure.

Budgets can be granted, limited, or unlimited maximum allowable budgets. The following actions are performed as parts of the problem preprocessing in order to simplify the problem solution:

1. Constraints 12 through 15 are eliminated if no budgetary constraints are imposed.
2. Constraint 12 is eliminated if the total budget is distributed by maintenance, rehabilitation, and replacement activities, because $B(t,TOTAL) = B(t,MN) + B(t,RH) + B(t,NB)$.
3. Constraints 13 through 15 are eliminated if the total budget is not distributed.
4. Constraints 16 and 17 are satisfied by including only improvement alternatives that can satisfy the minimum performance requirements.

The mathematical formulation of OPBRIDGE shows that the optimization is performed for each year independently.

ROUTINE MAINTENANCE: THE BASE ALTERNATIVE

The routine-maintenance alternative is considered to be essential, because it protects the bridges from accelerated deterioration. In a particular year, the load capacity deterioration and condition rating deterioration of those bridges that are not routinely maintained are accelerated by a multiplying factor of D_f ($D_f > 1.0$) compared with a factor of 1.0 for bridges that are routinely maintained. For this reason, routine maintenance is considered the base alternative that is provided if a major improvement alternative is not economical, not enforced by requesting immediate improvement for deficient bridges, or not possible because of budget limitation.

However, there are two cases in which only part of the routine maintenance is provided: (a) the budget can be entered as a distributed budget in which the maintenance budget is not large enough to perform all the routine maintenance required; and (b) the budget can be entered as a total budget that is not large enough to perform all the necessary routine maintenance. Of course, no major improvement alternative can be considered for any bridge in the second case. To make the routine maintenance alternative the base alternative in the mathematical formulation, four steps are needed:

1. An initial sharing routine-maintenance factor, FACMN1, is computed as follows:

$$\text{FACMN1} = \text{Minimum}(\text{BA/BR}, 1.0) \quad (20)$$

where BA is the budget available for routine maintenance and BR is the budget required adopting the routine maintenance alternatives for all bridges.

2. Each bridge is provided with an amount of routine maintenance dollars in Year t, AMCP(i,t), equal to FACMN1 multiplied by the routine maintenance dollars the particular bridge needs in Year t, AMC(i,t), that is,

$$\text{AMCP}(i,t) = \text{FACMN1} * \text{AMC}(i,t) \quad (21)$$

3. The following variables are redefined in this formulation. If the budget is distributed,

$$\text{IC}(i,\text{MN2},t) = \text{IC}(i,\text{MN2},t) - \text{AMCP}(i,t) \quad (22)$$

$$\text{B}(t,\text{MN}) = \text{B}(t,\text{MN}) - \text{BP}(t,\text{MN1}) \quad (23)$$

If the total budget is used,

$$\text{IC}(i,\text{MN2},t) = \text{IC}(i,\text{MN2},t) - \text{AMCP}(i,t) \quad (24)$$

$$\text{IC}(i,\text{RH},t) = \text{IC}(i,\text{RH},t) - \text{AMCP}(i,t) \quad (25)$$

$$\text{IC}(i,\text{NB},t) = \text{IC}(i,\text{NB},t) - \text{AMCP}(i,t) \quad (26)$$

$$\text{B}(t,\text{TOTAL}) = \text{B}(t,\text{TOTAL}) - \text{BP}(t,\text{MN1}) \quad (27)$$

where

$$\text{BP}(t,\text{MN1}) = \sum_{i=1}^{N_b} \text{AMCP}(i,t) \quad (28)$$

4. After the problem is solved, certain bridges might be recommended for improvement alternatives. Therefore, if FACMN1 < 1, the routine maintenance budget is reallocated among those bridges that were not selected for an improvement alternative.

The deterioration multiplying factor, D_f, is evaluated as follows:

$$D_f = 1.0 + 0.2 * (1.0 - \text{FACMN1}) \quad (29)$$

In Equation 29, the constant 0.2 is assumed to be the factor for deteriorating bridges if no routine maintenance is provided at all (i.e., a 20 percent increase in deterioration rate). If a bridge is provided with all the routine maintenance budget required (i.e., if FACMN1 equals 1.0), then D_f will also equal 1. On the other hand, if a bridge is provided with only 60 percent of the routine maintenance required, D_f will equal 1.08. Further, if routine maintenance is provided for a bridge during a certain year, the bridge load capacity and condition ratings at the end of the year are computed as follows:

$$\text{CE} = \text{CB} - \text{DY} * D_f \quad (30)$$

where

CE = load capacity or condition ratings at the end of the year;
CB = load capacity or condition ratings at the beginning of the year;
DY = deterioration of the load capacity or condition ratings during the year; and
D_f = deterioration factor computed from Equation 29.

APPROACH OVER HORIZON

The analysis is illustrated by the flowchart in Figure 7. The sequence of events is as follows:

1. The user enters budgets, objectives, and policies;
2. OPBRIDGE extracts data from the bridge data base and the cost and parameter file;

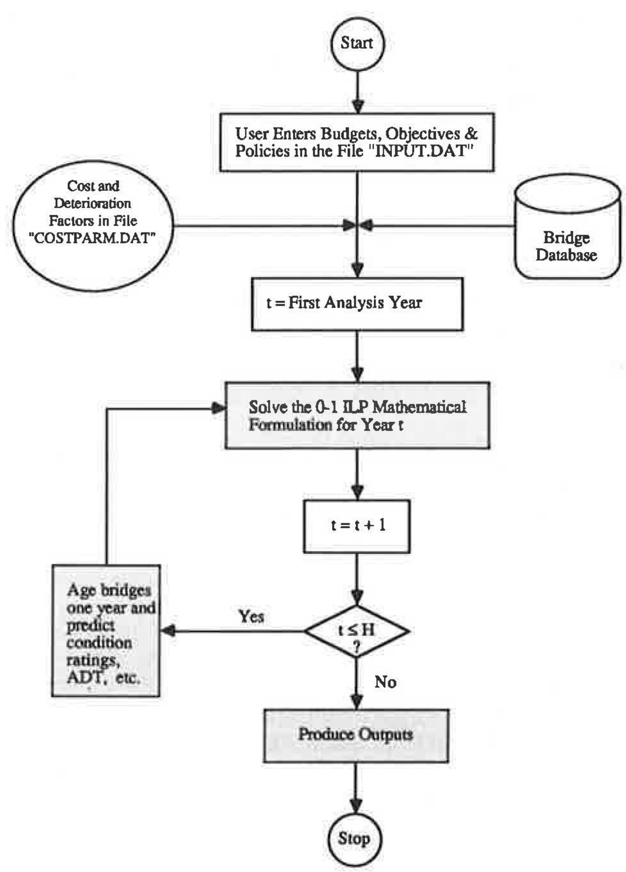

FIGURE 7 Flowchart for OPBRIDGE analysis.

3. OPBRIDGE optimizes decisions for every year in the analysis horizon. At the end of every year, OPBRIDGE ages the bridges 1 year and predicts condition ratings, ADT values, etc., allowing the system to do the analysis for the next year; and

4. Finally, OPBRIDGE produces detailed bridge-by-bridge output showing recommended current and future major actions, county-by-county output showing costs of major actions and budget required for each county, and tabular and graphical outputs showing future performance levels of the bridge system over the horizon, *H*.

SOLVING THE YEARLY BRIDGE OPTIMIZATION PROBLEM

The number of structures in North Carolina is almost 17,000. Approximately 2,600 pipes and culverts and 300 nonowned structures are not covered, leaving roughly 14,100 state-owned bridges. OPBRIDGE provides four possible actions per bridge—routine maintenance, major maintenance, rehabilitation, and replacement. Routine maintenance is the base alternative that is provided if other alternatives are not economical, are not enforced, or the budget is limited. Hence, there are a maximum of 14,100 * 3 = 42,300 0–1 decision variables. An average problem would have 25,000 to 30,000 0–1 decision variables. The numbers of constraints are as follows: (a) 14,100 multiple-choice constraints, one for each bridge; and (b) 3 or 1 budget requirement constraints, depending on whether or not the total budget is distributed, respectively. For the current state of the art in 0–1 integer linear programming, this problem is considered to be large.

The general-purpose branch-and-bound method, enumeration method, and cutting-planes method can solve only small to medium-sized (30 to 100 variables) 0–1 integer linear programming problems. If the number of variables and constraints becomes large (more than 100), then these methods become inefficient and in most cases even a good feasible solution (near optimal) may not be obtained (*7*).

Dynamic programming has also been used for solving the problem. However, dynamic programming, although fine for smaller problems (less than 50 variables), experiences degradation in efficiency as problem size increases (*8*).

For an algorithm to solve a large-scale knapsack problem, it should

1. Take advantage of the special structure of the problem;
2. Terminate in a finite number of steps—if it does not, then it should be able to generate a good feasible solution from the partial solution; and
3. Have reasonable computer storage requirements.

Nauss' algorithm and Ahmed's algorithm have been reported to satisfy these requirements. Nauss' algorithm uses branch and bound and an iterative procedure to calculate the optimum value of Lagrangian multiplier for arriving at an optimal solution of a knapsack problem with only one resource constraint (*8*). Ahmed mentioned that Nauss' algorithm is the most efficient algorithm developed to date for this type of problem (*7*, p. 13).

However, the knapsack formulation of the bridge problem has more than one resource constraint. Ahmed's algorithm initially uses the effective gradient concept by Senju and Toyoda (*9*) to solve a knapsack problem with more than one resource constraint (*7*). Senju and Toyoda (*9*) used the concept of effective gradient to design an algorithm for solving the multiconstrained knapsack problem. Briefly, the algorithm starts with an infeasible solution to the problem. It then computes the effective gradient of each variable whose value is equal to one (i.e., the variables that are in the solution). The variable with the smallest effective gradient is deleted and set equal to zero. The process is then repeated until feasibility is achieved. The whole procedure can then be repeated with the remaining (unused) capacities of the constraints. For details of this algorithm, see Senju and Toyoda (*9*). Ahmed (*7*) uses the concept of effective gradient to obtain an initial feasible solution for the 0–1 multiconstrained knapsack problem. Starting with this feasible solution, the algorithm switches to a ratio ranking procedure to tune in the solution, hence, hopefully obtaining a better feasible solution.

Senju and Toyoda (*9*, pp. B-196–B-207) proved numerically that the effective gradient concept can be applied satisfactorily for solving 0–1 integer linear programming problems. Ahmed (*7*, p. 51) tested his algorithm against another code, ILL1P–2, using 13 randomly generated test problems. The ILL1P–2 code uses the branch and bound and implicit enumeration technique. Therefore, the test problems were kept moderate in size so that the ILL1P–2 code can be applied. The ILL1P–2 code showed an average improvement of 0.258 percent in objective function values, compared to Ahmed's algorithm, which is insignificant for all practical purposes.

The algorithm was tested for use in OPBRIDGE and proved satisfactory for problems with a small number of bridges and alternatives. A problem with 25 bridges and approximately 3 alternatives each under a budgetary constraint was optimally solved by Farid et al. (*4*). Ahmed's algorithm (*7*) solved the same problem and achieved an objective function value of only 0.49 percent less than that of the optimum solution. The steps of Ahmed's algorithm (*7*) and the modifications made to speed up the process are described by Al-Subhi et al. (*1*).

SUMMARY AND CONCLUSIONS

A system-level optimization of bridge management decisions can be accomplished by 0–1 integer linear programming with multiple-choice constraints. Furthermore, the optimization is based on the objective of reducing overall costs to the ultimate owner, the user-taxpayer, the most defendable approach. Three improvement alternatives are assumed possible for a bridge at the beginning of every year—replacement, rehabilitation, and major maintenance. Routine maintenance, if provided, is assumed to protect the bridge against accelerated deterioration in varying degrees, but it does not raise the bridge condition ratings or user level of service. Two alternatives for two different bridges may have the same EUAC value. But, their impacts on reducing the current bridge annual maintenance and user costs are usually different. For this reason, maximizing the EUAC reduction for improvement alternatives over routine maintenance is used for optimizing economic decisions at the system level. The algorithm was programmed as OPBRIDGE and made operational at NCDOT as part of its decision support system. Analysis results for the North Carolina bridge inventory have been determined (*1*) and used to support funding requests.

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