Modeling Fatigue Loads for Steel Bridges

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Histograms of the gross vehicle weight (GVW) of trucks obtained from weigh-in-motion data for seven states are analyzed. These histograms are modeled by two bimodal distributions consisting of mixed pairs of lognormal distributions and a lognormal and a Type III largest extreme value distribution. Fatigue lifetimes for AASHTO categories A, B, C, and E details are calculated from these models of the distribution of GVW by approximating the Miner's stress as a linear function of the *m*th root of the *m*th expected moment of the GVW. The lifetimes based on the two models are compared with each other and with the results obtained by assuming a single lognormal distribution. The results show that the lifetimes estimated using the bimodal distributions differ little from each other but are significantly shorter than the lifetimes estimated from the single lognormal distribution. It was also noted that there are large differences between the estimated lifetimes of different AASHTO categories.

Load models appropriate for the estimation of bridge reliability due to fatigue life are different from those that are appropriate for the estimation of bridge reliability due to overloads. Although the occurrence of a single extraordinary load is important in the analysis of the ultimate capacity of a bridge, metal fatigue is concerned with the cumulative effect of loading at lower stress levels. As a result, a good fatigue model must be rich enough to model cumulative load effects.

The problem of modeling traffic loads for fatigue analysis has been studied by a number of researchers (*1–3*). Several fatigue load models developed from weigh-in-motion data compiled by Snyder et al. (*4*) are discussed and the consequences of assuming different models of the estimated lifetime of steel bridge components subject to fatigue loads due to trucks are examined.

Two distinct methods are used to estimate fatigue lifetimes (*5*). The first is the *S-N* approach, in which the lifetime in terms of stress cycles N is related to the mth power of the stress range S:

$$N = AS^{-m} \tag{1}$$

The second is the linear elastic fracture mechanics (LEFM) approach, in which the lifetime of a component is estimated from the relationship between the mth power of the stress intensity range ΔK and the rate of crack growth da/dN:

$$\frac{da}{dN} = C(\Delta K)^m \tag{2a}$$

$$= C[Y(a)^m S^m] \tag{2b}$$

In Equations 1 and 2 the exponent m and factors A and C are material constants. In Equations 2, $Y(a)$ is a function of the crack length a. Because of its simplicity, the first approach is used extensively for design calculations, for example, in the AASHTO specifications (*6*).

Whenever the stress range or stress intensity range cannot be assumed to be constant, the value for the stress range S in Equations 1 and 2 is modified to compensate for the time-varying loads. This is usually done by using an equivalent constant stress range such as the Miner's stress S_M, which has the form

$$S_M = \left(\frac{1}{N}\sum_{i=1}^{i=N} S_i^m\right)^{1/m} \tag{3}$$

where S_i is the stress range at the ith cycle and m is the exponent used in Equations 1 and 2. Because the number of cycles in fatigue problems is large and the stress range in each cycle is randomly distributed, the Miner's stress will approach the mth root of the mth expected moment of the underlying probability distribution for the stress range, or $[E(S^m)]^{1/m}$.

LOAD MODELS

The data analyzed here were collected by Snyder et al. (*4*) from weigh-in-motion studies performed in eight states in which calibrated highway bridges were used as scales. Each data set ranges from 1,377 to 6,547 observations, with a total of 24,179 observations. Figure 1 is a typical histogram of the gross vehicle weight (GVW) arbitrarily normalized so that a GVW of 1.0 is the weight of the AASHTO HS-20 design vehicle (72 kips). No information concerning the number of axles corresponding to the individual truck weights was included in the results of this study. The distribution of GVW is characterized by the two peaks or modes, which are a consequence of the distribution between heavier loaded trucks and lighter unloaded trucks. Although the lower mode appears to have a relatively wide peak, the upper mode is narrow and seems to have the shape characteristic of a negatively skewed distribution.

If one assumes that the mechanism for the dual peaks is due to two separate populations, loaded and unloaded trucks, an appropriate model for the GVW should be a mixed distribution with a probability density function (pdf) of the form

$$f_{GWV}(w) = pf_L(w) + (1 - p)f_H(w) \tag{4}$$

where $f_L(w)$ and $f_H(w)$ are the partial pdf's of light and heavy trucks, respectively, and p is the probability that an individual truck is a member of the light-truck population.

Two mixed distribution models were developed for each of the eight data sets. The first model (LN-LN) assumed that

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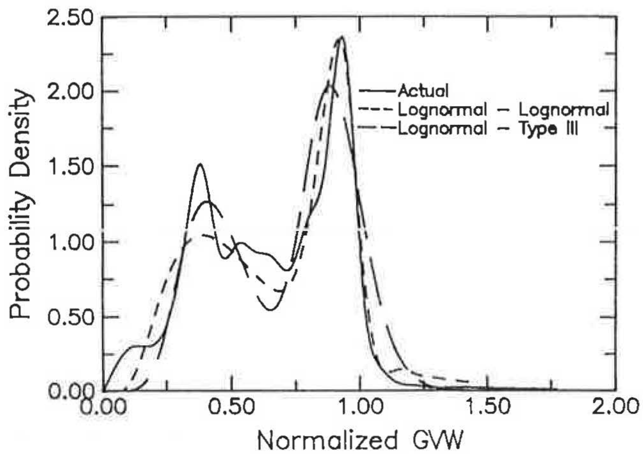

FIGURE 1 Comparison of estimated and measured histograms.

both the light and heavy trucks were distributed according to a lognormal distribution with a pdf of the form

$$f_Y(y) = \frac{1}{y(2\pi)^{1/2}\delta_Y} \times \exp\left\{-\tfrac{1}{2}\left[\frac{1}{\delta_Y}\ln(y/m_Y)\right]^2\right\} \quad y \geq 0 \tag{5}$$

where m_Y is the median of Y and δ_Y^2 is the variance of $\ln Y$.

In an attempt to model the left skewness of the distribution of the heavier trucks, the second model (LN-ET3) used a lognormal distribution for the light trucks and a Type III largest extreme value distribution for the heavy trucks with a pdf of the form

$$f_Y(y) = \frac{k}{w-u}\left(\frac{w-y}{w-u}\right)^{k-1} \times \exp\left[-\left(\frac{w-y}{w-u}\right)^k\right] \quad y \leq w \tag{6}$$

where k is the shape factor, u is the location parameter, and w is the upper limit of possible values.

The mean-squared error between the measured and mixed distributions LN-LN and LN-ET3 was used to measure goodness of fit:

$$\Sigma[f(w;\theta) - f_m(w)]^2 \tag{7}$$

where $f_m(w)$ is the measured density and θ is the vector of five or six parameters involved in the LN-LN and LN-ET3 models, $(p, m_{Y_1}, \delta_{Y_1}, m_{Y_2}, \delta_{Y_2})$ and $(p, m_{Y_1}, \delta_{Y_1}, k, u, w)$, respectively. Powell unconstrained optimization (7) was used for each set of data to find the optimal set θ resulting in the minimum of Equation 7.

The resulting parameters and the mean-squared errors are given in Tables 1 and 2 for the LN-LN and LN-ET3 models, respectively, for the distributions of normalized GVW for seven states tabulated by Snyder et al. The mean-squared errors associated with the LN-ET3 model are lower than those for the LN-LN model.

COMPARISON

To compare the effects of assuming each of the two postulated models, estimated lifetimes for AASHTO A, B, C, and E category details were calculated. Each detail was assumed to be designed for fatigue at the AASHTO maximum allowable stress range for the category. It is assumed that the stress range experienced by a detail is a linear function of the GVW of the truck causing the stress. If the fatigue criterion governs, the maximum allowable design fatigue stress range corresponds to the design truck weight and to the normalized truck weight of 1.0.

Using Equation 1 with the stress range equal to the Miner's stress $\approx [E(S^m)]^{1/m}$, the lifetime in years can be approximated by

$$L = \frac{AF_D^{-m}/E(W^m)}{\text{ADTT} \times 365} \tag{8}$$

where

A and m = material properties,
ADTT = average daily truck traffic,
L = detail life in years, and
F_D = fatigue design stress.

TABLE 1 ESTIMATED PARAMETERS FOR LN-LN MODEL

State	Lognormal 1 m_{y_1}	Lognormal 1 δ_{y_1}	p	Lognormal 2 m_{y_2}	Lognormal 2 δ_{y_2}	m.s.e
Arkansas	0.4491	0.3135	0.4293	0.8981	0.1323	1.552
California	0.3947	0.2638	0.5671	0.9660	0.1868	0.564
Georgia	0.3906	0.2551	0.5976	0.8383	0.1718	0.3478
Illinois	0.5236	0.3551	0.5077	0.9485	0.1935	0.1405
New York	0.4137	0.3129	0.6329	0.9654	0.1529	0.3187
Ohio	0.5038	0.3055	0.5792	1.0015	0.1255	0.2572
Texas	0.4265	0.3122	0.5290	1.0288	0.1857	0.5231

TABLE 2 ESTIMATED PARAMETERS FOR LN-ET3 MODEL

State	Lognormal			Type III			
	m_{y_1}	δ_{y_1}	p	w	u	k	m.s.e
Arkansas	0.5354	0.5404	0.6391	1.0578	0.8813	2.4144	0.9043
California	0.3808	0.2229	0.4798	1.2194	0.8054	1.6182	0.213
Georgia	0.3689	0.1601	0.3150	1.1520	0.5696	1.9122	0.07381
Illinois	0.4353	0.2449	0.2448	1.3958	0.7702	2.5764	0.07652
New York	0.3899	0.2587	0.5178	1.1728	0.7989	1.5873	0.1805
Ohio	0.4849	0.2759	0.5321	1.2504	0.9292	2.2981	0.1657
Texas	0.3908	0.1754	0.2622	1.2441	0.6717	1.3520	0.3285

TABLE 3 COMPARISON OF PREDICTED LIFETIMES

Property		Fatigue Sensitive Detail			
		A	B	C	E
F_D		24.0	16.0	11.0	5.0
m		2.31	2.55	3.10	2.80
A ($\times 10^9$)		6.07	3.72	12.17	1.04
State	Load Model	Lifetime in Years			
		A	B	C	E
Arkansas	LN-LN	19.53	16.55	40.19	61.42
	LN-ET3	17.97	14.77	32.49	52.58
	LN	23.59	21.12	58.99	83.53
California	LN-LN	20.85	17.34	39.92	62.90
	LN-ET3	23.26	19.70	47.37	72.85
	LN	27.97	25.40	72.99	85.37
Georgia	LN-LN	30.27	26.44	68.36	101.21
	LN-ET3	31.29	27.37	70.82	104.77
	LN	35.07	32.66	99.75	134.68
Illinois	LN-LN	16.66	13.81	31.75	50.00
	LN-ET3	17.67	14.79	34.82	54.14
	LN	18.96	16.60	43.95	64.03
New York	LN-LN	23.15	19.53	46.44	71.89
	LN-ET3	25.34	21.70	53.42	81.12
	LN	27.92	25.34	72.90	101.67
Ohio	LN-LN	17.95	14.99	35.01	54.66
	LN-ET3	18.96	15.95	37.97	58.66
	LN	19.64	17.26	46.05	66.79
Texas	LN-LN	16.41	13.40	29.59	47.64
	LN-ET3	29.26	24.49	37.64	73.40
	LN	21.86	19.32	51.59	75.12

The quantity $E(W^m)$ is the mth expected moment of the normalized GVW distribution.

$$E(W^m) = p\int_w^m f_L dw + (1 - p)\int_w^m f_H dw \quad (9)$$

The fatigue design stress F_D is a function of the detail type ranging from 2.0 ksi for cover plate terminus (Category E) to 24.0 ksi at web flange connections in rolled beams (Category A).

Shown in Table 3 are the estimated lifetimes based on the two double-mode models LN-LN and LN-ET3 developed here. For comparison purposes, lifetimes based on a single LN stress range distribution are also included in Table 3. For each of the seven states the lifetimes are estimated for four AASHTO detail categories A, B, C, and E. The ADTT was assumed to be 1,000 vehicles per day and the material constants are those tabulated by Nolan and Albrecht (*8*) from tests of a large number of typical fatigue-sensitive details in which runout effects were ignored.

CONCLUSIONS

The results show that, except for the data set from Texas, the estimated lifetimes are insensitive to which double-mode stress range model was used. In the case of Texas, the low central value for f_H in the LN-ET3 model, $u = 0.67$, compared with the high median for f_H in the LN-LN model, $m_{Y_2} = 1.03$, results in large differences in the estimated lifetimes. However, in all other states the estimated lifetimes differ by less than 10 percent, which, in light of the precision of such methods, is insignificant. On the other hand, estimated lifetimes based on the single-mode distribution are all longer than the estimates based on the LN-LN and LN-ET3 stress range models. The greatest difference was for Category C details, which have the steepest *S-N* relationship, with $m = 3.1$.

The estimated lifetimes are highly sensitive to design stresses and material properties. Although the *S-N* curves are significantly higher for Category A details than for Category E details, the use of significantly lower design stresses for Category E details causes estimated lifetimes for Category E details to be much longer than those for Category A details. The results in Table 3 ignore any run-out or fatigue limit effects, and as a result, all stress cycles cause damage. This may account for short lifetime estimates and possibly the large differences in the estimated lifetimes between details. However, the results point to possible large discrepancies between the fatigue lifetimes for various categories.

The results presented here suggest that there are discrepancies between the lifetimes estimated from single-mode stress distributions and those estimated from the double-mode models proposed here. The results also suggest that more realistic modeling of the stress range may be even more important for richer fatigue models such as *S-N* curves combined with fatigue limits or Paris crack growth relations with threshold stress intensity ranges.

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