Characterization of Aggregate Shape Using Fractal Dimension

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A fractal is a term used in geometry to describe an object the shape of which is intermediate between topological ideals. A fractal object is described by a fractal dimension. Such a parameter describes the deviation that a line, surface, or volume has from a topological ideal. For example, an ideal topological line has a dimension equal to 1, and an ideal plane has a topological dimension equal to 2. A fractal line, though, has a fractal dimension greater than 1, but less than 2, and a fractal surface has a fractal dimension greater than 2 but less than 3. As an experiment, the distance can be measured between two arbitrarily located points on a coastline. By using different rulers of progressively smaller sizes, the measured length will be found to increase as ruler size decreases. This epitomizes fractals. If the number of rulers required to travel between two points is called N and the ruler length is called y, then length L is equal to Ny. For a fractal, the plot of log (N) versus log (y) yields a straight line; moreover, fractal dimension is equal to the absolute value of the slope of this line. Initial experiments show that circumferential traces of aggregate are fractal lines, with dimensions between 1 and 1.3. In general, greater fractal dimensions are found for rough, irregular aggregate. Fractal dimensions approach 1 for circumferential tracks of smooth, rounded pieces and for flat, elongated pieces. Lower fractal dimension are also found for aggregate where shapes approach ideal, angular shapes, such as squares or triangles. The fractal dimension, therefore, appears promising for the characterization of aggregate.

Mineral aggregates are critical components of construction materials and are used by themselves as low-traffic-volume road surfaces and base layers for higher trafficked pavements. Aggregates make up approximately 95 percent of the weight in asphalt concrete. Thus, the characteristics of mineral aggregates can dictate the performance of the structures in which they are used.

As important as aggregates are to the performance of construction materials, there are relatively few methods to quantify their contribution to performance. This is particularly true with respect to the surface characteristics such as particle shape, roughness, and surface charge. Those attributes are important to the interaction between the grains and to the interaction between the aggregate and whatever binder may be present in the material. Most tests that provide information on aggregate surface quality are subjective or are based on arbitrarily defined parameters.

In asphalt concrete, particle shape has been related to the optimum binder content, the quantity of air voids, the workability, the shear resistance, and the tensile strength of the mixture (*1–3*). Generally, it is desirable to have a somewhat angular shape in asphalt mixtures. Flat or elongated particle shapes are undesirable.

The surface texture or roughness is dependent on the degree to which the aggregate is polished or dull. It is usually described in terms of being very rough, rough, smooth, or polished. The texture can affect the fatigue resistance, air voids, binder content, tensile strength, and stability of asphalt mixtures (*4,5*).

Experiments are subsequently presented that attempt to quantify aggregate surface shape by using the concept of fractal dimension. In describing the geometry of aggregate by using the concept of fractals, it is hoped that a quantitative method is developed for the characterization of concrete aggregate shape that is more easily applied than existing methods.

SUMMARY OF EXISTING METHODS

Aggregate shape can be defined by methods such as those developed by Krumbein (*6*) or Rittenhouse (*7*). The Krumbein roundness number of a particle is expressed as

$$\text{roundness} = r/R$$

where r is the average radius of the corners of the projected image of the particle and R is the radius of the maximum inscribed circle of the projected image.

The Krumbein number can range from 0 to 1, though the typical range is from 0.1 (angular) to 0.8 (well rounded). Rittenhouse defined sphericity as the ratio of the diameter of a circle with an area equal to that of the projected area of the particle to the diameter of the smallest circumscribing circle of the particle. The range for sphericity is from 0 to 1, though the practical range is from 0.45 (elongated) to 0.97 (very spherical). Charts have been developed for visual comparison of both these methods.

Mather (*8*) proposed that the roughness of aggregate particles could be described in terms of the arithmetic average deviation of the actual surface from the surface mean. As was discussed earlier, surface texture is usually subjectively rated as being very rough to polished.

ASTM has a test method (D 3398) in which the particle index of an aggregate may be determined as an indication of the overall particle shape and texture. This method relies on the calculation of voids in the aggregate system at defined levels of compaction. These void quantities are then used in an equation that gives the particle index. The levels of com-

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paction used in this test were arbitrarily chosen, and there is no actual measurement of the aggregate surface geometry.

A NEW APPROACH: FRACTAL CHARACTERIZATION

Concept of Fractals

Mandelbrot (*9*) illustrated the use of fractals in determining the length between two points (*A* and *B*) on the west coast of Great Britain, which is very irregular and complex (see Figure 1). The length of the coastline between *A* and *B* may be measured with a ruler of arbitrary length *y*. Beginning at point *A*, the ruler is placed end to end until point *B* is reached (see Figure 2). A segment smaller than *y* remains at the end and is designated *f* (see Figure 3). The length between *A* and

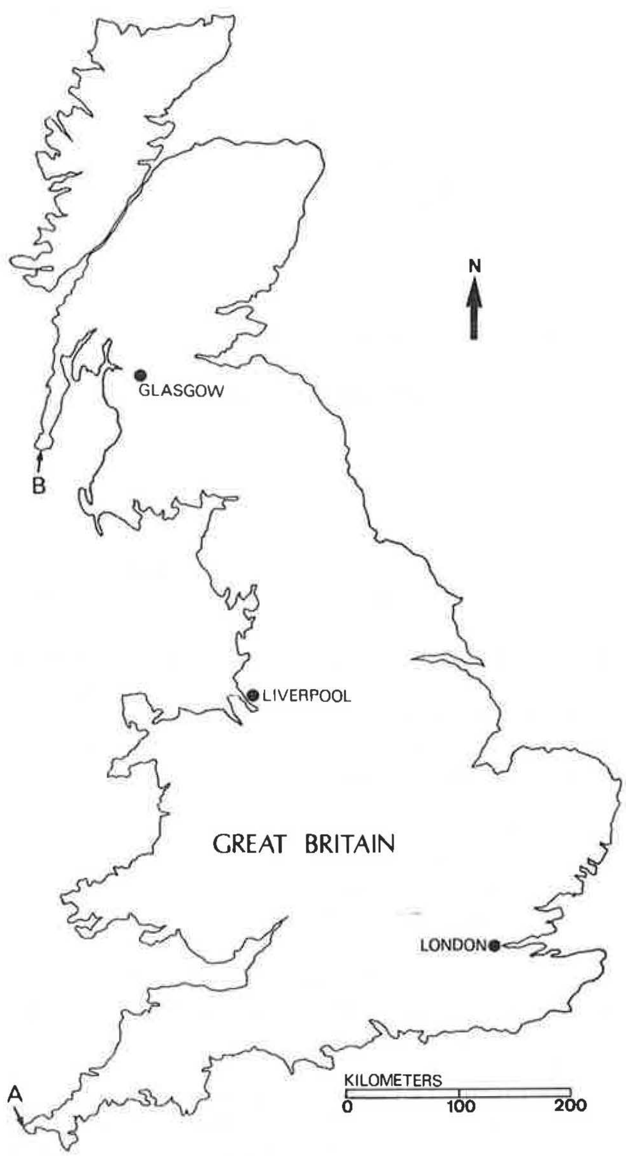

FIGURE 1 Outline of the coast of Great Britain. Two arbitrarily located points, *A* and *B*, are shown.

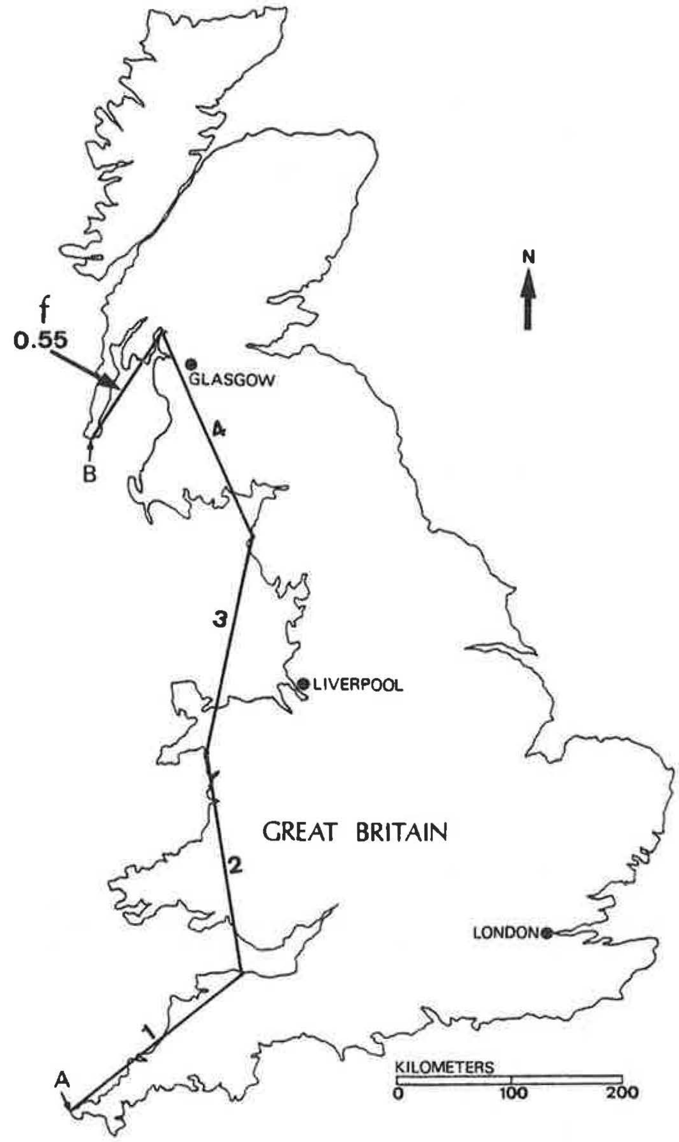

FIGURE 2 Attempt to measure the length between points *A* and *B* by using a 200-km ruler.

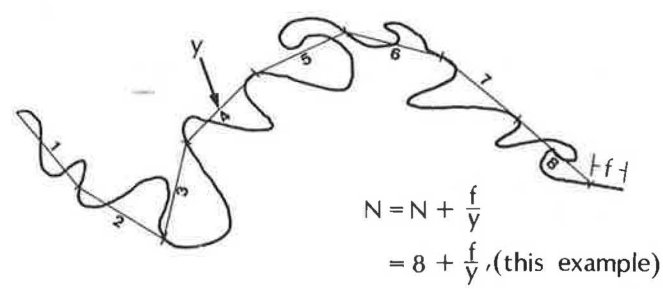

FIGURE 3 Demonstration of the concept of the remainder, *f*, left over when measuring the length by using a fixed-ruler length.

B can be calculated as

$$L = Ny + f \tag{1}$$

where y is the ruler length and N is the number of rulers placed end to end between the two points. Expressed another way,

$$L = \left[N + (f/y)\right]y \tag{2}$$

If a shorter ruler were used to repeat the measurement of length between A and B, the total length measured with the shorter ruler would exceed that measured with the longer one. The shorter ruler would sample bays and inlets that were bridged with the longer ruler. Thus, the scale of the ruler is important.

The relationship between total length and ruler length can be expressed as

$$L = \left[N + (f/y)\right]y^D \tag{3}$$

where D is the fractal dimension (*10*). This parameter is also known as the Hausdorff-Besicovitch dimension as defined by Equation 3. It is also referred to as the divider, or compass, method fractal dimension, to acknowledge its calculation procedure.

For a straight line, D has a value of 1. The topological dimension of a straight line is also unity. Therefore, the topological and fractal dimensions are equal. A fractal line, however, is where the fractal dimension is greater than unity.

Equation 3 can be rewritten to become

$$Ly^{-D} = N + f/y \tag{4}$$

Normalizing Equation 4 so that $L = 1$,

$$y^{-D} = N + f/y \tag{5}$$

Taking the log of Equation 5 yields

$$-D \log(y) = \log (N + f/y) \tag{6}$$

The value of D may be obtained as the negative slope of the plot of log $(N + f/y)$ versus log (y) for several different ruler lengths y (see Figure 4). Even where L of Equation 3 is not equal to 1, its value would not affect the slope of the plot.

In the example of the British coastline, suppose that three different rulers were used to measure the length between points A and B in Figure 1. Those rulers, along with the resultant $N + f/y$ values, are presented in Table 1. Figure 4 is the log-log plot of those results indicating that the slope is -1.3 and, thus, D is 1.3. Therefore, the western coast of Great Britain is a fractal with a fractal dimension of 1.3.

The coastal segment is an example of a deterministic fractal in that the fractal dimension is calculated deterministically. The western coast of Great Britain is also a self-similar fractal, where appearance (bays, spits, etc.) is similar regardless of the scale at which the coast is examined.

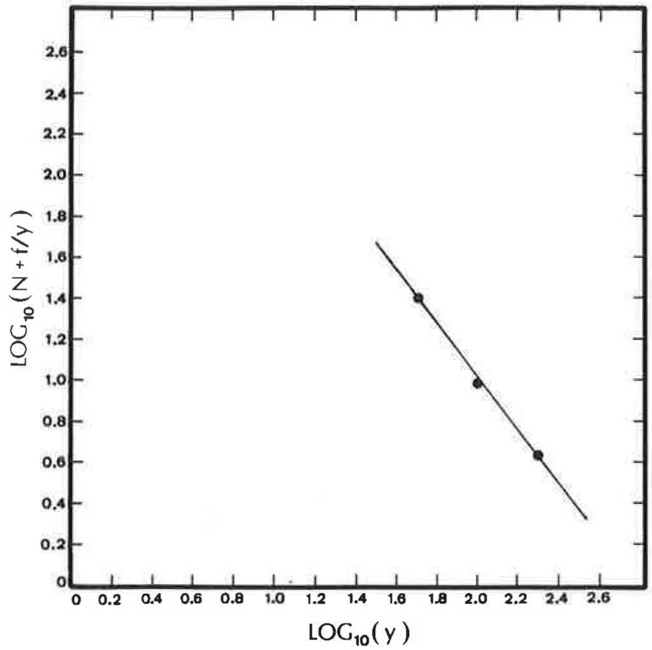

FIGURE 4 Log-log plot of results in Table 1. Slope of plot is −1.3. Thus, fractal dimension is 1.3.

TABLE 1 ATTEMPTS TO MEASURE LENGTH *A* AND *B* (FIGURE 1) BY USING THREE DIFFERENT RULERS

y	N + f/y	L: (N + f/y)y
200 km	4.55	910 km
100 km	10.75	1075 km
50 km	28.25	1412.5 km

Note: y values, in km, refer to the scale shown in Figure 1.

Fractal Characterization of Aggregate

Peleg and Normand (*11*) apply the concept of the fractal dimension to the characterization of instant coffee particles. A fractal dimension is calculated for the silhouette of each particle following the procedure presented in Figure 5. The objective is simply the characterization of particle irregularity that has relevancy for the dissolution of the particle. The current study seeks to characterize aggregate shape by using fractal dimension. Because aggregates are particles similar in shape and profile to instant coffee particles, the Peleg and Normand study is a model for the application of fractals to the characterization of aggregate.

Four example aggregates are presented in Figures 6–9. Those aggregates were photographed against contrasting backgrounds to obtain silhouettes. Dividers opened to different ruler sizes and walked around the perimeter of each silhouette yielded the results indicated in Table 2. This procedure follows Figure 5. It is noted in Table 2 that flat and elongated pieces have the lowest fractal dimension (i.e., the fractal dimension is 1.0). In contrast, water-rounded limestone and crushed pieces have greater fractal dimensions.

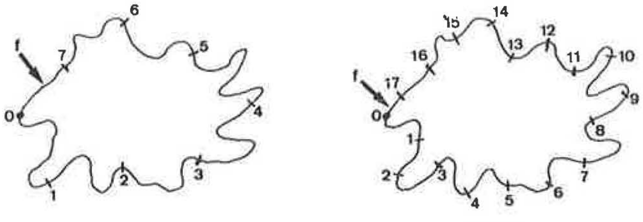

FIGURE 5 Richardson profiles. Two profiles show circumference measured with two different ruler lengths y [from Peleg and Normand (*11*)].

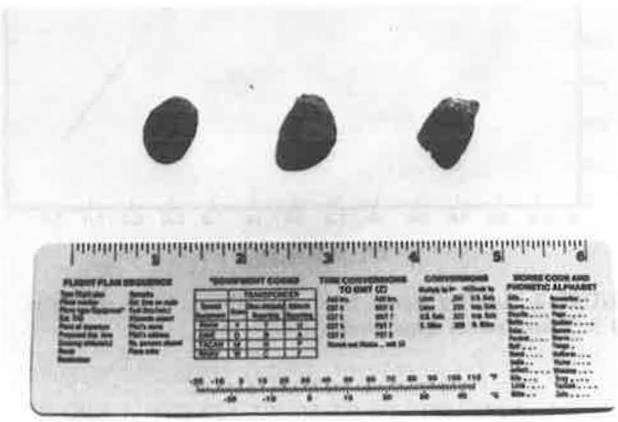

FIGURE 6 Rounded aggregate pieces.

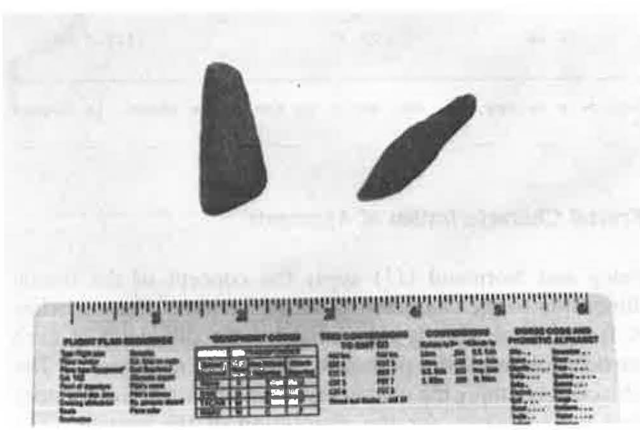

FIGURE 7 Flat and elongated aggregate pieces.

Using photographs of silhouettes poses a problem. The silhouette of a flat, circular object is identical to that of a marble. Each silhouette will yield a fractal dimension of 1.0. On edge, the flat, circular object is rectangular (which may or may not yield a fractal dimension of 1.0). Therefore, it is perhaps desirable to photograph aggregate pieces from several perspectives to examine the fractal dimension for the entire piece.

The fractal dimension of each aggregate is determined in Table 2 by using only three ruler lengths y. This small sample size was chosen because dividers were used to walk the perimeter of each silhouette (Figures 6–9). The fractal dimension will vary somewhat for such a small sample size, but, provided ruler lengths are selected greater than the minimum resolution of the photograph, this variability will be small.

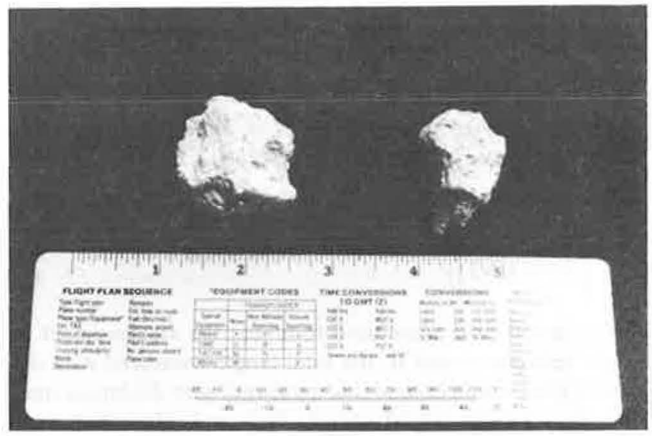

FIGURE 8 Crushed aggregate pieces.

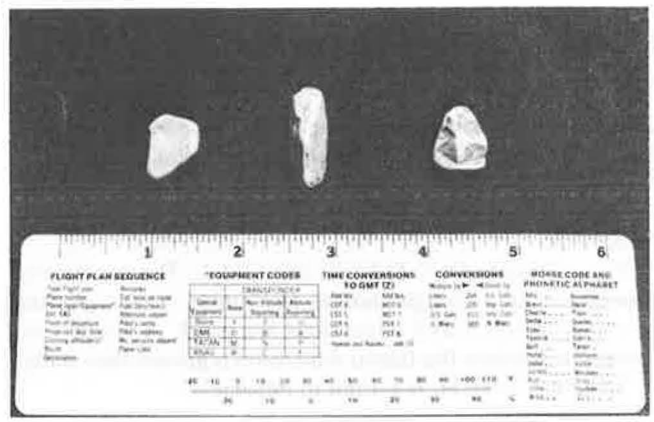

FIGURE 9 Water-rounded limestone pieces.

Application of the fractal technique in the manner proposed here requires the use of a camera. This is a practical method, nevertheless. Photographs are simply obtained of aggregate silhouettes (by photographing an aggregate piece against a contrasting background). Dividers are then used to measure the circumference of the silhouette. Finally, a fractal dimension is calculated on the basis of the results described previously. This is the total number of steps involved in this method.

To demonstrate further the usefulness of the fractal dimension for aggregate characterization, two photographs are chosen from Krynine and Judd [(*12*), Figures 8.16, 8.17, pp. 323–324]. Those figures contrast rounded aggregate (Figure 8.16) with harsh (crushed, angular) aggregate (Figure 8.17). Each figure is a photograph of nine samples. Figures 10 and 11 here indicate outlines for those pieces of aggregate.

Those figures also show the fractal dimension calculated for the outline of each sample. Table 3 presents the summary of measurements used to calibrate each fractal dimension. The mean fractal dimension for the nine samples in Figure

TABLE 2 FRACTAL DIMENSION FOR EXAMPLE AGGREGATES

y in.	$N + f/y$
Rounded Pieces (Figure 6)	
½	4.45
¼	9.00
⅛	18.00
D = 1.017	
Flat and Elongated Pieces (Figure 7)	
½	7.8
¼	16.0
⅛	30.0
D = 0.983	
Crushed Aggregate (Figure 8)	
½	8.7
¼	17.4
⅛	39.0
D = 1.083	
Water-Rounded Limestone (Figure 9)	
½	3.6
¼	7.8
⅛	16.0
D = 1.067	

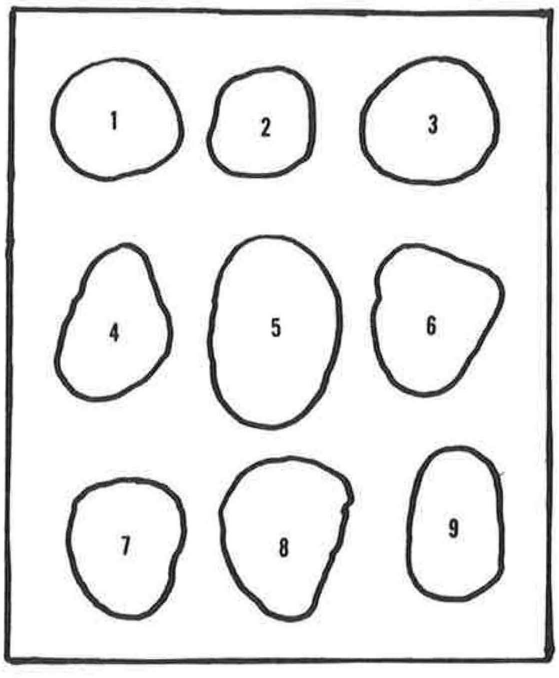

FIGURE 10 Traces (outlines) of nine samples of rounded aggregate [from Krynine and Judd (*12*) Figure 8.16, p. 323]. Numbers inside the outlines are sample numbers used in Table 3.

10 is 1.056, and the mean fractal dimension for the samples in Figure 11 is 1.077.

It may be useful to determine whether the fractal dimensions for Figures 10 and 11 are significantly different. If the null hypothesis is posed that 1.056 is equal to 1.077 (i.e., that the mean fractal dimensions are equal), and a t-statistic is

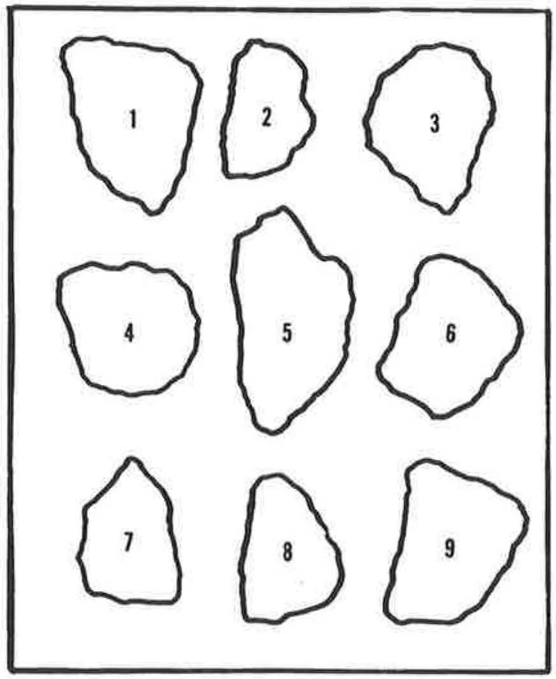

FIGURE 11 Traces (outlines) of nine samples of aggregate displaying various degrees of angularity [from Krynine and Judd (*12*), Figure 8.17, p. 324]. Numbers inside the outlines are sample numbers used in Table 3.

used to test this hypothesis, the t-statistic for the two samples is -1.70. If further the alternative hypothesis is posed that 1.056 is less than 1.077, then by using table values for the t-statistic for a one-tailed test, the null hypothesis is not rejected at the 5 percent significance level (the table t value is -1.75) but the null hypothesis is rejected and the alternative hypothesis is accepted at the 10 percent significance level (the table t value is -1.34). Further, by using a nonparametric Wilcoxon rank sum test, the sum of ranks for fractal dimensions in Figure 10 is 68, and for Figure 11 the rank sum is 103. For a one-tailed test at the 5 percent significance level, the table values for the lower sum and upper sum are 66 and 105, respectively. The sum for Figure 10 is 68, just inside this interval, so the null hypothesis that both samples are equal is not quite rejected. The t-test and the Wilcoxon rank sum test show that the difference between the fractal dimensions for the two aggregate groups, rounded and harsh, is almost significant for the 5 percent level and certainly significant at the 10 percent level. It does appear that the fractal dimension characterizes aggregate shape: the more angular (harsh) the aggregate is, the greater is its fractal dimension.

DETAILED APPLICATION OF FRACTAL DIMENSION TO CHARACTERIZATION OF AGGREGATE

A broader experiment is now complete where the concept of fractal dimension was applied to 30 different samples of aggregate. A fractal dimension for each aggregate sample is calculated as follows:

TABLE 3 FRACTAL DIMENSION RESULTS FOR FIGURES 10 AND 11

Sample	y(cm)	N + f/y	Sample	y(cm)	N + f/y
1	1.0	6.30	1	1.0	7.70
	0.7	9.29		0.7	11.43
	0.3	21.30		0.3	29.00
	D = 1.006			D = 1.101	
2	1.0	5.20	2	1.0	5.70
	0.7	8.00		0.7	8.79
	0.3	18.67		0.3	22.00
	D = 1.050			D = 1.115	
3	1.0	6.60	3	1.0	7.40
	0.7	9.71		0.7	10.64
	0.3	22.50		0.3	26.50
	D = 1.014			D = 1.063	
4	1.0	6.70	4	1.0	6.90
	0.7	9.70		0.7	10.29
	0.3	24.00		0.3	25.50
	D = 1.061			D = 1.083	
5	1.0	8.3	5	1.0	8.60
	0.7	12.20		0.7	13.00
	0.3	29.80		0.3	31.67
	D = 1.060			D = 1.077	
6	1.0	6.90	6	1.0	7.40
	0.7	10.29		0.7	10.86
	0.3	25.30		0.3	26.00
	D = 1.076			D = 1.041	
7	1.0	6.40	7	1.0	6.15
	0.7	9.57		0.7	9.36
	0.3	23.70		0.3	22.67
	D = 1.084			D = 1.076	
8	1.0	7.10	8	1.0	6.20
	0.7	10.64		0.7	9.57
	0.3	26.00		0.3	23.30
	D = 1.074			D = 1.091	
9	1.0	6.40	9	1.0	7.80
	0.7	9.64		0.7	11.50
	0.3	23.70		0.3	27.70
	D = 1.083			D = 1.050	
	Mean D = 1.056			Mean D = 1.077	
	Variance = 0.00082			Variance = 0.00056	

1. The aggregate sample is photographed along with a ruler scale against a contrasting background (such as in Figures 6–9).
2. Calipers (dividers), set at different ruler lengths, are walked around the perimeter of the sample silhouette. The ruler length is determined from the scale shown in the photograph.
3. A fractal dimension is calculated on the basis of the results from the second step (as is done in Tables 2 and 3). The three steps follow what is illustrated in Figure 5.

Results for those 30 samples are presented in Table 4. A summary of the results for this table is as follows:

Sample	*No. of Samples*	*Avg Fractal Dimension*
Rounded aggregate	10 (5, 6, 7, 9, 12, 13, 16, 17, 18, 25)	1.052
Rectangular prof	12 (2, 8, 10, 11, 14, 15, 20, 21, 26, 27, 29, 30)	1.120
Square profiles	2 (3, 4)	1.068
Triangular prof	5 (1, 22, 23, 24, 28)	1.110
Elongated pieces	1 (19)	1.00

This summary shows that the fractal dimension is a useful technique for characterizing aggregate shape. Angular pieces, such as rectangular, square, or triangular pieces, on average, have larger fractal dimensions than do rounded pieces.

CONCLUSIONS

Existing methods for characterizing aggregate shape determine the deviation of a particle from an ideal sphere. A fractal is any line, surface, or object that differs from a topological ideal, such as a sphere. The fractal dimension contributes to existing methods for concrete aggregate characterization. The greater the fractal dimension of a concrete aggregate silhouette, the more different this particle is from a topological ideal, which may be a square, rectangle, triangle, or sphere. Hence, the fractal dimension is sensitive to concrete aggregate shape deviation from any ideal shape and not just a sphere. In this manner, the fractal dimension contributes to existing shape characterization methods.

This fractal technique is not proposed as a replacement for existing techniques. The Rittenhouse ratio describes roundness: the closer the ratio is to 1, the more like a sphere the particle is. The Krumbein ratio describes angularity. A square particle, for example, has a low ratio because the four angular corners have minute radii.

A square, though, is a good object to use in describing the difference between the fractal dimension and the Krumbein ratio. A perfect square has a fractal dimension of 1 (i.e., its perimeter has this fractal dimension) but has a Krumbein ratio less than 0.1. Suppose that on each of the four sides of the

TABLE 4 FRACTAL DIMENSION RESULTS FOR 30 AGGREGATE SAMPLES

Sample ID #	Fractal Dimension	Description
1	1.03	triangular profile
2	1.07	rectang profile
3	1.06	square profile
4	1.075	square profile
5	1.00	rounded profile
6	1.00	round elong prof
7	1.00	rounded profile
8	1.00	rectang profile
9	1.10	round profile; one broken edge
10	1.14	rounded rectangle
11	1.17	rectang profile
12	1.00	round elong prof
13	1.08	rounded profile
14	1.15	rectang profile
15	1.00	rectang profile
16	1.175	round elong prof
17	1.155	rounded profile
18	1.00	rounded profile
19	1.00	elong rectangle
20	1.275	rectang profile
21	1.15	rectang profile
22	1.22	triangle with one rounded edge
23	1.00	triangular prof
24	1.07	triangular prof
25	1.00	rounded profile
26	1.07	rectang profile
27	1.09	rectang profile
28	1.22	triangular prof
29	1.24	rectang profile
30	1.11	rectang profile

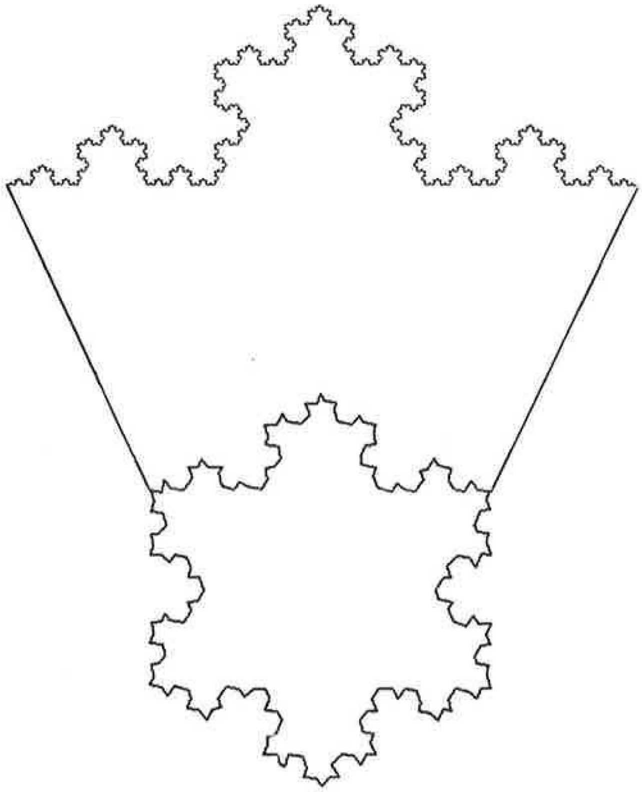

FIGURE 12 Artificial Koch curve constructed by using different sizes of isoceles triangles. The partial curve on the top represents an enlarged section. The Krumbein ratio would describe the global shape of this object. The fractal dimension describes not only the shape but also the finer detail, such as is shown in the top, more detailed portion of the curve. This finer detail relates to particle roughness.

square a triangle is affixed where the apex points outward. Moreover, let the apex of the triangle have the same angularity as each of the four original corners of the square. This object will have the same or similar Krumbein ratio, but the fractal dimension will increase (i.e., it will be greater than 1.0).

The fractal dimension, therefore, is sensitive to the number of asperities present on the circumference of a particle. Likewise, it can account for the presence/distribution of particle features as they grow smaller in size (i.e., transitioning from shape size features to surface roughness features). By contrast, the Krumbein and Rittenhouse ratios are established only when the particle(s) are viewed as a whole and therefore are limited to the "shape size" features. This is best seen in Figure 12, a figure of an artificial Koch curve constructed by using multiple scales of triangles. Its fractal dimension is 1.26 (*10*). The Krumbein ratio for Figure 12, though, would be low because of the high angularity of this object.

In summary, where the Rittenhouse ratio described form, the Krumbein ratio describes angularity and the fractal dimension describes the presence or absence of multiple scales of asperities on the circumference of a particle. This relates to the roughness and texture of the particle surface.

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