Repeated Load Model for Subgrade Soils: Model Applications

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A load-deformation model for subgrade soils where total cumulative axial strains are correlated with applied stresses and number of repetitions is used to study the behavior of subgrade soils under repeated loads and to investigate the load repetition effects on subgrade modulus and shear strength. Predictions of allowable stresses for different limiting strains are also established, and limiting criteria in terms of applied stresses and number of repetitions to failure are compared with other subgrade criteria available in the literature. Model predictions indicate that the subgrade modulus increases for repeated stresses lower than a "threshold stress" and assumes a constant value with increased number of load repetitions. For higher stresses, the modulus decreases, indicating a strain-softening behavior and a corresponding increase in the rate of accumulation of axial strains. On the other hand, cohesion and friction associated with failure under repeated loads decrease with increase in number of repetitions to failure. The variation of repeated stress level with number of repetitions to failure determined by the proposed model is compared with subgrade criteria suggested by other investigators. Stress predictions by using those criteria are generally conservative for number of repetitions greater than about 10^3. For smaller number of repetitions, those criteria could be nonconservative. Moreover, permanent strain predictions by those criteria for number of load repetitions greater than 10^4 could range from 20 percent to 25 percent of the permanent strains at failure as determined by the proposed model.

The repeated load behavior of subgrade soils is generally considered to be of major significance on the performance of pavement structures. Present subgrade models provide correlations between repeated stresses and resilient or recoverable strains (*1–3*) and between repeated stresses and permanent or plastic strains (*4–6*). Those correlations are used to predict pavement response and assess pavement performance in terms of fatigue and rutting. However, because those conditions are limited to stresses and resilient strains or stresses and permanent strains, their incorporation in numerical algorithms such as the finite-element method will lead to solutions that do not satisfy kinematic conditions between displacements and total strains. Moreover, available subgrade models do not account for subgrade failure under repeated loads and therefore are limited in their assessment of pavement stability. Raad et al. (*7–9*) have indicated that the shear strength of pavement layers have a significant effect on whether the accumulation of strains under repeated load applications will proceed at an increasing rate or whether it will cease and thus lead to a shakedown condition and a stable response.

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In a companion paper by Raad and Zeid in this Record, a load deformation model for subgrade soils is developed where total cumulative axial strains are correlated with applied stresses and number of load repetitions. The concept of constant failure strain independent of load history is presented and used in the proposed model.

In this paper, the proposed model is applied to study the behavior of subgrade soils under repeated loads. Specifically, the following points are investigated:

1. The effects of load repetitions on subgrade modulus and shear strength;
2. The relation between repeated stress state, limiting strain, and number of repetitions to failure; and
3. The variation of repeated stress level with number of load repetitions to failure as determined by the proposed model in comparison with subgrade criteria suggested by other investigators.

PROPOSED MODEL

A repeated load model for subgrade soils has been proposed and is based on results of static triaxial tests, slow cyclic tests, and repeated load tests for a compacted silty clay. Details of model development are presented elsewhere (see companion paper in this Record). According to this model, the behavior of the compacted silty clay in terms of axial strain ϵ_a, repeated load stress level q_r (defined as the ratio of repeated deviator stress to the strength obtained from a standard triaxial test at a strain rate of 0.5 percent/min), and number of load repetitions N depends on the magnitude of the applied stress level relative to the "threshold stress level" q_{rt}. The "threshold stress level" corresponds to the stress level below which the accumulation of axial strains will eventually cease and lead to a stable response and above which progressive accumulation of axial strains will occur and cause unstable response and ultimate failure. The failure strain in this case is dependent on confining pressure, dry density, and compaction moisture content but is independent of stress history. The "threshold stress level" for the repeated load tests performed lies between 0.80 and 0.90.

For repeated stress level values smaller than q_{rt},

$$q_r = \frac{\epsilon_a}{a_L + s_L \log N} \quad (1)$$

where a_L and s_L are material parameters.

For repeated stress level values greater than q_{rl},

$$q_r = \frac{\epsilon_a}{a_h + b_h \epsilon_a} \quad (2)$$

where

$$b_h = B_h + S_h \log N \quad (3)$$

and a_h, B_h, S_h are material parameters. The axial strain ϵ_a in Equations 1 and 2 is expressed in percent.

A summary of shear strength characteristics and model parameters for different dry density γ_d, compaction moisture content m, and confining pressure σ_3 is presented in Tables 1 and 2, respectively.

Incorporating the concept of a constant strain at failure into the proposed model allows assessment of upper-bound stress levels above which the subgrade will exhibit progressive increase in strains with number of load repetitions and below which the subgrade will have a stable response and the accumulation of strains will eventually cease. Those limiting stresses could be used to define failure criteria under repeated loads. The need for such criteria in advanced numerical analysis of pavement systems has been addressed by Raad and Figueroa (*3*) and more recently by Raad et al. (*7–9*). Moreover, the proposed model could be used to estimate the magnitude of total strains in the subgrade and, therefore, the permanent strains if the resilient strains are known.

MODEL APPLICATIONS

Load Repetition Effects on Subgrade Modulus and Shear Strength

The effects of repeated load applications on subgrade modulus were determined by using the proposed model defined in

TABLE 1 SHEAR STRENGTH CHARACTERISTICS OF THE SILTY CLAY

Compaction Properties	Cohesion C (psi)	Angle of Friction ϕ(degrees)	Undrained compressive strength σ_{df} (psi)	
			$\sigma_3 = 0$	$\sigma_3 = 14.5$ psi
$\gamma_d = 129.5$ lb/cu ft m = 7 %	39.0	32	140	173
$\gamma_d = 129.5$ lb/cu ft m = 10 %	23.0	32	83	117

NOTES

- Shear strength properties were determined using strain-controlled undrained triaxial tests with rate of applied strain equal to 0.50 percent per minute.
- σ_{df} is equal to the difference of major principal stress σ_1 and minor principal stress σ_3 at failure.

TABLE 2 PARAMETERS USED IN THE PROPOSED SUBGRADE REPEATED-LOAD MODEL

		Subgrade repeated-load model: Low Stresses ($q_r \leq 0.80$)		High Stresses ($q_r > 0.80$)			Failure Strain ϵ_f (%)
		a_L	s_L	a_h	B_h	S_h	
Dry of optimum compaction	$\gamma_d = 129.5$ lb/cu ft m = 7 % $\sigma_3 = 0$	2.78	0.231	1.96	0.322	0.0586	2.82
	$\gamma_d = 129.5$ lb/cu ft m = 7 % $\sigma_3 = 14.5$ psi	2.74	0.160	1.85	0.371	0.0588	2.91
Wet of optimum compaction	$\gamma_d = 129.5$ lb/cu ft m = 10 % $\sigma_3 = 0$	5.25	0.648	4.30	0.0253	0.170	6.00
	$\gamma_d = 129.5$ lb/cu ft m = 10 % $\sigma_3 = 14.5$ psi	5.71	0.950	3.90	0.380	0.163	9.02

Equations 1, 2, and 3. In this case, the modulus E after N repetitions was compared with an initial modulus value E_i corresponding to N_i repetitions. The subgrade modulus E after N repetitions of a given deviator stress σ_d is defined as the ratio of the repeated deviator stress to the total axial strain ϵ_t (resilient strain plus permanent strain) experienced by the subgrade during the Nth repetition. The modular ratio E/E_i could then be used as a relative measure of subgrade modulus with number of load repetitions. For a given repeated stress level, the modular ratio is expressed as

$$\frac{E}{E_i} = \frac{\epsilon_{ti}}{\epsilon_t} \tag{4}$$

where ϵ_{ti} and ϵ_t are total axial strains during repetitions N_i and N, respectively. The strain ϵ_t could be related to the total accumulated axial strain ϵ_a and resilient strain ϵ_r by the simple relation

$$\epsilon_t = \frac{d\epsilon_a}{dN}\Delta N + \epsilon_r \tag{5}$$

where ΔN is equal to 1 and ϵ_r is assumed constant for a given repeated stress level.

For stress level values less than the "threshold value" q_{rl}, it could be shown that

$$\frac{E}{E_i} = \frac{\dfrac{q_r\, s_L}{N_i} + \epsilon_r}{\dfrac{q_r\, s_L}{N} + \epsilon_r} \tag{6}$$

and for q_r greater than q_{rL}

$$\frac{E}{E_i} = \left[\frac{(a_h S_h / N_i)}{\left(\dfrac{1}{q_r} - B_h - S_h \log N_i\right)^2} + \epsilon_r\right] \div \left[\frac{(a_h S_h / N)}{\left(\dfrac{1}{q_r} - B_h - S_h \log N\right)^2} + \epsilon_r\right] \tag{7}$$

The resilient strain ϵ_r could be determined from the following relations that define resilient modulus M_R in terms of repeated deviator stress σ_d:

For dry of optimum compaction conditions (γ_d = 129.5 lb/ft^3, m = 7 percent),

$$\log M_R = 4.56 - 0.118 \log \sigma_d \tag{8}$$

For wet of optimum compaction conditions (γ_d = 129.5 lb/ft^3, m = 10 percent),

$$\log M_R = 4.36 - 0.221 \log \sigma_d \tag{9}$$

where M_R and σ_d are expressed in pounds per square inch.

The variation of modular ratio E/E_i with number of load repetitions is shown in Figures 1 and 2. It is interesting to observe that for stresses smaller than the threshold value, subgrade strain hardening will occur and thus lead to an increase in subgrade modulus with number of stress repetitions. Conversely, for stresses larger than the threshold value, initial

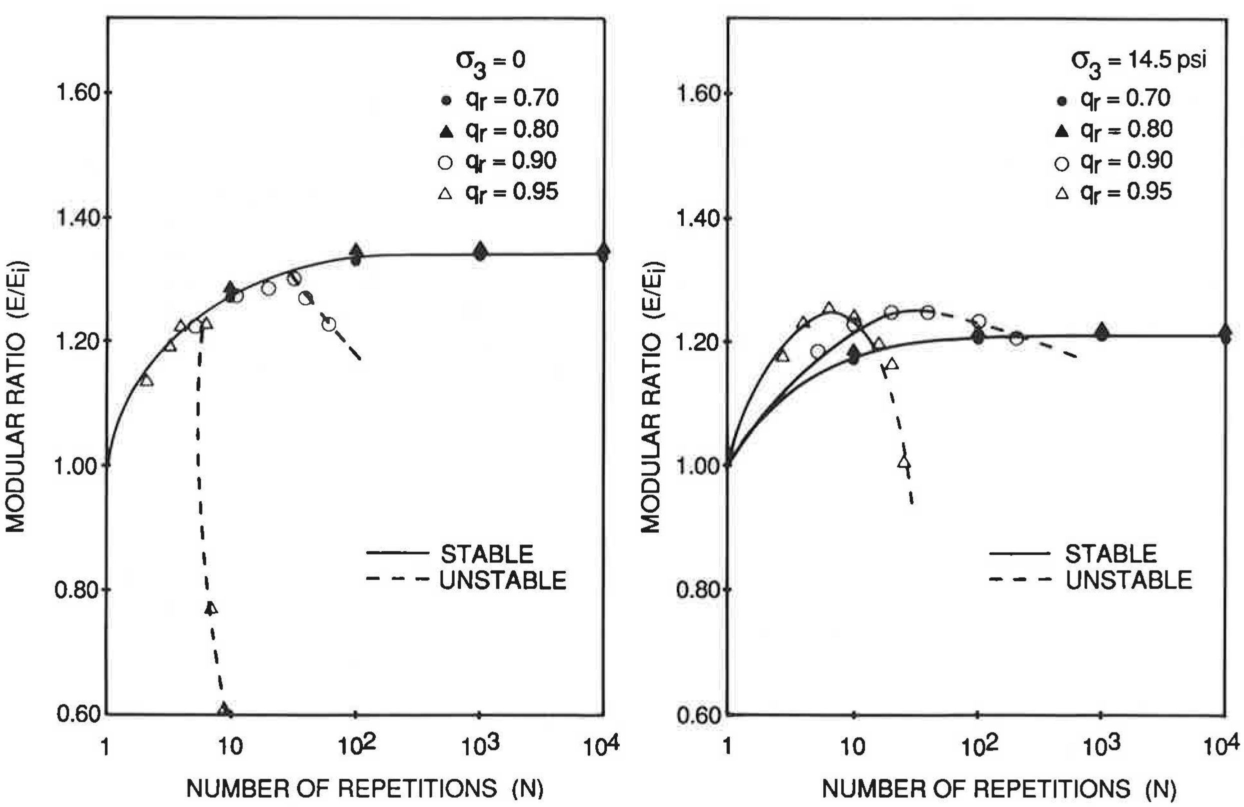

FIGURE 1 Stiffening and softening effects on subgrade under repeated loads (γ_d = 129.5 lb/ft^3, m = 7 percent).

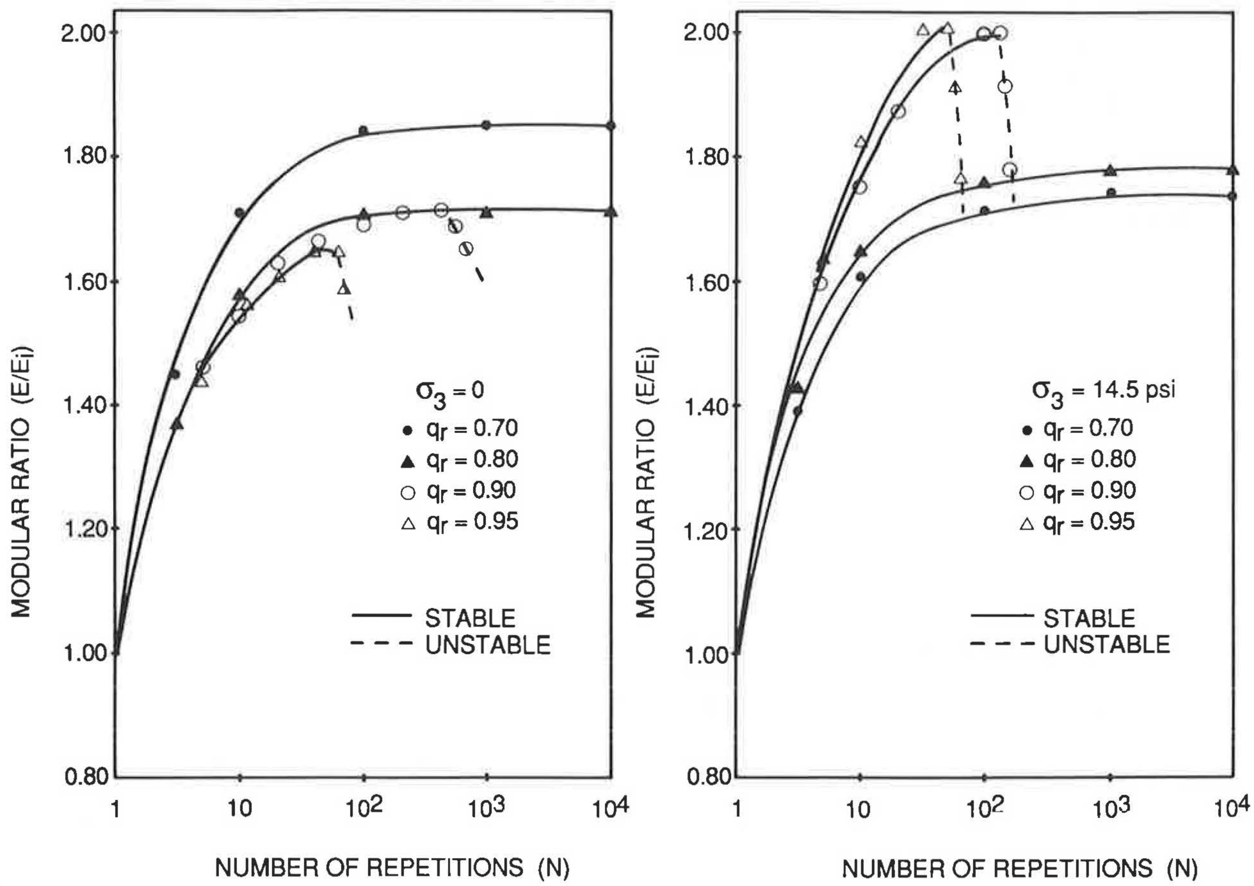

FIGURE 2 Variation of axial strain and axial strain rate with number of stress repetitions (σ_3 = 14.5 psi, γ_d = 129.5 lb/ft^3, m = 10 percent).

strain hardening is observed followed by strain softening as a result of unstable response caused by increased rate of strain accumulation. Strain-hardening effects are more pronounced for specimens with higher compaction moisture content. This could probably be attributed to local over-consolidation effects in the microstructure of the wet clay when subjected to repeated loads. However, at magnitudes of repeated loads larger than the "threshold value," the rate of accumulation of axial strains will eventually become higher and the corresponding modulus will therefore be lower.

The effect of strain hardening and strain softening on subgrade modulus and its relationship to subgrade stability can have significant practical applications. For example, if nondestructive testing equipment such as the falling weight deflectometer is used to determine the onset of subgrade softening through successive applications of loads with increasing magnitude, then a subgrade limiting load could be estimated. Progressive accumulation of strains will occur with load repetitions above this load, and below it the accumulation of strains will cease and lead to a stable subgrade response.

The proposed model was used to determine the change of shear strength parameters, the cohesion C, and angle of friction ϕ with number of load repetitions to failure. The failure strain ϵ_f is substituted in Equation 2, and the corresponding limiting principal stresses were computed for different number of repetitions to failure. Cohesion and friction associated with failure under repeated loading were compared with cohesion C_s and angle of friction ϕ_s obtained from standard strain-controlled triaxial tests at strain rate of 0.5 percent/min. Results are presented in Figure 3. A degradation effect of both cohesion and friction is observed with increase in number of repetitions to failure. The decrease appears to be more significant, particularly for cohesion, for specimens with higher compaction moisture content. This could probably be a result of higher pore-water pressure build up under repeated loading.

Limiting Criteria

Correlations between repeated stress level and number of repetitions for different limiting values of axial strain ϵ_a were derived for the compacted silty clay by using the proposed model and assuming a threshold stress level q_{rt} equal to 0.80. Those relations are presented in Figures 4 and 5. The influence of confining pressure on those limiting relations is more significant for specimens compacted wet of optimum than for dry of optimum compaction. The proposed model could also be used to determine the stress state in terms of p and q associated with a given limiting strain and a selected number

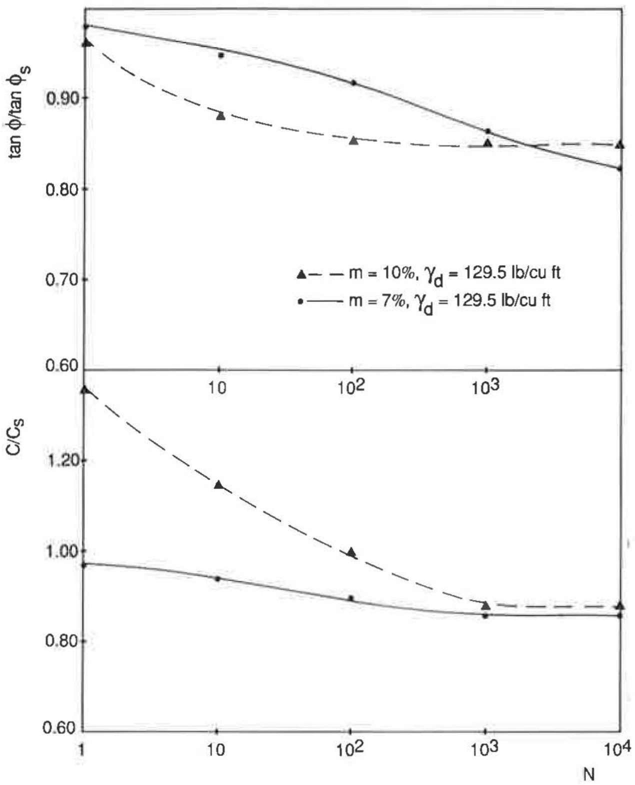

FIGURE 3 Variation of subgrade cohesion and friction with number of load repetitions.

of load repetitions. In this case,

$$p = \frac{\sigma_1 + \sigma_3}{2} \tag{10}$$

$$q = \frac{\sigma_1 - \sigma_3}{2} \tag{11}$$

where σ_1 and σ_3 are major and minor principal stresses, respectively.

The variation of p and q for different limiting strain values corresponding to number of repetitions greater than 10^4 is illustrated in Figures 6 and 7. Limiting strain criteria could be selected in this case in relation to anticipated subgrade performance under long-term repeated loading. For example, stress points higher than those defined by the p-q relationship for limiting axial strain ϵ_a equal to the failure strain ϵ_f would result in the progressive accumulation of subgrade strains with number of load repetitions and thus lead to unstable response. Conversely, for stress points located below the limiting p-q line, the response is stable and subgrade strains will eventually cease to accumulate with increased number of load repetitions. The variation of cohesion C angle of friction ϕ corresponding to number of repetitions greater than 10^4 and limiting strain ϵ_a between ϵ_f and 0.5 ϵ_f is presented in Figure 8. Values for the angle of friction, expressed as tan ϕ, lie essentially between 0.90 tan ϕ_s and 0.75 tan ϕ_s, whereas the cohesion varies in the range of $0.90C_s$ to $0.40C_s$.

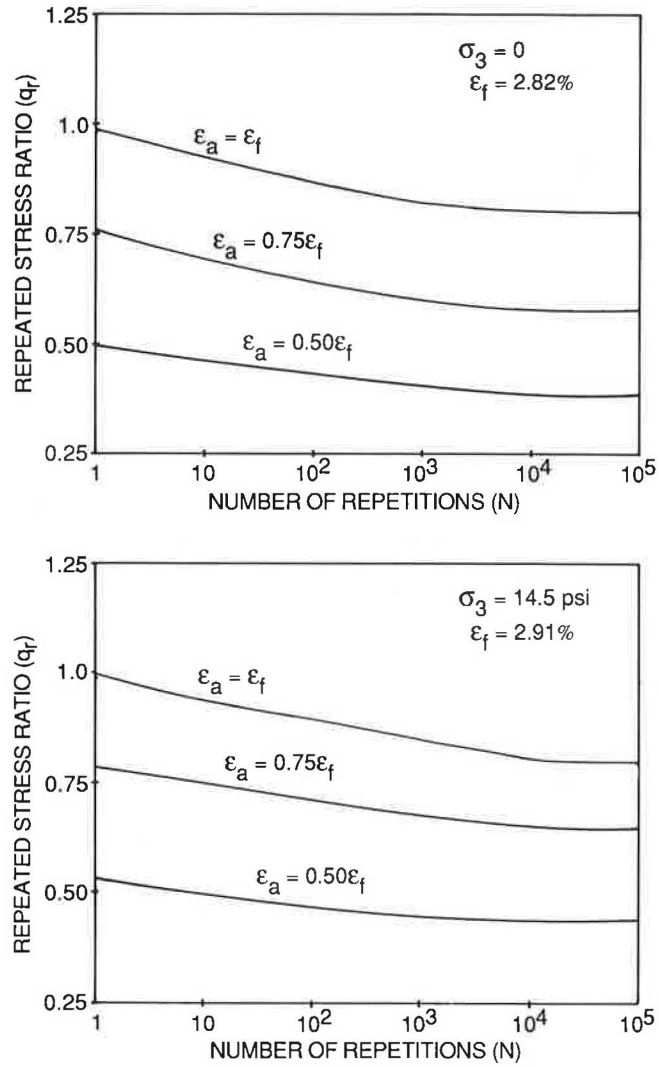

FIGURE 4 Allowable stress level for different subgrade limiting strains (γ_d = 129.5 lb/ft³, m = 7 percent).

At high moisture content and confining pressure or both, compacted clays may in some cases not exhibit a definite peak stress when tested in standard strain-controlled triaxial mode. The axial strains will progressively build up at a decreasing rate, and the test is generally stopped when the axial strain reaches 20 percent. Under repeated loading conditions, such clays may experience continuous increase in total axial strain ϵ_a and gradual decrease of the rate of strain $d\epsilon_a/dN$. In this case, a failure strain according to the definition used earlier in this paper does not exist. However, a limiting strain that corresponds to the maximum allowable value in relation to subgrade performance can be used instead. Additional research is needed to characterize the behavior of such clays and to develop improved constitutive models under repeated loads.

Comparisons with Other Models

The variation of repeated stress level with number of load repetitions to failure for the compacted silty clay as predicted

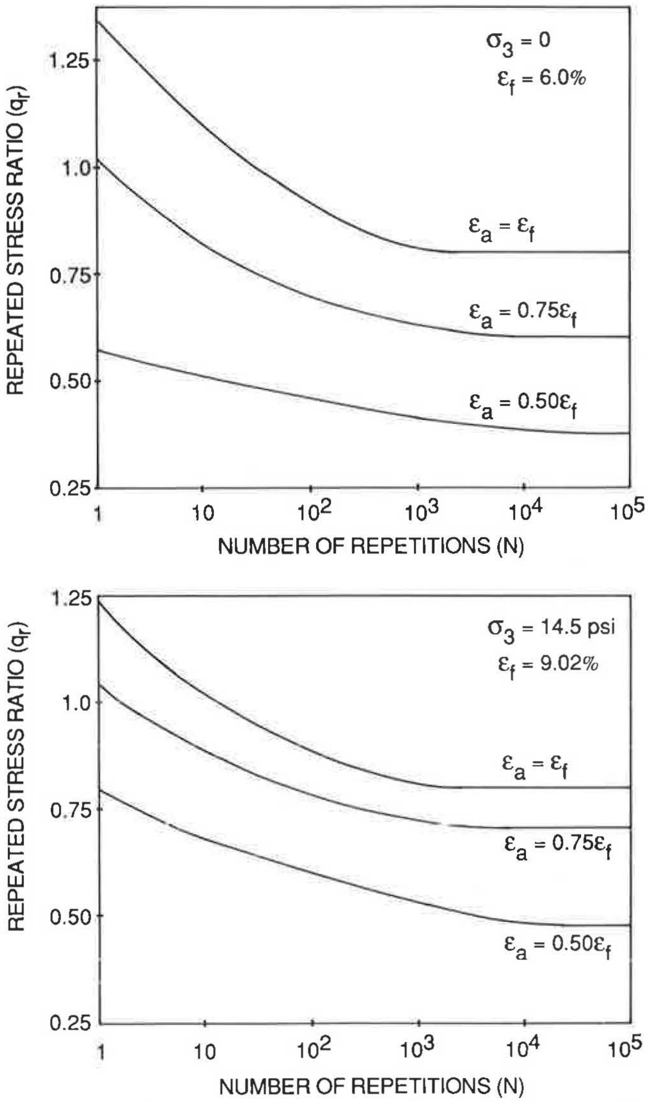

FIGURE 5 Allowable stress level for different subgrade limiting strains (γ_d = 129.5 lb/ft³, m = 10 percent).

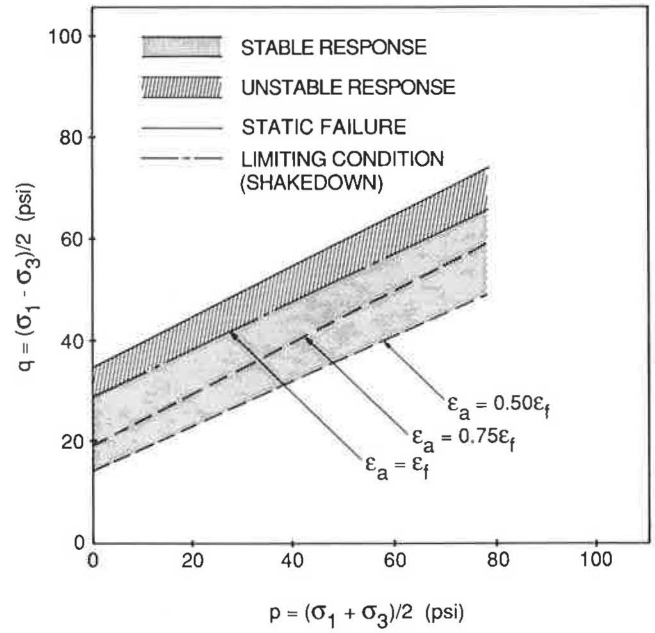

FIGURE 6 Subgrade *p-q* relations for different limiting strains (γ_d = 129.5 lb/ft³, m = 7 percent).

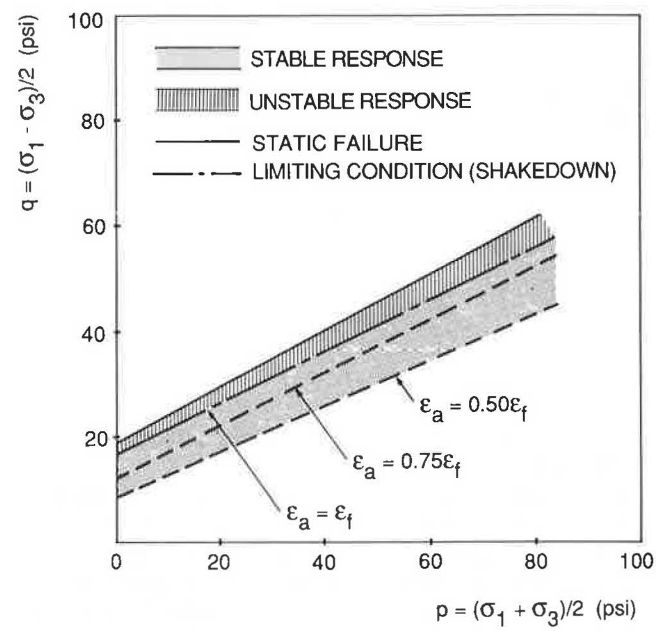

FIGURE 7 Subgrade *p-q* relations for different limiting strains (γ_d = 129.5 lb/ft³, m = 10 percent).

by the proposed model is compared with results obtained by using the Shell criterion (*10*) and other criteria developed by Darter and Devos (*11*), Poulsen and Stubstad (*12*), and Chou et al. (*13*). Comparisons presented in Figures 9 and 10 show that the Shell criterion and Darter and Devos criterion are not conservative when used for a number of load repetitions smaller than 10^2 to 10^3 and fairly conservative for a larger number of repetitions. Conversely, results obtained from criteria proposed by Poulsen and Stubstad and Chou et al. yield consistently conservative predictions. The permanent strain ϵ_p exhibited by the compacted silty clay when loaded according to those criteria is compared with values predicted by the proposed model (Table 3). Results indicate that the permanent strain is significantly higher for specimens with higher compaction moisture content. This implies that the use of existing subgrade criteria does not necessarily result in equal performance levels for different subgrade conditions. According to the criteria in Table 3, the permanent strains corresponding to load repetitions greater than 10^4 will range essentially between 0.4 percent and 2 percent in comparison with permanent strains at failure in the range of 2 percent to 8 percent as predicted by the proposed model.

SUMMARY AND CONCLUSIONS

A load-deformation model for subgrade soils where total cumulative axial strains are correlated with applied stresses

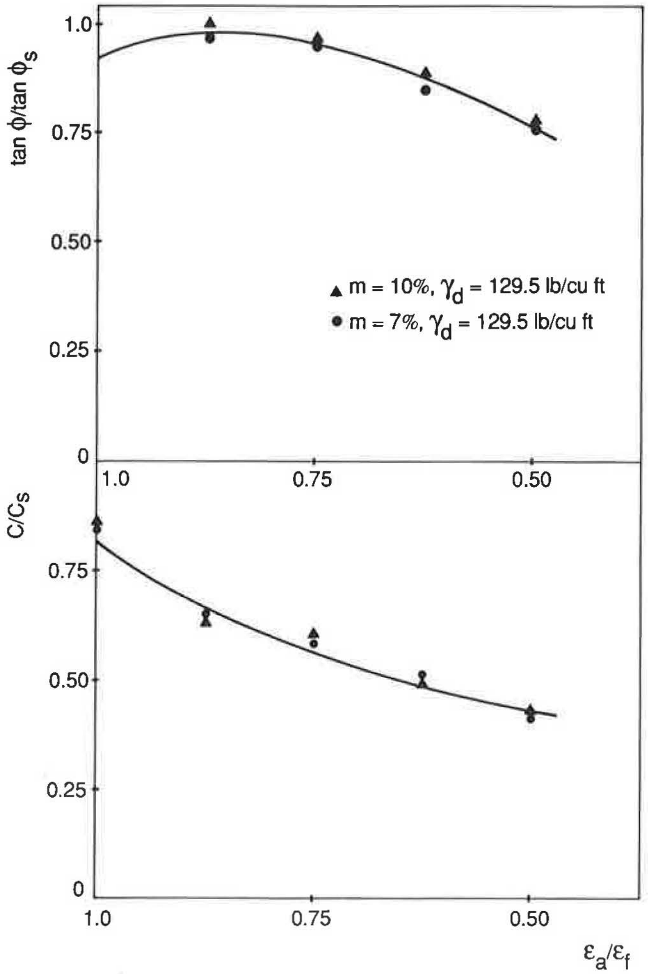

FIGURE 8 Influence of strain limits on subgrade cohesion and friction for *N* greater than 10^4.

and number of load repetitions was used to investigate load repetition effects on subgrade modulus and shear strength. Predictions of allowable stresses for different limiting strains were also established, and limiting criteria in terms of applied stresses and number of repetitions to failure were compared with other subgrade criteria in the literature.

Strain hardening and strain softening of the subgrade can occur, depending on the magnitude of the repeated stress relative to the "threshold stress." The subgrade modulus, defined as the ratio of repeated stress to total strain per load repetition, increases for repeated stresses lower than the "threshold stress" and maintains a constant value with increased number of load repetitions. For higher stresses, however, the modulus will eventually decrease, indicating a strain-softening behavior. If nondestructive testing equipment were to be used to determine the onset of subgrade softening through successive applications of load with increasing magnitude, then an estimate of subgrade limiting load can be determined. The accumulation of strains will cease with number of load repetitions below this value, and progressive accumulation of strains will occur above it.

Cohesion and friction associated with failure under repeated loading conditions decrease with the increase in number of

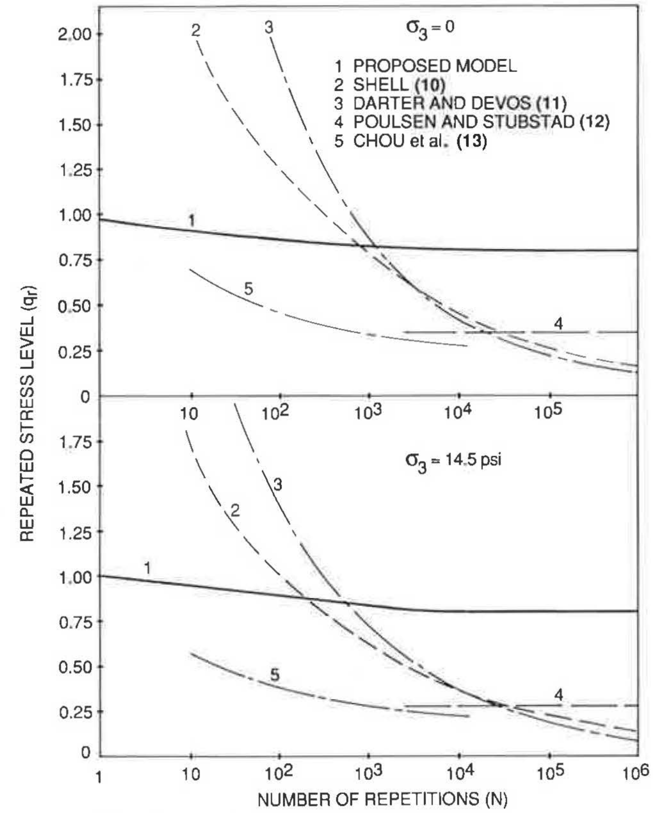

FIGURE 9 Comparisons of predicted repeated stress level at failure by using different models (γ_d = 129.5 lb/ft^3, *m* = 7 percent).

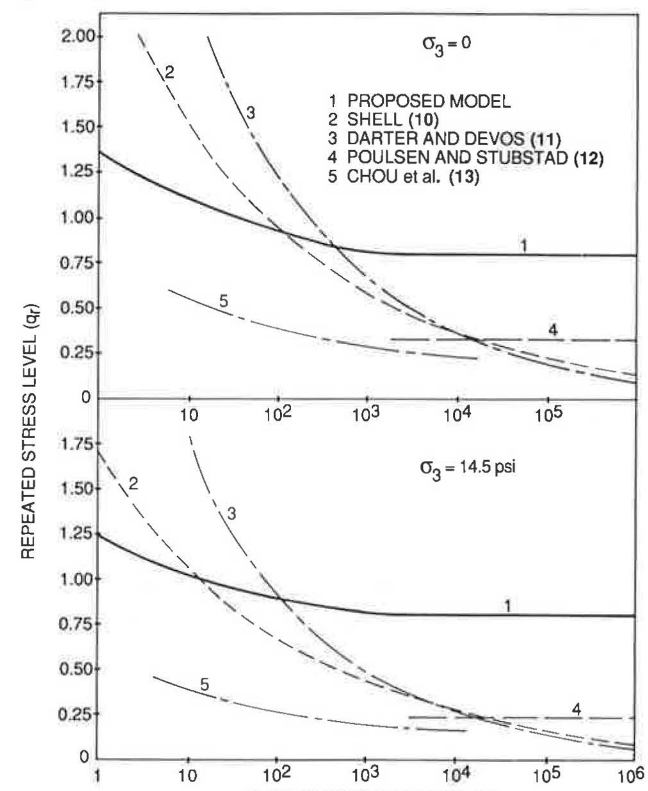

FIGURE 10 Comparisons of predicted repeated stress level at failure by using different models (γ_d = 129.5 lb/ft^3, *m* = 10 percent).

TABLE 3 COMPARISONS OF PERMANENT STRAIN PREDICTIONS BY DIFFERENT MODELS

	Dry of optimum (γ_d = 129.5 lb/cu ft, m = 7%)		Wet of optimum (γ_d = 129.5 lb/cu ft, m = 10%)	
	ϵ_p (%)		ϵ_p (%)	
	N = 10^4	N = 10^6	N = 10^4	N = 10^6
Proposed Model	2.23 - 2.28	2.23 - 2.28	5.27 - 7.91	5.27 - 7.91
Shell (10)	0.94 - 1.39	0.35 - 0.50	2.19 - 2.54	0.86 - 1.0
Darter & Devos (11)	0.89 - 1.29	0.18 - 0.36	2.11 - 2.39	0.70 - 0.80
Poulsen and Stubstad(12)	0.74 - 1.08	0.74 - 1.08	2.02 - 2.41	2.02 - 2.41
Chou et al. (13)	0.60 - 0.87	-	1.45 - 1.71	-

repetitions to failure. The decrease is more significant for specimens with the same dry density but higher moisture content.

Allowable repeated stress states in terms of major and minor principal stresses (i.e., *p-q* relations) were determined for different limiting strain values corresponding to number of load repetitions greater than 10^4. Similar relations can be used in advanced numerical analysis of pavement structures for the purpose of developing improved understanding of pavement behavior.

The variation of repeated stress level with number of repetitions to failure determined by the proposed model was compared with subgrade criteria suggested by other investigators. Stress predictions by those criteria are generally conservative for number of repetitions greater than about 10^3. For smaller number of repetitions, those criteria could be nonconservative. Moreover, results indicate that the permanent strains that occur in the subgrade, when loaded according to any of those criteria, is greatly influenced by compaction moisture content. In this case, the permanent strains corresponding to load repetitions greater than 10^4 will range between 0.4 percent and 2 percent in comparison with permanent strains at failure in a range of 2 percent to 8 percent as predicted by the proposed model.

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