Sight Distance Requirements for Symmetrical- and Unsymmetrical-Crest Vertical Curves

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Major changes in vehicular design have occurred in the United States during the last 25 years that have affected both the computation and measurement of stopping sight distance (SSD). First, a brief review of the changes in the design parameters of symmetrical-crest vertical curves, as reflected in the 1984 AASHTO manual, is presented. Second, an analytical approach computing the length of unsymmetrical curves to provide SSD requirements, for which no design guidelines are currently available, is presented. Unsymmetrical curves may be warranted in special situations with constrained geometrics, e.g., freeway ramps, grade separation structures, multiple control points, etc. A parameter (γ) is introduced as an indicator of nonsymmetry in the computation of length of unsymmetrical-crest curves. For values of γ less than unity, the procedure presented results in a longer curve length than that used for a symmetrical curve, with the maximum length occurring at $\gamma = 0.38$ for driver's eye and object heights of 3.5 and 0.5 ft, respectively. Overall, unsymmetrical curves required longer lengths than those currently used for symmetrical curves. A complete procedure for setting up unsymmetrical curves to meet SSD requirements is presented along with a technique to locate the highest point. Finally, recommendations are made for further research for formalizing additional design guidelines for unsymmetrical curves.

Sight distance considerations constitute a key element of highway design. The ability of the motorist to see a sufficient distance ahead to perceive potential hazards and to make proper decisions is a major factor in the safe and efficient operation of highways (*1,2*). Motorists must not be trapped in situations where they have neither sufficient time nor distance to take evasive actions. Further, traffic engineers must recognize the importance of interface between vehicular and human factors in the design of highways.

Vehicular design has undergone significant changes in dimensions and operating characteristics during the last 25 years. Also, substantial changes in the mix of vehicles between passenger cars and heavy vehicles have occurred during the last two decades. Lastly, highway users differ in their physical stature and in their psychological attributes. Today's driving population in most countries has somewhat of a different distribution of age groups and sex compared with the early 1960s. Highway design practices should recognize these changes and should attempt to incorporate their effect into the design parameters.

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THE IMPORTANCE OF SIGHT DISTANCE

AASHTO discusses three types of sight distance for consideration in highway design (*2*). There are stopping sight distance (SSD), passing sight distance (PSD), and decision sight distance (DSD). In addition, the importance of sight distance at intersections is mentioned in the AASHTO manual. Of the three types, SSD constitutes the single most important design criterion for highways.

Current highway design standards dictate that at any point on a given roadway, a minimum SSD must be provided. Failure on the part of the roadway designer to provide the minimum SSD may expose the motorist to undue hazards and increase the likelihood of accidents. The provision of minimum PSD although considered desirable will result in inordinately long vertical curves and in high construction costs. DSD is important only in special situations in which there is a likelihood of error in information reception, decision making, or control actions. Clearly, consideration of SSD (rather than PSD or DSD) is of utmost importance in the design of crest curves (*3*). SSD is incorporated in the design of unsymmetrical crest vertical curves. In order to provide continuity, a brief synopsis of symmetrical vertical curves is also presented.

GEOMETRY OF CREST VERTICAL CURVES

The purpose of vertical curves that join two intersecting grade lines of railroads or highways is to smooth out the changes in vertical motion. Vertical curves are designed to contribute to safety, comfort, and appearance of the roadway. These curves are generally parabolic in nature and can be either symmetrical or unsymmetrical. Symmetrical curves are those with equal tangents at the point where the curve is divided equally at the vertical point of intersection (VPI) of the two tangents (Figure 1). The point on the left tangent line where the curve starts is termed the vertical point of curvature (VPC) and the corresponding point where the curve ends on the right tangent is called the vertical point of tangency (VPT).

The majority of vertical curves constructed in the United States are symmetrical in nature. Standards for incorporating sight distance requirements for symmetrical crest curves were originally developed by AASHO in 1965 (*1*) and updated in 1984 (*2*). The rate of change r of slope of a symmetrical vertical curve remains unchanged throughout the length of the curve. The constant value of r is a characteristic feature of the parabolic nature of the curve.

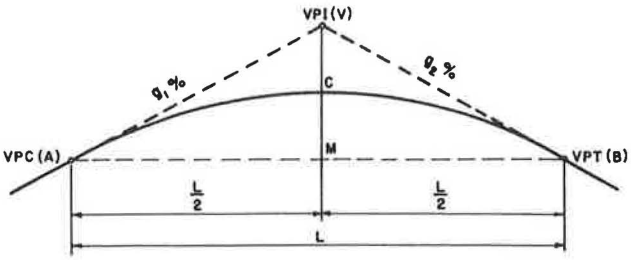

FIGURE 1 Symmetrical-crest vertical curve.

An unsymmetrical curve is characterized by two unequal tangents, resulting in an unequal division of the curve at the VPI (Figure 2). The rates of change of the slope of the two sections of the curve under the two tangents are different and the point under the VPI forms the transition between the two rates. Although unsymmetrical curves are far less common than symmetrical ones, AASHTO states "it is possible that an unsymmetrical curve will fit more closely certain imposed requirements than the usual symmetrical equal-tangent curve" (2). Unsymmetrical curves may be warranted in situations with constrained geometrics—roadways with multiple control points, freeway ramps, and grade-separated structures where a minimum vertical clearance between two roadbeds must be provided. Few guidelines are currently available on how to incorporate sight distance requirements in unsymmetrical-crest vertical curves. However, unsymmetrical curves should be used more frequently than before, but because AASHTO does not provide guidelines this analysis can be used to design an unsymmetrical curve should one be needed.

Two aspects of SSD are important from the roadway designer's viewpoint: (a) computation of SSD at various speeds, and (b) measurement of roadway length at crest curves to ensure the provision of SSD. Both computation and measurement represent the use of analytic techniques requiring basic assumptions on vehicular dimensions and geometric features of the roadway. Major changes in vehicular design have occurred in the United States during the last 25 years that have affected both the computation and measurement of SSD. Historically, the critical dimensions (i.e., length, breadth, and height) of passenger cars have decreased because of safety standards, energy consumption, and driver preferences. Also, advances in automotive technology have brought about major changes in vehicular dynamics, including acceleration and deceleration characteristics, speed attainability over gradient sections, and maneuverability around sharp curves (4).

First, the changes in the design parameters of symmetrical vertical curves are reviewed to incorporate changes in vehicular design as reflected in the 1984 AASHTO manual (2).

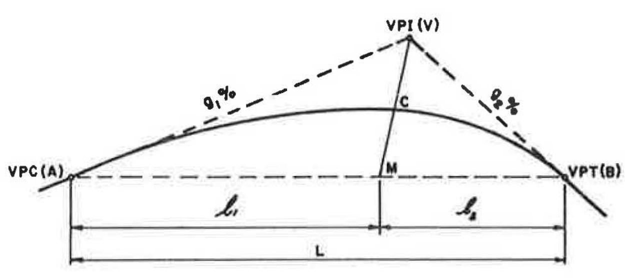

FIGURE 2 Unsymmetrical-crest vertical curve.

Second, the lengths of unsymmetrical curves to provide SSD requirements are computed.

COMPUTATION OF SSD

Equation 1 is used to calculate SSD values as the sum of reaction distance and braking distance. Included are two parameters, reaction time t_r and pavement friction f, and one major variable, speed V.

$$S = 1.47Vt_r + \frac{V^2}{30(f \pm G)} \quad (1)$$

where

S = SSD,
V = speed (mph),
t_r = perception-reaction time (sec),
f = coefficient of friction between tires and pavement, and
G = percent of grade divided by 100.

Perception-Reaction Time

The 1965 AASHO manual (1) recommended a design value of 2.5 sec for perception-reaction time (including 1.5 sec for perception and 1.0 sec for reaction) under emergency braking conditions for a typical driver (1,5,6). The value of 2.5 sec is based on limited experimental data collected many years ago. Recent studies (2) indicate that "a reaction time of 2.5 sec is considered adequate for more complex conditions than those of the various studies, but it is not adequate for the most complex conditions encountered by the driver." Woods (7) reported that "perception reaction time is substantially less than the 2.5 second value currently used. . . . The distribution of perception reaction time has an extreme value of 3 seconds." However, the 1984 AASHTO manual (2) has retained the earlier assumed value of 2.5 sec for the purpose of computing SSD.

Pavement Friction

Values of coefficient of friction (f) in Equation 1 for wet pavement as used in the 1965 AASHO manual (1) were developed almost 30 years ago (8). The 1984 AASHTO manual (2) recommends f values that are slightly lower than those used in the past. These revised values recognize recent studies that indicate that there is a wide variation in pavement friction that reflects the effect of surface texture on stopping distances (4).

Speed

The 1965 AASHO manual (1) assumed that top speeds were somewhat lower on wet pavements than on the same surface under dry conditions. On the basis of this assumption, the average running speed rather than design speed was used in computing SSD values. This assumption was questioned in later years as many motorists were found to drive as fast on

TABLE 1 COMPARISON OF MINIMUM SSD VALUES FOR WET PAVEMENT BETWEEN 1965 (*1*) AND 1984 (*2*) AASHTO GUIDELINES (t_r = 2.5 sec)

Design Speed	1965 AASHO			1984 AASHTO			1983 AUTHORS' RECOMMENDATION		
	Assumed Speed	Coef. of Friction	SSD	Assumed Speed for Condition	Coef. of Friction	SSD	Assumed Speed for Condition	Coef. of Friction	SSD
(MPH)	(MPH)	(f)	(feet)	(MPH)	(f)	(feet)	(MPH)	(f)	(feet)
30	28	0.36	200	28-30	0.35	200-225	29	0.35	180
40	36	0.33	275	36-40	0.32	275-325	38	0.31	300
50	44	0.31	350	44-50	0.30	400-475	47	0.28	440
60	52	0.30	475	52-60	0.29	525-650	56	0.27	600
70	58	0.29	600	58-70	0.28	625-850	64	0.26	760

wet pavements (*9*). The 1984 manual suggests the use of design speed rather than average running speed. Table 1 presents a comparison of SSD values computed using the two versions of the AASHTO manual (*1,2*). Also included in Table 1 is a set of SSD values suggested in 1983, by Khasnabis and Reddy (*8*), before publication of the 1984 AASHTO manual (*2*).

The higher values indicate the effect of design speed and revised friction (f) values.

Symmetrical-Crest Curves

The expression given by Hickerson (*10*) for computing the length of a symmetrical-crest vertical curve is as follows:

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2} \quad \text{if } S < L \tag{2}$$

where

- L = required length of vertical curve (ft),
- S = required sight distance (SSD in this case) (ft),
- A = algebraic difference in grades (percent),
- h_1 = driver's eye height (ft), and
- h_2 = object height (ft).

Equation 2 can be rewritten as

$$L = KA \tag{3}$$

where

$$K = \frac{S^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2} \tag{4}$$

In Equation 3, L is the length of crest curve needed for each percent algebraic change in grade A and is a convenient expression of the design control. For each design speed V (which determines the value of S) and a given combination of driver's eye height (h_1) and object height (h_2), K is thus the rate of vertical curvature (or the length per unit value of A) needed to provide the required SSD. Situations in which $S > L$ are somewhat uncommon and are not considered critical from the point of view of design.

Eye Height (h_1)

The 1965 AASHO guidelines (*1*) recommended an eye height value of 3.75 ft (45 in.) on the basis of the actual observation of vehicular dimensions up to the mid-1960s. Passenger vehicle design trends have resulted in lower vehicular heights and thus in lower driver's eye height values (*9,11,12*). Eye height has finally stabilized and little change, if any, is likely to occur during the next decade.

The 1984 AASHTO manual (*2*) recognizes these changes and calls for using a driver eye height of 3.5 ft for the design of crest vertical curves. In their 1983 study, Khasnabis and Reddy (*8*) suggested the use of 3.5 ft: "It appears that height of 3.5 feet (as opposed to the currently used 3.75 ft) would better represent the eye height of a majority of passenger cars in the United States." Even this reduction of eye height is considered by some experts to be insufficient to reflect a majority of passenger cars in the United States. For example, Woods (*7*) reported a 95 percentile value of 3.23 ft and has shown that the assumed value of 3.5 ft at a design speed of 70 mph used to compute the length of crest vertical curves would result in an α level of 25 percent (corresponding to Type 1 error).

Object Height (h_2)

In 1940, AASHO selected a design object length of 4 in. on the basis of optimizing the trade-off between object height and the required length of vertical curve (*11*). This 4-in. object height represents a compromise solution between construction cost and the need to maintain a clear sight line to the pavement, which is evident from the fact that this requirement was changed to 6 in. in 1965 and has been retained at 6 in. in 1984. To quote the 1984 manual (*2*), a 6-in. height is representative of the "lowest object that can be perceived as a hazard by a driver in time to stop before reaching it." Khasnabis and Reddy (*8*) discussed the implication of object height and demonstrated that even a small reduction in eye height will result in a substantially longer crest curve.

Required Length of Symmetrical Curve

For h_1 = 3.5 ft and h_2 = 0.5 ft, Equation 4 can be rewritten as

$$K = \frac{S^2}{1{,}329} \tag{5}$$

TABLE 2 COMPARISON OF CREST VERTICAL CURVE LENGTHS ON THE BASIS OF SSD REQUIREMENTS

Design Speed (MPH)	K = Rate of Vertical Curvature Length (ft) per Percent of A		
	1965 AASHO	1984 AASHTO	1983 AUTHORS' RECOMMENDATION
30	28	30-38	24
40	55	60-80	68
50	85	110-160	146
60	160	190-310	271
70	255	290-540	435

In Table 2, a set of K values is presented for use in comparing crest curve lengths between the 1965 and 1984 AASHTO guidelines (*1,2*) and the 1983 recommendation of Khasnabis and Reddy (*8*). Higher values of K for 1984 in Table 2 reflect the consequence of increased SSD values as shown in Table 1 plus a reduction in h_1 from 3.75 to 3.5 ft.

UNSYMMETRICAL-CREST CURVES

Unsymmetrical vertical curves are not common, but as AASHTO mentions, "on certain occasions, because of critical clearance or other controls, the use of unsymmetrical vertical curves may be required" (*2*,p.305). Figure 2 shows that the points of intersection of the vertical line through the VPI with the curve at C and with the long chord at M are not the midpoints of the curve or the chord, respectively. An unsymmetrical curve is not divided into two equal halves around the VPI. Thus, unlike a symmetrical curve with a constant rate of change in the slope, an unsymmetrical curve is characterized by two rates of change in slope—one for the left portion of the curve from the VPC to the VPI and another for the right section of the curve from the VPT to the VPI. This difference is explained in the derivations of Appendix A.

Length Requirement of Unsymmetrical Curves

In Figure 2, let

l_1 = length of curve AC (left section) (ft), and
l_2 = length of curve BC (right section) (ft)

so that

$$L = l_1 + l_2$$

where L = length of entire curve (ft). Setting

g_1 = percent grade of left tangent AV, and
g_2 = percent grade of right tangent BV,

the algebraic difference in percent grade of the tangents is

$$A = |g_2| + |g_1|$$

and $\gamma = l_1/l_2$. Deviation of the value of γ from unity is a measure of the degree of nonsymmetry.

Thus, it can be shown (Appendix A) that

$$L = \frac{AS^2}{200(\sqrt{h_1\gamma} + \sqrt{h_2/\gamma})^2} \tag{6}$$

Alternatively,

$$l_2 = \frac{AS^2\gamma}{200(\gamma\sqrt{h_1} + \sqrt{h_2})^2(1 + \gamma)} \tag{7}$$

$$l_1 = \gamma l_2, \tag{8}$$

so that L can be computed from

$$L = l_1 + l_2. \tag{9}$$

Note that $\gamma > 1$ when $l_1 > l_2$ or $0 < \gamma \leq 1$, when $l_1 \leq l_2$.

In practice, γ is likely to be within the range of 0.25 to 2 and when γ equals unity, the curve becomes symmetrical. Further, deviation of the γ value from unity (in either direction) is an index of the degree of nonsymmetry. The length of the unsymmetrical curve L needed to provide a required sight distance S can be computed from Equation 6. Alternatively, Equation 6 can be written as

$$L = K^1A \tag{10}$$

where

$$K^1 = \frac{S^2}{200(\sqrt{h_1\gamma} + \sqrt{h_2/\gamma})^2} \tag{11}$$

K^1 is the length of crest curve needed for each percent of algebraic change in grade A. For each design speed V (which determines the value of S), a combination of h_1, h_2 (eye and object heights), and known γ value (degree of nonsymmetry), K^1 is the rate of unsymmetrical vertical curvature needed to provide the required SSD.

Alternatively, Equations 7–9 can be rewritten as

$$k_2^1 = \frac{S^2\gamma}{200(\gamma\sqrt{h_1} + \sqrt{h_2})^2\,(1 + \gamma)} \tag{12}$$

$$k_1^1 = \gamma k_2^1 \tag{13}$$

$$K^1 = k_2^1 + k_1^1 \tag{14}$$

where

k_2^1 = rate of vertical curvature of the right section,
k_1^1 = rate of vertical curvature of the left section, and

K^1 = rate of unsymmetrical vertical curvature as defined before.

If r_1 and r_2 are the rates of change of slope of the left portion (from VPC to VPI) and the right portion (from VPT to VPI), respectively,

$$r_1 = \frac{A}{L}\frac{l_2}{l_1} \quad (15)$$

and

$$r_2 = \frac{A}{L}\frac{l_1}{l_2} \quad (16)$$

By contrast, a symmetrical curve has a constant rate of change of slope r from the VPC to the VPT equal to A/L.

If the object of the analysis is to derive estimates of k_1^1 and k_2^1 separately, Equations 12 and 13 can be used for such purposes. Table 3 presents a set of K^1 values, using Equation 11 and the same data used in Table 2, plus the additional parameter γ. At $\gamma = 1$, the K^1 values in Table 3 are similar to those in Table 2. Tables 4 and 5 present the values of k_1^1, k_2^1, and K^1 as computed by Equations 12–14 for the same set of γ values. Note that at $\gamma = 1$, k_1^1 and k_2^1 are the same, representing a special case of a symmetrical curve.

Design Guidelines

The K^1 values have also been plotted in Figures 3 and 4 for the low and high values of SSD for specific design speeds, for assumed values of γ ranging from 0.25 to 2.0. Because guidelines for length requirements for unsymmetrical curves are currently unavailable, Figures 3 and 4 can be used to compute the length of unsymmetrical-crest curve for a specific value of γ.

From data presented in Tables 3–5, as well as Figures 3 and 4, length requirements of unsymmetrical curves (for the assumed value of h_1 and h_2) exceed those of symmetrical curves when the numerical value of γ is less than unity. However, the change in the K^1 value as a result of a reduction in the value of γ is not monotonic. For example, the reduction in γ from 1.0 to 0.5 brings about a significant increase in the value of K^1. However, a reduction in γ from 0.5 to 0.25 causes a small reduction in K^1 because of the nature of the mathematical function (Equation 11) used for computing K^1.

Calculus was used to identify the value of γ at which K^1 is maximized. By taking the first derivative of Equation 11 with

TABLE 3 K^1 VALUES FOR DIFFERENT SPEEDS AND γ VALUES EXPRESSED AS RATE OF CURVATURE (IN FEET PER UNIT VALUE OF A) FOR UNSYMMETRICAL CURVES

Design Speed (MPH)	SSD (Ft)	γ Value									
		0.25		0.50		1.00		1.50		2.00	
		Low	High	Low	High	Low	High	Low	High	Low	High
30	200-225	36	46	37	47	30	38	24	31	20	26
40	275-325	68	96	70	98	57	79	46	64	38	53
50	400-475	145	204	148	209	120	170	97	137	81	114
60	524-650	249	382	255	392	207	318	167	257	139	213
70	625-850	354	654	362	670	294	544	237	439	197	365

TABLE 4 k_1^1, k_2^1, K^1 VALUES FOR DIFFERENT SPEED AND γ VALUES (IN FEET PER UNIT VALUE OF A) FOR UNSYMMETRICAL CURVES—LOW END

Design Speed (MPH)	SSD (Ft)	γ Value														
		0.25			0.50			1.00			1.50			2.00		
		k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1
30	200	7	29	36	12	25	37	15	15	30	14.6	9.7	24	13	7	20
40	275	14	55	69	23	47	70	28.5	28.5	57	27.6	18.4	46	25	13	38
50	400	29	116	145	49	99	148	60	60	120	58	39	97	54	27	81
60	525	50	200	250	85	170	255	103.5	103.5	207	100	67	167	93	46	139
70	625	71	283	354	121	241	362	147	147	294	142	95	237	132	66	198

TABLE 5 k_1^1, k_2^1, K^1 VALUES FOR DIFFERENT SPEED AND γ VALUES (IN FEET PER UNIT VALUE OF A) FOR UNSYMMETRICAL CURVES—HIGH END

Design Speed (MPH)	SSD (Ft)	γ Value														
		0.25			0.50			1.00			1.50			2.00		
		k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1	k_1^1	k_2^1	K^1
30	225	9	37	46	16	31	47	19	19	38	19	12	31	17	9	26
40	325	19	77	96	33	65	98	39.5	39.5	79	38	26	64	35	18	53
50	475	41	163	204	70	139	209	85	85	170	82	55	137	76	38	114
60	650	77	306	383	131	261	392	159	159	318	154	103	257	142	71	213
70	850	131	524	655	224	446	670	272	272	544	263	176	439	243	122	365

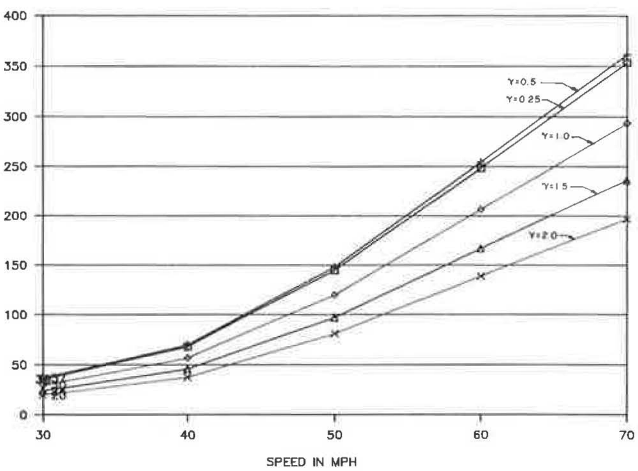

FIGURE 3 K^1 values for different speeds for lower SSD values.

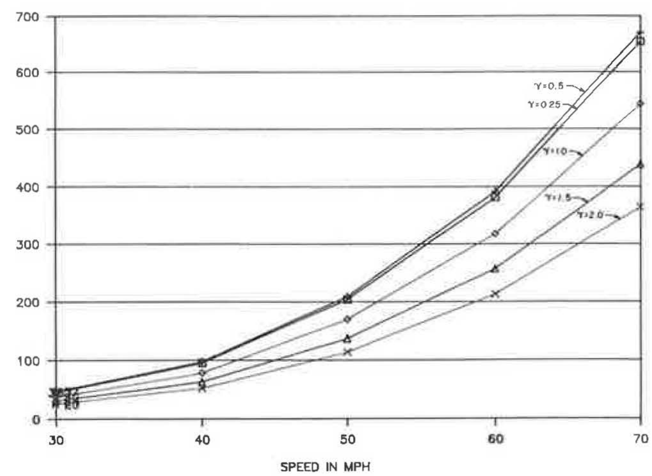

FIGURE 4 K^1 values for different speeds for higher SSD values.

respect to γ and setting it equal to zero, K^1 is maximized at $\gamma_{critical} = \sqrt{h_2}/\sqrt{h_1}$ and for the assumed values of $h_1 = 3.5$ ft and $h_2 = 0.50$ ft, this $\gamma_{critical}$ value is 0.38.

In an effort to further demonstrate the relationship between K^1 and γ, K^1 values for different speed data for γ values ranging from 0.20 to 0.60 are presented in Table 6. From this table, the curve length is indeed maximized when γ approaches 0.40, which closely approximates the γ value of 0.38 derived by calculus.

Figures 3 and 4 indicate that unsymmetrical curves, if designed as symmetrical curves according to current AASHTO guidelines, may not provide safe stopping distances for γ parameters less than unity. Additionally, the required length is maximized at $\gamma = 0.40$. For example, for a design speed of 50 mph, the lower range of the K value of a symmetrical curve is 110 ft (Figure 3 and Table 2). For an unsymmetrical curve for $\gamma = 0.50$, the corresponding value is 148 ft (Table 3 and Figure 2). Further, for the unsymmetrical curve for $\gamma = 0.50$, k_1^1 and k_2^1 are estimated as 49 and 99 ft, respectively, making a total length (K^1) of 148 ft (Table 4). Clearly, more research is needed before specific guidelines could be formalized in this respect.

For γ parameters exceeding unity, the reverse might appear to be true, but K^1 values for γ exceeding unity should not be used for design purposes because the derivation of K^1 values is based on assumed direction of travel from the left to the right, with eye height h_1 and object height h_2 located at left and right, respectively. Because a majority of two-lane rural highways are for two-way travel, clearly the directions of h_1 and h_2 could reverse themselves from those assumed in the derivation. Thus, the curves in Figures 3 and 4 to the right of the line representing $\gamma = 1$ (symmetrical curves) ought to be used with extreme caution, only when the roadway is for one-way travel. For all other cases, the γ value should be computed as

$$\gamma = (l_1/l_2) \text{ or } (l_2/l_1)$$

whichever is smaller, and the curves to the left of the line representing $\gamma = 1$ should be used. Thus, the curves in Figures 3 and 4, to the right of the line representing $\gamma = 1$ (or K^1 values for γ values exceeding unity) are purely for academic interest with little practical value. This situation is somewhat analogous to SSD values computed for dry pavements, although

TABLE 6 K^1 VALUES (IN FEET PER UNIT VALUE OF A) FOR VARIOUS VALUES OF γ AND DESIGN SPEEDS FOR UNSYMMETRICAL VERTICAL CURVES

γ Value	Design Speed (MPH)									
	30		40		50		60		70	
	Low	High	Low	High	Low	High	Low	High	Low	High
0.20	34	34	65	90	137	193	236	361	334	618
0.30	37	37	70.5	99	149	210	257	394	364	674
0.40	38	38	71.4	100	151	213	260	399	369	682
0.50	37	37	70	99	148	209	255	392	362	670
0.60	36	36	68	95	143	202	247	379	350	647

for all practical considerations wet-pavement conditions prevail in highway design.

Highest Point on a Crest Curve

Both for symmetrical and unsymmetrical curves, the turning point can be under the vertical point of intersection (VPI) of the two tangents, which is likely to happen only in specific situations; more often than not, the turning point is likely to be located either to the right or the left of the VPI.

The procedure for locating the turning point of a vertical curve includes taking the first derivative of the expression for computing the elevation of the curve (expressed in terms of x, the distance from the VPC), setting it equal to zero, and solving for x. The procedure results in the following equation for symmetrical curves:

$$X_{TP} = \frac{g_1 L}{|g_1| + |g_2|} = \frac{g_1 L}{A} \qquad (17)$$

where $A = |g_1| + |g_2|$.

The highest point of a symmetrical curve is more likely to be located either to the left or right of the VPI, depending on whether $|g_1|$ is less or more than $|g_2|$, respectively. Only when $|g_1|$ equals $|g_2|$ is the highest point exactly under the VPI, i.e., X_{TP} equals $L/2$ in Equation 17.

Following the same procedure and approaching both from the left tangent (VPC) and the right tangent (VPT) for an unsymmetrical curve, the turning point(s) can be located using the following expressions, as derived in Appendix B:

$$X_{TPL} = \frac{g_1 L}{A} \cdot \frac{l_1}{l_2} \qquad X_{TPL} \leq l_1 \qquad (18)$$

or

$$X_{TPR} = \frac{g_2 L}{A} \cdot \frac{l_2}{l_1} \qquad X_{TPR} \leq l_2 \qquad (19)$$

where X_{TPL} equals the location of the turning point measured from the left (VPC), and X_{TPR} equals the location of the turning point measured from the right (VPT). The purpose of the inequalities in expressions 18 and 19 is to ensure that the turning point is contained within the prescribed length of the l_1 or l_2 value. Only one of these two equations will prevail in most cases. Only under a specific set of geometric combination of lengths and grades will the turning point for unsymmetrical curves be under the VPI. This condition will happen only when the points X_{TPL} and X_{TPR} converge under the VPI as derived as follows:

$$X_{TPL} = g_1 \frac{L}{A} \cdot \frac{l_1}{l_2} = l_1 \qquad (20)$$

or

$$\frac{g_1 L}{A l_2} = 1 \qquad (21)$$

and

$$X_{TPR} = \frac{g_2 L}{A} \cdot \frac{l_2}{l_1} = l_2 \qquad (22)$$

or

$$\frac{g_2 L}{A l_1} = 1 \qquad (23)$$

Equations 21 and 23 are complements of each other, representing the rare case when the turning point, whether approached from the left or the right, will be under the VPI. Last, when $|g_1|$ equals $|g_2|$, and l_1 equals l_2, both X_{TPL} and X_{TPR} are equal to $L/2$. In this case, the vertical curve is a symmetrical curve (see Equation 15).

APPLICATION

For example, given $g_1 = +3$ percent, $g_2 = -4$ percent, $\gamma = 0.5$; design speed $V = 50$ mph; and elevation of the VPC $= E_{VPC} = 100$ ft at Station 50 + 00; it may be required to

1. Compute the length of the unsymmetrical curve for SSD condition,
2. Construct the complete vertical curve, and
3. Locate the highest point and its elevation.

From Table 1, at a design speed of 50 mph, the range of SSD is between 400 and 475 ft. The lower value is used for this solution. From Table 4, at a design speed of 50 mph, k_1^1 = 49 ft and k_2^1 = 99 ft; approximately, k_1^1 = 50 ft and k_2^1 = 100 ft. $A = |g_1| + |g_1| = 7$, so that $l_1 = 7 \times 50$ ft = 350 ft, and $l_2 = 7 \times 100$ ft = 700 ft, so that $L = l_1 + l_2$ = 1,050 ft.
[Horizontal distances are measured in stations (100 ft) and vertical distances are feet.]

The results are as follows:

$$E_{VPC} = E_A = 100 \text{ ft} \quad \text{(given)}$$

$$E_{VPI} = E_{VPC} + g_1x_1 = 100 + 3 \times 3.5 = 110.5 \text{ ft}$$

$$E_{VPT} = E_{VPI} - g_2x_2 = 110.5 - 4 \times 7 = 82.5 \text{ ft}$$

$$r_1 = \frac{A}{L} \cdot \frac{l_2}{l_1} = \frac{-7}{10.5} \times \frac{7}{3.5} = -1.333$$

$$r_2 = \frac{A}{L} \cdot \frac{l_1}{l_2} = \frac{-7}{10.5} \times \frac{3.5}{7} = -0.3333$$

In order to construct the curve from the left,

$$E_{x1} = E_{VPC} + g_1x_1 + {}^1\!/_2 r_1 x_1^2 \qquad (0 \le x_2 < l_1)$$

$$= 100 + 3x_1 - {}^1\!/_2 \times 1.333 \times x_1^2 \quad (0 \le x_1 \le 3.5) \qquad (24)$$

In order to construct the curve from the right,

$$E_{x2} = E_{VPT} + g_2x_2 - {}^1\!/_2 r_2 x_2^2 \quad (0 \le x_2 < l_2)$$

$$= 82.5 + 4x_2 - {}^1\!/_2 \times 0.33 \times x_2^2 \quad (0 \le x_2 \le 7) \qquad (25)$$

The complete construction of the curve is presented in Table 7 using Equations 24 and 25. In Column 5 of Table 7, the changes in change in elevation for every station are presented. The values approximate −0.33 (r_1) and −1.33 (r_2) for the left and the right portions of the curve, respectively, because the rates of change of the slope for the left section and right section are constant, being equal to r_1 and r_2, respectively. Further, the change in change in elevation per station length is equal to the rate of change of slope.

In order to locate the turning point,

$$X_{TPL} = \frac{g_1L}{A} \cdot \frac{l_1}{l_2} = \frac{3 \times 10.5}{7} \times \frac{3.5}{7} = 2.25$$

(which is alright, because it is less than 3.5). Also,

$$X_{TPR} = \frac{g_1L}{A} \cdot \frac{l_2}{l_1} = \frac{4 \times 10.5}{7} \times \frac{7}{3.5} = 12$$

(which is too large, because it exceeds 7).

Thus, the turning point is located within the l_1 regime (left portion of the curve) at x_1 = 2.25, as its elevation is

$$Ex_1 = 100 + 3x_1 - {}^1\!/_2 \times 1.333x_1^2$$

$$= 100 + 3 \times 2.25 - {}^1\!/_2 \times 1.333 \times (2.25)^2 = 103.37 \text{ ft}$$

CONCLUSION

Changes in design parameters of symmetrical-crest vertical curves have been reviewed in an effort to incorporate changes in vehicular design, and a procedure for computing length requirements of unsymmetrical-crest curves was presented for which no design guidelines are currently available. The following conclusions were obtained.

Symmetrical Curves

The 1984 AASHTO manual (*2*) recommends the use of slightly longer SSD values than those in the 1965 AASHO manual (*1*). These longer lengths are the results of changes in the assumed value of pavement friction and the use of a range of speed. During the last 30 years, there has been a gradual reduction in the height of passenger cars with a smaller reduction in vehicular eye heights. The new AASHTO procedure incorporating these changes results in longer vertical curves. The required length of vertical curves is much more sensitive to object height than to eye height. However, the original object height of 6 in. has been retained in the 1984 AASHTO manual (*2*).

Unsymmetrical Curves

Parameter γ introduced as an indicator of nonsymmetry can be incorporated into the computation of lengths of unsymmetrical curves. For values of γ less than unity, the procedure presented results in longer curve lengths than those used for symmetrical curves, with the maximum length occurring at $\gamma_{critical}$ = 0.38. For values of γ exceeding unity, the procedure results in shorter curve lengths. However, caution is recommended to the highway engineer in the use of the parameter exceeding unity. For two-way travel, because the direction of eye and object are interchangeable, the use of the smaller of the two values of γ (l_1/l_2 and l_2/l_1) is recommended for computing the length of unsymmetrical curves.

TABLE 7 CONSTRUCTION OF UNSYMMETRICAL VERTICAL CURVE GIVEN g_1 = 3 PERCENT, g_2 = −4 PERCENT, l_1 = 350 ft, l_2 = 700 ft, AND γ = 0.5

Station (1)	X1 or X2 (Station) (2)	Elevation on Curve Ex1 or Ex2 ft (3)	Change in Elevation ft/Station (4)	Change in Change in Elevation ft/Station^2 (5)
50+00	0 (VPC)	100		
50+50	0.50	101.33		
51+50	1.50	102.99	1.66	
52+50	2.50	103.33	-0.34	-1.32 (r_1)
53+50	3.50 (VPI)	102.33	-1.00	-1.34 (r_1)
53+50	7(VPI)	102.33		
54+50	6	100.49	-1.84	
55+50	5	98.33	-2.16	-0.32 (r_2)
56+50	4	95.83	-2.50	-0.34 (r_2)
57+50	3	92.99	-2.84	-0.35 (r_2)
58+50	2	89.83	-3.16	-0.32 (r_2)
59+50	1	86.33	-3.50	-0.34 (r_2)
60+50	0 (VPT)	82.50	-3.83	-0.33 (r_2)

Thus, the procedure suggested will always result in longer curve lengths for unsymmetrical curves than those that are currently used for symmetrical curves. Further research is needed before specific design guidelines for unsymmetrical curves can be formalized.

REFERENCES

1. *A Policy on Geometric Design of Rural Highways*. AASHO, Washington, D.C., 1965.
2. *A Policy on Geometric Design of Highways and Streets*. AASHTO, Washington, D.C., 1984.
3. S. Khasnabis, T. R. Reddy, and S. P. Mani. The Impact of Small Cars and other Changing Factors on Stopping Sight Distance Requirements. In *ITE Annual Meeting Compendium*, Chicago, 1982, pp. 212–218.
4. *NCHRP Report 270: Parameters Affecting Stopping Sight Distance*. TRB, National Research Council, Washington, D.C., 1984.
5. G. Johansson and K. Rumar. Drivers' Brake Reaction Times, *Human Factors*, Vol. 13, No. 1, Feb. 1971, pp. 23–27.
6. G. T. Taoka. Statistical Evaluation of Brake Reaction Time. Presented at the Annual Meeting of the ITE, Chicago, Aug. 1982.
7. D. L. Woods. Toward a More Rational Approach to Highway Vertical Curve Design. Presented at 68th Annual Meeting of the Transportation Research Board, Washington, D.C., Jan. 1988.
8. S. Khasnabis and T. R. Reddy. A Reevaluation of Crest Vertical Curve Length Requirements. *Transportation Quarterly*, Vol. XXXVII, No. 4, Oct. 1983, pp. 567–582.
9. E. Farber. Driver Eye Height Trends and Sight Distance on Vertical Curves. In *Transportation Research Record 855*, TRB, National Research Council, Washington, D.C., 1982, pp. 27–33.
10. T. F. Hickerson. *Route Location and Design*. McGraw-Hill, New York, 1964.
11. J. C. Glennon. Effect of Sight Distance on Highway Safety. *State-of-the-Art Report 6, On Relationship Between Safety and Key Highway Features*, TRB, National Research Council, Washington, D.C., 1987, pp. 64–73.
12. S. Khasnabis and R. R. Tadi. Impact of Changes in Vehicular Design on Highway Safety. *Proc., National Conference on Transportation System Studies*, Indian Institute of Technology, Bombay, Dec. 1988, pp. 122–128.

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APPENDIX A

LENGTH OF AN UNSYMMETRICAL VERTICAL CURVE

Setting r_1 equal to rate of change of slope of the left portion of the curve from A to V and r_2 equal to rate of change of slope of the right portion of the curve from B to V, Hickerson (*9*) has shown that

$$r_1 = \frac{A}{L} \cdot \frac{l_2}{l_1} \text{ and } r_2 = \frac{A}{L} \cdot \frac{l_1}{l_2}$$

Because $l_1/l_2 = \gamma$, $r_1 = A/L\gamma$ and $r_2 = A\gamma/L$. [Horizontal distances are measured in units of stations (100 ft) and vertical distances in feet.]

By the parabolic law, offsets vary as the square of the distance. Hence, from Figure A1,

$$h_1 = {}^1\!/_2 r_1 d_1^2 \text{ and } h_2 = {}^1\!/_2 r_2 d_2^2$$

hence,

$$d_1^2 = \frac{2h_1}{r_1} \text{ and } d_2^2 = \frac{2h_2}{r_2}$$

Substituting the values of r_1 and r_2,

$$d_1^2 = \frac{2h_1 L\gamma}{A} \text{ and } \frac{2h_2 L}{A\gamma}$$

Therefore,

$$\begin{aligned} S &= d_1 + d_2 \\ &= \sqrt{\frac{2h_1 L\gamma}{A}} + \sqrt{\frac{2h_2 L}{A\gamma}} \\ &= \sqrt{\frac{2L}{A}} \times (\sqrt{h_1\gamma} + \sqrt{h_2/\gamma}) \end{aligned}$$

Squaring,

$$S^2 = \frac{2L}{A} \times (\sqrt{h_1\gamma} + \sqrt{h_2/\gamma})^2$$

Solving for L,

$$L = \frac{AS^2}{2(\sqrt{h_1\gamma} + \sqrt{h_2/\gamma})^2}$$

When both horizontal and vertical distances are measured in feet, this expression can be rewritten as

$$L = \frac{AS^2}{200(\sqrt{h_1\gamma} + \sqrt{h_2/\gamma})^2}$$

To compute the length l_1 and l_2 separately, refer again to Figure A1, from which it can be shown (*9*) that

$$r_1 = \frac{A}{L} \cdot \frac{l_2}{l_1} \text{ and } r_2 = \frac{A}{L} \cdot \frac{l_1}{l_2}$$

$$e = {}^1\!/_2 r_1 l_1^2 = {}^1\!/_2 r_2 l_2^2 = \frac{A}{2L} \cdot l_1 l_2$$

[Horizontal distances are measured in units of stations (100 ft) and vertical distances are measured in feet.] Because offsets vary as the square of the distance,

$$h_1 = {}^1\!/_2 r_1 d_1^2 \text{ and } h_2 = {}^1\!/_2 r_2 d_2^2$$

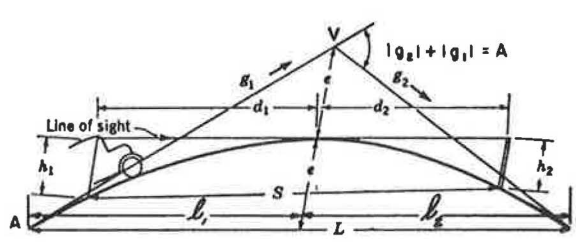

FIGURE A1 Sight distance over vertical curve when S < L (*10*).

or

$$\left(\frac{h_1}{e}\right) = \left(\frac{d_1}{l_1}\right)^2 \text{ and } \left(\frac{h_2}{e}\right) = \left(\frac{d_2}{l_2}\right)^2.$$

or

$$d_1 = l_1\sqrt{\frac{h_1}{e}} \text{ and } d_2 = l_2\sqrt{\frac{h_2}{e}}$$

Now,

$$S = d_1 + d_2 = \frac{1}{\sqrt{e}}(l_1\sqrt{h_1} + l_2\sqrt{h_2})$$

Because

$$L = l_1 + l_2 = l_2(1 + \gamma)$$

it follows that

$$e = \frac{A}{2L}\cdot l_1 l_2 = \frac{A\gamma l_2^2}{2l_2(1 + \gamma)} = \frac{A\gamma l_2}{2(1 + \gamma)}$$

Thus,

$$S = \sqrt{\frac{2(1 + \gamma)}{A\gamma l_2}}\cdot(\gamma l_2\sqrt{h_1} + l_2\sqrt{h_2})$$

$$= \sqrt{\frac{2l_2(1 + \gamma)}{A\gamma}}\cdot(\gamma\sqrt{h_1} + \sqrt{h_2})$$

or

$$S^2 = \frac{2l_2(1 + \gamma)}{A\gamma}\cdot(\gamma\sqrt{h_1} + \sqrt{h_2})^2$$

$$l_2 = \frac{AS^2\gamma}{2(1 + \gamma)(\gamma\sqrt{h_1} + \sqrt{h_2})^2}$$

When both horizontal and vertical distances are measured in feet, this expression can be rewritten as

$$l_2 = \frac{AS^2\gamma}{200(1 + \gamma)(\gamma\sqrt{h_1} + \sqrt{h_2})^2}$$

and by definition,

$$l_1 = \gamma l_2$$

because

$$L = l_1 + l_2$$

APPENDIX B

TURNING POINT OF AN UNSYMMETRICAL CURVE

It can be shown from Figures B1 and B2 that

$$Ex_1 = E_A + g_1x_1 + {}^1\!/_2 r_1x_1^2$$

when measured from the left

$$Ex_2 = E_B + g_2x_2 + {}^1\!/_2 r_2x_2^1$$

when measured from the right where Ex_i = elevation at point x_i on the curve, with $0 < x_1 \leq l_1$ (from the left) and $0 < x_2 \leq l_2$ (from the right).

To locate the turning point, set

$$\frac{dEx_i}{dx_i} = 0$$

to yield $g_1 + \gamma_1x_1 = 0$ and $g_2 + \gamma_2x_2 = 0$. At the turning point, $x_1 = X_{TPL}$ and $x_2 = X_{TPR}$, where X_{TPL} = location of turning point from the left and X_{TPR} = location of turning point from the right.

Thus,

$$X_{TPL} = \frac{g_1}{r_1}$$

$$= \frac{g_1L}{A}\cdot\frac{l_1}{l_2}$$

and

$$X_{TPR} = \frac{g_2}{r_2}$$

$$= \frac{g_2L}{A}\cdot\frac{l_2}{l_1}$$

(because $r_1 = Al_2/Ll_1$ and $r_2 = Al_1/Ll_2$).

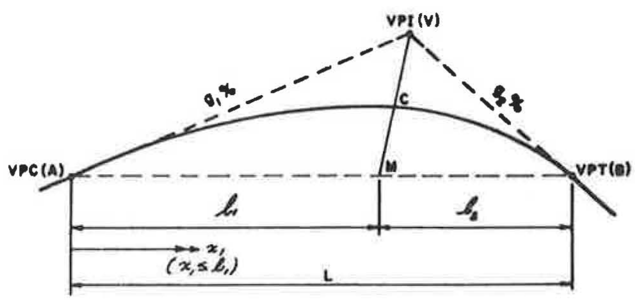

FIGURE B1 Unsymmetrical vertical curve with distance measured from the left.

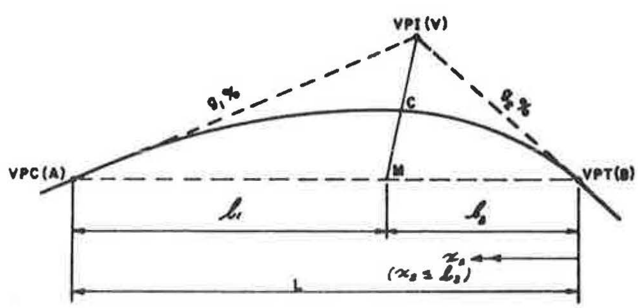

FIGURE B2 Unsymmetrical vertical curve with distance measured from the right.