Non-Euclidean Metrics in Nonmotorized Transportation

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An investigation into the use of simple, real-world, non-Euclidean metrics for pedestrian and bikeway planning is described. The nature of non-Euclidean geometry is described and general applications are presented. Sensitivity of mode choice with respect to time is explained. The concept of geometric delay is discussed. In addition, principles of non-Euclidean metrics are applied to three areas: taxicab geometry, efficiency of alternative network designs, and plastic space. Results indicate that the application of non-Euclidean metrics is a useful tool in planning human-powered transportation facilities. Further areas of research that will enhance and broaden the use of non-Euclidean metrics are identified.

Engineers and planners are well acquainted with the Euclidean metric (or geometry) through their formal education. Although the Euclidean metric appears to provide a good method for measuring the natural world, non-Euclidean metrics can be a valid tool for understanding the artificial man-made world. In fact, non-Euclidean metrics can be more useful to traffic engineers and urban planners than are Euclidean metrics (*1*), because the man-made world consists of a variety of different geometric forms. The usual distance measure, which is generally assumed and taken for granted, is not really what human beings perceive when they traverse city blocks.

Use of a non-Euclidean metric for pedestrian and bikeway planning is investigated in a preliminary way. First, the nature of non-Euclidean metrics is described and compared with that of the Euclidean metric, including some applications. Second, sensitivity of mode choice with respect to time is explained. The validity of considering time as more realistic than distance is examined by considering the phenomenon of transportation gaps occurring in most transportation systems in advanced countries (*2*). Third, the concept of geometric delay in Euclidean and non-Euclidean systems is discussed. Finally, three typical applications of non-Euclidean metrics are described—taxicab geometry, efficiency of alternative geometric networks, and the concept of plastic space (*3*).

NON-EUCLIDEAN METRICS AND SPATIAL ARRANGEMENT

A metric space comprises a set of points and a positive, subadditive distance function that relates every pair of points. The Euclidean distance relation $d_E(i,j)$ is the most frequently used metric in engineering and urban planning. This distance is the length of the shortest possible path joining a pair of points (i,j) and only one such path can exist for each pair of points. In two dimensions, the Euclidean metric is given by the expression

$$d_E(i,j) = \left[\sum_{k=1}^{2} (x_{ik} - x_{jk})^2 \right]^{1/2} \tag{1}$$

When the points are characterized in m dimensions, Equation 1 becomes

$$d_E(i,j) = \left[\sum_{k=1}^{m} (x_{ik} - x_{jk})^2 \right]^{1/2} \tag{2}$$

When the points lie in a three-dimensional space, $m = 3$. This Euclidean space also has special significance for human-powered transportation because, if the third dimension is altitude, greater human effort may be expended in using it, for example, in going up grades.

The Euclidean metric is a special case ($p = 2$) of the Minkowski p metrics defined in m-dimensional space by the relation

$$d_E^p(i,j) = \left[\sum_{k=1}^{m} |x_{ik} - x_{jk}|^p \right]^{1/p} \tag{3}$$

where p is any positive integer.

For $p = 1$ and $m = 2$, the distance

$$d_T(i,j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| \tag{4}$$

(the sum of intervals along each axis) is of great interest in civil engineering (Figure 1). This two-dimensional metric is generally known as the taxicab, pedestrian, Manhattan, or city-block metric (*3*).

The taxicab metric (or pedestrian metric) is intuitively appealing to transportation engineers, urban planners, and infrastructure designers. Particularly important to professionals are questions connected with the optimum location of apartments, industrial complexes, phone booths, sidewalks, and other facilities. In fact, any problem connected with a grid pattern of streets can be solved in terms of the taxicab metric. Krause (*1*) has investigated several real-world problems with taxicab geometry.

Radial, circumferencial, triangular, hexagonal, octagonal, and n-directional street patterns also yield examples of specific metrics that are in use in the optimum location problem (*3*).

As shown in Figure 1, in two-dimensional Euclidean space the locus of all points of distance R from the origin O defines

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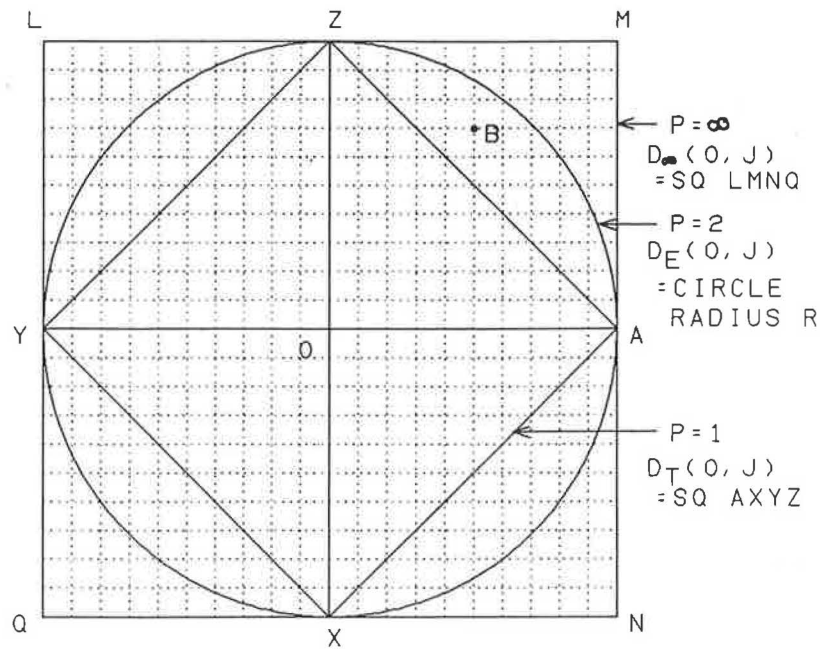

FIGURE 1 Various Minkowski metrics.

a circle of radius R. However, if the taxicab metric is adopted the locus of points equidistant from O describes square $AXYZ$ of diagonal of length $2R$. In the metric for $p = \infty$, another square $LMNQ$ whose sides are each $2R$ can be defined that represents the locus of all points equidistant from O.

Although in Figure 1 OA appears to be longer than OB, in the taxicab metric OB is longer than OA. In other words,

$$d_E(O,A) > d_E(O,B)$$

$$d_T(O,A) < d_T(O,B)$$

Also, given, for example, that $d_p(A,B) = 6$, $d_p(B,C) = 10$, and $d_p(A,C) = 8$, the configurations of these distances will appear as shown in Figure 2 for the taxicab (left) and Euclidean (right) metrics, respectively. The differences between the two parts of the figure illustrate the fact that in the taxicab metric a pedestrian does not take the shortest route but follows a grid street pattern in going from one point to another.

In general, the spatial arrangement or locational pattern of objects (such as man-made structures) may be used to determine a convenient metric for the space.

CONSTRAINTS IN HUMAN-POWERED TRANSPORTATION

If one critically observes the range of the transportation function, a hypothesis can be made that in practice three modes of transportation dominate the overall hierarchy of transportation available to people: walking for short distances, cars for medium distances, and airplanes for long distances. Transportation planners are well aware of the refusal distance of the average pedestrian, usually 400 m (or ¼ mi). Beyond 400 m, the majority of pedestrians demand some kind of a mechanical device or system to transport them. For instance,

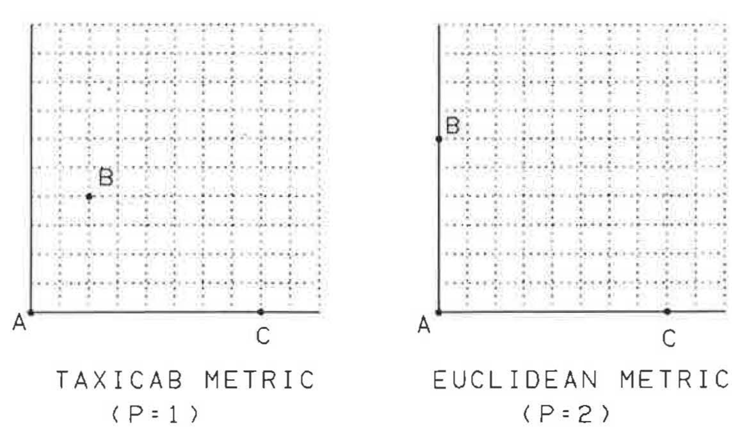

FIGURE 2 Taxicab and Euclidean metrics.

a pedestrian who needs to travel a distance of 4 km (2.5 mi) will not agree to spend 50 min walking. A faster means of transport will be sought. Ample evidence exists to show that the trip maker's choice of mode is not based on cost alone, but rather on travel time. Conceptually, distance is related to time (*2*), leading to the importance of delay (transportation delay or geometric delay), a topic discussed later.

Table 1 indicates that when the time of travel is doubled, distance covered increases tenfold, whereas speed of travel increases fivefold. This phenomenon, which was studied by Bouladon (*2*), generally produces the three dominant modes: walking, driving a car, and flying by plane. At the same time, the phenomenon produces pronounced transportation gaps as shown in Figure 3. Of course, these data are not uniform for all populations but depend on the aggregate level of economic development prevalent in the society under examination (*2*). In general, $t = 6.6d^{0.3}$ where t = time (min) and d = distance (km), or $t = 7.6d^{0.3}$ where t = time (min) and d = distance (mi).

On the one hand, the notion of transportation gaps can be easily dismissed on the grounds that evidently there is no market available for a particular mode in the transportation hierarchy that results in a gap. On the other hand, it may prove most advantageous to understand the real needs of the trip maker and the boundary conditions that society and the built environment have imposed on different modes in this hierarchy.

Bouladon (*2*) has demonstrated that the spectrum of transportation modes can be divided into roughly five areas, as shown in Figure 3. When demand for transport (vertical axis) is plotted against the speed, time, or distance measure (horizontal axis), the transportation range is well covered by pedestrians, cars, and air transport. Every mode is competent and efficient over a certain distance, time, and speed range, according to the technology and economics inherent in the design of the system under study. Because time is the sensitive variable, the challenge for the transportation engineer is to somehow reduce the time needed to walk or bicycle a discrete distance. Reductions in time can best be achieved through examining ways to improve pedestrian networks and decrease time delays for pedestrians and bicyclists. Note that among the various modes of transportation available, the pedestrian mode has the largest demand.

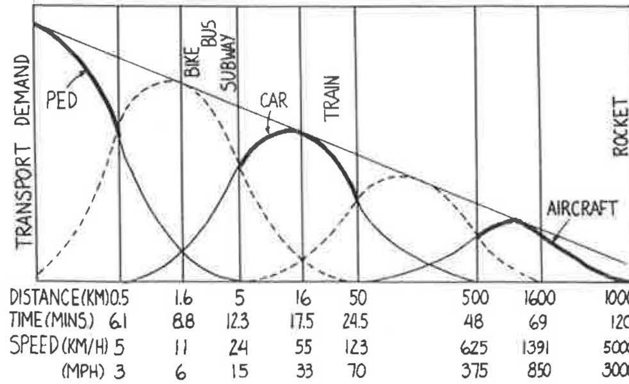

FIGURE 3 Transportation gaps.

TABLE 1 GENERAL TRANSPORT CONCEPT: DISTANCE, TIME, AND SPEED RELATIONSHIPS

Distance, *d* mi	Distance, *d* km	Time, *t* min	Theoretical transport speed mph	Theoretical transport speed km/h	Transport alternative
0.25	0.4	5	3	4.8	Walking
0.62	1	6.6	5.6	9.1	Bus or Bicycle
2.5	4	10	15	24	Subway/Light Rail
6.2	10	13.2	28	45.5	Car on City Street
25	40	20	70	120	Car on Freeway
62	100	26.4	141	228	Train
250	400	40.0	375	605	Small Plane
620	1000	52.5	706	1140	Jet

$t = 6.6d^{0.3}$ t = time in minutes and d = distance in km.

$t = 7.6d^{0.3}$ t = time in minutes and d = distance in miles.

Geometric Delay

The concept and measurement of delay for motorized transport is well established, particularly with the adoption of the 1985 *Highway Capacity Manual* (*4*) although Webster (*5*) used this concept as early as 1966 in the context of delay created by traffic signals. Geometric delay, similar to signal delay, is connected with the geometrics of the layout of transportation facilities, such as that experienced while negotiating rotary intersections. Several researchers have measured geometric delays for vehicles on highway and street networks. However, little investigation has been carried out to measure the magnitude of geometric delay for pedestrians and bicyclists using city streets (*6*).

Geometric delay in the context of Euclidean and non-Euclidean distance needs to be examined by investigating the time taken by a pedestrian to traverse a distance from, say, A to J in a typical grid street network shown in Figure 4 where the city blocks are L ft square and the streets are W ft curb to curb. Three cases are considered.

- Case 1. If there were no buildings or traffic to obstruct the pedestrian, the shortest path between A (origin) and J (destination) would be the straight line connecting A and J. If the blocks were 400 by 400 ft, and the streets 40 ft wide, the Euclidean distance d_E would be

$$d_E = [(2L + W)^2 + (3L + 3W)^2]^{1/2} = 1{,}565 \text{ ft}$$

Assuming walking speed = 4 ft/sec, Euclidean walking time t_E = 1,565/4 = 391 sec.

- Case 2. If the walking domain of a pedestrian consists of all the sidewalks adjacent to the buildings and also, if there is no vehicular traffic, street furniture, guardrails, or traffic control devices to obstruct movement, then the pedestrian can follow any path within the walking domain to reach the destination, to minimize walking distance d_G and, naturally, walking time. This concept is referred to as the geometrical minimum walking time (t_G). Referring to Figure 4,

$$\begin{aligned} d_G &= AB + BC + CE + EG + GI + IJ \\ &= 40 + 400 + 402 + 402 + 402 + 400 \\ &= 2{,}046 \text{ ft} \end{aligned}$$

and assuming a walking speed of 4 ft/sec,

$$t_G = 2{,}046/4 = 512 \text{ sec}$$

- Case 3. However, if a pedestrian's freedom of movement is constrained by motor vehicles, street furniture, guardrails, and traffic control devices, the pedestrian would have to conform to traffic laws and follow the taxicab or non-Euclidean path as shown in Figure 4.

$$\begin{aligned} d_T &= AB + BC + CD + DE + EF + FG + GH + HI + IJ \\ &= 4W + 5L \\ &= 2{,}160 \text{ ft} \end{aligned}$$

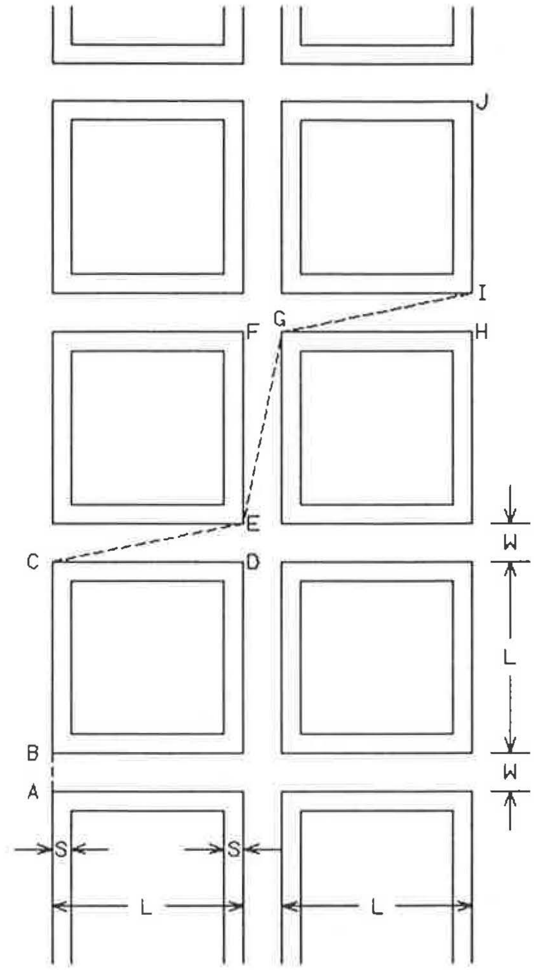

FIGURE 4 Geometric delay for pedestrians.

and

$$t_T = 2{,}160/4 = 540 \text{ sec}$$

In addition, this pedestrian would have to wait at four street crossings for the Walk signal of, say, 30 sec (4 × 30 = 120 sec). Thus, although d_T = 2,160, t_T = 540 + 120 = 660 sec, which could be considered as the practical minimum walking time.

Thus, geometric delay is the practical minimum walking time (using legal paths) minus the geometrical minimum walking time previously defined. Summarizing the results of the three typical cases, t_E = 391 sec; t_G = 512 sec; and t_T = 660 sec, in which t_G and t_T are 31 and 69 percent higher than t_E, respectively, and t_T is 29 percent higher than t_G. These calculations amply demonstrate the fact that geometric delay represents a significant component of the pedestrian's travel time.

Naturally, these values will vary depending on the relative location of origins and destinations on the street network, size of blocks and streets, and pedestrian signal cycle lengths.

Therefore, a reasonable expectation would be for transportation engineers and urban planners to minimize pedestrian and bicyclist geometric delay from all known sources, or at least to be cognizant of geometric delay.

APPLICATIONS

Taxicab Geometry

Of the many applications of non-Euclidean metrics, the one that has been investigated most is taxicab geometry, which has special significance in city planning. The only assumption in taxicab metrics is that the street system be in the form of a square or rectangular grid.

In many cities around the world, particularly in China, India, and North America, the spatial pattern of streets is usually a grid and the use of taxicab geometry is applicable. Spatial separation is more realistic using the taxicab metric $d_T(i,j)$ rather than the Euclidean metric $d_E(i,j)$, and the following elementary applications of taxicab metric are useful:

1. Trade, catchment, or market areas can be demarcated on maps by replacing $d_E(i,j)$ by $d_T(i,j)$. Several examples of this application have been used in the past. For example, if two roommates having jobs at locations *A* and *B* decide to find an apartment such that the sum of the distance they have to walk to work is no more than 18 blocks, they could demarcate a taxicab ellipse as shown in Figure 5 and seek an apartment within its confines.
2. Facility location using the taxicab metric is also applicable. Figure 6 shows an example of facility *P* whose location minimizes the distance $d_T(P, A) + d_T(P, B) + d_T(P, C) + d_T(P, D)$. All points within the crosshatched area make this condition possible.
3. The combination of walking distances and ideal location of mass transit (or subway stations) is another interesting area of investigation. Here, of course, the possibility could exist that the subway system is not running parallel to the street system. Bus systems can also be investigated in a similar way.

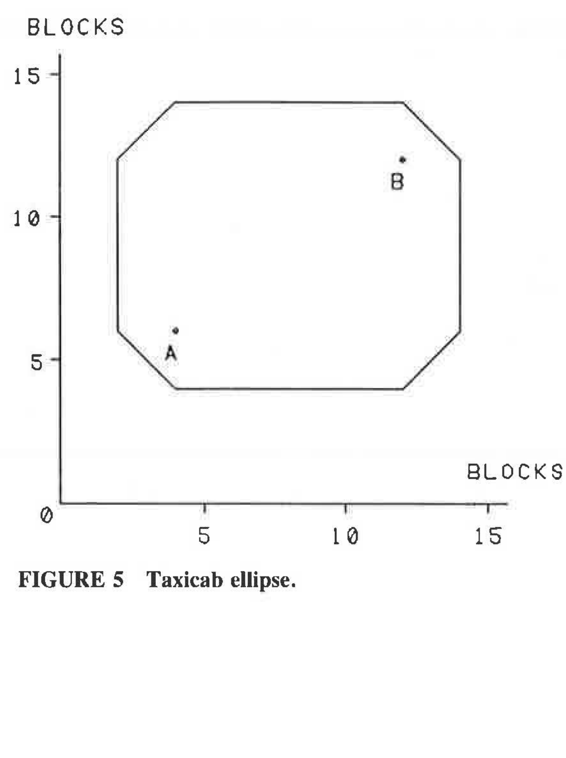

FIGURE 5 Taxicab ellipse.

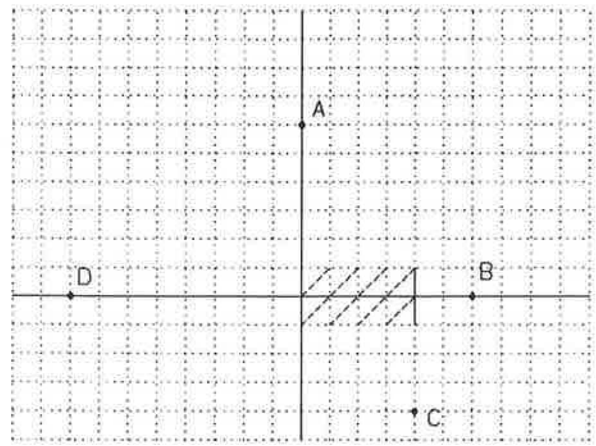

FIGURE 6 Taxicab metric.

EFFICIENCY OF ALTERNATIVE NETWORK DESIGNS

As previously discussed, a close parallel exists between the geometrics of pedestrian movement on sidewalks and motor vehicles on streets. Certain forms of delay incurred by pedestrians are also similar to vehicular delays (*6*). In these comparisons, time-distance and cost-distance immediately come to mind. For pedestrians and bicyclists, the question of minimizing time-distance is paramount.

In street systems across the world, alternative plans range from radial, circumferencial, or grid patterns for traditional cities with high-density centers, to highly complex combinations for the modern expansions of existing cities. Naturally, different geometric patterns of streets are associated with different patterns of relative accessibility.

Holroyd and others (*7,8*) have examined several different street systems as shown in Figure 7. These systems (consisting of circular cities with a 1-mi radius) make use of internal or external ring roads whereas others rely on radial, rectangular, and other polygonal networks. Efficiency of these networks is evaluated for internal movement when both origin and destination are inside the city. Average walking or biking distance between random pairs of points (origins and destinations) is presented in Table 2. Also, Figure 8 shows network street length or cost versus average trip length. If the entire city were paved (hypothetically) and a pedestrian were able to move in Euclidean space with no obstructions, then trip length would be 0.905 mi for a city with a radius of 1 mi. This (direct) case cannot be represented by a point on Figure 8. Random trip origin and destination point pairs are the same in all eight cases.

A broad conclusion that can be deduced from this experiment (Figure 8) is that the best results are obtained by adopting a street system with a radial, rectangular (grid), or internal ring configuration, because such systems minimize the average trip length as well as the total length of the street network.

The search for efficient geometrical designs for transportation networks is long standing. Elevated pedestrian bridges and underground pedestrian tunnels (underpasses) together with skywalks, such as those provided at Cincinnati, Spokane,

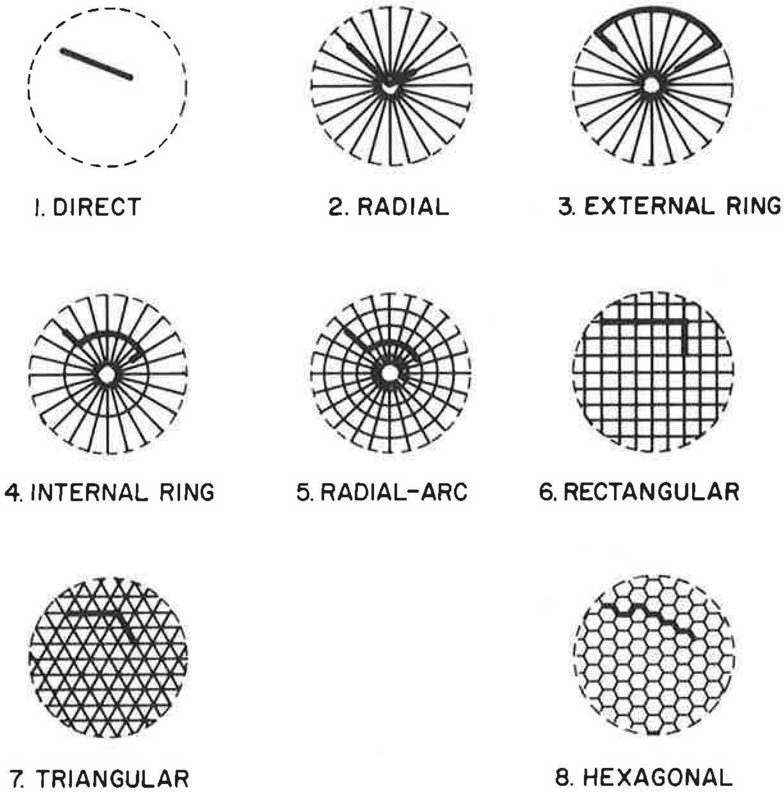

FIGURE 7 Alternative routing systems.

TABLE 2 AVERAGE LENGTH OF TRIPS IN A CIRCULAR CITY

Configuration	Trip Length	Trip Length $a = 1$ mile	Street Length (miles)
1. Direct	$0.905a$	0.905	∞
2. Radial	$1.333a$	1.333	24
3. Ext. Ring	$2.237a$	2.237	30.28
4. Int. Ring	$\frac{4a}{3} - \frac{(4-\pi)b}{2} + \frac{4b^2}{3a^2}$	1.445	27.14
5. Radial-arc	$1.104a$	1.104	42.84
6. Rectangular	$1.153a$	1.153	28
7. Triangular	$0.998a$	0.998	43
8. Hexagonal	$1.153a$	1.153	45

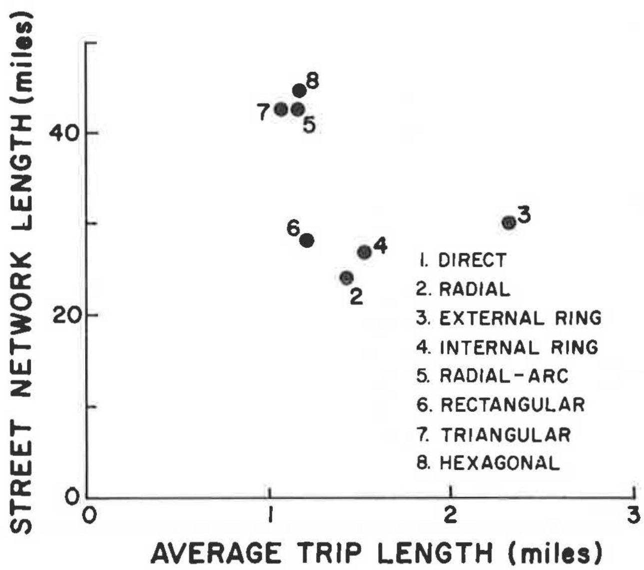

FIGURE 8 Average trip length versus street length.

Calgary, and Minneapolis, serve to reduce walking distance and reduce accidents.

CONCEPT AND USE OF PLASTIC SPACE

Several researchers in recent years, particularly Forer (*9*) and Ewing (*10*), have examined the relationships of time and distance in defining the use of plastic space. For pedestrians and bike users, the consideration of time is more realistic in space than just mileage.

Correspondence between time space and geographical space is examined for the Washington State University (WSU) campus. A matrix of travel times to 25 crucial points on the campus map of WSU was recorded by using the sidewalks and legal pedestrian crossings by the shortest path. A similar matrix of these same 25 points was recorded by driving on the campus, also by the shortest paths along the campus streets. Figures 9–11 represent the geographical space, pedestrian walking time space, and vehicular driving space of WSU campus, respectively. Transformations of space-stretching and space-

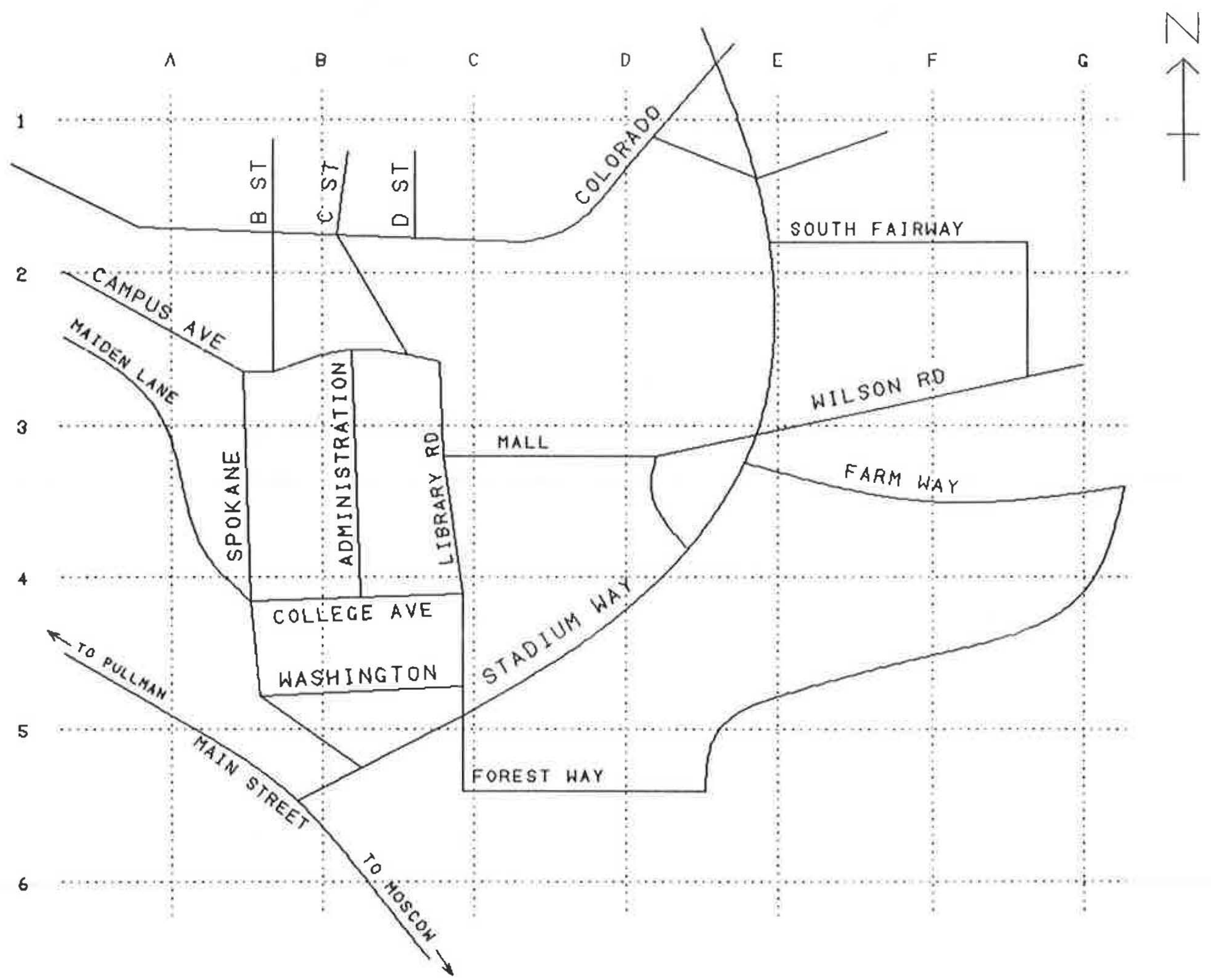

FIGURE 9 WSU campus in geographical space (1 in. = 1,000 ft).

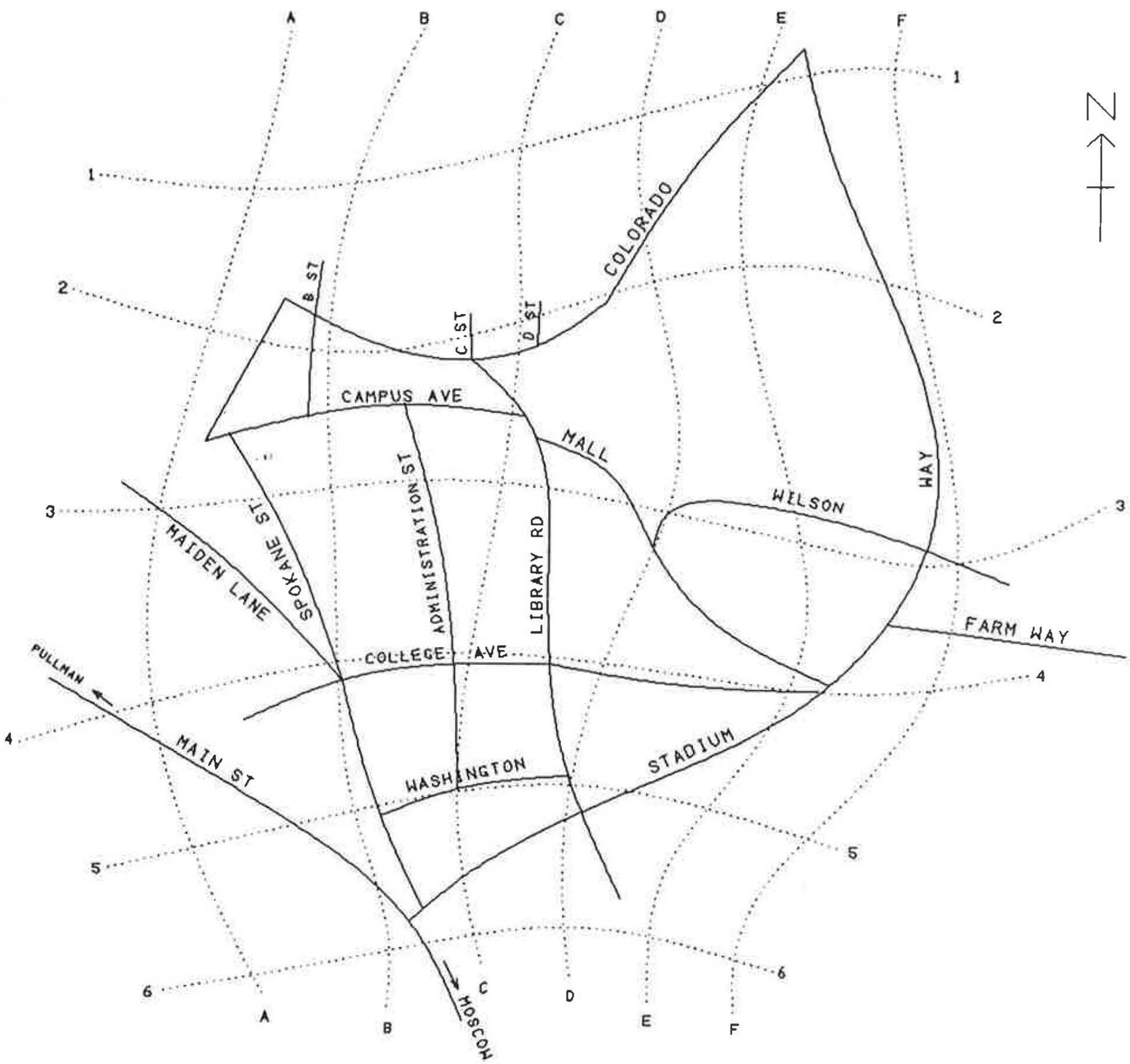

FIGURE 10 WSU campus in pedestrian time space (0.5 in. = 1 min).

shrinking because of the use of different modes is evident by comparing these three illustrations. The WSU campus is particularly efficient for the pedestrian mode as compared with the vehicular mode and because of the provision of several automobile-free zones.

The concept of plastic space has important applications in exploring the efficient forms of spatial reorganization. Plastic space also has the potential for use in policy in terms of decision making regarding future investments. For example, additions of streets and sidewalks to an existing network could be configured by comparing Euclidean and non-Euclidean metrics applied to a street system to find the impact of such an addition. Transportation planners may find this approach of use in studying the potential impact of technical and economic proposals. Weir (*11*) has applied this method at State College, Pennsylvania, with success.

Plastic space would be particularly useful in parts of the city where pedestrian movement is predominant for several good reasons, one of which may be to reduce pedestrian delay and possibly also to increase safety. Provision of suitable footbridges over streets, skywalks, and automobile-restricted zones may be considered. Designing streets and pedestrian walkways more sensitively is really what is called for.

CONCLUSION AND RECOMMENDATIONS

Three aspects of human-powered transportation have been described—general nature of non-Euclidean metrics, sensitivity of the pedestrian and bicycle mode to travel time, and basic concepts of geometric delay. Some practical examples

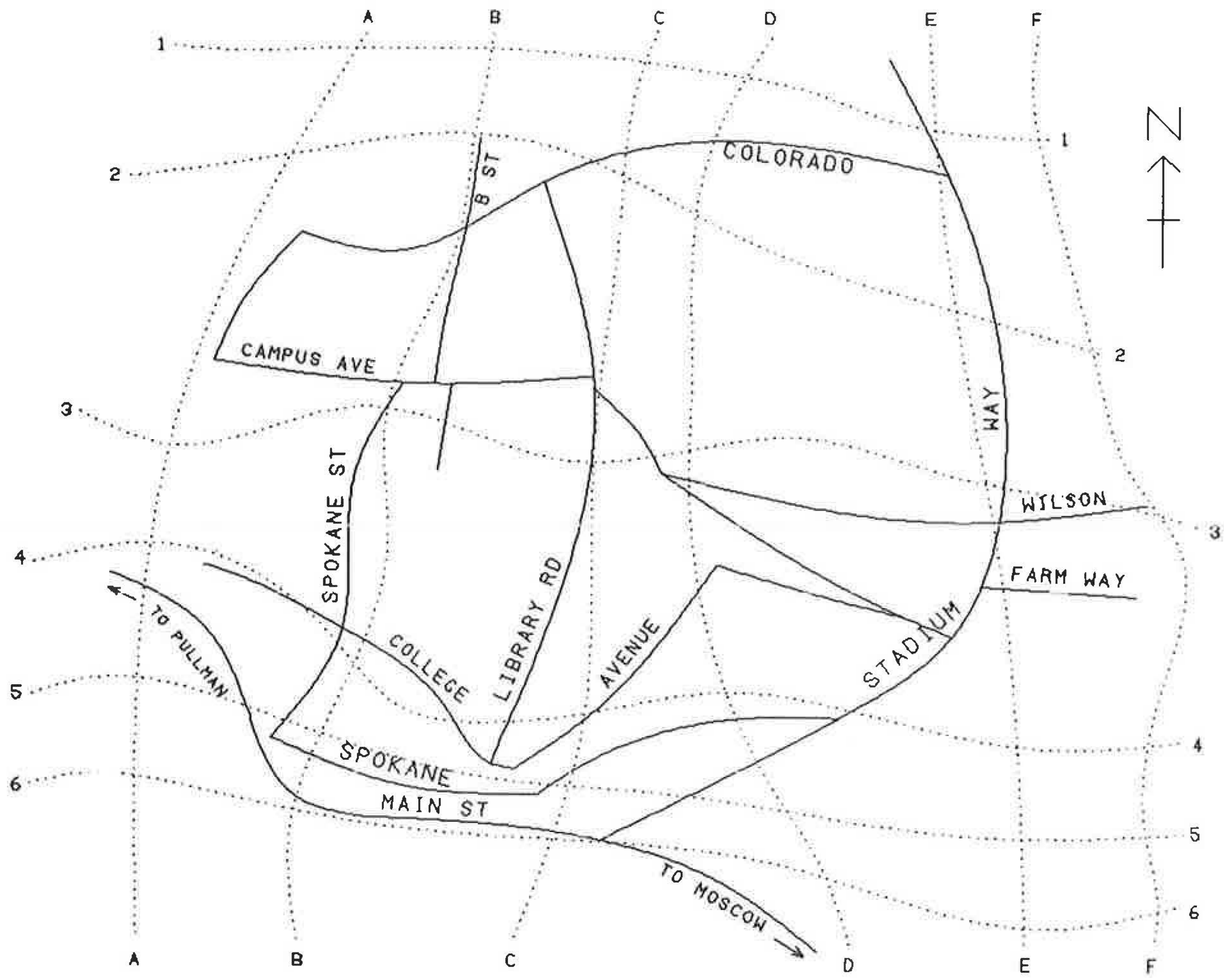

FIGURE 11 WSU campus in vehicular time space (1.25 in. = 1 min).

of applying these and other concepts to the human-powered mode have also been described.

Knowing the principles and laws governing human-powered transportation is not enough. One must be capable of applying these principles in planning for pedestrians and bicycle users in a world competing vigorously for time and space. From this standpoint, the ideas expressed are just the beginning of an ongoing research.

Areas of research stemming from this investigation include the following:

1. A study of geometric delay for pedestrians and bicycle users relative to different geometric networks and ways in which such delay can be minimized (in the same way that traffic engineers minimize delay for motor vehicles) needs to be undertaken;
2. A study of plastic space for designing and evaluating plans for pedestrian and bicycle users movement networks (facilities) particularly for minimizing walking and bicycle-riding distances appears to be essential where nonmotorized transportation is significant;
3. The question of improving pedestrian and bikeway safety relative to the motor vehicle, through a thorough examination of street geometry, traffic control devices, skywalks, and underground passageways, should form part of any street expansion or alteration system; and
4. The science of facilities location for different geometric network configurations, particularly for intermodal efficiency, for example, applied to pedestrian and subway systems would prove beneficial.

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