Subarea Focusing with Combined Models of Spatial Interaction and Equilibrium Assignment

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Subarea focusing is a means of reducing computational requirements of large transportation networks when only a small portion of the region is affected by a project. Relatively unimportant links are eliminated from the network and distant zones are aggregated, whereas full detail is retained in the area of greatest impact. Errors in forecasts that can occur through reductions in numbers of zones—the principal means of saving computer time and memory—are examined. On the basis of results from the highway network for Wausau, Wisconsin, general procedures and relationships are developed for determining the amount of error that can occur from subarea focusing.

The recent development of software for the analysis of transportation impacts in small areas (i.e., SubArea Focusing and Quick Response System II) has raised some important methodological issues. First, it is not known whether subarea focusing is a valid concept for a meaningfully broad class of transportation problems. Second, if the concept is valid, there is little information about when and how it can be applied. And third, subarea focusing needs further testing with the newest generation of travel forecasting models—those that join a spatial interaction model to a traffic assignment model to find a combined equilibrium solution.

Subarea focusing is primarily a way to reduce the data preparation costs and computation requirements of travel forecasting with large transportation networks. It applies to transportation projects that affect only a small portion of the region. The network is redrawn so that the area of primary impact is shown in considerable detail (small zones and many links) and distant sections of the network are shown in much less detail (large zones and only the most important links). If a comprehensive network is already available for a region, computer programs, such as the SubArea Focusing package (*1*), can be used to automate the process. Otherwise, the planner must exercise judgment in selecting zone sizes and link densities.

The key assumption of subarea focusing is that forecast errors will be small if their sources are spatially distant from the area of primary impact. For example, a study of single-route transit ridership (*2*) indicated that zones on connecting routes could be quite coarse without affecting the forecast. This result stemmed from peculiarities of the structure of transit networks and may not apply to other types of travel.

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If it can be applied properly, subarea focusing has important advantages. It can reduce (perhaps only slightly) data preparation time and it can considerably reduce computation requirements. Large networks could fit into smaller computers and most of the computation time could be eliminated.

Two rules of thumb apply to computation time. First, it is approximately proportional to the number of links in the network; second, it is approximately proportional to the square of the number of zones. Both rules have important exceptions; for instance, the spatial interaction model described later has computation times that are nearly proportional to the cube of the number of zones. If computation time were the only issue, then planners should be much more interested in eliminating zones than links.

In deciding whether subarea focusing is beneficial, the planner must keep in mind that any modification to an unfocused network will cause an error in the forecast—the bold assumption is that the unfocused network approximates truth. Of course, forecasts with unfocused networks are themselves rampant with errors (*3,4*). A 30 percent average root-mean-square (RMS) error in highway link volumes is not unusual. If the errors in highway link volumes caused by subarea focusing are small relative to the errors already inherent in the forecast, then subarea focusing makes sense. If subarea focusing noticeably distorts the forecast, then it should be avoided.

As will be shown later, determining the error caused by subarea focusing is a major undertaking. If planners were required to ascertain the error before performing their forecast, any value of subarea focusing would be lost. A more efficient strategy would be to first develop general relationships that could be used to estimate errors due to subarea focusing. Ideally, these relationships should be specific to the particular unfocused network. A second solution would be to adopt relationships developed for other networks, such as the relationships derived in this paper.

In order to better understand errors from subarea focusing, simulations were run on a network from Wausau, Wisconsin. The simulations were performed using the Highway Land Use Forecasting Model II (HLFM II). HLFM II simultaneously produces both a land use forecast and a traffic forecast. HLFM II was selected over traditional travel forecasting models (e.g., UTPS or QRS II) because its spatial interaction step is more sensitive to variations in zone size and shape. Errors measured from HLFM II should be larger than errors from models that only provide a traffic forecast.

OVERVIEW OF THE SIMULATION STEPS

The spatial interaction step in HLFM II is a version of the Lowry-Garin model (*5,6*) of land use. The Lowry-Garin model remains one of the most popular methods of land use forecasting because of its consistency with economic base theory, its similarity to traditional theories of travel demand, and its straightforward method of solution. The Lowry-Garin model states that workers will locate their residences proximate to their workplaces and that services will locate proximate to their markets. Two service categories are defined: services for residences and services for businesses. The important exogenous variables in the model are the zonal locations of basic employment (e.g., factory workers), the travel times between all pairs of zones in the region, and measures of the ability of any zone to attract population or service employment. HLFM II's specific implementation of the Lowry-Garin model is briefly described in the Appendix.

HLFM II also forecasts traffic by estimating the numbers of trips between each pair of zones and assigning the trips to the network. HLFM II implements several forms of traffic assignment, some of which can produce a combined equilibrium solution. A network is considered to be in equilibrium when (*a*) numbers of trips between pairs of zones are consistent with travel times between those zones, (*b*) link volumes are consistent with link travel times, and (*c*) all trips are assigned to a shortest path between the origin zone and the destination zone. The method of traffic assignment adopted for this study is a hybrid of the Evans algorithm of elastic-demand traffic assignment (*7*) and a form of incremental assignment that has been derived from Frank-Wolfe decomposition (*8–10*). Tests of this algorithm on several networks have shown that it consistently converges to the equilibrium solution in about the same time as Frank-Wolfe decomposition (*11,12*). This algorithm is referred to as elastic-demand incremental assignment.

It has been shown (*11*) that the error in link volumes from insufficient iterations with the Frank-Wolfe decomposition family of algorithms varies approximately with the reciprocal of the number of iterations. In an elastic-demand assignment each iteration consists of a complete pass through the spatial interaction step, an all-or-nothing assignment, and a step in which the model finds an average of traffic volumes from all previous iterations. Thus, a convergence error of less than 0.5 percent could be achieved with approximately 250 iterations. This number greatly exceeds common practice. However, the convergence error must be kept small if the effects of zone restructuring are to be accurately measured.

FOCUSING THE WAUSAU NETWORK

As shown in Figure 1, the Wausau network is already focused on the central business district (CBD). All streets within the CBD are represented, and zone sizes in the CBD are about 0.2 mi^2. The remaining zones become larger with increasing distance from the CBD; the largest zones range from 3 to 10 mi^2. The Wausau network contains just 36 zones and 9 external stations. The network simulates the p.m. peak hour.

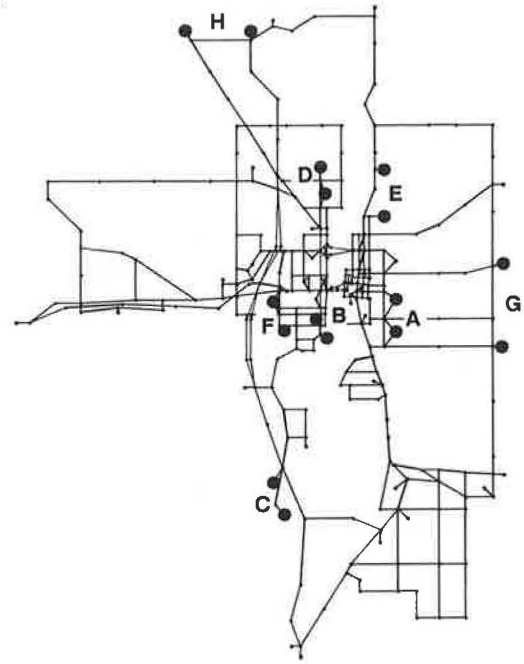

FIGURE 1 Base Wausau network showing the eight zone pairs.

Obtaining Base Networks

A complete understanding of errors due to zone restructuring requires both equilibrium and all-or-nothing assignments. Consequently, it was necessary to prepare a base network that would give results between the two assignment methods that were reasonably compatible. The base network was obtained by running the original Wausau network through 20 iterations of elastic-demand incremental assignment. The near-equilibrium link travel times from this simulation were used as the starting point for all remaining simulations.

It was expected that simulations involving elastic-demand incremental assignment would require 250 iterations to eliminate the problem of convergence error. This assumption was checked by comparing two separate assignments on the base network, one at 250 iterations and one at 1,000 iterations. Convergence error from the 1,000-iteration assignment can be considered insignificant. The RMS difference in link volumes between the two assignments was 0.6 vehicle/hr, or about 0.14 percent—better than expected.

Comparison Networks

Comparison networks were created to determine the sensitivity of the model to small amounts of zonal restructuring and to determine how errors increase as larger numbers of zones are eliminated. A total of 15 comparison networks were created; all were small variations on the base network. Each of the first eight networks combined a single pair of adjacent zones, thereby eliminating one zone from the total for the network. The pairs of zones are shown in Figure 1 and are labeled A through H. All were outside but at varying distances

TABLE 1 RMS ERRORS (VEHICLES PER HOUR) OF ELIMINATING A SINGLE ZONE FROM THE WAUSAU BASE NETWORK

Network	Elastic-Demand Incremental All Links	Elastic-Demand Incremental CBD Links	All-or-Nothing All Links	All-or-Nothing CBD Links
A	20.7	30.1	25.6	40.1
B	42.2	6.5	44.2	6.9
C	17.2	0.6	17.5	0.4
D	30.2	5.9	45.8	6.1
E	55.9	5.1	44.6	3.5
F	30.3	6.7	33.1	14.1
G	4.5	6.1	5.1	6.9
H	26.3	0.4	26.3	2.6

from the CBD. Two of the zone pairs, G and H, were sets of external stations.

The next four networks (identified here as Networks AB, CH, EG, and DF) were combinations of the previous eight. Each of these networks eliminated two zones from the base network. For example, Network AB collapsed both Zone Pair A and Zone Pair B.

Two networks (ABCH and EGDF) were combinations of the previous four. Both of these networks eliminated four zones. The last network was a combination of the previous two. It eliminated eight zones and is referred to as ABCHEGDF.

Determining the Error

The base network and all the comparison networks were each run twice—once through 250 iterations of elastic-demand incremental assignment and once through all-or-nothing assignment. The RMS errors in directional link volumes were computed for each comparison network. Only links that represented real streets were included; centroid connectors and other artificial network elements were ignored.

The RMS difference between the two base assignments (all-or-nothing and elastic-demand incremental) was 114 vehicles/hr, or about 26.5 percent. This difference was smaller than expected, perhaps because the base network had already been processed through 20 iterations of elastic-demand incremental assignment.

The RMS errors for the first eight comparison networks are shown in Table 1. The errors are reported for both assignment methods and for the set of all links and for the set of links within the CBD. As expected, there was a substantial difference between the RMS errors across all links and the RMS errors for only the CBD links. In general the RMS errors were less for the CBD links; the major exceptions were Zone Pair A (immediately adjacent to the CBD) and Zone Pair G (relatively small external stations).

An unexpected result, but a highly useful one, is the similarity between errors from the elastic-demand incremental assignment and errors from the all-or-nothing assignment. This similarity suggests that it is unnecessary to perform lengthy equilibrium assignments solely to determine the errors from subarea focusing. All-or-nothing assignments will suffice, provided that the network has been prepared with near-equilibrium link travel times.

Eliminating a single zone from a network introduces a disturbance to every link with positive volumes. Some links are affected more than others, particularly links that are near the eliminated zone. If two spatially separated zones are eliminated, it could be surmised that the two disturbances would behave as if they were independently distributed random variables. Because the nature of the disturbance is essentially unknown, this hypothesis must be verified empirically.

Table 2 shows the results from the remaining seven networks, each of which collapsed multiple zone pairs. Listed in Table 2 are the actual RMS errors and the estimated RMS errors, calculated by assuming that the disturbances from eliminating a single zone are independently distributed random variables. For example, the estimated error for Network AB was found from the errors measured in Network A and Network B; the estimated error for Network ABCH was found from the measured errors in Networks A, B, C, and H.

TABLE 2 RMS ERRORS OF ELIMINATING MULTIPLE ZONES FROM THE WAUSAU BASE NETWORK

Network	Elastic-Demand Incremental Measured	Elastic-Demand Incremental Expected	All-or-Nothing Measured	All-or-Nothing Expected
Across All Links				
AB	45.2	47.0	48.9	51.1
CH	31.4	31.4	31.9	31.6
EG	43.2	56.1	45.0	44.9
DF	42.4	42.8	56.2	56.5
ABCH	55.0	56.5	58.6	60.1
EGDF	60.6	70.6	72.0	72.2
ABCHEGDF	82.5	89.9	93.1	93.9
Across CBD Links				
AB	30.4	30.8	37.7	40.7
CH	0.6	0.7	0.6	2.6
EG	5.9	8.0	7.0	7.7
DF	8.1	8.9	15.2	15.4
ABCH	30.4	30.8	37.6	40.8
EGDF	10.2	12.0	19.5	17.2
ABCHEGDF	32.2	33.1	45.4	44.3

NOTE: Data are in vehicles per hour.

The estimated errors agree closely with the measured errors. This agreement holds for both assignment methods and for the set of all links and the set of CBD links. The ability to make the assumption of independence is important. It permits estimation of the effect of eliminating an arbitrary number of zones from knowledge of the effect of eliminating a single zone.

PATH DEPENDENCE

The elimination of a single zone affects a large number of paths. Each path will carry the disturbance, but the paths are not assigned as if they were independently distributed random variables. Links closest to the eliminated zone are assigned more paths than links farther away; some links (e.g., those with high capacity) are unusually attractive to paths. Thus, some links are consistently affected more than others—disproportionately influencing the RMS error. The probabilistic dependence of path assignment is important to the concept of subarea focusing.

Tables 3 and 4 illustrate the levels of path dependence in the Wausau base network with all-or-nothing assignment. Statistics are provided for six zones, one zone from Pairs A through F. (Zone Pairs G and H were external stations; HLFM II could not generate path dependence data for external stations.) For each zone, Table 3 shows the standard deviation of the number of paths that are assigned to any link. The zones were treated as being origins only, so there were 44 paths in each case. Any given link could be assigned as many as 44 paths or as few as 0 paths. This standard deviation is calculated separately for the set of all links and for the set of CBD links. Table 3 also indicates the mean number of paths that are assigned to each link.

The number of paths on a link would be binomially distributed if paths were randomly and independently assigned to the links (they are not). Take, for example, the assignment of paths from one of the zones in Pair A to CBD links. With a mean assignment of 1.901 paths, the probability that a link would be randomly and independently assigned any given path is 0.0432 (i.e., 1.901/44). Consequently, the standard deviation from the binomial distribution is 1.349, much smaller than the actual value of 6.016 from Table 3. The difference can be attributed to the dependence between paths.

A useful measure of this dependence is the covariance between paths. Assuming paths are identically (but not independently) distributed, the covariance can be found from the following relationship:

$$\sigma^2_{\text{measured}} = \sigma^2_{\text{binomial}} + m(m - 1)C \quad (1)$$

where

C = the covariance,
$\sigma^2_{\text{measured}}$ = the measured variance,
$\sigma^2_{\text{binomial}}$ = the variance from the binomial distribution, and
m = the number assigned paths.

TABLE 3 STANDARD DEVIATIONS AND MEANS OF NUMBERS OF PATHS ASSIGNED TO LINKS IN THE WAUSAU NETWORK FOR A SINGLE ORIGIN ZONE

	All Links		CBD Links	
Represented Zone Pair	St. Dev.	Mean	St. Dev.	Mean
A	3.232	0.858	6.016	1.901
B	1.863	0.608	1.492	0.656
C	3.119	0.750	1.107	0.590
D	2.506	0.682	1.398	0.508
E	3.041	0.860	1.641	0.721
F	1.945	0.636	1.988	0.984

TABLE 4 PROBABILITIES AND COVARIANCES OF PATHS ASSIGNED TO LINKS IN THE WAUSAU NETWORK FOR A SINGLE ORIGIN ZONE

	All Links		CBD Links	
Represented Zone Pair	Probability	Covar.	Probability	Covar.
A	0.0195	0.0051	0.0423	0.018
B	0.0135	0.0015	0.0149	0.00084
C	0.0170	0.0048	0.0134	0.00034
D	0.0155	0.0030	0.0115	0.00077
E	0.0195	0.0044	0.0164	0.0011
F	0.0145	0.0017	0.0224	0.0016
All	0.0169	0.0036	0.0203	0.0039
All Except A	--	--	0.0157	0.00091

Applying Equation 1 to the previous example yields a covariance of 0.018. The probability of a path being assigned to a link and the path covariances are shown in Table 4. The covariances vary greatly from zone to zone. For example, among CBD links the covariance for paths from Zone A is about an order of magnitude larger than the others—demonstrating that Zone A, which is quite close to the CBD, would be a poor choice for elimination.

As discussed in the next section, the path covariance is particularly helpful in estimating errors in new networks.

ESTIMATING ERRORS IN EXISTING NETWORKS

Every network has unique error characteristics. Ideally, the following procedure should be followed:

1. Obtain an unfocused network for the region.
2. Run this network through a sufficient number of iterations of the chosen assignment method to ascertain near-equilibrium link travel times.
3. Develop a base forecast by running an all-or-nothing assignment on the network with the near-equilibrium link travel times.
4. Choose a sample of zone pairs that are suitably distant from the impact area.
5. Collapse each zone pair to a single zone and run an all-or-nothing assignment.
6. Measure the RMS error on the links in the impact area by comparing their volumes with the base assignment.
7. Find an average RMS error for the sample of zones with

$$\varepsilon^S = \left(1/n \sum_{i=1}^{n} \varepsilon_i^2\right)^{1/2} \qquad (2)$$

where n is the number of samples.

8. Determine the effect of the subarea focusing with

$$\varepsilon^R = q^{1/2}\, \varepsilon^S \qquad (3)$$

where q is the number of zones to be eliminated.

9. Eliminate the desired number of zones and eliminate links that had zero assigned volumes in Step 2.

Experience with the Wausau network suggests that the error estimates obtained from this procedure should be good. Although time consuming, this procedure could be simultaneously performed for many potential subareas, thereby satisfying all future subarea focusing needs.

EXTRAPOLATING ERRORS TO OTHER NETWORKS

The procedure in the previous section requires an unfocused network of reasonable quality, but focused networks are often developed from scratch. In such cases, it is impossible to determine an average error before the network is built. The only option is to extrapolate errors from networks developed elsewhere.

Some simple relationships can be derived to help anticipate the amount of error caused by subarea focusing in a nonexistent network. To do this, the following assumptions can be made.

1. Zones are identical.
2. The elimination of any zone causes the same disturbance to the subset of links.
3. The magnitude of that disturbance is proportional to the number of trips in a single path.
4. For any path, the assignment of paths to a given link is a random Bernoulli process.
5. The number of links in any path for a given network is a constant.
6. The path covariance is a constant.

Each of these assumptions could be made more realistic, but it is unlikely that extremely precise relations would be beneficial given the limited knowledge of subarea focusing errors.

The number of assigned paths from any zone, m, is one less than the number of zones, z. From Equation 1

$$\sigma^2_{\text{zone}} = (z - 1)p(1 - p) + (z - 1)(z - 2)C \qquad (4)$$

where

σ^2_{zone} = the variance of paths from a single zone assigned to a link,
p = the probability that a path is assigned to a link, and
C = the path covariance.

The error from a single zone reduction would be

$$\varepsilon^S = k\, T/z^2\, \sigma_{\text{zone}} \qquad (5)$$

where T is the total number of vehicle trips in the network and k is an empirical constant. Substituting Equation 4 into Equation 5 produces

$$\varepsilon^S = k\, T/z^2\, [(z - 1)p(1 - p) + (z - 1)(z - 2)C]^{1/2} \qquad (6)$$

The effect of a reduction in q zones is

$$\varepsilon^R = k\, T/z^2\, \{q[(z - 1)p(1 - p) + (z - 1)(z - 2)C]\}^{1/2} \qquad (7)$$

The value of k can be determined from Equations 1 and 6 and from data similar to those in Tables 1 and 4. For example, k can be computed for the Wausau network on CBD links. On the basis of the all-or-nothing assignment for Zone Pairs B through F, Equation 1 yields an average error from a single zone reduction of 7.7. The covariance for these same zone pairs is 0.00091 (Table 4), and the probability of a path being assigned to a link is 0.0157. The Wausau network has 45 zones and external stations, and it has about 24,700 vehicle trips in the p.m. peak hour. Substituting into Equation 6 and solving for k gives a value of 0.41.

Extrapolating the results between networks requires an assumption about the value of k and the value of the path covariance. These values can be ascertained from any other network by repeating the same procedure that was carried out on the Wausau network. The probability that a path is assigned to a link can be found from the whole path length (as measured in links) and the total number of links:

p = (path length)/(number of links)

The path length can be conveniently approximated by dividing the sum of volumes on all links in the network by the total number of vehicle trips.

For example, a 50-zone reduction of the East Brunswick network (*11*) is contemplated. This network has 129 zones, a total of 939 one-way equivalent links, 25,600 vehicle trips in the a.m. peak hour, and a total link volume of 540,000 vehicles. Therefore, the average path length is 21.1 links, and the probability of a path being assigned to a link is 0.0225. Adopting the values of k and the path covariance from the Wausau CBD and substituting into Equation 7 gives an RMS error of 19 vehicles/hr, or about 3 percent of the average directional volume.

The path covariance is strongly related to the distance between the subarea and the eliminated zones. The adopted path covariance should be extracted from a network as similar as possible in size and structure.

Although not addressed in this study, it would be particularly helpful if path covariance for a single link could be expressed as a function of distance to any given zone. With this information, simple guidelines could be established (e.g., do not eliminate zones within x mi of the subarea), and the need for sensitivity analysis could be reduced.

ERRORS IN ESTIMATES OF POPULATION

The major purpose of an HLFM II simulation is to obtain estimates of the spatial distribution of population and employment. Table 5 gives the RMS errors in population for each of the Wausau comparison networks. The RMS errors were computed for the 24 zones that were not touched by any of the zone reductions. The errors were quite small (less than 1 percent of the average zonal population of 1,885), and the RMS error does not appear to be strongly related to the number of zones eliminated. Population forecasts for almost any subarea can be expected to be insensitive to zone size and structure in the remainder of the region.

TABLE 5 RMS ERRORS IN POPULATION FOR 24 ZONES IN THE WAUSAU NETWORK

Network	RMS Error
A	4.3
B	6.0
C	1.4
D	8.2
E	14.9
F	2.8
G	0.8
H	0.9
AB	8.1
CH	1.3
EG	1.0
DF	7.9
AGCH	7.8
EGDF	8.4
ABCHEGDF	10.9

CONCLUSIONS

The major advantage of subarea focusing is a reduction of computer requirements when handling large networks. The greatest reductions can be achieved by aggregating zones within the network; eliminating links offers little advantage, considering the potential for damage to the forecast. Zones can be successfully aggregated, depending on the unfocused zone structure, the nature of the links within the subarea, the relationship between the subarea and the rest of the network, and the intended use of the forecast. Before subarea focusing is attempted, planners must determine how it will affect the quality of the forecast. This impact can be ascertained by performing trial simulations on the unfocused network or by extrapolating information about subarea focusing errors from networks of similar structure.

Subarea focusing offers considerable promise for land use simulations. Forecasts of demographic variables within subareas are relatively insensitive to the structure of the zones in the remainder of the region. Furthermore, subarea focusing overcomes two serious problems with land use models: (*a*) the requirement that the whole region be included within the zone system and (*b*) the extreme computer requirements of networks with large numbers of zones.

APPENDIX—IMPLEMENTATION OF LOWRY-GARIN MODEL

The model used for this research has been implemented as HLFM II. This program runs on MS-DOS and OS/2 microcomputers. HLFM II contains a wide variety of options for data preparation and execution, which were not invoked in this study. The following description of the Lowry-Garin model is confined to those features actually used. Provisions in HLFM II for expanded capabilities are noted as appropriate.

The Garin version of the Lowry model is a series of matrix equations that forecasts the distribution of population and employment in an urban area. The Lowry-Garin model recognizes only four land use activities: residential, basic industries, service industries for population, and service industries for businesses. Basic industries (i.e., industries that receive their income from outside the urban area) are assumed to be fixed at known locations. The Lowry-Garin model attempts to maintain proximity of workers' residences to their workplaces and to maintain proximity of service industries to their respective markets (either residences or other business, depending on the type of service activities).

The Lowry-Garin model is derived here by constructing an employment conservation equation. Let E be a vector of total employment (each element, e_i, of E being the total employment of the ith zone), E_B be a vector of basic employment, E_R be a vector of service employment required by residences, and E_W be a vector of service employment required by workers (i.e., businesses). Total employment is the sum of its three components:

$$E = E_B + E_R + E_W \quad \text{(A-1)}$$

Each of the three vectors on the right-hand side of Equation A-1 represents the spatial distribution of a sector of employment in the urban area. Basic employment, E_B, is the only explicit exogenous variable in the Lowry-Garin model. Employment serving residences, E_R, and employment serving workers, E_W, are dependent on trip-making patterns, the transportation system, and existing land use.

Employment in industries that serve workers, E_W, is calculated by distributing service employees around all employment locations as given by the vector E. Define h_{ij} as the conditional probability that an employee in Zone j is served by another employee in Zone i. Denote this matrix of conditional probabilities as H. Also define f as the number of service employees required for each employee, averaged across the whole urban area. Then

$$E_W = fHE \tag{A-2}$$

HLFM II permits specifying a different f at each zone, but this was not done for the tests in this paper.

A similar relation can be constructed for employees serving the entire population. Define b_{ij} as the conditional probability that an individual who lives in j is served by an employee in i. This conditional probability matrix is B. Also define g as the number of employees that serve each individual, averaged across the whole urban area. Then, as in Equation A-2,

$$E_R = gBP \tag{A-3}$$

where P is the population vector containing elements, p_i, each of which is the population in Zone i. The variable g could have been set separately for each zone but was not. Population distribution is computed from total employment. Define a_{ij} as the conditional probability that an individual working in j lives in i. Let A be the matrix of these conditional probabilities. Also define q_i as the ratio of population to employees in residental Zone i. Furthermore, let

$$Q = [\delta_{ij}\, q_i] \tag{A-4}$$

where δ_{ij} is the Kronecker delta. Note that Q is a diagonal matrix. Populations of all the zones are found from

$$P = QAE \tag{A-5}$$

Consequently, from Equations A-3 and A-5,

$$E_R = gBQAE \tag{A-6}$$

Substituting Equations A-2, A-3, and A-6 into Equation A-1 reduces the employment conservation equation to one with terms for only total employment, E, and basic employment, E_B.

$$E = E_B + gBQAE + fHE \tag{A-7}$$

Equation A-7 can be solved for the spatial distribution of total employment (E) in terms of basic employment.

$$E = (I - gBAQ - fH)^{-1}\, E_B \tag{A-8}$$

The spatial distribution of population can be computed from Equation A-5, and the spatial distributions of employment in the two service sectors, E_R and E_W, are directly computed from Equations A-2 and A-6.

The three conditional probability matrices (A, B, and H) are computed from singly constrained trip distribution equations with an exponential deterrence function. For example, the A matrix can be found by

$$a_{ij} = w_i \exp(-\beta c_{ij}) / \sum_i w_i \exp(-\beta c_{ij}) \tag{A-9}$$

where

c_{ij} = the generalized cost of travel between Zones i and j,
w_i = the attractiveness for residential Zone i, and
β = a calibrated parameter.

The model uses residential-developable area for w_i. The generalized cost of travel is computed as if trips followed the shortest path between pairs of zones in the urban area. That is,

$$c_{ij} = (1 + y/v)\, t_{ij} + t_{ij}^z \tag{A-10}$$

where

t_{ij} = the portion of shortest travel time that occurs on streets,
y = the monetary cost per minute,
v = the value of time, and
t_{ij}^z = the portion of shortest travel time that occurs on centroid connectors.

Adjustments to Residential Attractiveness

Land area does not formally appear in either the Lowry-Garin model or in the trip distribution equations. Nonetheless, land area has been introduced into the model by making residential attractiveness (w_i in Equation A-9) equal to residential-developable area and by making service attractiveness equal to service-developable area. The same parcel of land may be included in both measures of attractiveness. The trip distribution equations will assign activities to zones roughly in proportion to these developable areas. If a zone is almost fully occupied by service activities, then only a few people should be able to live there. The model can be instructed to reduce residential trip attractiveness in response to large allocations of service employees to a zone. This adjustment is handled iteratively. The residential attractiveness at iteration n (after the first iteration) is based on the amount of service allocated at iteration $n - 1$. Specifically,

$$W^n = W^1 - z(E_R^{n-1} + E_W^{n-1}) \tag{A-11}$$

where

W^n = the vector of residential attractiveness at iteration n,
W^1 = the vector of residential attractiveness at the first iteration as specified in the data input step,
z = area per service employee,
E_R^{n-1} = the vector of the number of employees that serve residences as calculated on iteration $n - 1$, and
E_W^{n-1} = the vector of the number of employees that serve other employees as calculated in iteration $n - 1$.

Residential attractiveness at any iteration (W^n) is constrained to be greater than or equal to zero. In addition, the adjustment to residential attractiveness (the second term of Equation A-11) cannot exceed the service-developable area. This procedure for adjusting residential attractiveness effectively incorporates the notion of land capacity into the forecasts. HLFM II permits a different value of area per service employee in each zone.

In addition, HLFM II can be forced to allocate a specific population to any given zone. This constraint is satisfied by making adjustments to residential attractiveness in the trip distribution equations.

Adjustments to Service Attractiveness

The Lowry-Garin model does not provide for agglomeration even though Lowry did recognize that certain zones would attain high levels of service activities. The original Lowry model permitted the planner to set the minimum amount of service employment that could occur in a zone.

Agglomeration can be partially handled by making upward adjustments of service attractiveness from those set by the planner. The underlying assumption is that agglomeration will occur in zones that have the highest potential to attract a disproportionately large share of customers. These zones will tend to be central to the region or near large concentrations of population. At least two iterations of the Lowry-Garin model are required—the first iteration establishes the "potential" and the second iteration distributes the residence-serving and employment-serving trips according to this potential. The measure of potential is continually updated as the number of iterations is increased. The method is somewhat complex and was not invoked in the assignment tests.

HLFM II can be instructed to allocate a specific number of service employees to any given zone. This constraint is satisfied by making adjustments to service attractiveness within the trip distribution equations.

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