Combined Trip Distribution and Assignment Model Incorporating Captive Travel Behavior

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Most of the previous literature on combined trip distribution and assignment problems has focused on the logit and entropy distribution models. Though these models are satisfactory for many applications, they are incapable of handling situations in which the observed trip patterns are represented by both compulsory (captive) and discretionary (free) travel behavior. Consequently, the use of a dogit distribution formula in the construction of a combined trip distribution and assignment model is suggested. The new version of the combined model can itself be reformulated as an equivalent mathematical programming problem so that the equilibrium conditions on the network and the dogit destination demand functions can be derived as the Kuhn-Tucker conditions of the proposed programming problem. Moreover, this equivalent mathematical program turns out to be a convex programming problem with linear constraints, a great advantage from the computational aspect. Numerical experiment indicates that this behaviorally sound combined model can be used in a realistic application at a reasonable cost and within a reasonable time period.

The transportation planning process as currently carried out consists of four major stages: trip generation, trip distribution, modal split, and trip assignment. Planners customarily treat the four stages sequentially as a set of independent problems. The potential drawbacks of this approach are twofold. First, actual interactions among stages are not explicitly accounted for, and biased demand predictions usually result. Second, as far as traffic equilibrium is concerned, the estimates of traffic flows are not always consistent, and, in general, do not converge to a stable solution. These deficiencies suggest that some or all of the stages in the transportation planning process be handled simultaneously or combined.

With this in mind, a combined trip distribution and assignment model is considered in which the solutions of interzonal trips and link flows are solved jointly. Because trip generation and modal split are not treated here, the proposed model should be applied exclusively on the automobile network, and the model requires the total number of automobile trips originating at each zone as input. Generalization of the model to include trip generation and modal split will be the subject of future research.

To some extent, the proposed model may be considered a variation of the combined trip distribution and assignment models that were studied by others (*1–3*). The major difference between the previously developed models and the proposed model is that the latter uses the dogit model (instead of logit- and entropy-type models) to find how observed trips are distributed among the various destinations. The principal reason to employ the dogit model in distribution analysis is because at any time an observed trip pattern is typically composed of at least two types of trips: (*a*) compulsory trips (e.g., work, business, school, etc.) and (*b*) discretionary trips (e.g., shopping, recreation, etc.). Compulsory trips are those that will be made even in the worst conditions. Thus, frequency, destination, and mode are nearly, if not absolutely, fixed. On the other hand, discretionary trips are less regular both in time and space (mainly because they are less economically motivated and are more sociologically and psychologically motivated). For example, these trips may be suppressed by inclement weather, crowded highways, or substitutes that are offered for them.

Because both compulsory and discretionary trips will exist in many travel situations, some people in an urban area are captive to one or more specific aspects of travel, and some are free to make one or more choices. Despite this fact, most existing demand models have failed to explicitly distinguish between captive and free travel behavior. For example, the well-known logit model is structured so that each individual is assumed to exercise a choice for each travel decision to which the model is applied. A clear disadvantage of this assumption is that when the captive travel behavior is observed, the logit estimation will yield errors in parameter estimates and demand predictions. One way to alleviate the estimation problem is to carry out the model calibration with data only for people who have free choice rather than with mixed data. This approach, however, requires a careful preparation of the calibration data, and, most significant, leads to inability to detect the effects of certain transportation policies and actions that may remove captivity for some people or make others captive.

Thus, to take into account captive and free travel behavior, a combined trip distribution and assignment model will be developed in which trip distribution is given by a behaviorally more realistic dogit model. To fulfill this objective, a dogit distribution model will be formulated on the basis of its original individual probabilistic form. After the dogit-type stochastic trip distribution is combined with a deterministic user network equilibrium, an equivalent minimization problem is proposed for which the Kuhn-Tucker conditions include the usual user-equilibrium equations for the basic network and the dogit demand functions for the interzonal trips. Because this equivalent minimization problem turns out to be a convex programming problem with linear constraints, it can be solved

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efficiently by available algorithmic approaches. Finally, a numerical example is presented to demonstrate that the proposed combined model and methodology can be used in a realistic application at a reasonable cost and within a reasonable time period.

MODEL DEVELOPMENT

In this section a new version of the combined trip distribution and assignment model is presented in which the trip distribution is given by a dogit model. To explain the theories and assumptions underlying the proposed combined model, trip distribution and trip assignment models are first described separately and then combined into a single formulation.

The dogit model, as derived independently (*4,5*), is a special case of mixed probability discrete choice (or random utility) models. The functional form of the dogit model is

$$P_{jt} = \frac{\sigma_j}{1 + \sum_k \sigma_k} + \frac{1}{1 + \sum_k \sigma_k} \frac{e^{V_{jt}}}{\sum_k e^{V_{kt}}} \tag{1}$$

where

P_{jt} = the probability that an individual t randomly drawn from the population will choose the jth of K alternatives,
σ_j = a nonnegative parameter associated with the jth alternative, and
V_{jt} = the systematic utility of the jth alternative.

The dogit model has two distinct features. First, when σ_k, $k \in K$, is not equal to zero for all alternatives in the model, the ratio of the probabilities of choosing any two alternatives will depend on the attributes of all alternatives and hence be unconstrained by the Independence from Irrelevant Alternatives (IIA) property. Furthermore, if $\sigma_k = 0$ holds for some alternatives, the dogit model also allows some pairs of alternatives to exhibit the IIA property, while allowing the remaining pairs to be free from the IIA restriction.

Second, because the parameters σ_k in the model can represent the likelihood of an individual randomly drawn from the population being captive to particular alternatives, the dogit model can distinguish between compulsory (captive) and discretionary (free) travel behavior. To see this, assume that the population under study can be partitioned into $K + 1$ groups (see Figure 1), where K is the number of available alternatives. The jth of K groups represents a group of individuals captive to the jth alternative, whereas the last [i.e., $(K + 1)$th] group represents a group of individuals not captive to any alternative. The first term on the right-hand side of Equation 1 can be interpreted as the probability that a randomly drawn individual comes from the jth group, in which case the individual is captive to the jth alternative, and thus the probability of the jth alternative being chosen is clearly 1. The second term on the right-hand side of Equation 1 has two parts. The part involving the σ vector can be interpreted as the probability that a randomly drawn individual comes from the last group, and the other is the logit probability model of choosing the jth alternative given that the individual has free choice. The probabilities that a randomly drawn individual may also come from the other groups in K are ignored because in these cases the probabilities of the jth alternative being chosen are zero.

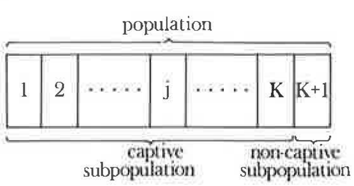

FIGURE 1 Partitioned population.

On the basis of Equation 1, the trip distribution model in this paper is specified as

$$T_{ij} = O_i[E(P_{j|i})]$$
$$= O_i \left[\frac{\sigma_{ij}}{1 + \sum_k \sigma_{ik}} + \frac{1}{1 + \sum_k \sigma_{ik}} \frac{e^{V_{ij}}}{\sum_k e^{V_{ik}}} \right] \tag{2}$$

where

T_{ij} = the number of trips from Origin Zone i to Destination Zone j,
O_i = the fixed and known number of trips leaving from Origin Zone i,
σ_{ij} = a nonnegative parameter representing the odds that population in Origin i is captive to the jth destination,
V_{ij} = the systematic utility function associated with the i-j pair,
$P_{j|i}$ = the probability that a randomly selected individual who originates at Zone i will choose the jth destination, and
$E(P_{j|i})$ = the expected value of P, which will be interpreted as the share of the population originating at i that is attracted to the jth destination.

The trip distribution model formulated in Equation 2 needs some explanation. In discrete choice modeling, the utility function relates the choice probability $P_{j|i}$ to a vector of variables that may include individual characteristics and transportation attributes. Thus, the aggregation process is usually required because the intent is to expand the individual choice estimates to an entire population or subpopulation to obtain a forecast of aggregate shares of alternatives. However, the aggregation process can be ignored if the utility function in the dogit model do not include variables that vary across individuals. This helps explain the reason why the trip distribution model in Equation 2 has a form similar to that in Equation 1, and the expected value of $P_{j|i}$ can be interpreted as the share of the population originating at i that is attracted to Destination j.

To interpret the model in Equation 2, T_{ij} can be viewed as the expected number of trips from Zone i to Zone j in the daily peak-load period, and during that period, the number of compulsory work trips from i to j is given by

$$\frac{O_i \sigma_{ij}}{1 + \sum_k \sigma_{ik}} \tag{3}$$

The number of discretionary nonwork trips from i to j is given by

$$\frac{O_i}{1 + \sum_k \sigma_{ik}} \frac{e^{V_{ij}}}{\sum_k e^{V_{ik}}} \tag{4}$$

In addition, because the captivity parameters σ_{ij} can be expressed as

$$\frac{\sigma_{ij}}{1} = \frac{O_i \dfrac{\sigma_{ij}}{1 + \sum_k \sigma_{ik}}}{O_i \dfrac{1}{1 + \sum_k \sigma_{ik}}}$$

they are simply the ratios of the number of compulsory trips from i to j and the number of discretionary trips leaving i.

The dogit distribution model constructed in Equation 2 is theoretically more general than the logit distribution model used by Expression 3. This is because by setting all captivity parameters equal to zero (i.e., $\sigma_{ik} = 0 \; \forall \; k \in K$), the logit distribution model can be obtained as a special case of the dogit distribution model. Similarly, the dogit distribution model is more general than the standard entropy distribution model used elsewhere (*2,6,7*), because the latter model is a limiting case of the logit distribution model [see the proof by Safwat and Magnanti (*8*)].

For convenience, the utility functions in Equation 2 will be specified as

$$V_{ij} = M_j - \theta u_{ij} \tag{5}$$

where

M_j = a measure of attractiveness (a constant) associated with the jth destination,
u_{ij} = the travel cost over the shortest path connecting the i-j pair, and
θ = the parameter associated with u_{ij}.

Trip Assignment Model

Given a network of links and nodes and a trip table listing trips between all pairs of zones, the trip assignment problem is concerned with the allocation of the trips to the network links. In this paper, the driver's behavior on a road network is assumed to follow Wardrop's first principle. The principle states that, at equilibrium, the average travel cost on all used paths connecting any given i-j pair will be equal, and the average travel cost will be less than or equal to the average travel cost on any unused path (*9*). In the transportation literature, the flows that satisfy this principle are said to be a user-equilibrium or user-optimal flow pattern. The mathematical expression equivalent to user equilibrium can be stated as follows:

$$(c_p^{ij} - u_{ij})h_p^{ij} = 0 \qquad \forall \; i, j, p \tag{6}$$

$$(c_p^{ij} - u_{ij}) \geq 0 \qquad \forall \; i, j, p \tag{7}$$

where

c_p^{ij} = the average travel cost of Path p between Origin i and Destination j,
u_{ij} = the minimum (or equilibrium) travel cost between i and j, and
h_p^{ij} = the flow on Path p connecting the i-j pair.

Equations 6 and 7 imply that users' behavior is deterministic. Route choice might be treated more realistically as stochastic, as was trip distribution choice. Nevertheless, the deterministic assumption is reasonable for congested network systems (*10*). The major reason is that as congestion increases, the differences between alternative routes are more accurately perceived by users, and, hence, the route choice approaches the deterministic equilibrium solution.

Now, if Equations 6 and 7 are combined with the conservation of flow conditions

$$\sum_p h_p^{ij} = T_{ij} \qquad \forall \; i, j \tag{8}$$

and the corresponding nonnegativity constraints

$$h_p^{ij} \geq 0 \qquad \forall \; i, j, p \tag{9}$$

they constitute a quantitative statement of user-equilibrium conditions.

It is well known (*11*) that these equilibrium conditions can be interpreted as the Kuhn-Tucker conditions for an equivalent minimization problem, which is

$$\min Z(f) = \sum_a \int_0^{f_a} c_a(w)dw \tag{10}$$

subject to Constraints 8 and 9, and a definitional constraint,

$$f_a = \sum_i \sum_j \sum_p \delta_{ap}^{ij} h_p^{ij} \qquad \forall \; a \tag{11}$$

where

f_a = the flow on Link a,
$c_a(f_a)$ = the average travel cost per trip on Link a for Flow f, and
δ_{ap}^{ij} = 1 if Link a belongs to Path p from i to j, and 0 otherwise.

Combined Trip Distribution and Assignment Model

The equilibrium trip assignment model described above is used for a fixed distribution of trips. In this case, because demand is constant, travelers will not alter their destination even when faced with the additional costs that travel to a specific destination entails. This counterintuitive travel behavior leads to consideration of a combined trip distribution and assignment model in which the trip flows of origin-destination (O-D) pairs will respond to changing network flow conditions. The proposed equilibrium distribution assignment model combines a dogit-type stochastic trip distribution with deterministic network equilibrium and is specified as follows:

$$T_{ij} = O_i\left(\frac{\sigma_{ij}}{1 + \sum_k \sigma_{ik}} + \frac{1}{1 + \sum_k \sigma_{ik}} \frac{e^{M_j - \theta u_{ij}}}{\sum_k e^{M_k - \theta u_{ik}}}\right) \quad \forall\, i, j \tag{12}$$

$$(c_p^{ij} - u_{ij})h_p^{ij} = 0 \quad \forall\, i, j, p \tag{13}$$

$$c_p^{ij} - u_{ij} \geq 0 \quad \forall\, i, j, p \tag{14}$$

This equilibrium model will be called the combined dogit trip distribution and assignment (CDDA) model in the following discussion. Equations 12, 13, and 14, when combined with the flow conservation conditions

$$\sum_p h_p^{ij} = T_{ij} \quad \forall\, i, j \tag{15}$$

the two corresponding nonnegativity constraints

$$h_p^{ij} \geq 0 \quad \forall\, i, j, p \tag{16}$$

$$T_{ij} \geq \sigma_{ij} O_i/(1 + \sum_k \sigma_{ik}) \quad \forall\, i, j \tag{17}$$

and one definitional constraint (Equation 11), constitute a quantitative statement of user equilibrium conditions for the CDDA model. The equilibrium conditions (Equations 11–17) state that at equilibrium, a set of O-D trip flows and path flows must satisfy the following requirements:

1. The O-D trip flows satisfy a distribution model of the dogit form (Equation 12).
2. The path flows are such that the user-equilibrium criterion is satisfied (Equations 13 and 14).
3. The O-D trip flow between i and j equals the total trip flows generated from i (resulting from summation over j on both sides of Equation 12).
4. The flows on all paths connecting each i-j pair equal the O-D trip flow between i and j (Equation 15).
5. Each path flow is nonnegative (Equation 16).
6. Each O-D trip flow is equal to or larger than its corresponding O-D compulsory trip flow (Equation 17).
7. The definitional relationship between path and link flows is satisfied (Equation 11).

All used paths between each O-D pair must have equal path costs. These costs represent the minimum path costs, and the O-D trip flows are in equilibrium determined by the minimum path costs.

EQUIVALENT MINIMIZATION PROBLEM

To solve the equilibrium of the CDDA model, the approach is to show that an equivalent minimization problem (EMP) exists whose solutions satisfy the equilibrium conditions (Equations 11–17). Consider the following minimization problem:

$$\min Z(T, f) = G(T) + F(f) \tag{18}$$

such that

$$\sum_j T_{ij} = O_i \quad \forall\, i \tag{19}$$

$$\sum_p h_p^{ij} = T_{ij} \quad \forall\, i, j \tag{20}$$

$$h_p^{ij} \geq 0 \quad \forall\, p, i, j \tag{21}$$

$$T_{ij} \geq \sigma_{ij}\, O_i/(1 + \sum_k \sigma_{ik}) \quad \forall\, i, j \tag{22}$$

where

$$G(T) = \frac{1}{\theta} \sum_i \sum_j \left[\left(T_{ij} - \frac{\sigma_{ij} O_i}{1 + \sum_k \sigma_{ik}}\right) \times \ln\left(T_{ij} - \frac{\sigma_{ij} O_i}{1 + \sum_k \sigma_{ik}}\right) + \left(T_{ij} - \frac{\sigma_{ij} O_i}{1 + \sum_k \sigma_{ik}}\right) \times \ln\left(\frac{1 + \sum_k \sigma_{ik}}{O_i}\right) - T_{ij} M_j - T_{ij}\right] \tag{23}$$

$$F(f) = \sum_a \int_0^{f_a} c_a(w)dw \tag{24}$$

$$f_a = \sum_i \sum_j \sum_p h_p^{ij} \delta_{ap}^{ij} \quad \forall\, a \tag{25}$$

In this formulation, the objective function (Equation 18) comprises two sets of terms. The first set, $G(T)$, has as many terms as the number of O-D pairs in the network. Each term, $G_{ij}(T)$, is a function of the number of trips T_{ij} distributed from a given origin i to a given destination j. The second set, $F(f)$, has as many terms as the number of links in the network. Each term, $F_a(f)$, is a function of the flows over all paths that share a given link a [as implied by the link-path incidence relationships (Equation 11)].

Equations 19 and 20 are flow conservation constraints, which state, respectively, that the number of trips distributed from i to all j's must equal the number of trips generated from i and that the number of trips on all paths connecting each i-j pair must equal the number of trips distributed from i to j. Equation 21 represents the flow nonnegativity constraints required to ensure that the solution of the program is physically meaningful. Constraint 22 is required to ensure that the objective function is well defined. (Because $O_i \geq 0$ and $\sigma_{ik} \geq 0$ for all k, Contraint 22 implies that T_{ij} is greater than or equal to a nonnegative constant.) Finally, the link-path incidence relationships (Equation 25) express the link flows in terms of the path flows [i.e., $f = f(h)$].

The importance of the EMP (Equations 18–25) is that even with mild assumptions imposed on the problem data, it is a convex program, which has a unique solution that is equivalent to equilibrium on the proposed CDDA model. The formal proof of this result is given elsewhere (*12*), but it is worth mentioning that the equivalence between the EMP and the CDDA model can be established by examining the optimality conditions of the EMP.

PARAMETER CALIBRATION AND SOLUTION ALGORITHM

Dogit Model Estimation

The dogit distribution model specified in Equations 2 and 5 contains several free parameters that must be estimated, that is, σ, M, and θ. The dogit model parameters will be estimated using the maximum likelihood (ML) method. By assuming that all the values of variables in the utility function are the same for each traveler in a particular zone and that the sample is composed of zonal values, the likelihood function of the observed sample is given by

$$L = \frac{T!}{\prod_i \prod_j T_{ij}!} \prod_i \prod_j \bar{P}_{j|i}(\sigma, M, \theta)^{T_{ij}} \tag{26}$$

where T is the total number of travelers (equal to $\Sigma_i \Sigma_j T_{ij}$) and $\bar{P}_{j|i} = T_{ij}/O_i$. The ML estimates of the parameters are obtained by taking the derivatives of the logarithm of L with respect to σ, M, and θ, equating the derivatives to zero, and solving for σ, M, and θ.

Three observations are made concerning dogit model calibration. First, an identification problem will result when calibrating the dogit distribution model. In general, if N zones are encompassed in the model, there are only $N^2 - N$ parameters that can be identified in the estimation.

Second, the number of parameters that must be estimated increases rapidly as the number of zones increases. This can be a serious problem in typical network analysis, where the observed number of O-D pairs is usually large. The usual way to reduce computational costs is to impose some reasonable constraints on the captivity parameters (*12*).

Third, because the log likelihood function of Equation 26 lacks concavity (*13,14*), the estimation must proceed with great care because multiple maxima or saddle points or both may be encountered. In this case, the appropriate way to check the global maximum may be to carry out several searches, using a different initial point each time. The main purpose of this process is to search for alternative maxima and establish a convincing case for the global maximum.

Evans Algorithm

Once the parameter values are obtained, the EMP can be readily solved by appropriate algorithmic procedures to yield the desired equilibrium on the CDDA model. Because the EMP is virtually a convex programming problem, the well-known Frank-Wolfe (*15*) and Evans (*2*) algorithms can be used to solve the equilibrium problem.

In this study, the Evans algorithm is selected for three reasons. First, in contrast to the Frank-Wolfe algorithm, in which only one destination is loaded at each iteration, the Evans algorithm ensures that every destination will be loaded with trips from each origin at every iteration. Thus, the Evans algorithm is more efficient than the Frank-Wolfe algorithm for the usual combined trip distribution and assignment problems.

Second, the objective function of the subproblem in the Frank-Wolfe algorithm is derived from the linearized objective function of the EMP (Equation 18). The objective function in the Evans subproblem, on the other hand, involves a partial linear approximation in the sense that the link cost functions in Equation 18 are linearized but the remaining functions are not. As Florian (*16*) has remarked, the feasible direction derived from partial linear approximation is better than linear approximation because the subproblem in the Evans algorithm used for finding the direction of descent is closer to the original EMP. The solution of the Evans algorithm would, therefore, require fewer iterations than the Frank-Wolfe algorithm.

Third, each iteration of the Evans algorithm computes an exact solution for equilibrium conditions of the CDDA model, whereas in the Frank-Wolfe algorithm, none of the equilibrium conditions is met until final convergence. This has an important implication in large-scale network applications because it is often unlikely that either the Evans or the Frank-Wolfe algorithm will be run to exact convergence because of the high computational costs involved.

NUMERICAL EXAMPLE

In this section the CDDA model and its associated methodology will be applied to a hypothetical transportation system. Two computer programs were written; one was used to estimate the parameters of the dogit trip distribution model, and the other was for the solution algorithm used to obtain simultaneous prediction of equilibrium in the CDDA model.

The hypothetical highway network, which consists of 6 nodes and 20 links, is shown in Figure 2. Two of the nodes are assumed to be intermediate, and the rest are origins or destinations (or both) defining 16 O-D pairs, as shown in Table 1. Table 1 also gives the travel demand associated with each O-D pair. Table 2 gives the following information for each link: name of the "from" node, name of the "to" node, link capacity, and free-flow (uncongested) travel time. [The travel time (t_a) is used as a proxy for the cost variable (c_a).]

To implement the CDDA model, the utility function (V_{ij}) in the trip distribution model must be specified as well as the link performance function [$c_a(f_a)$] in the trip assignment model. For the Evans algorithm to work, the link performance function must be monotonic increasing. To this end, a standard function developed by the U.S. Bureau of Public Roads (*17*) is used. The performance function has the following form:

$$t_a = t_a^0 \left[1 + \alpha\left(\frac{f_a}{p_a}\right)\right]^\beta \quad \forall a \tag{27}$$

where

t_a = congested travel time on Link a,
t_a^0 = uncongested (free-flow) travel time on Link a,

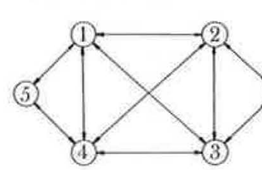

FIGURE 2 Hypothetical network example.

TABLE 1 LIST OF O-D PAIRS

O-D No.	Origin	Destination	Observed O-D Trip Flows
1	1	1	40.0
2	1	2	100.0
3	1	3	30.0
4	1	4	150.0
5	2	1	100.0
6	2	2	50.0
7	2	3	170.0
8	2	4	30.0
9	3	1	130.0
10	3	2	280.0
11	3	3	50.0
12	3	4	210.0
13	4	1	170.0
14	4	2	200.0
15	4	3	350.0
16	4	4	60.0

TABLE 2 HYPOTHETICAL LINK DATA

Link No.	From	To	Link Capacity	Free-Flow Travel Time
1	1	2	100.0	22.0
2	1	3	100.0	43.0
3	1	4	150.0	24.0
4	1	5	200.0	12.0
5	2	1	100.0	13.0
6	2	3	110.0	23.0
7	2	4	50.0	34.0
8	2	6	200.0	12.0
9	3	1	190.0	25.0
10	3	2	150.0	29.0
11	3	4	300.0	24.0
12	3	6	200.0	15.0
13	4	1	150.0	16.0
14	4	2	300.0	23.0
15	4	3	400.0	19.0
16	4	5	100.0	12.0
17	5	1	100.0	12.0
18	5	4	50.0	12.0
19	6	2	300.0	15.0
20	6	3	200.0	12.0

f_a = the flow (volume) on Link a,
p_a = the "practical" capacity of Link a, and
α, β = parameters whose usual values are 0.15 for α and 4 for β.

For convenience, the utility function in the trip distribution model is assumed to be linear in the parameters and has the form

$$V_{ij} = -\theta_1 u_{ij} + \theta_2 M_j \tag{28}$$

where

u_{ij} = travel cost over the shortest path connecting the i-j pair,
M_j = attractiveness of Zone j, and
θ_1, θ_2 = parameters to be estimated.

The M_js originally specified in Equation 5 were a set of alternative-specific constants. But for the sake of reducing the number of parameters to estimate, the M_js are treated as the explanatory variable in Equation 28 and associated with a single parameter θ_2. The above treatment is purely for simplicity in this study, but in practical applications the omission of constant terms in the utility function should be avoided whenever possible. This is because the inclusion of constant terms not only can compensate for sampling and measurement errors, but also can capture the mean effects of unobserved or unmeasured variables that describe the unique characteristics of the choice alternatives.

For convenience, the attractiveness of Zone j in Equation 28 is measured by the employment density (employees per acre) in Zone j. If other measures, such as population and area, are used for the representation of zonal attractiveness, the variable M_j must be replaced with a linear function of those measures with unknown parameters inside a log operation (*18*).

Given the data of O-D trip flow (Table 1), travel cost over the shortest path connecting each O-D pair (Table 3), and zonal employment density (Table 3), the calibration results for the dogit distribution model are shown in Table 4, and the estimated O-D compulsory and discretionary trip flows are shown in Table 5.

Given the estimated coefficient values and the data provided in Tables 1 and 2, equilibrium on the hypothetical transportation system can be predicted. The prediction procedure was required to stop when the changes in O-D trip flows and link flows between successive iterations were negligible or when the number of iterations reached 20. The final equilibrium results in Table 6 are those of the 12th iteration.

As indicated in Table 6, the equilibrium results of the example appear reasonable in that (*a*) the predicted number of trips between each *i-j* pair is larger than the corresponding estimated number of compulsory trips, implying that existing O-D compulsory trip flows will remain unchanged regardless of congestion potentially occurring on the network; (*b*) there are no positive flows on paths with higher than the minimum perceived costs, indicating that the user optimization principle is well satisfied; and (*c*) predicted O-D trip flows and minimum O-D path costs have values similar to those observed.

The value of the objective function in the Evans algorithm consistently decreased from one iteration to the next in all the runs. In particular, the improvement in the value of the

TABLE 3 HYPOTHETICAL DATA FOR MODEL CALIBRATION

Origin	Destination	Observed O-D Minimum Path Travel Time	Observed Employment Density
1	1	30.0	25.0
1	2	25.0	35.0
1	3	45.0	40.0
1	4	25.0	25.0
2	1	15.0	25.0
2	2	30.0	35.0
2	3	25.0	40.0
2	4	35.0	25.0
3	1	25.0	25.0
3	2	30.0	35.0
3	3	50.0	40.0
3	4	25.0	25.0
4	1	20.0	25.0
4	2	25.0	35.0
4	3	20.0	40.0
4	4	50.0	25.0

TABLE 4 DOGIT ESTIMATION RESULTS

Model Parameters	Estimated Coefficient Values
θ_1	-0.12
θ_2	0.08
σ_{11}	NI*
σ_{12}	0.27
σ_{13}	NI*
σ_{14}	0.96
σ_{21}	0.33
σ_{22}	NI*
σ_{23}	0.84
σ_{24}	NI*
σ_{31}	0.30
σ_{32}	0.96
σ_{33}	NI*
σ_{34}	0.69
σ_{41}	0.45
σ_{42}	0.51
σ_{43}	0.69
σ_{44}	NI*
log-likelihood at zero	-2938.943
log-likelihood at convergence	-2600.262

Note: NI* = not identifiable.

TABLE 5 COMPARISON BETWEEN OBSERVED AND ESTIMATED O-D TRIP FLOWS

O-D Pairs	Observed O-D Total Trip Flows	Estimated O-D Total Trip Flows	Estimated O-D Compulsory Trip Flows	Estimated O-D Discretionary Trip Flows
1-1	40.0	40.72	24.34	16.38
1-2	100.0	99.29	32.85	66.44
1-3	30.0	33.33	24.33	9.00
1-4	150.0	146.66	116.81	29.85
2-1	100.0	100.33	44.94	55.39
2-2	50.0	47.61	27.24	20.38
2-3	170.0	169.79	114.40	55.39
2-4	30.0	32.26	27.25	5.01
3-1	130.0	126.61	63.81	62.80
3-2	280.0	280.90	204.19	76.71
3-3	50.0	52.92	42.54	10.38
3-4	210.0	209.57	146.76	62.81
4-1	170.0	172.30	123.16	49.14
4-2	200.0	199.61	139.58	60.03
4-3	350.0	352.01	188.84	163.17
4-4	60.0	56.08	54.74	1.34

objective function during the first five iterations was substantial and tended to be insignificant during the following iterations. This result may suggest that a reasonably accurate solution can be obtained in no more than 10 iterations. Finally, because the hypothetical example had a much smaller number of links and nodes than real-life networks, it is somewhat difficult to extrapolate the CPU time necessary to run the CDDA model in actual applications. Nevertheless, because the CDDA model adds few simple arithmetic operations in the Evans algorithm, the computational time should be comparable with the time required to solve any of the existing combined models.

TABLE 6 DOGIT EQUILIBRIUM RESULTS

Trip Distribution

Origin	Destination	Predicted O-D Trip Flow	Predicted Minimum O-D Path Cost
1	1	40.33	F*
1	2	98.43	25.09
1	3	34.31	44.06
1	4	146.92	24.89
2	1	98.44	14.83
2	2	46.16	F*
2	3	172.77	24.19
2	4	32.63	33.90
3	1	129.22	24.77
3	2	276.54	30.99
3	3	52.93	F*
3	4	211.31	24.89
4	1	172.56	20.20
4	2	209.51	23.82
4	3	341.86	20.60
4	4	56.08	F*

Trip Assignment

Link No.	From	To	Predicted Link Flow	Predicted Link Cost
1	1	2	98.43	25.09
2	1	3	30.51	44.06
3	1	4	106.01	24.89
4	1	5	44.72	12.00
5	2	1	98.44	14.83
6	2	3	84.22	24.19
7	2	4	32.63	33.90
8	2	6	88.55	12.07
9	3	1	129.22	24.77
10	3	2	123.49	30.99
11	3	4	211.31	24.89
12	3	6	153.06	15.77
13	4	1	172.56	20.20
14	4	2	209.51	23.82
15	4	3	345.66	20.60
16	4	5	0.00	12.00
17	5	1	0.00	12.00
18	5	4	44.72	13.15
19	6	2	153.06	15.15
20	6	3	88.55	12.07

Note: F* = fixed intrazonal path cost.

CONCLUSIONS

The most important feature of the proposed combined trip distribution and assignment model is that the equilibrium O-D trip flows satisfy a dogit model that is able to describe users' compulsory and discretionary travel behavior in response to performance on a transportation network. Thus, the proposed model should be more sound behaviorally than any of the other combined trip distribution and assignment models reported in the literature. Moreover, because the model can itself be reformulated and solved by an EMP and because this problem is a convex programming problem with linear constraints, it can be solved efficiently by several algorithmic approaches that are available for such problems. In particular, when applying the Evans algorithm to the equilibrium problem, the proposed combined model should be usable in a realistic application at a reasonable cost and within a reasonable time period. To verify this expectation, future research should focus on the application of the model to a real large-scale network.

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REFERENCES

1. J. A. Tomlin. A Mathematical Programming Model for the Combined Distribution-Assignment of Traffic. *Transportation Science*, Vol. 5, 1971, pp. 122–140.
2. S. P. Evans. Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment. *Transportation Research*, Vol. 10, 1976, pp. 37–57.
3. S. Minis. Equilibrium Traffic Assignment Models for Urban Networks. S.M. thesis. Massachusetts Institute of Technology, Cambridge, 1984.
4. M. Ben-Akiva. Choice Models with Simple Choice Set Generation Processes. Working paper. Massachusetts Institute of Technology, Cambridge, 1977.
5. M. J. I. Gaudry and M. G. Dagenais. The Dogit Model. *Transportation Research*, Vol. 13B, 1979, pp. 105–111.
6. S. Erlander. Accessibility, Entropy and the Distribution and Assignment of Traffic. *Transportation Research*, Vol. 11, 1977, pp. 149–153.
7. M. Florian and S. Nguyen. A Combined Trip Distribution, Modal Split and Trip Assignment Model. *Transportation Research*, Vol. 12, 1978, pp. 241–246.
8. K. N. Safwat and T. L. Magnanti. A Combined Trip Generation, Trip Distribution, Modal Split, and Trip Assignment Model. *Transportation Science*, Vol. 18, 1988, pp. 14–30.
9. J. D. Wardrop. Some Theoretical Aspects of Road Traffic Research. *Proc., Institution of Civil Engineers*, Vol. II, No. 1, 1952, pp. 325–378.
10. Y. Sheffi and W. Powell. A Comparison of Stochastic and Deterministic Traffic Assignment over Congested Networks. *Transportation Research*, Vol. 15B, 1981, pp. 53–64.
11. M. J. Beckmann, C. B. McGuire, and C. B. Winsten. *Studies in the Economics of Transportation*. Yale University Press, New Haven, Conn., 1956.
12. Y. L. Chu. A Combined Trip Distribution and Assignment Model with Dogit Destination Demand Functions. Ph.D. dissertation. University of Pennsylvania, Philadelphia, 1989.
13. J. Swait and M. Ben-Akiva. Incorporating Random Constraints in Discrete Models of Choice Set Generation. *Transportation Research*, Vol. 21B, 1987, pp. 91–102.
14. M. J. I. Gaudry and M. J. Wills. Testing the Dogit Model with Aggregate Time-Series and Cross-Sectional Travel Data. *Transportation Research*, Vol. 13B, 1979, pp. 155–166.
15. M. Frank and P. Wolfe. An Algorithm for Quadratic Programming. *Naval Research Logistics Quarterly*, Vol. 3, 1956, pp. 95–110.
16. M. Florian. Nonlinear Cost Network Flow Models in Transportation Analysis. *Mathematical Program Study 26*, 1986, pp. 167–196.
17. *Highway Capacity Manual*. U.S. Bureau of Public Roads, 1950.
18. A. Daly. Estimating Choice Models Containing Attraction Variables. *Transportation Research*, Vol. 16B, 1982, pp. 5–15.

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