Evaluation of the Subbase Drag Formula by Considering Realistic Subbase Friction Values

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A modification of the reinforcement formula that considers the realistic frictional characteristics of subbase types is presented. The objective of this study is not to abandon the current formula but to arrive at a better formula, one that considers the field observations. Rational reinforcement design is important because the amount of reinforcement affects the restraint on the movement of a pavement section, or slab, and the long-term performance. This study was the result of a need to revise the reinforcement formula based on the subbase drag theory. The reinforcement formula was modified in accordance with the experimental results obtained concerning subbase frictional resistance. The modification was necessary to include the actual characteristics of subbase friction in the reinforcement design formula for both continuously and jointed reinforced concrete pavements. The new formula reflects the experimental results concerning subbase friction. It represents the actual components of frictional resistance at the interface: adhesion, bearing, and shear. The implementation of information from this study will result in more rational reinforcement design. The formula calculates the steel requirement for the middle of the slab; in other words, the calculated value is the maximum requirement, and the locations between the free end and the middle of the slab will require less reinforcement. Further experimental study is necessary to calibrate the new formula.

In the revised 1985 AASHTO Guide for Design of Pavement Structures (*1*), the friction factor is used to develop a nomograph for estimating the required steel percentage for both continuously reinforced concrete pavement (CRCP) and jointed reinforced concrete pavement (JRCP) for transverse and longitudinal reinforcement, respectively. As mentioned in this guide, this parameter corresponds to the coefficient of friction. For various subbase materials, the recommended and currently used friction factor varies from 0.9, for natural subgrade, to 1.8, for stabilized subbases. With the use of this factor it is assumed that the amount of subbase frictional force is directly proportional to the weight of the slab.

The first studies involving frictional resistance measurement started as early as 1924. The main objective of those experiments was to observe the relationship between the friction value and the subbase type. The observed values were obtained before the use of stabilized subbases. According to the results of recent research performed at the Center for Transportation Research of The University of Texas at Austin (*2,3*), the frictional resistance is primarily a function of the subbase type and the magnitude of resistance is independent of slab thick-

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ness. Thus, all the JRC pavements laid over cement-stabilized subbases experienced excessive cracks, regardless of the thickness of the slab.

OBJECTIVE

The primary objective of this study was to demonstrate the limitations of subgrade drag theory by considering new data. The next objective was to rederive the subgrade drag equation to more precisely reflect the subbase frictional analysis observed in the field. New findings on frictional restraint at the interface of the slab and subbase required a revision of the subbase friction concept. A formula was developed for the design of both CRCP transverse reinforcement and JRCP longitudinal reinforcement (*1*). The derivation of the present formula is based on the classical friction concept. In other words, the resultant friction force is mainly a function of slab weight, not of the frictional characteristics of the subbase. In reality, the frictional resistance consists of three components: adhesion, bearing, and shear at the interface (*2*). For each subbase type, the relative effect of each component is different. That is why the failure plane for unbounded subbase occurs at the interface, whereas for bounded subbase, it occurs within the subbase.

LIMITATIONS OF THE CURRENT FORMULA

The steel requirement is mainly dependent on the thickness of the slab, not the subbase type, because the assumed friction factors are more or less the same for various subbase types. This is one of the limitations of the current reinforcement formula that was observed when it was compared with the results of a previous study (*2*). From experience it can be concluded that there is a significant difference in applied frictional resistance among the various subbase types. A slab on a cement-stabilized subbase experiences more cracks than one on an asphalt-stabilized subbase.

DERIVATION

The derivation of the reinforcement formula is the result of equating the force in steel at the middle of the slab, assuming that a crack has formed at that location, to the developed frictional force. Although the current formula is based on the same equilibrium, it lacks the representation of the actual

subbase frictional characteristics. Among the subbase types, only untreated clay fits the classical friction pattern. The purpose of this paper is to explain the reason for the modification of the nomograph (and the formula of the nomograph) by using the recent findings reported elsewhere (*2,3*).

The current formula is

$$P_{sold} = \frac{L\mu}{2f_s} \times 100 \tag{1}$$

where

P_{sold} = percent steel required by using the old formula,
μ = friction factor,
L = length of the slab (ft), and
f_s = maximum allowable stress of steel (psi).

The derivation of this formula is given later, but a modification is necessary, because the subbase is the major parameter for the resulting frictional resistance. The current approach in the AASHTO guide assumes that the magnitude of frictional resistance is independent of the subbase type.

The friction factor values for certain subbase materials are included in Table 1. Table 1 also shows the push-off test results for the same subbase types from Project 459 (*2,3*).

The equilibrium condition of the slab after the first crack formation (approximately $L/2$ from the free end) is illustrated in Figure 1. The resultant friction force should be balanced with the load in steel. The equilibrium is formed as

$$P = P_F$$

where

P = force carried by the steel for a unit width of slab (lb) and
P_F = frictional force applied to the unit width of slab (lb).

The frictional force, simply, equals the contact area of one-half of the slab times the frictional resistance:

$$P_F = \frac{L}{2} \tau_F B$$

where

τ_F = frictional resistance (psi) and
B = width of the pavement (ft).

TABLE 1 SUBBASE FRICTION VALUES

Subbase Type	μ-Value from Guide	μ-Value[a] Measured	Frictional Resistance (psi)
Cement-Treated granular base	1.8	52.3	15.40
Flexible Subbase	1.5	5.0	3.37
Asphalt-stabilized granular base	1.8	3.8	2.20
Lime-treated clay	1.8	2.9	1.70
Untreated clay	0.9	1.9	1.10

[a]Based on 1st cycle of push-off tests

Converting the units of L and B from feet to inches, to be compatible with the rest of the equation, results in

$$P = \frac{144 \, L \, \tau_F \, B}{2} \tag{2}$$

where

P = the load of the longitudinal steel (lb),
τ_F = the frictional resistance (psi),
L = length of the slab (ft), and
B = width of the slab (ft).

We know that

$$P = A_s f_s \tag{3}$$

where

A_s = area of the longitudinal steel (in.²) and
f_s = maximum allowable stress of the longitudinal steel (psi).

By definition, percent of steel is the ratio of steel area to the concrete area, i.e.,

$$P_s = \frac{A_s}{A_c} \times 100 \tag{4}$$

or

$$A_s = \frac{P_s A_c}{100}$$

Since

$$A_c = 12 \, D \, B$$

then

$$A_s = \frac{12 \, P_s \, D \, B}{100} \tag{5}$$

If Equation 5 is inserted into Equation 3 and Equation 3 is equated to Equation 2, the following equation is obtained:

$$\frac{P_{snew} D \, 12 \, B \, f_s}{100} = \frac{144 \, L \, \tau_F \, B}{2} \tag{6}$$
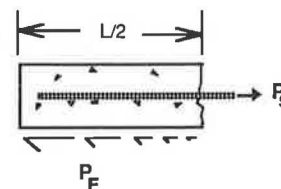

FIGURE 1 Equilibrium of forces after the first crack occurs.

where
P_{snew} = percent steel required by using the new formula.
Rewriting Equation 6 in terms of P_{snew},

$$P_{\text{snew}} = \frac{7200 \, L \, \tau_F \, B}{12 \, D \, Bf_s}$$

The width of the slab (B) is assumed to be the unit width, and it is automatically dropped from the formula. All the dimensions were converted to pound-inch units. After con-

version, the P_{snew} formula becomes

$$P_{\text{snew}} = \frac{600 \, L \, \tau_F}{Df_s} \qquad (7)$$

RESULTS

P_{snew} and P_{sold} for various subbase materials are compared in Figures 2 and 3. The data for these plots are given in Table
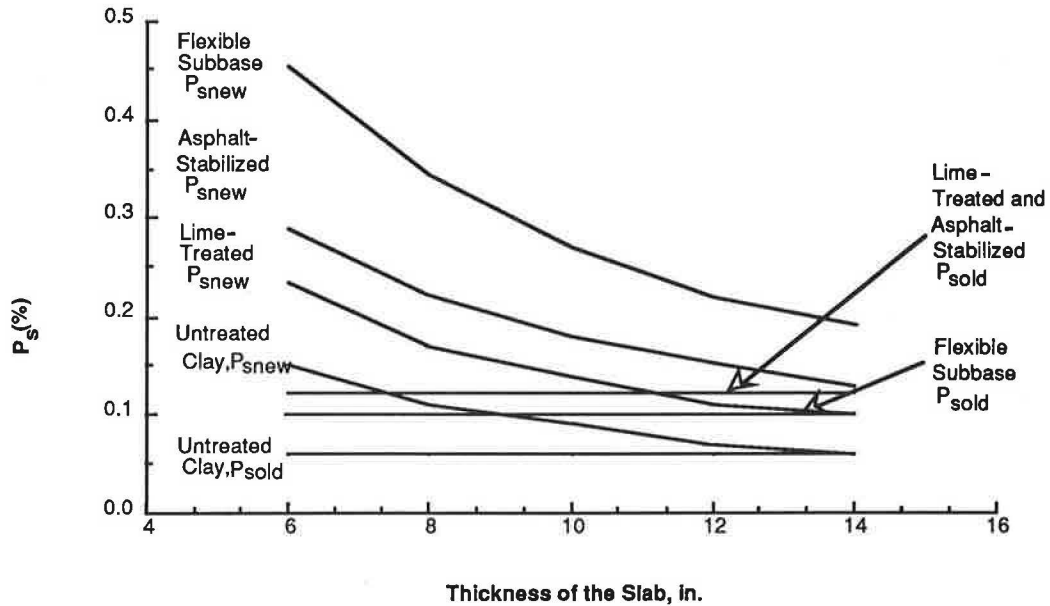

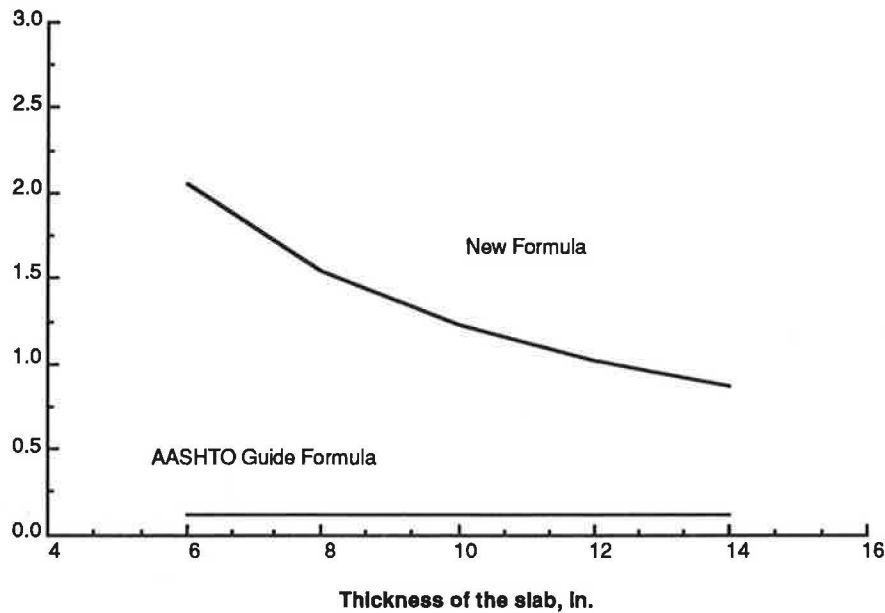**FIGURE 2 Steel percentage requirement of four different subbase materials, by using P_{snew} and P_{sold}.**

FIGURE 3 Steel percentage requirement of cement-treated base, by using P_{snew} and P_{sold}.

TABLE 2   DATA SHEET FOR P_{snew} VERSUS P_{sold} (f_s = 65,000 PSI)

Subbase Type	Length of Slab	μ	τ	D_{equiv}	P_{sold}[a] (%)	D=6	P_{snew} D=8	% D=10	D=12	D=14
Untreated	25	0.90	1.10	14.70	0.020	0.060	0.046	0.036	0.030	0.026
Clay	60				0.060	0.145	0.112	0.088	0.073	0.063
	80				0.080	0.194	0.150	0.117	0.097	0.084
	100				0.100	0.242	0.187	0.146	0.121	0.104
Asphalt-	25	1.80	2.20	14.70	0.050	0.121	0.094	0.073	0.060	0.052
Stabilized	60				0.119	0.290	0.224	0.176	0.146	0.125
	80				0.158	0.387	0.299	0.234	0.194	0.167
	100				0.198	0.484	0.374	0.293	0.242	0.209
Flexible	25	1.50	3.37	26.96	0.041	0.185	0.143	0.112	0.093	0.080
Subbase	60				0.099	0.445	0.344	0.269	0.222	0.192
	80				0.132	0.593	0.458	0.358	0.296	0.256
	100				0.165	0.741	0.573	0.448	0.371	0.320
Lime-	25	1.80	1.72	11.50	0.050	0.095	0.073	0.057	0.047	0.041
Treated	60				0.120	0.227	0.175	0.140	0.113	0.098
	80				0.160	0.307	0.234	0.183	0.151	0.131
	100				0.198	0.378	0.292	0.229	0.189	0.163
Cement-	25	1.80	15.35	102.30	0.050	0.844	0.652	0.510	0.422	0.364
Stabilized	60				0.119	2.026	1.566	1.225	1.013	0.875
	80				0.158	2.702	2.087	1.633	1.350	1.167
	100				0.198	3.377	2.610	2.041	1.688	1.458

[a] Percent reinforcement predicted by using the old formula.

2. As can be observed, P_{snew} is higher than P_{sold} for all the materials (for all the thicknesses), except for the slab on lime-treated base (LTB). For slab on LTB, P_{sold} becomes higher than P_{snew} if the slab thickness is 12 in. or higher. The old formula does not vary with thickness, whereas the new one does (Figure 4).

DISCUSSION OF RESULTS

As can be observed in Figures 2 and 3, P_{sold} becomes larger than P_{snew} when the thickness is greater than a certain value. A close examination of the steel percentage formula is necessary to see whether the old and new reinforcement formula can be combined.

Assume that we have a certain kind of subbase with a given frictional characteristic (i.e., that τ and μ are known). Then let us equate Equation 1 to Equation 7:

$$P_{sold} = P_{snew} \tag{8}$$

If we use the right-hand sides of both of the equations, Equation 8 becomes

$$\frac{L\mu}{2f_s} \times 100 = \frac{6L\,\tau_F}{Df_s} \times 100 \tag{9}$$

Rewriting Equation 9 in terms of D, and calling the thickness D_{equiv}, gives

$$D_{equiv} = 12\frac{\tau_F}{\mu} \tag{10}$$

which creates an equation for calculating the equivalent thickness. Any thickness less than D_{equiv}, the thickness calculated from Equation 10, for the given subbase types will create a higher P_{snew}, whereas greater thicknesses will create the opposite case, $P_{sold} > P_{snew}$, as illustrated in Figures 2 and 3.

In the derivation of the new steel percentage formula, frictional resistance was assumed to be constant throughout the length of the slab. The maximum frictional resistance occurs at the free end of the slab and then gradually decreases to zero at the center of the slab. In other words, assuming the maximum friction for the complete length of the slab will result in higher reinforcement values. Therefore, in reality, the representative frictional resistance should be less than that used in the derivation. However this requires an extensive use of the computer program for both material and environmental conditions. If an average frictional resistance, calculated by using the computer program, is used in the formula, it will be variable even for the same subbase type. This is mainly because of the dependency of frictional resistance on the slab movement. Therefore, the use of the above assump-
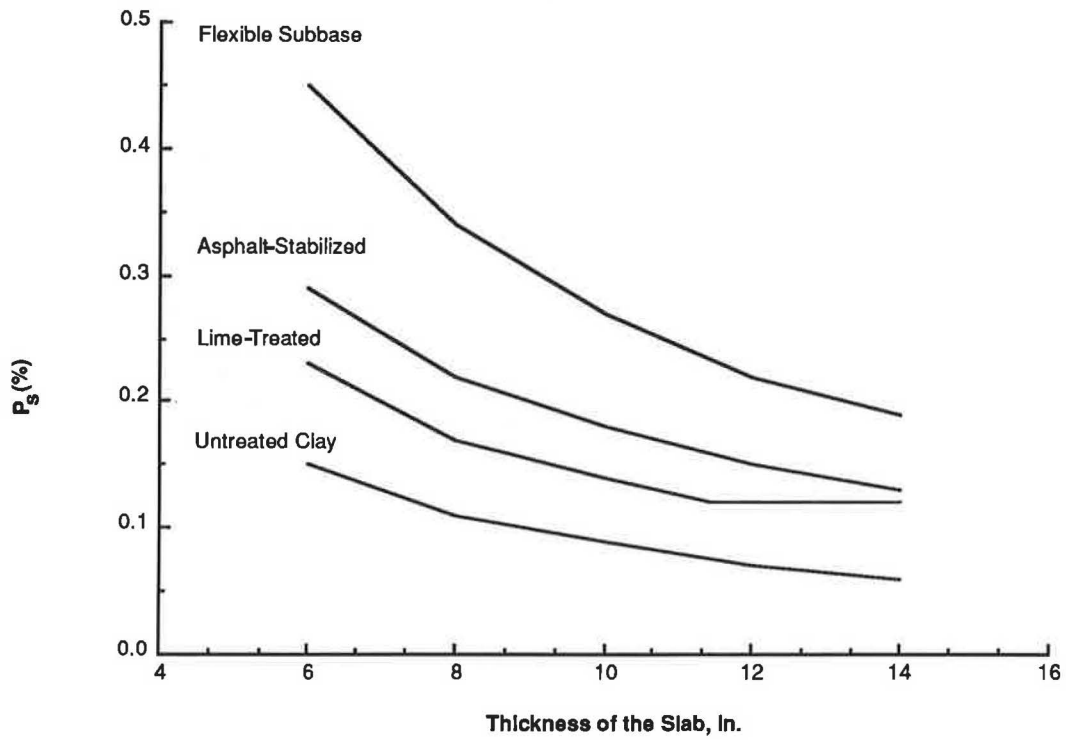
FIGURE 4 Combination of AASHTO P_s formula with the new P_s formula.
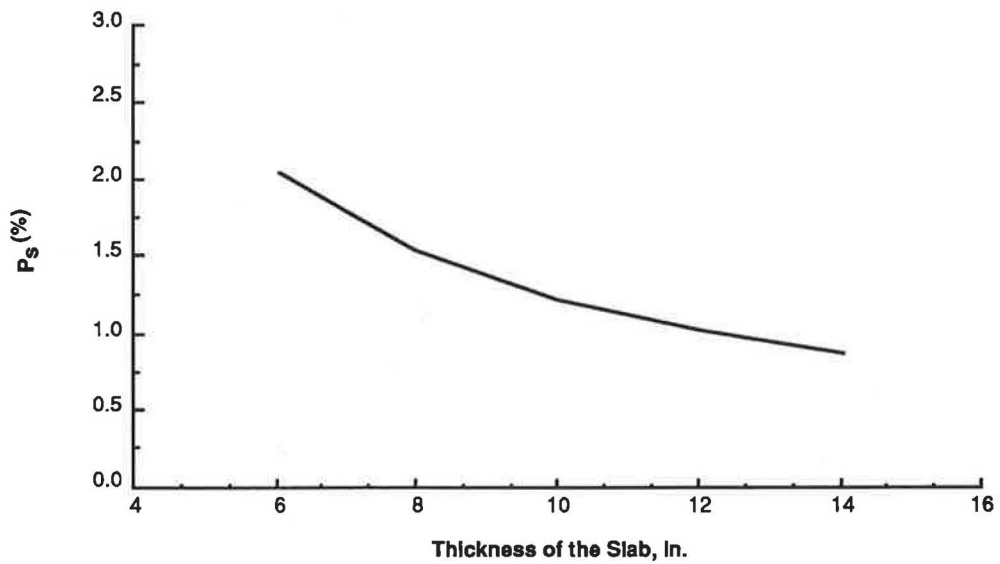

FIGURE 5 Combination of AASHTO P_s formula with the new P_s formula for cement-treated base.

tion is accepted as long as the formula is used as a guideline for the reinforcement design. Another important point is the high steel percentage requirement of the slab laid on cement-stabilized subbase (Figure 5). A solution to this problem is to use a bond breaker, which is, in fact, the asphalt-stabilized subbase illustrated in Figure 4. The use of a bond breaker will lower the steel percentage requirement considerably.

CONCLUSIONS

1. Previous subgrade drag theory is incorrect, because, instead of representing the frictional characteristics of the subbase, the friction factor, which resulted in a friction force, is merely a function of the slab thickness.

2. The rederived formula represents much better than the previous formula the actual frictional resistance of a range of available subbase types.

3. The use of the new formula yields an approximate result. The variation of friction along the slab length requires use of a program for calculating the friction force and, in turn, increasing the accuracy of the required steel percentage.

4. The P_{snew} formula is inadequate for D for thicknesses larger than D_{equiv}. The equivalent thickness is a function of the friction coefficient and the frictional resistance.

RECOMMENDATIONS

1. An interactive algorithm should be developed (like the one in the JRCP computer program) to calculate the actual frictional resistance corresponding to the slab movement. The current formula will be used until the interactive program is available.

2. The use of the P_{snew} formula yields higher tensile stresses in concrete because of the amount of restraint to relative movement, for $D < D_{equiv}$, so the use of P_{snew} will increase the number of cracks. This development requires further study to observe the effect on long-term pavement performance.

3. The combined P_{sold} and P_{snew} (Figures 4 and 5) for various thicknesses will not be a good substitute for the AASHTO Guide equation (Equation 1). Especially for cement stabilized subbases, it is not economically feasible to use up to 2 percent steel reinforcement to keep the cracks tighter. The high expense can be avoided by using a bond breaker to lower the subbase frictional resistance to a reasonable value.

4. A reasonable frictional resistance value, one that is lower than the maximum one, should be selected for the subbase type under consideration. In other words, the magnitude of frictional resistance is a function of slab movement. It varies from maximum at the free end to zero at the middle of the slab. Therefore, the maximum frictional resistance used in the formula is always higher than it should be.

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