MICH-PAVE: A Nonlinear Finite Element Program for Analysis of Flexible Pavements

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A nonlinear mechanistic finite element program called MICH-PAVE has been developed for use on personal computers to aid in the analysis and design of flexible pavements. The program has three major features. First, it utilizes a newly developed flexible boundary concept for pavement analysis. Second, it uses performance models for the prediction of fatigue life and rut depth, utilizing results of the mechanistic analysis (stresses, strains, and other pavement variables). Third, it is "user-friendly." The program was developed for the Michigan Department of Transportation and is in the public domain. The methodology used, and the features of the program, are described.

More and more in recent years pavement design is being based on mechanistic analysis. The migration from empirical methods to mechanistic analysis has been facilitated by the availability of relatively inexpensive microcomputers that can be used in daily practice. Early computer programs for mechanistic analysis (such as CHEV5L, BISAR, ELSYM5, etc.) modeled pavements as being composed of linear elastic layers and computed deflections, stresses, and strains within a pavement arising from a wheel load. In these programs each pavement layer is assumed to extend infinitely in the horizontal directions, allowing the three-dimensional problem to be reduced to an axisymmetric two-dimensional problem. Because of the linear elastic assumption, multiple wheel loads can be analyzed by superposing the results from single wheel loads.

The main drawbacks of these linear elastic layer programs are that

- They cannot model the nonlinear resilient behavior of granular and cohesive soils;
 - They normally assume weightless pavement material;
- They may yield tensile stresses in granular material, which cannot physically occur; and
- They do not account for "locked-in" stresses from compaction during construction.

To overcome these shortcomings, nonlinear analysis programs based on the finite element method have been developed (e.g., ILLI-PAVE). However, because of the large memory and computational effort requirements, these programs primarily have been implemented on mainframe computers. Fur-

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ther, the interaction of the user and these programs, in terms of data input and interpretation of the output, is not "friendly," making their use in daily practice undesirable.

For most state highway agencies, a "user-friendly" flexible pavement program that can be used for the design or rehabilitation of flexible pavements in daily practice is desired. This program should consider all the major factors affecting the design or rehabilitation of pavements. To achieve this, a research study was sponsored by Michigan Department of Transportation (MDOT). The main goal of this research was to review existing analysis and design methods and then to develop a user-friendly nonlinear finite element program that can be used on personal computers in daily practice. Since current personal computers have limited memory capacities, the traditional finite element method, which requires a large amount of memory, cannot be suitably implemented on them unless accuracy is sacrificed. To overcome this shortcoming, Harichandran and Yeh (1) proposed a new technique of placing a relatively shallow finite element mesh on a flexible boundary. This technique substantially reduces the memory and computational requirements of the nonlinear finite element method without sacrificing accuracy. In this research, this technique is implemented with user-friendly input and output features, to develop a nonlinear finite element program for the analysis and design of flexible pavements. The program has been named MICH-PAVE.

MATERIAL NONLINEARITY

There are many nonlinear material models that may be used in mechanistic analysis. Four suitable models—the hyperbolic, the resilient modulus, the shear and volumetric stress-strain (also called the contour model), and the third-order hyperelastic models—were reviewed (2-7). Of these, the resilient modulus model was chosen as the most suitable at the present time. This choice is based on the applicability of the model to repeated loading patterns experienced by pavements, and on the relative ease of determining the model parameters by state highway agencies. The resilient modulus model characterizes the resilient stress-strain properties of soils through a stress-dependent modulus and a constant Poisson ratio.

For granular soils, the resilient modulus is characterized as

$$M_r = K_1 \theta^{K_2} \tag{1}$$

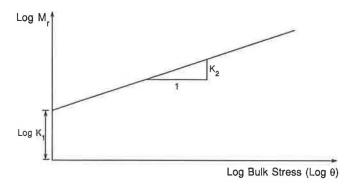


FIGURE 1 Resilient modulus model for granular material.

where M_r = resilient modulus (psi), $\theta = \sigma_1 + \sigma_2 + \sigma_3$ = bulk stress (psi), and K_1 and K_2 are material constants. This relationship is illustrated in Figure 1.

For cohesive soils, the resilient modulus is expressed through the bilinear relationship

$$M_r = \begin{cases} K_2 + K_3[K_1 - (\sigma_1 - \sigma_3)] & \text{for } K_1 > (\sigma_1 - \sigma_3) \\ K_2 + K_4[(\sigma_1 - \sigma_3) - K_1] & \text{for } K_1 \le (\sigma_1 - \sigma_3) \end{cases}$$

where $(\sigma_1 - \sigma_3)$ = deviator stress (psi), and K_1 , K_2 , K_3 , and K_4 are material constants. This relationship is illustrated in Figure 2.

If the finite element method is used with only the resilient modulus model, it will converge extremely slowly. Therefore, Raad and Figueroa (8) applied the resilient modulus model with the Mohr-Coulomb failure criterion. The Mohr-Coulomb failure criterion is used to modify the principal stresses of each element in the granular layers and roadbed soil after each iteration so as not to exceed the Mohr-Coulomb failure envelope. Thompson (9) used this algorithm in the ILLI-PAVE program. A similar algorithm was developed for MICH-PAVE.

FEATURES OF THE MICH-PAVE PROGRAM

The MICH-PAVE program is capable (depending on the user's choice) of performing either linear or nonlinear finite element analysis of flexible pavements. It assumes axisymmetric loading conditions arising from a single circular wheel load on pavement layers of infinite horizontal extent. The various features of the program are described in this section. Technical details and results from various sensitivity analyses can be found elsewhere (10,11).

Mesh Generation and Flexible Boundary

The finite element method needs to satisfy some basic requirements for the mesh, such as the location of the side and bottom boundaries, the size and shape of the elements, and the distribution of the elements in the various regions. To establish criteria for locating the boundaries in a finite element mesh,

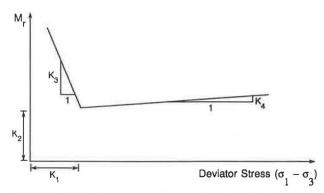


FIGURE 2 Resilient modulus model for cohesive material.

Duncan et al. (12) showed comparisons between displacements and stresses computed using the finite element method and those computed using elastic half-space and layered system analysis.

For an elastic half-space subjected to a uniform circular load, displacements and stresses computed by the finite element method compare well with those determined from the Boussinesq solution, when

- the bottom boundary in the finite element mesh is fixed at a depth of about 18 radii of the loaded area; and
- the vertical side boundary is located at a distance of about 12 radii from the center and constrained from moving radially.

For a three-layered system, a reasonable comparison between the two procedures can be obtained if the bottom boundary in the finite element mesh is moved to a depth of about 50 radii, while maintaining the side boundary at 12 radii.

Previous experience has shown that stresses based on quadrilateral elements will be accurate provided that the length-to-width ratio for the elements does not exceed five to one. Furthermore, smaller elements should be used close to the loaded area, and progressively larger elements may be used in the regions away from the loaded region.

Based on the above considerations and experience, the following mesh generation rules were used. In the radial direction, a mesh with a total width of 10 radii was divided into four zones. The first zone, between 0 and 1 radius, is equally divided into four elements; the second zone, between 1 radius and 3 radii, is equally divided into four elements; the third zone, between 3 radii and 6 radii, is equally divided into three elements; and the fourth zone, between 6 radii and 10 radii, is equally divided into two elements. The MICH-PAVE program will automatically generate the default values of the finite element mesh along the radial and vertical direction. A typical mesh is shown in Figure 3.

In the MICH-PAVE program, a flexible boundary is used instead of a fixed bottom boundary. Therefore, a mesh that is 50 radii deep is not required. Normally when a mesh depth of about 10 radii is used, sufficiently accurate results will be obtained. In general, if the flexible boundary is located too close to the top of the roadbed soil, the displacements may still be accurate, but the stresses may not be. If the boundary is placed too deep, the primary advantages of using the flexible boundary are lost. The flexible boundary is usually placed

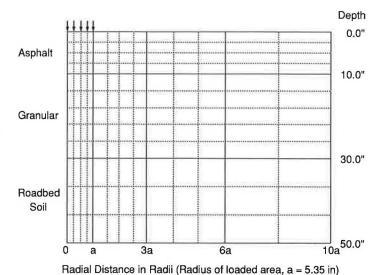


FIGURE 3 Typical finite element mesh.

at about 12 in. below the upper surface of the roadbed soil, or at a depth of 50 in., whichever is greater. In the MICH-PAVE program, the depth at which the flexible boundary is placed is specified by the user, by inputting the depth of the roadbed soil to which stress and strain calculations are required. If the boundary is placed at a depth of less than about 50 in., then the stresses still will be accurate, but the vertical deflections may be overestimated.

Modulus of Half-Space Below Flexible Boundary

The half-space below the flexible boundary is assumed to be homogeneous and linear elastic. The boundary is therefore always placed within the last layer (roadbed soil). Although the modulus of the roadbed soil below the boundary may in reality be stress dependent and vary from one location to another, in most pavement sections the stresses from wheel loads are substantially diminished at the level of the roadbed soil. Thus, the use of a constant modulus below the boundary has a negligible effect on the stresses and displacements above the boundary. In MICH-PAVE, the modulus used for the half-space below the boundary is the average moduli of the finite elements immediately above the boundary. To avoid undesirable "edge effects," the elements closest to the right vertical boundary are not used in computing this average.

Gravity and Lateral Stresses

The MICH-PAVE program includes the effect of gravity and lateral stresses arising from the weight of the materials. At any location within the pavement, the vertical gravity stress (σ_{σ}) is computed as the accumulation of the layer thicknesses multiplied by the appropriate unit weights.

The lateral stress (σ_h) is calculated from the coefficient of earth pressure at rest (K_0) and vertical gravity stress, as

$$\sigma_h = K_0 \sigma_g \tag{3}$$

where

 $K_0 = 1 - \sin \phi$ for cohesionless soil and gravel; $K_0 = 1 - 0.95 \sin \phi$ for cohesive soil; and

 ϕ = angle of internal friction.

To approximately account for "locked-in" stresses caused by compaction, the user can input a value for K_0 higher than the coefficient of earth pressure at rest.

Iterative Solution and Convergence Criterion

MICH-PAVE performs nonlinear analysis by the following steps:

- 1. Initially, the wheel load at the surface is assumed to spread over a 2:1 region (i.e., the radius of the loaded area is assumed to increase by 1 in. for every 2 in. of depth). The stresses arising from this assumed distribution of the wheel load are combined with gravity stresses, to compute the initial resilient moduli for each of the finite elements in granular and cohesive materials.
- 2. A linear analysis is performed, and a more accurate stress distribution is computed.
- 3. For those elements that exceed the Mohr-Coulomb failure criterion, the computed stresses are adjusted according to the procedure outlined by Raad and Figueroa (8).
- 4. A new value of the resilient modulus is computed for each element based on the latest stresses.
- 5. Convergence in the resilient moduli is checked by computing the relative error, $\varepsilon = \sum (M_{r,i} - M_{r,i-1})^2 / \sum M_{r,i-1}^2$, where $M_{r,i}$ are the current resilient moduli, $M_{r,i-1}$ are the previous resilient moduli, and the summations are taken over all nonlinear elements.
- 6. Steps 2 through 5 are repeated until the relative error ε is less than 0.001.

7. Displacements, strains, and stresses are then computed at all specified locations, and the fatigue life and rut depth of the pavement are estimated.

Interpolation and Extrapolation of Stresses and Strains at Layer Boundaries

For a pavement section in which various layers are fully bonded (i.e., when no slip occurs at layer boundaries), quantities such as the vertical and shear stresses and the radial and tangential strains should be continuous across layer interfaces. However, because of the low-order interpolation functions chosen in the finite element approach, these quantities are not continuous across element boundaries. Thus, if these quantities are estimated by finite element approach at two adjacent points across an interface, the results will show an apparent discontinuity that is an artifact arising from the error in the finite element method. This result is undesirable and can be overcome by using linear interpolation to estimate these quantities at an interface from those at the middle of the adjacent elements. For example, if σ_1 is the stress at the center of an element immediately above the interface and σ_2 is the stress at the center of an element immediately below the interface, then the stress at the interface obtained by linear interpolation is

$$\sigma = \sigma_1 + \frac{\sigma_2 - \sigma_1}{z_2 - z_1} (z - z_1) \tag{4}$$

where z_1 and z_2 are the depths of the points at which σ_1 and σ_2 are evaluated, and z is the depth of the interface.

Since the finite element approach gives accurate estimates of stresses and strains at the center of elements, but can yield significant error at element edges, even those stresses and strains at the interfaces that are discontinuous across the interface can be in significant error. For quantities such as the radial and tangential stresses that are discontinuous across an interface, it is possible to estimate their values at one side of the interface by linear extrapolation of the values at the center of two elements on that side of the interface. For example, if σ_1 and σ_2 are stresses at the center of two consecutive elements below (or above) an interface, then the linearly extrapolated stress at the lower (or upper) side of the interface is also given by Equation 4, where z_1 and z_2 are the depths of the points at which σ_1 and σ_2 are estimated, and z is the depth of the interface.

Finally, based on prior knowledge of the solution (vertical and shear stresses must be zero at the surface, except for the vertical stress below the wheel load, which must be identical to the tire pressure), the surface stresses are arbitrarily set to their proper values in MICH-PAVE.

The improvement in accuracy obtained through interpolation and extrapolation is illustrated by considering a typical pavement section for linear analysis using the MICH-PAVE and CHEV5L programs. (Results from CHEV5L are accurate for linear analysis and are used as the benchmarks.) In this example, a wheel load of 9,000 lb and a tire pressure of 100 psi were used, with material having the following material properties: Layer 1—AC, E=300,000 psi, $\nu=0.4$; thickness = 8 in.; Layer 2—Base, E=20,000 psi, $\nu=0.38$, thickness = 12 in.; and Layer 3—Roadbed soil, E=8,000 psi, $\nu=0.45$, semi-infinite depth.

A comparison of stresses computed by CHEV5L, and by MICH-PAVE before and after extrapolation, are given in Table 1. Since the materials are assumed to be weightless, gravity stresses are neglected in both programs. Similar comparisons for stresses computed before and after interpolation, and adjustment of surface stresses, are given in Table 2. The plus and minus symbols used for depth indicate points just above and below interfaces. It is clear that interpolation and extrapolation improve the accuracy of these stresses. Similar improvements are obtained for strains.

Equivalent Resilient Moduli for Linear Analysis

In some cases it may be desirable to compare the results of nonlinear finite element analysis with those from a linear analysis or those of elastic layer programs such as CHEV5L, or to utilize the speed or multiple-wheel load capabilities of existing linear analysis programs. In such cases it would be desirable to input to the linear program an equivalent resilient modulus for each layer that reflects the true stress-dependent variation of the modulus within the layer. The equivalent moduli can be estimated from a nonlinear finite element analysis, using the various resilient moduli in each finite element.

MICH-PAVE computes an equivalent resilient modulus for each pavement layer that is obtained as the average of the moduli of the finite elements in the layer that lie within an assumed 2:1 load distribution zone, as shown in Figure 4. The average moduli of those elements within the regions ABGH, BCFG and CDEF are taken as the equivalent moduli for layers 1, 2 and 3, respectively.

TABLE 1 COMPARISON OF RADIAL AND TANGENTIAL STRESSES AT 67 IN. FROM THE CENTER OF THE LOADED AREA

Depth (in)	Radia	al stress	(psi)	Tangential stress (psi)				
	Before	After	CHEV5L	Before	After	CHEV5L		
0	-199.02	-167.91	-172.8	-199.02	-167,91	-173.0		
8(+)	124.65	106,31	111.6	124.65	106,31	111.9		
8(-)	1.84	0,35	0.22	1.84	0.35	0.24		
20(+)	2,53	2,49	3,52	2.53	2,49	3,53		
20(-)	-0.66	-0.47	-0,23	-0.66	-0,47	-0.23		

Depth	Verti	cal stress	(psi)	Shear stress (psi)				
(in)	Before	After	CHEV5L	Before	After	CHEV51		
0	143.57	100.0	99.7	2.62	0,0	0.0		
8(+)	0.0	12.70	12.25	3.29	2,69	0,55		
8(-)	9.15	12,70	12.25	0,65	2.69	0.55		
20(+)	3.03	3.10	3,36	0.12	0,11	0.07		

3.36

3 10

TABLE 2 COMPARISON OF VERTICAL AND SHEAR STRESSES AT 67 IN. FROM THE CENTER OF THE LOADED AREA

The use of equivalent resilient moduli obtained through nonlinear analysis utilizing the MICH-PAVE program, in a linear program (CHEV5L), is investigated through typical examples.

20(-)

Tables 3 and 4 show the material properties of a 12-in. fulldepth asphalt concrete (AC) section, and a three-layer section with 3 in. of AC and 12 in. of granular material on roadbed soil (RS). The material properties shown in the tables, including the modulus, Poisson ratio (ν) , lateral earth pressure coefficient (K_0) , material constants $(K_1, K_2, K_3, \text{ and } K_4)$ and density have been obtained previously (5). A cohesion of 6.45 psi and an angle of friction of 0 degrees were used for the roadbed soil in the full-depth section. For the three-layer section, c = 0 psi and $\phi = 45$ degrees were used for the granular material and c = 6.45 psi and $\phi = 0$ degrees were used for the roadbed soil. For the granular material and roadbed soil, the equivalent resilient moduli are also shown in the tables. In the MICH-PAVE analysis, the flexible boundary was placed beneath the roadbed soil, at depths of 52 in. and 60 in. for the full-depth asphalt and three-layer sections, respectively.

The sections were analyzed for a wheel load of 9,000 lb and a tire pressure of 100 psi. Figures 5 and 6 show comparisons of surface deflections for nonlinear analysis using MICH-PAVE, and the linear analysis using CHEV5L with the equiv-

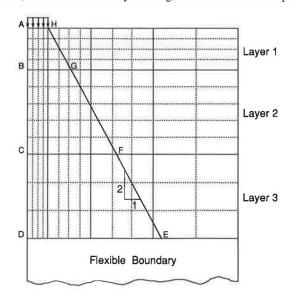


FIGURE 4 Elements used to compute equivalent resilient moduli.

alent moduli. Figures 7 and 8 show comparisons of the vertical and radial stresses at 0.5 in. from the center of the loaded area. Deflections and stresses computed by the nonlinear finite element program ILLI-PAVE are also shown in these figures.

0.07

The results indicate that the use of equivalent resilient moduli obtained from nonlinear analysis, in a subsequent linear analysis, gives reasonably similar displacements. Notable differences are obtained in the stresses, because the linear analysis using CHEV5L neglects the weight of the material. In particular, linear analysis can give tensile stresses in granular soils.

The stresses computed using MICH-PAVE and ILLI-PAVE are almost identical. However, ILLI-PAVE tends to give lower surface deflections than MICH-PAVE. Linear analysis, for which accurate results can be obtained through CHEV5L, has shown that displacements obtained by using a flexible boundary (as in MICH-PAVE) are slightly greater and more accurate than those obtained by using a deep fixed boundary (as in ILLI-PAVE) (10,11). Using a deep finite element mesh tends to produce a "stiffening" effect, resulting in smaller displacements.

For the three-layer pavement, the variation of modulus within the granular layer and roadbed soil is shown in Table 5 for the first eight elements from the left boundary. The depth and radial distance to the middle of the elements are tabulated. The finite element mesh was similar to that shown in Figure 3, except that the thicknesses of the layers are as given in Table 4. It is apparent that the modulus decreases with depth and radial distance within the granular layer. This is because the bulk stress (θ) decreases with depth and radial distance; hence, according to Equation 1, the resilient modulus follows the same pattern. Within the roadbed soil, however, the modulus increases with depth and radial distance. This increase is because the deviatoric stress $(\sigma_1 - \sigma_3)$ decreases with distance away from the wheel load; hence, according to Equation 2, the modulus must also do the same. In MICH-PAVE, the modulus of the half-space below the flexible boundary is taken to be the average modulus of the elements immediately above the boundary. Care must be taken in MICH-PAVE to use a thickness for the roadbed soil that is sufficiently deep; otherwise the modulus of the half-space below the flexible boundary may turn out to be too small, yielding large displacements. It is recommended that the depth from the pavement surface to the flexible boundary (which is determined by the user inputs of layer thicknesses including that of the roadbed soil) be about 50 in. Larger depths have no appreciable effect on displacements.

TABLE 3 MATERIAL PROPERTIES FOR THE FULL-DEPTH AC SECTION

Layer Type	Thickness (inches)	Modulus (psi)	iv.	Ko	K.1	К2	K3	K.a	Density (pcf)
AC	12	500,000	. 4	. 67					150
RS	40	8,753*	,45	.82	6.2	3021	1110	-178	115

^{*} Equivalent resilient modulus

TABLE 4 MATERIAL PROPERTIES FOR THREE-LAYER SECTION

Layer Type	Thickness (inches)	Modulus (psi)	ν	Ko	К1	K ₂	К ₃	K ₃	Density (pcf)
AC	3	500,000	-4	.67					150
Granular	12	22,543*	.38	₂ 60	5,000	. 5			140
RS	45	7,435*	-,45	82	6,2	3021	1110	-178	115

^{*} Equivalent resilient moduli

Fatigue and Rut-Depth Predictions

Results from the nonlinear mechanistic analysis, together with other parameters, are used as input to two performance models (fatigue life and rut depth) derived on the basis of field data (13), to predict the fatigue life and rut depth of flexible pavements. The models relate the fatigue life and rut depth to the number of equivalent 18-kip single-axle loads, surface deflection, moduli and thicknesses of the layers, percent air voids in the asphalt, tensile strain at the bottom of the asphalt layer, average compressive strain in the asphalt layer, kinematic viscosity of the asphalt binder, and average annual air temperature. Fatigue life and rut depth are useful design parameters on which to base the selection of a suitable pavement section.

User-Friendly Features

MICH-PAVE is designed with menus, data-entry forms, and on-screen plots, to facilitate easy input of data and interpretation of results. It also has extensive error-trapping features. Menus facilitate the selection of various stages of a typical pavement analysis, such as the specification of new data, mod-

ification of existing data, analysis, plotting of results, and subtasks within these. The spreadsheet-like data-entry forms allow input data to be entered or edited easily with no format requirements. When possible, suitable data for typical pavements are suggested on the screen. New or changed data are immediately checked for errors, and the user is prompted for corrections. Output from the analysis, such as stresses, strains, and displacements at specified locations, is saved in a file and may also be plotted on the screen. These features make the program easy to use in daily practice.

The program has been structured and optimized for speed, within the memory limitations of present personal computers (640 KB RAM). The total analysis time for typical three- and four-layer pavements is about 2 and 3.5 minutes, respectively, on an IBM AT-compatible computer with an 80287 math coprocessor.

PROGRAM AVAILABILITY

The MICH-PAVE program is in the public domain and may be obtained on request from Larry Heinig, Michigan Department of Transportation, Division of Testing and Research, P.O. Box 30049, Lansing, Mich. 48909. A user's manual is also available (14).

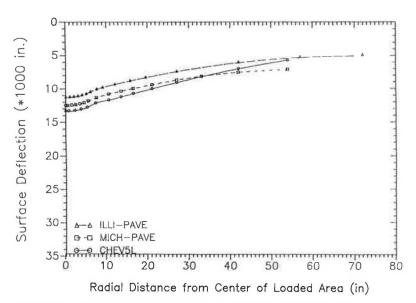


FIGURE 5 Comparison of surface deflection (12-in. full-depth AC).

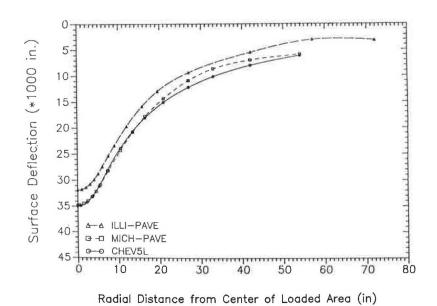


FIGURE 6 Comparison of surface deflection (3-in. AC and 12-in. base).

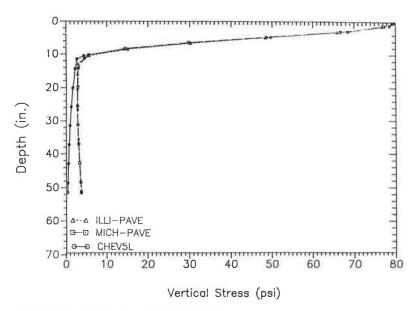


FIGURE 7 Comparison of vertical stress (12-in. full-depth AC).

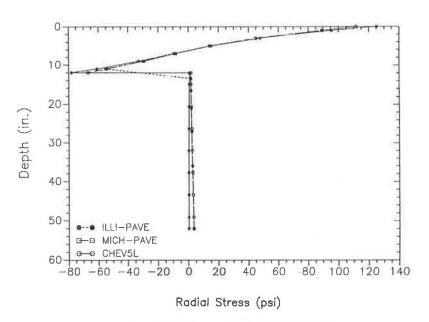


FIGURE 8 Comparison of radial stress (12-in. full-depth AC).

TABLE 5 VARIATION OF MODULUS (PSI) IN GRANULAR AND ROADBED SOIL LAYERS

T	Depth	Radial Distance from Center of Wheel Load (inches)									
Layer	(in.)	0,67	2.01	3.35	4,68	6.69	9.37	12.04	14.72		
	4.2	36,800	35,400	32,200	27,800	22,100	15,300	12,000	10,300		
	6.6	31,600	30,500	28,600	26,100	20,400	16,000	12,700	10,900		
Gran.	9.0	25,800	25,200	24,200	22,700	20,300	15,500	13,200	11,500		
	11.4	21,400	21,100	20,600	19,700	18,500	16,100	13,000	11,900		
	13.8	18,200	18,100	17,800	17,400	16,800	15,500	14,000	12,100		
RS	22.5	5,640	5,660	5,690	5,750	5,860	6,040	6,290	6,580		
	37.5	7,340	7,350	7,360	7,380	7,430	7,520	7,630	7,750		
	52.5	8,250	8,250	8,260	8,260	8,270	8,300	8,340	8,380		

CONCLUSIONS

The features and methodology used by a state-of-the-art nonlinear finite element program for the analysis of flexible pavements, MICH-PAVE, are described. The analysis accounts for material nonlinearity, the unbound nature of granular soils, and "locked-in" lateral stresses arising from compaction. By utilizing a flexible bottom boundary in the finite element model, the memory and computational effort required by nonlinear analysis are drastically reduced, allowing the program to be used on personal computers. The program is designed to be user friendly, making it suitable for use in daily practice.

Results from the MICH-PAVE program have been compared with those from CHEV5L and ILLI-PAVE. For linear analysis, MICH-PAVE and CHEV5L give similar strains and displacements. The stresses computed by MICH-PAVE, however, account for the weight of pavement materials that are neglected by CHEV5L. For nonlinear analysis, MICH-PAVE and ILLI-PAVE give very similar stresses, but displacements computed by ILLI-PAVE are smaller than those computed by MICH-PAVE. It is believed that the use of a flexible boundary in MICH-PAVE alleviates the "stiffening" effect that results when a deep finite element mesh with a fixed boundary is used.

A method of obtaining a constant equivalent modulus for a pavement layer with a stress-dependent resilient modulus is outlined. The equivalent modulus, estimated from the results of nonlinear analysis, may be used with linear analysis programs to yield surface deflections that would be similar to those computed by nonlinear analysis.

Results of the mechanistic analysis are used as input to two pavement performance models developed using field data. The models employ computed stresses and strains, as well as the average annual temperature and the kinematic viscosity and air voids of the asphalt binder.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to Michigan Department of Transportation (MDOT) for providing the financial support. The contribution of MDOT personnel, and the suggestions made by participants of the 4-week NHI course held at Michigan State University in February 1989, are greatly appreciated.

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Publication of this paper sponsored by Committee on Flexible Pavement Design.