Probabilistic Modeling of Flexible Pavements

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A number of uncertainties and random factors play a role in the deterioration process of pavements under the effect of traffic. A probabilistic approach can make allowance for the stochastic nature of this process and provide a better rationale for the design methods. An extension in this direction of the empirical design method derived from the AASHO road tests for flexible pavements with unbounded granular layers is presented. The probabilistic development addresses two aspects of the problem. First, the reliability assessment of a pavement section is obtained by considering the Shook and Finn design equation as a function of two random variables—the expected number of traffic loads and the California bearing ratio (CBR) of the subgrade soil. The formulation is a second-order, second-moment development of this equation. It requires knowledge of only the means and standard deviations of the random variables. This model compares favorably with a statistical analysis of the AASHO test results. A sensitivity analysis indicates that the CBR variability has a dramatic influence on the pavement reliability. The uncertainty on the expected traffic loads has little effect on the reliability for a large number of axle loads. Hence, the mean value of the expected traffic load is a sufficient estimate of this parameter. Second, an analytical model is formulated for the coefficients of equivalence of the (unbounded) granular materials of the base and subbase courses, using the theory of stochastic stress propagation in particulate media. It is shown that these coefficients are representative of the ability of the granular course to spread the applied load in a diffusion process. They are expressed as functions of the angle of internal friction of the material, and a modified formulation is derived for the equivalent thickness of the pavement.

A number of uncertainties and random factors play a role in the deterioration process of pavements under the effect of traffic. A probabilistic approach can make allowance for the stochastic nature of this process and provide a better rationale for the design methods. This paper presents an extension in this direction of the empirical design method derived from the AASHO road tests for flexible pavements with unbounded granular layers. The Shook and Finn model (*1*), which accounts for the bearing capacity of the subgrade, was used.

The probabilistic development addresses two aspects of the problem:

1. The reliability of a pavement section as a function of the variability of its subgrade bearing capacity and expected traffic load; and
2. The fundamental relationship between the coefficients of equivalence of granular materials of the pavement and their mechanical properties.

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BACKGROUND

A basic equation resulting from the AASHO road tests (*2*) expressed the decrease of the trafficability index (p) of a given flexible pavement structure as a function of the number of applied loads:

$$p = c_0 - (c_0 - c_1) \cdot (W/\rho)^{\beta} \tag{1}$$

where

c_0 and c_1 = initial and final values of the trafficability index,
W = number of applied equivalent axle loads, and
ρ = maximum capacity of the structure, in number of axle loads.

The empirical coefficients ρ and β in Equation 1 were related to the mechanical properties of the pavement structure through its equivalent thickness, written in the case of a three-layer structure resting on a semiinfinite subgrade as

$$D = a_1 D_1 + a_2 D_2 + a_3 D_3 \tag{2}$$

where D_1, D_2, and D_3 are the respective thicknesses of the asphaltic concrete wearing course, gravel base course, and sandy gravel subbase (Figure 1). The regression coefficients, a_1, a_2, and a_3 are the coefficients of equivalence of the materials. They represent the relative contribution of each material to the overall strength of the structure. This concept of equivalent thickness implies that two structures built with different materials and course thicknesses, but exhibiting identical D values, would have the same rate of decrease of their trafficability. The validity of this statement should, however, be considered in light of the particular conditions under which the tests were performed. These are, for example, related to the climatic context, the available subgrade bearing capacity, the drainage conditions, and the types of structures experimented. This later aspect is particularly noteworthy because the failures observed in the AASHO road tests occurred often by loss of bearing capacity of the subgrade.

It was shown by Skok and Finn (*3*) that a strong correlation exists between the vertical stress on the subgrade calculated according to the elastic multilayer theory and the equivalent thickness found from the empirical model of the AASHO tests. This hypothesis was corroborated by additional parametric studies (*4,5*). Therefore, it seems reasonable to consider the propagation of vertical loads in the subgrade as the dominant factor of the observed mechanisms.

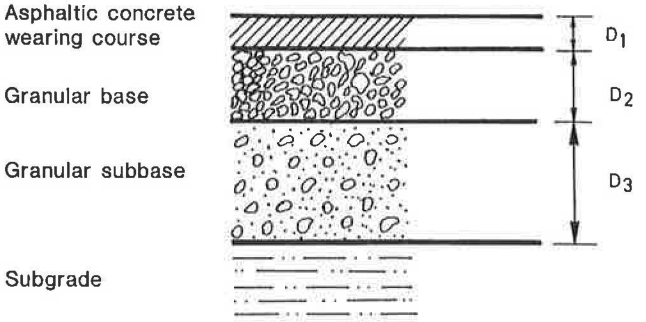

FIGURE 1 Typical four-layer system of flexible pavement tested in AASHO road tests.

RELIABILITY-BASED DESIGN

A measure of the bearing capacity of the subgrade was introduced into the AASHO empirical model by Shook and Finn (*1*), thus enabling the design method to be applied to geotechnical conditions differing from those of the AASHO tests. Their modified model related the necessary equivalent thickness of a pavement structure to the number of equivalent axle loads, and the CBR of the subgrade, as

$$D = f(W, L_1, L_2) \cdot (CBR_0/CBR)^{0.4} \tag{3}$$

In Equation 3, CBR_0 denotes the bearing ratio measured during the AASHO tests, and CBR is the actual bearing ratio of the considered subgrade. The function f resulted from a regression analysis:

$$f = -20.5 + 2.07 \ln W + 0.669 L_1 + 0.0932 L_1 L_2 \tag{4}$$

where

L_1 = axle load and
L_2 = 0 for single axle, 1 for double axle.

In the above equations, D should be expressed in inches (1 in. = 25.4 mm) and L_1 in kip (1 kip = 4.448 kN). Equation 4 does not include the safety factor originally applied by Shook and Finn to the number of axle loads.

A major uncertainty involved in the design of pavement structures arises from the assessment of the subgrade bearing capacity because of the large scattering encountered in CBR measurements. It is not unusual to observe coefficients of variation of this parameter (i.e., the ratio of the standard deviation over the mean value) in the range of 20 to 40 percent. For example, according to European road design codes (e.g., as noted by others [*6*]), a roadbed can be considered a homogeneous section even if the coefficient of variation of its bearing ratio is as high as 50 percent. Uncertainty is also involved in predicting the number of traffic loads (W) to be carried by a given road section during its expected life.

Second-Moment Approximation

The framework of a probabilistic model was suggested recently (*7*) to assess the reliability of a road section when the above uncertainties are considered. A comprehensive presentation of this approach is given herein.

The reliability of a pavement section, i.e., the probability of its survival under prescribed operating conditions, is defined as the probability that the equivalent thickness D required at any time during the life of the pavement will be less than the design value D^*. From another standpoint, provided that the CBR is a spatially uncorrelated process in the longitudinal direction, the probability of D exceeding D^* can be interpreted as the fraction of the considered pavement section length that will be unable to carry the expected number of axle loads for the whole lifetime of the pavement.

Seldom are the complete statistical distributions of engineering properties accessible. Fortunately, significant information is contained in the first moment about the origin of a random variable, the mean, and its second moment about the mean, the variance. Considering the necessary equivalent thickness (D) as a function of the two random variables (CBR and W), its mean and variance can be expressed as functions of the means and variances of the random variables. A second-moment representation of D is obtained by expanding Equation 3 into Taylor series truncated after the quadratic terms. A detailed description of this method can be found elsewhere (*8,9*). The following approximations result for the mean and variance of D, respectively:

$$\mu_D = D(\mu_W, \mu_{CBR}) + \frac{1}{2}\frac{\partial^2 D}{\partial W^2}(\mu_W, \mu_{CBR}) \cdot s_W^2 + \frac{1}{2}\frac{\partial^2 D}{\partial CBR^2}(\mu_W, \mu_{CBR}) \cdot s_{CBR}^2 \tag{5}$$

$$s_D^2 = \left(\frac{\partial D}{\partial W}(\mu_W, \mu_{CBR})\right)^2 \cdot s_W^2 + \left(\frac{\partial D}{\partial CBR}(\mu_W, \mu_{CBR})\right)^2 \cdot s_{CBR}^2 \tag{6}$$

where μ and s denote, respectively, the mean and standard deviation of the variables written in subscripts.

Introducing Equation 3 in Equations 5 and 6 gives

$$\mu_D = \frac{1}{(\mu_{CBR})^{0.4}} \cdot (1.531 f(\mu_W) - 1.585 V_W^2 + 0.429 f(\mu_W) V_{CBR}^2) \tag{7}$$

$$s_D^2 = \frac{1}{(\mu_{CBR})^{0.8}} \cdot (10.043 V_W^2 + 0.374 f^2(\mu_W) V_{CBR}^2) \tag{8}$$

with

$$f(\mu_W) = -20.5 + 2.07 \ln(\mu_W) + 0.669 L_1 + 0.0932 L_1 L_2 \tag{9}$$

where V_{CBR} and V_W are the coefficients of variation of the CBR and number of axle loads, respectively.

Based on the knowledge of only its two first statistical moments, the function D may be modeled as a normal (Gaussian) variate. However, considering that the necessary equivalent thickness can take only finite positive values, the use of

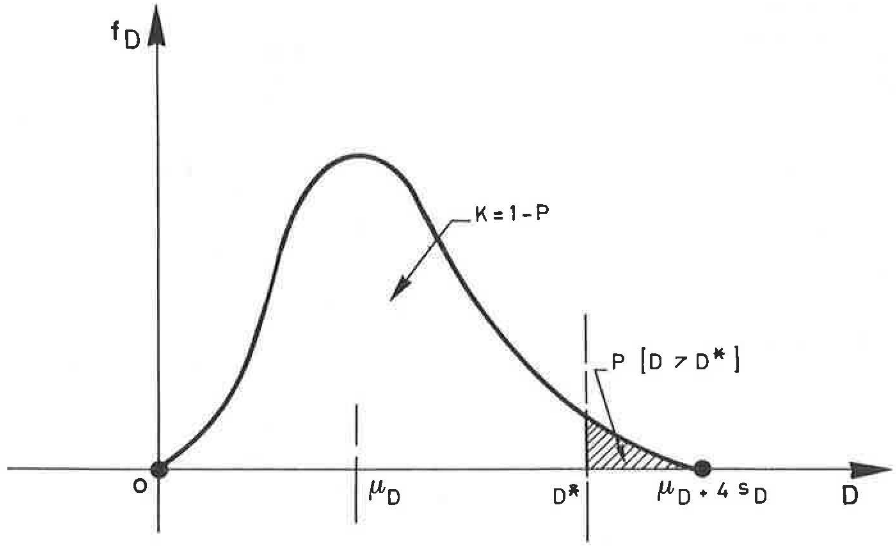

FIGURE 2 Beta distribution model of the necessary equivalent thickness (D) where μ_D is the mean value of D, s_D is the standard deviation of D, D^* is the design value of D for a reliability K, and P is the probability of distress.

the Beta distribution is more appropriate in this case. In effect, the versatility of the Beta distribution function allows for the modeling of bounded random variables. A comprehensive discussion on the engineering application and benefit of this type of distribution function can be found elsewhere (*10*). For the purpose of the reliability computations, the lower and upper bounds of the Beta distribution for D were selected as zero and four times its standard deviation above the mean, respectively. A typical distribution for D is shown in Figure 2 with a sketch of the probability of distress, $P[D > D^*]$. The reliability is given by

$$k = 1 - P[D > D^*] \tag{10}$$

From a design standpoint, the value of the required level of reliability should be selected and, using the distribution of D, the corresponding value of the design equivalent thickness computed. It involves finding the value D^* corresponding to a selected shaded area in Figure 2 (easy-to-use charts or computer software exists to perform this simple operation for the Beta distribution).

Parametric Study

A parametric study was carried out using the proposed model, to investigate the sensitivity of the required thickness for the asphaltic concrete wearing course (D_1) to the variability of the random variables (CBR and W). The wearing course thickness (D_1) was computed according to Shook and Finn (*1*) as

$$D_1 = \frac{D^* - D_2 - 0.75D_3}{2} \tag{11}$$

where D_2 and D_3 are the base and subbase course thicknesses, respectively.

The data used in the parametric study are summarized in Table 1, and the results are presented in Figure 3*a* and *b*. The thickness needed for the wearing course increases rapidly with the coefficient of variation of the subgrade CBR and with the required reliability of the pavement. For example, for a desired reliability level of 95 percent, D_1 must be increased from about 2 in. to more than 3 in. if the coefficient of variation of the CBR increases from 20 to 40 percent. On the other hand, the uncertainty on the number of axle loads has practically no influence on the design (horizontal lines of D_1 versus V_W for constant V_{CBR} in Figure 3*b*). This is a conse-

TABLE 1 DATA USED IN THE PARAMETRIC STUDY

Thickness:
D_2 = 5.9 in. (15 cm)
D_3 = 9.8 in. (25 cm)
CBR of Subgrade:
μ_{CBR} = 10
V_{CBR} = 40% when V_W variable
Expected Traffic:
L_1 = 18 kip
L_2 = 0 (single axle)
μ_W = 10^7 axles
V_W = 15% when V_{CBR} variable

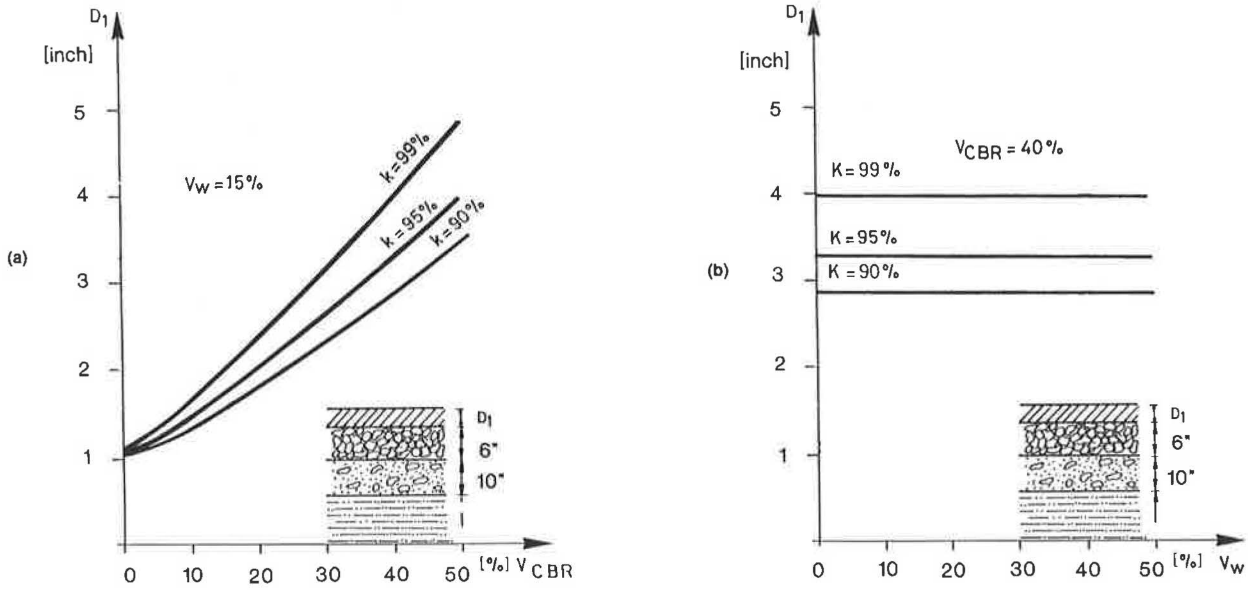

FIGURE 3 Parametric study: influence of the variability of the CBR (*a*) and expected traffic (*b*) on the required thickness (see data in Table 1) where V_{CBR} is the coefficient of variation of the bearing ratio of the subgrade, V_W is the coefficient of variation of the expected traffic loads, and D_1 is the thickness of the wearing course required for a reliability *K*.

quence of the logarithmic form of Equation 4. Thus, the proposed model may be simplified by representing the expected traffic load (W) in Equations 7 and 8 by its mean value only (μ_W). This simplification leads to the following expressions for the mean and standard deviation of the necessary equivalent thickness:

$$\mu_D = \frac{f(\mu_W)}{(\mu_{CBR})^{0.4}} \cdot (1.531 + 0.429\ V_{CBR}^2) \qquad (12)$$

$$s_D = \frac{0.612}{(\mu_{CBR})^{0.4}} \cdot f(\mu_W) \cdot V_{CBR} \qquad (13)$$

Application to the AASHO Road Tests

The statistics of the subgrade in situ *CBR* reported for the AASHO road tests indicated a mean value of 2.9 and a coefficient of variation of 35 percent (Figure 4). On this basis, the proposed probabilistic method can be applied to the case of the AASHO tests, using the coefficients of equivalence of the materials recommended by Shook and Finn (*1*), i.e., $a_1 = 2$, $a_2 = 1$, and $a_3 = 0.75$.

The results are summarized in Figure 5 and compared with the experimental data for the range of applied traffic loads. The theoretical curve obtained for a reliability of 95 percent leads to design values of D that are 4 in. larger than the experimental mean values. From their statistical analysis of the test results, Shook and Finn also concluded that to cover the experimental data points with a 95 percent confidence level, the average regression line had to be shifted by an amount of 4 in.

COEFFICIENTS OF EQUIVALENCE FOR THE GRANULAR MATERIALS

In the previous section, it was shown how the reliability of a flexible pavement structure can be evaluated for a given value of its equivalent thickness. In practice, the design of a structure at a given level of reliability requires not only the assessment of its necessary equivalent thickness, but also the determination of the resulting thicknesses for the individual layers. An empirical linear relationship, Equation 2, was derived from the AASHO road tests to relate these quantities to the equivalent thickness. Numerical values fitting the AASHO test data were proposed for the coefficients of equivalence of the materials, such as in Equation 11. However, the use of these empirical values for materials differing from those of the experimented structures is questionable. To date, the coefficients of equivalence have not been related explicitly to the engineering properties of the materials. An analytical model is formulated herein to accomplish this.

Theory of Stochastic Stress Diffusion

The conventional approach to assess the transmission of traffic load through a pavement system is to compute stresses by using the theory of linear elasticity. Such solutions require the assumption of an equivalent continuum. Furthermore, unless anisotropy is explicitly introduced, the elastic solutions are unable to account for the state of compaction of the granular materials and its effect on the distribution of stresses in the pavement layers.

In the present model, the transmission of applied load through the pavement layers is described using the theory of stochastic

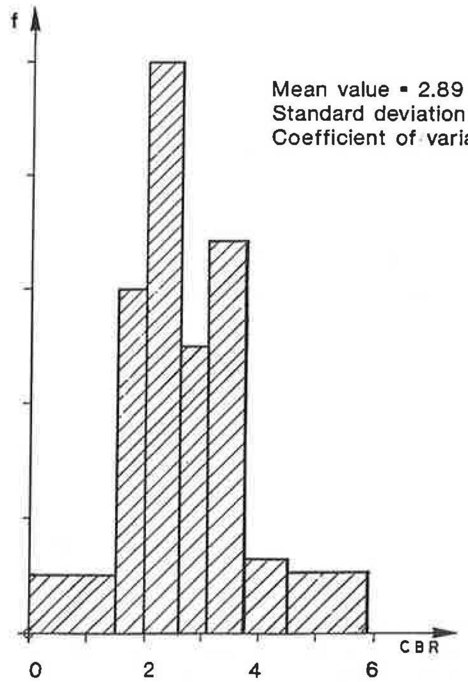

FIGURE 4 Statistics of the subgrade CBR in the AASHO road tests.

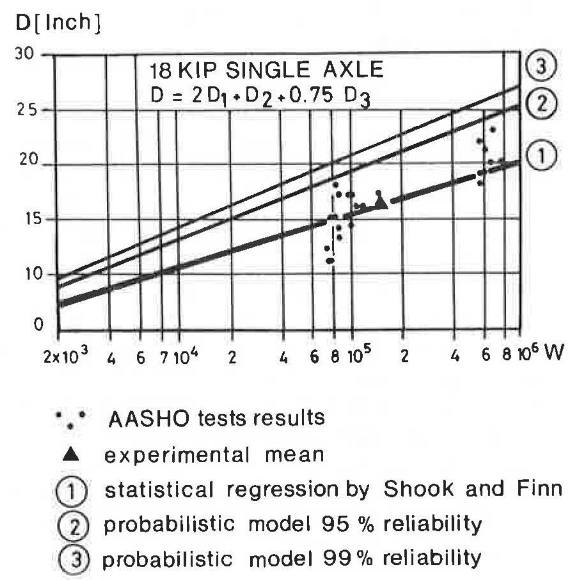

FIGURE 5 Application of the reliability-based model to the AASHO road tests (modified after Shook and Finn [*1*]). *D* is equivalent thickness and *W* is number of applied axle loads.

stress diffusion in particulate media developed by Sergeev (*11*) and Harr (*12*). A detailed formulation of the theory can be found elsewhere (*13,14*). Instead of assuming continuity and homogeneity of the medium, its particulate and inherently random nature is recognized, and boundary value problems are solved by using probabilistic arguments. The transmission of applied loads is the result of the propagation of contact forces between particles. This can be modeled as the progression with depth of a vertical force undergoing random lateral fluctuations when proceeding from a particle to its neighbor. For two-dimensional plane strain conditions, this model leads to a diffusion-type equation:

$$\frac{\partial \bar{S}_z}{\partial z} = C \frac{\partial^2 \bar{S}_z}{\partial x^2} \tag{14}$$

where $\bar{S}_z(x,z)$ is the expected vertical stress at a point (defined by the coordinates x and z), and C is the coefficient of diffusion that governs the rate at which the particulate material spreads the applied surface load. The coefficient C can be expressed as

$$C = \nu \cdot z \tag{15}$$

where the coefficient of diffusivity (ν) is a state parameter of the material. From a structural viewpoint, ν is influenced by such factors as stress-induced anisotropy of the granular assembly, intergranular friction properties, and state of compaction of the medium.

For a concentrated surface load (P) the solution of Equation 14 is a bell-shaped Gaussian density function (Figure 6):

$$\frac{\bar{S}_z(x,z)}{P} = \frac{1}{z\sqrt{2\pi\nu}} \exp\left(\frac{-x^2}{2\nu z^2}\right) \tag{16}$$

These results were generalized for distributed applied loads, axisymmetric or tridimensional geometry, and multilayer soil systems (*12*).

Let us consider a system consisting of three courses of particulate material of thicknesses H_1, H_2, and H_3, resting on a semiinfinite subgrade (Figure 7*a*). The expected vertical stress at each interface under the load is denoted by $\bar{S}_{zi}$. By definition, the equivalent thickness $\bar{H}_1$ of layer 1 is the thickness of material 2, which can be substituted to the thickness H_1 of material 1 without modifying the distribution of vertical stresses in layer 2. In other words, the vertical stress $\bar{S}_{z2}$ in the equivalent structure shown in Figure 7*b* ($\bar{H}_1$, H_2, H_3) is equal to the stress at the same level in Figure 7*a*. The equivalent thickness of layers 1 and 2, $\bar{H}_2$ is defined in the same way, as is

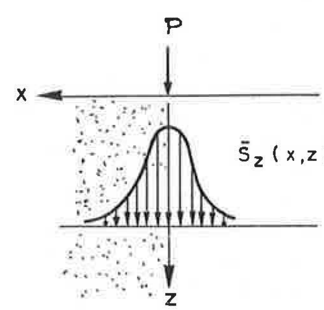

FIGURE 6 Vertical stress distribution in a semi-infinite particulate medium under a concentrated surface load.

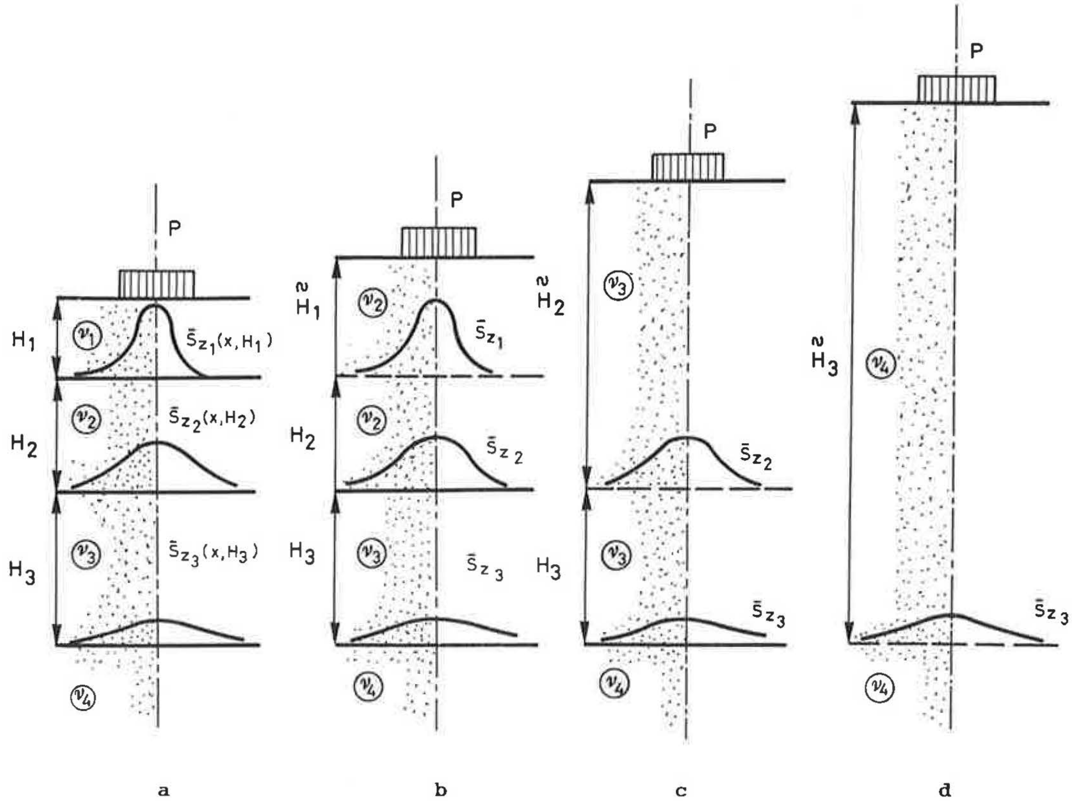

FIGURE 7 Concept of equivalent thickness in a stratified particulate medium where $\bar{S}_{zi}$ is the vertical stress at interface i and ν_i is the coefficient of stress diffusivity in layer i.

also the equivalent thickness of layers 1, 2, and 3 (Figure 7*c* and *d*):

$$\tilde{H}_3 = H_1 \sqrt{\frac{\nu_1}{\nu_4}} + H_2 \sqrt{\frac{\nu_2}{\nu_4}} + H_3 \sqrt{\frac{\nu_3}{\nu_4}} \qquad (17)$$

where ν_1, ν_2, ν_3, and ν_4 are the coefficients of stress diffusivity of the layers.

For a system of N layers, a recurrence relationship can be derived (*12*):

$$\tilde{H}_{N-1} = H_1 \sqrt{\frac{\nu_1}{\nu_N}} + H_2 \sqrt{\frac{\nu_2}{\nu_N}} + \ldots + H_{N-1} \sqrt{\frac{\nu_{N-1}}{\nu_N}} \qquad (18)$$

According to this model, $\tilde{H}_3$ represents the thickness of subgrade material that can be substituted for the other courses without modifying the state of vertical stress in the subgrade. Thus, for a given subgrade material characterized by its coefficient of diffusivity (ν_4) systems with the same value of $\tilde{H}_3$ are equivalent from the standpoint of the vertical stress distribution applied to the subgrade. This condition is written:

$$H_1\sqrt{\nu_1} + H_2\sqrt{\nu_2} + H_3\sqrt{\nu_3} = \tilde{H}_3\sqrt{\nu_4} = \text{constant} \qquad (19)$$

or, more generally

$$\frac{\sqrt{\nu_1}}{b} \cdot H_1 + \frac{\sqrt{\nu_2}}{b} H_2 + \frac{\sqrt{\nu_3}}{b} H_3 = \text{constant} \qquad (20)$$

where b is an arbitrary constant.

Application to a Pavement System

There is conceptual and formal analogy between the equivalent thickness of a pavement and the equivalent thickness defined in the theory of stochastic stress diffusion for a stratified particulate medium. Identifying Equation 20 to Equation 2 leads to

$$D = a_1 D_1 + \frac{\sqrt{\nu_2}}{b} D_2 + \frac{\sqrt{\nu_3}}{b} D_3 \qquad (21)$$

where

a_1 = empirical coefficient of equivalence related to the asphaltic concrete wearing course;
ν_2 and ν_3 = coefficients of stress diffusivity of the granular base and subbase courses;
D_1, D_2, D_3 = thicknesses of the courses in inches; and
b = constant.

Recalling that the stochastic theory of stress propagation was at first derived for cohesionless particulate materials, the wearing course asphaltic concrete in a flexible pavement does not conform strictly to the hypotheses of the theory in its present state of development. Thus, in Equation 21, the contribution of the wearing course is represented by an empirical coefficient, whereas the analytical solution is applied to the granular material courses.

Conceptually, the coefficient of diffusivity of a cohesionless soil (ν) is related to the coefficient of lateral pressure and is influenced by the stress history and the shear strength of the granular material. For loose, normally consolidated granular soils, it was shown that ν can be approximated by the coefficient of earth pressure at rest (K_0) and thus related to the angle of internal friction of the material (*13,14*).

In the present case of a pavement structure with compacted granular materials, the coefficient of stress diffusivity is approximated by the coefficient of passive earth pressure K_p:

$$\nu = tg^2\,(45^\circ + \phi'/2) \tag{22}$$

where ϕ' is the effective angle of internal friction. Therefore, D in Equation 21 becomes

$$D = a_1 D_1 + \frac{1}{b}\tan(45^\circ + \phi_2'/2)D_2 + \frac{1}{b}\tan(45^\circ + \phi_3'/2)D_3 \tag{23}$$

Comparison with the AASHO Road Test Results

The granular materials of the AASHO road tests were a crushed gravel for the base course (stabilized base courses are not considered in this study) and a sandy gravel with added fines for the subbase. For a comparison with the proposed model, in absence of published shear test results, the friction angle of these materials is estimated to range between 40 and 45 degrees for the base course, and between 25 and 35 degrees for the subbase.

From Equations 2 and 23, the ratio a_3/a_2 of the equivalence coefficients is

$$a_3/a_2 = \tan\left(45^\circ + \frac{\phi_3'}{2}\right) \bigg/ \tan\left(45^\circ + \frac{\phi_2'}{2}\right) \tag{24}$$

As shown in Table 2, theoretical and experimental estimates for this ratio are close. Furthermore, the proposed model is in agreement with the value of a_3/a_2 obtained by Shook and Finn (*1*) for $\phi_2' = 43^\circ$ and $\phi_3' = 30^\circ$ (Figure 8). These values introduced into Equation 23 give a value of 2.3 for the constant b leading to

$$D = 2D_1 + 0.435\tan(45^\circ + \phi_2'/2)\,D_2 + 0.435\tan(45^\circ + \phi_3'/2)\,D_3 \tag{25}$$

with D, D_1, D_2, and D_3 in inches. Equation 25 is a generalized form of the Shook and Finn (*1*) expression for the equivalent thickness and can be integrated into the reliability-based model developed in the previous section.

TABLE 2 COEFFICIENTS OF EQUIVALENCE OF THE GRANULAR COURSES a_3/a_2 FOR THE AASHO ROAD TESTS

Computed using the proposed model

ϕ'_2	ϕ'_3	a_3/a_2
40°	25°	0.73
	30°	0.81
	35°	0.90
45°	25°	0.65
	30°	0.72
	35°	0.80

Empirical values from the tests

reference	a_3/a_2
HRB [2] without seasonal ponderation	0.71
Shook and Finn [1]	0.75
HRB [2] with seasonal ponderation	0.79
Painter [15]	0.81

CONCLUSIONS

A probabilistic extension of the empirical method derived from the AASHO road tests for the design of flexible pavements has been proposed. The reliability assessment of a pavement section is obtained by considering the Shook and Finn (*1*) design equation as a function of two random variables, the expected number of traffic loads, and the CBR of the subgrade soil. The formulation is a second-order, second-moment development of this equation, which requires knowledge of only the means and standard deviations of the random variables. The model compares favorably with that from a statistical analysis of the AASHO test results. A sensitivity analysis indicates that the CBR variability has a large influence on the pavement reliability. The uncertainty on the expected traffic loads has little effect on the reliability when a large number of axle loads is considered. Hence, the mean value of the expected traffic load is a sufficient estimate for this parameter.

An analytical model has been formulated for the coefficients of equivalence of the (unbounded) granular materials of the base and subbase courses, using the theory of stochastic stress propagation in particulate media. It is shown that in the context of this theory, these coefficients represent the property of the granular course to spread the applied load in a diffusion process. A modified formulation is derived for the equivalent thickness of the pavement, and the coefficients are simply expressed as function of the angles of internal friction of the materials.

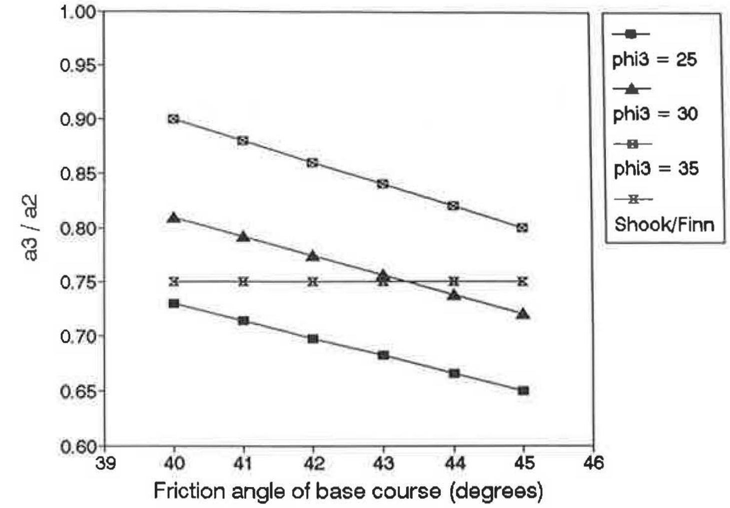

FIGURE 8 Coefficients of equivalence of the granular courses a_3/a_2. Comparison of theoretical and empirical models (see also Table 2).

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