Simultaneous Optimization of Signal Settings and Left-Turn Treatments

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Two apparent weaknesses in current signal setting methodology are addressed, namely the identification of the optimum number of phases and the optimization of left-turn phasing (e.g., protected, permissive, or both). The proposed method integrates these two elements into the signal timing process for isolated intersections, which also entails the optimization of cycle length and splits. The method directly considers the effect of minimum green times, practical cycle lengths, and permissive left-turn capacity models in an attempt to reach an optimal decision. Although primarily applicable to pretimed signal control the logic may also be adapted to single-ring actuated controllers. The scope of the method is limited to intersection geometries with exclusive left-turn lanes on all approaches and no overlap phasing.

The state of the art in signal-setting methodology, which has not appreciably changed over the past two decades, is principally based on the original work by Webster and Cobbe (*1*), who developed formulations for cycle lengths and green splits to minimize overall intersection delay. This method allocates green times to critical movements to equalize their volume-to-capacity (*v/c*) ratio. This concept is still applied in the United States for isolated intersections in the 1985 version of the *Highway Capacity Manual* (*2*). A similar approach has been adopted by Akcelik in the *Australian Capacity Manual* (*3,4*). In the SIGCAP program, Allsop proposes a different concept in which signal settings are derived that maximize intersection reserve capacity (*5,6*). However, this criterion invariably leads to the implementation of the maximum cycle length, a rather poor choice when delays are considered. Delay (and stop) minimization constitutes another approach to timing signals. Yagar's model (*7*) and Allsop's SIGSET model (*8*) apply this criterion to isolated intersections. In Yagar's model, delays are expressed in terms of piecewise linear functions of red times to maintain linearity in problem formulation. Reljic's model allows the user to select an optimization criterion from a host of intersection measures of performance including total vehicle delay, total number of stops, total fuel consumption, total person-delays, or queue lengths (*9*). Optimization is carried out as a sequential minimization without constraints belonging to the penalty methods (*10*). The delay criterion is also fundamental in the setting of traffic signal systems such as TRANSYT (*11*), SIGOP (*12*), and MITROP (*13*).

A key element in optimizing signal settings is the treatment of left-turn movements. Inadequate left-turn capacity in a through phase may cause excessive delays for left turns as well as through traffic on the same approach and may even promote unsafe maneuvering by drivers into gaps of the opposing stream. In such cases, provision of protected left-turn phasing must be considered. Yet, a review of existing guidelines for left-turn phasing (*14–17*) revealed that whereas other considerations exist for implementing protected phasing (such as accidents, conflicts, geometry, and operating speed), overall signal optimization as a means for improving left-turn operation is seldom considered. For example, inadequate left-turn capacity on a given approach may be the result of poor green split allocation rather than excessive left-turn demand. In this case, provision of a protected phase is unwarranted and may even degrade overall intersection performance. Available signal-setting models widely used in the United States lack a phase optimization capability. For example, TRANSYT cannot optimize the number of phases or left-turn treatment, both of which must be specified by the user. PASSER–II–84 (*18*) and MAXBAND (*19*), two arterial optimization packages, can only model protected left turns. This constraint was later relaxed in the PASSER II–87 version (*20*). The Signal Operations Analysis Package (SOAP–84) has the ability to compare various left-turn phasing options but the options must also be specified a priori by the user (*21*). Furthermore, the model performs an iterative optimization of cycle, phasing, and splits, and thus does not guarantee a global optimum solution to the problem. In summary, all the reviewed signal-setting methods lacked a systematic approach in which left-turn phasing treatments are integrated into the overall signal optimization scheme. This need is shown to be evident both at the intersection and systems level.

PROBLEM DEFINITION

The review suggests that the use of rigorous mathematical modeling techniques is best suited for the solution of the problem at hand. Not only is global optimality ensured, but solution algorithms are already in place to perform the optimization (*22*). Moreover, standard solution reports of these algorithms provide the user with a wealth of information in post-optimality analyses, such as the value of slack variables and sensitivity analysis to model parameters. Yet, the method described does not represent a first attempt at using mathematical programming techniques for signal-timing purposes. In addition to the referenced models by Allsop (*5*), Yagar (*7*), and Reljic (*9*), models by Sakita (*23*) and Improta and Cantarella (*24*) deserve further detailed discussion because of their similarity to the approach suggested. The first model uses linear programming to determine either the minimum or optimum cycle length at a signalized intersection subject to

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green time constraints that permit movements to operate below capacity. Application of the optimum cycle time assumes Webster's formula (in which the optimum cycle length ≈ twice the minimum cycle). Furthermore, only one saturation flow rate is allowed per movement; thus, treatment of left turns is restricted to protected phasing only (with overlap phasing allowed). Improta and Cantarella (*24*) developed a binary-mixed-integer linear program for the optimization of signal settings. Their work relies on the identification of incompatible movement groups each of which is assigned an exclusive phase. A group may consist of one or several movements, but no one movement may be assigned to more than one group. This limitation again restricts the modeling of protected and permissive phasing because a left turn movement may be compatible with an opposing flow in one phase (permissive) but not in another (protected). However, a key feature of Improta's (*24*) model is its ability to determine the optimum number of phases and phase sequencing because all phases are group-oriented. Objective functions may include cycle length minimization, maximization of reserve capacity [similar to Allsop's (*5*)], or delay minimization [similar to Yagar's (*7*)]. Cantarella and Improta (*25*) later suggested the use of graph theory methods for problem formulation and solution. A key limitation of signal settings derived from minimum-delay models is their high sensitivity to the form of delay model that is used, especially at medium-to-high v/c ratios. In a sense, one is not certain whether settings that are based on Webster's (*1*), HCM (*2*), or TRANSYT-7F (*11*) formulas are compatible. On the other hand, little disagreement exists among researchers in the United States and overseas on how to express saturation flows, v/c ratios, and minimum greens. The solution described is an initial attempt at overcoming some of these inconsistencies among the various delay models.

Two rather simple but fundamental principles of signal timing drive the problem formulation, namely:

- Selection of shortest-possible cycle length, and
- Selection of minimum sufficient number of phases.

In summary, problem formulation involves determining cycle length, splits, and left-turn treatments that allow each intersection movement to operate at or below a prespecified v/c ratio threshold. This approach is similar to cycle length formulation in the 1985 HCM, which is aimed at sustaining a maximum intersection v/c ratio (*2*). The settings must satisfy the constraints on the range of cycle length and minimum greens, using the shortest-possible cycle and smallest number of signal phases.

This formulation both incorporates continuous variables (splits, minimum greens, lost times) as well as discrete variables (number of phases and cycle length in increments of 5 sec, as is typical in traffic engineering applications). Thus, a mixed-integer linear formulation is appropriate to characterize the problem. Figure 1 shows intersection movements (top) and phase designation (bottom) notation used in the formulation. The scope of the model is limited to intersections with exclusive left-turn lanes on all approaches and to nonoverlap phasing patterns. Thus, the optimum number of phases may vary from two to four. In addition, right turns are treated as through movements, although this assumption can be relaxed by varying the movement saturation flow rates for shared right and through lanes.

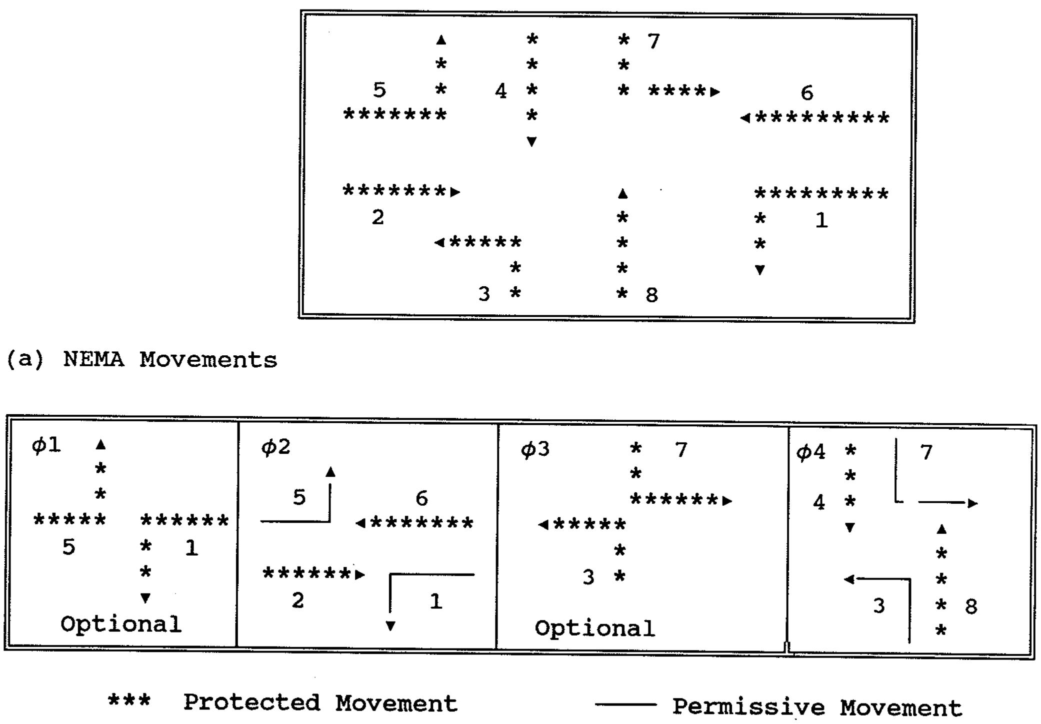

FIGURE 1 NEMA movement (*top*) and phase designations (*bottom*).

GLOSSARY OF TERMS

The following variables are used in the formulation:

C = cycle length (sec);
C_{max} = maximum cycle length (sec);
C_{min} = minimum cycle length (sec);
δC = allowable cycle length increment (sec);
f_i = flow rate for Movement i;
g_j = effective green time for Phase j (sec);
g^j_{min} = minimum effective green for Phase j (sec);
i = movement index, $i = 1, 2, \ldots, 8$;
I_j = binary variable for Optional Phase j, which takes value 1 if Phase j is selected, 0 otherwise (j = 1 or 3 from Figure 1);
j = phase index, $j = 1, 2, \ldots, 4$;
J_k = binary variable for cycle frequency, which takes value 1 if cycle frequency k is optimal, 0 otherwise;
k = cycle frequency = $1/C$ (sec^{-1});
k_j = shadow cycle frequency for Optional Phase j, which takes value k if Phase j is selected, 0 otherwise (j = 1 or 3);
k_{max} = maximum cycle frequency = $1/C_{min}$ (sec^{-1});
k_{min} = minimum cycle frequency = $1/C_{max}$ (sec^{-1});
l = lost time per phase (or per movement because no overlaps are considered) (sec);
S_{ij} = saturation flow rate for Movement i in Phase j (zero if Movement i is not serviced by Phase j);
τ_j = green split ($g_j * k$) for Phase j;
X_i = threshold v/c ratio for Movement i; and
z = number of left turns allowed per cycle in the clearance interval.

FORMULATION

Objective Function

The objective is to minimize the cycle length C subject to the given constraints. In order to maintain linearity throughout the formulation, cycle frequency k is maximized instead. Thus,

$$\text{Objective Function: } \max k \qquad (1)$$

Minimum Green Constraints

Assuming that the actual and effective greens to each phase are numerically equivalent, then the minimum green constraint is expressed as

$$g_j \geq g^j_{min} \qquad j = 2 \text{ and } 4 \qquad (2)$$

Multiplying both sides in Constraint 2 by k yields

$$\tau_j - kg^j_{min} \geq 0 \qquad (3)$$

Considering the case for $\phi 1$ and $\phi 3$, the minimum green constraint applies only when these phases become part of the optimum solution. Hence,

$$\tau_j \geq kg^j_{min} - (1 - I_j) \qquad j = 1 \text{ and } 3 \qquad (4)$$

in which Constraint 4 is active when $I_j = 1$. When no protected phases are selected, $I_j = 0$ and Constraint 4 is no longer binding.

Constraints on v/c

Each of the eight movements in Figure 1 is constrained to operate at or below a prespecified v/c ratio X_i. Through movements are assumed to operate at a constant (but not necessarily equal) saturation flow rate. Left turns, on the other hand, discharge at three different rates during the protected, permissive, and clearance phases of the cycle. Table 1 presents the saturation flow matrix corresponding to the movements and phases shown in Figure 1.

TABLE 1 SATURATION FLOW MATRIX

Movement (i)	Phase (j) 1	2	3	4
1	Su11	So12	0	0
2	0	ST22	0	0
3	0	0	Su33	So34
4	0	0	0	ST44
5	Su51	So52	0	0
6	0	ST22	0	0
7	0	0	Su73	So74
8	0	0	0	ST84

Legend:

Su_{ij}= Unopposed left turn saturation flow rate for movement i in ϕj.

So_{ij}= Opposed left turn saturation flow rate for movement i in ϕj.

ST_{ij}= Thru saturation flow rate for movement i in ϕj.

Thus for through movements, this set of constraints is expressed as

$$X_i * ST_{ij} * \tau_j \geq f_i \qquad i = 2, 4, 6, \text{ and } 8; j = 2 \text{ and } 4 \tag{5}$$

Left-turn capacity is expressed as the sum of three terms:

- Protected phase capacity $= Su_{ij} * \tau_j$ $\qquad i = 1, 3, 5, \text{ and } 7$ (6)

in which τ_j is the protected phase split ($\phi 1$ or $\phi 3$).

- Permissive phase capacity $= So_{ij} * \{ST_{opp} * \tau_j - f_{opp}\}/\{ST_{opp} - f_{opp}\}$ (7)

in which ST_{opp} is the saturation flow rate for the opposing through movement; f_{opp} is the opposing through flow rate; and τ_j denotes the permissive phase split ($\phi 2$ or $\phi 4$). For the intersection configuration in Figure 1, this term is expressed strictly as a linear function of τ_j.

- Clearance phase capacity $= 3{,}600\, zk$ (8)

The final form of the v/c constraint for left turns is expressed as

$$X_i[Su_{ij} * \tau_{ju} + So_{ij} * (ST_{opp} * \tau_{jo} - f_{opp})/(ST_{opp} - f_{opp}) + 3{,}600zk] \geq f_i$$

$$i = 1, 3, 5, \text{ and } 7; j = 1, 2, 3, \text{ and } 4 \tag{9}$$

in which τ_{ju} = unopposed green split; τ_{jo} = opposed green split; So_{ij} = opposed left turn saturation flow rate generally expressed in terms of the opposing flow rate (f_{opp}).

Cycle Length Constraint

In most traffic signal applications, the length of the cycle is typically set in 5 sec (or in general δC) multiples. For a feasible cycle range $C_{min} \leq C \leq C_{max}$, this constraint can be expressed as

$$C = C_{min} * J_1 + (C_{min} + \delta C) * J_2 + (C_{min} + 2\delta C) * J_3 + \ldots + C_{max} * J_{max} \tag{10}$$

and

$$J_1 + J_2 + \ldots + J_{max} = 1 \tag{11}$$

in which $J_1, J_2, \ldots, J_{max}$ are all binary variables.

Expressing these relationships in terms of k yields the constraint

$$k = \sum_{m=0}^{(C_{max}-C_{min})/\delta C} [1/(C_{min} + m\delta C)] * J_{m+1} \tag{12}$$

such that

$$\sum_m J_{m+1} = 1 \qquad m = 0, \ldots, (C_{max} - C_{min})/\delta C \tag{13}$$

Phase Split Constraints

The phase lengths must satisfy the requirement

$$\sum_j (g_j + l_j) = C \tag{14}$$

In this formulation, total lost time per cycle cannot be determined before optimization because this value will depend on the ultimate number of phases. Therefore, introduction of a set of artificial variables defined as shadow cycle frequencies becomes necessary to capture the additional lost time generated in the event $\phi 1$ or $\phi 3$, or both, are selected. Thus, the constraints

$$k_j \geq k - (1 - I_j) \qquad j = 1 \text{ and } 3 \tag{15}$$

and

$$K_j \leq I_j \qquad j = 1 \text{ and } 3 \tag{16}$$

will ensure that if ϕ_j is selected, then k_j must have a nonzero value. In addition, if ϕ_j is omitted, then from Constraint 16 K_j is set to zero. Equation 14 can now be rewritten in terms of splits as

$$\left\{\sum_j \tau_j\right\} + 2lk + \left\{\sum_{j=1,3} lk_j\right\} = 1 \tag{17}$$

Because the objective is to maximize k, then Constraint 17 will attempt to drive both τ_j and k_j to their minimum. Because τ_js are also constrained by the v/c ratio thresholds (Equations 5–9), then only the k_js can be driven to their minimum value k, thus ensuring that the additional lost times caused by $\phi 1$ and $\phi 3$ are properly accounted for in the formulation.

NUMERICAL EXAMPLE

Consider the peak period flows shown in Figure 2. Through movements operate in two traffic lanes, whereas left turns operate from exclusive lanes of adequate length. Assume that the measured or estimated saturation flow rate for through movements is ST_{ij} = 3,200 vehicles per hour per green (vphg) (for i = 2, 4, 6, and 8), and that for unopposed left-turn movements is Su_{ij} = 1,400 vphg (i = 1, 3, 5, and 7). For opposed left turns, assume the simple linear model So_{ij} = 1,400 − f_{opp} (vphg) (i = 1, 3, 5, and 7). Use of other opposed left-turn saturation flow rate models (*26*) would not violate the linearity of the constraints so long as left turns occur from exclusive lanes. Lost time per phase is taken at 3 sec. The following design criteria are established:

1. $X_i \leq 0.90$ for i = 1, 3, 5, 7 (left turns);
2. $X_i \leq 0.85$ for i = 2, 4, 6, 8 (through movements);
3. $g(\phi 1 \text{ or } \phi 3) \geq 5$ sec;
4. $g(\phi 2 \text{ or } \phi 4) \geq 10$ sec;
5. $40 \leq C \leq 150$ sec in 5-sec multiples; and
6. z = one vehicle per cycle.

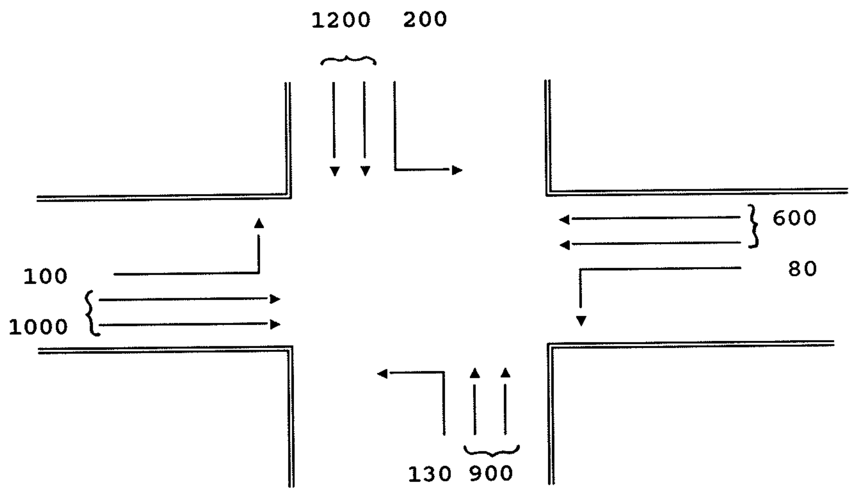

FIGURE 2 Movement flow rates in vehicles per hour.

Determine cycle length, splits, and left-turn treatment that meet the specified objectives. Next, perform a sensitivity analysis of the optimum solution to (a) lost time per phase (*l*); (b) design *v/c* ratios X_i; and (c) assumed left-turn capacity in the clearance interval (*z*).

Formulation

- Objective function: max *k*
- Minimum green constraints

$$\phi 1: \tau_1 \geq 5k - (1 - I_1) \quad (18)$$

$$\phi 2: \tau_2 \geq 10k \quad (19)$$

$$\phi 3: \tau_3 \geq 5k - (1 - I_3) \quad (20)$$

$$\phi 4: \tau_4 \geq 10k \quad (21)$$

- Maximum *v/c* ratio constraints

Movement 1: $0.9 * 1{,}400\tau_1 + 0.9 * (1{,}400 - 1{,}000)(3{,}200\tau_2 - 1{,}000)/(3{,}200 - 1{,}000) + 0.9 * 3{,}600k \geq 80$ (22)

which can be rewritten as $1{,}260\tau_1 + 524\tau_2 + 3{,}240k \geq 244$ (23)

Movement 2: $2{,}720\tau_2 \geq 1{,}000$ (24)

Movement 3: $1{,}260\tau_3 + 288\tau_4 + 3{,}240k \geq 238$ (25)

Movement 4: $2{,}720\tau_4 \geq 1{,}200$ (26)

Movement 5: $1{,}260\tau_1 + 886\tau_2 + 3{,}240k \geq 266$ (27)

Movement 6: $2{,}720\tau_2 \geq 600$ (28)

Movement 7: $1{,}260\tau_3 + 626\tau_4 + 3{,}240k \geq 376$ (29)

Movement 8: $2{,}720\tau_4 \geq 900$ (30)

- Phase split and shadow frequency constraints

$$\tau_1 + \tau_2 + \tau_3 + \tau_4 + 6k + 3k_1 + 3k_3 = 1 \quad (31)$$

$$k_1 \geq k - (1 - I_1) \quad (32)$$

$$k_3 \geq k - (1 - I_3) \quad (33)$$

$$k_1 \leq I_1 \quad (34)$$

$$k_3 \leq I_3 \quad (35)$$

- Cycle length constraints

$$k = \sum_{m=0}^{22} \{1/(40 + 5m)\} J_{m+1} \quad (36)$$

subject to

$$\sum_{m=0}^{22} J_{m+1} = 1 \quad (37)$$

- Binary variables constraints

$$I_1, I_3, J_1, J_2, \ldots, J_{23} = 0 \text{ or } 1 \quad (38)$$

This binary-mixed-integer linear program was solved with the branch-and-bound solution algorithm available in the Linear Interactive Discrete Optimizer (LINDO) (*27*) package. This algorithm was used to generate the optimum solutions and conduct post-optimality analyses.

RESULTS

The optimization run results are presented in Table 2. As presented, the model recommends a 3-phase signal operation including a protected left-turn phase for Movements 3 and 7. This phase was timed at its minimum value of 5 sec. Optimum cycle length is 85 sec. In Table 3, a capacity analysis of the eight movements is presented. Several observations are noted:

TABLE 2 SIGNAL SETTINGS RESULTS SUMMARY

Phase	$\phi 1$	$\phi 2$	$\phi 3$	$\phi 4$	Lost Time	Total
Split	0	.3927	.059	.4412	.1071	1
Green	0	33.5	5	37.5	9	85s

TABLE 3 MOVEMENT CAPACITY ANALYSIS

Movement (i)	f_i	Hourly Capacity in					v/c
		$\phi 2$	$\phi 3$	$\phi 4$	Yellow	c(Total)	
1 *	80	47	0	0	42	89	0.90
2	1000	1256	0	0	0	1256	0.80
3 *	130	0	83	21	42	146	0.89
4 *	1200	0	0	1412	0	1412	0.85
5	100	202	0	0	42	244	0.41
6	600	1256	0	0	0	1256	0.48
7	200	0	83	111	42	236	0.85
8	900	0	0	1412	0	1412	0.64

* Movement operates at or close to threshold volume to capacity ratio.

1. For Movements 3 and 7, available permissive phase capacity in the green and clearance intervals was smaller than the demand volume; thus, a protected phase was needed. A shorter protected phase time would have sufficed because both movements operate below their threshold v/c ratio (albeit slightly).
2. Left-turn movements directly affect the signal timing plan, with Movements 1 and 3 considered critical. Only (through) Movement 4 is considered critical in the final analysis.
3. Breakdown of left-turn capacity by phase for all movements indicated that 23 percent is developed in protected phasing, 54 percent in permissive phasing, and 23 percent in the clearance interval. This breakdown highlights the significant effect of the latter assumption on left-turn capacity estimates.
4. By restraining the cycle length to multiples of 5 sec, the optimal solution generally overestimates the optimal cycle and green times and results in lower v/c ratios for most movements.
5. Split optimization in the model does not follow the simple rule that the split (τ) is proportional to the critical flow ratio (f/S) because the value of S is not defined for protected or permissive left-turn movements. No attempt is made to allocate the left-turn demand to each of the protected or permissive phases.
6. Although the model has no explicit mechanism for recommending strictly protected phasing, a simple review of the optimization results allows the user to investigate this treatment. This investigation can be done by dividing the total left-turn demand by the protected phase capacity (presented in Table 3). If the resulting v/c ratio is less than the threshold value for that movement, then a protected phase should be sufficient to accommodate the demand. This situation may arise when the minimum green for the protected phase is long enough to service the entire left-turn demand. As stated earlier, this scenario did not apply for any of the left-turn movements in this example.

No feasible solution could be obtained for the example problem when left turns were allowed to proceed strictly in protected phasing, even at the maximum cycle length of 150 sec. The value of using the permissive left-turn capacity thus confirms the findings of previous studies in that respect (*28*).

PARAMETRIC ANALYSIS

Sensitivity of the optimum solution to various model parameters is explored. The analysis is performed by varying one set of parameters at a time, keeping all other parameters fixed at their default values. The base case (Case 1) is identical to the numerical example. Alternative cases are developed as follows:

1. Cases 2, 3, and 4: through movements threshold *v/c* ratios assume values of 0.90, 0.95, and 1.00, respectively;
2. Cases 5, 6, and 7: left-turn threshold *v/c* ratios assume values of 0.85, 0.95, and 1.00, respectively;
3. Cases 8, 9, and 10: number of permitted left turns in the clearance interval assumes values of 0.5, 1, and 2 vehicles per cycle, respectively; and
4. Cases 11, 12, and 13: lost time per phase assumes values of 3.25, 2.5, and 2.0 sec, respectively.

A comparison of the results for each case is presented in Table 4.

The effect of increasing through movement *v/c* ratio is evident. Optimum cycle length decreased with each increase in the *v/c* ratio (Cases 2, 3, and 4). However, the optimum number of phases remained at three because of the effective reduction in the permissive left-turn phase capacity as opposing flows are allowed to operate closer to their capacity. On the other hand, because of reduced cycle length, the number of left turns discharged in the clearance interval is increased. In effect, these two factors cancel each other and the left-turn treatment is essentially unchanged. A better understanding of this trade-off can be seen in Figure 3. In this figure, the percentage of left-turn capacity (for Movements 1, 3, 5, and 7 combined) generated in the protected, permissive, and clearance interval is plotted against the through-movement threshold *v/c* ratio. As the latter value increases, the contribution of the permissive phase is reduced from a high of 54 percent (at X_t = 0.85) to a low of 29 percent (at X_t = 1.0). On the other hand, the capacity share is virtually identical for the protected and clearance phases ranging from a low of 23 percent (at X_t = 0.85) to a high of 35.5 percent (at X_t = 1.0). The overall left-turn capacity, also shown in Figure 3, ranged from 715 vehicles per hour (vph) (at X_t = 0.85) to 840 vph (at X_t = 1.0), an increase of 17.5 percent that is primarily attributable to the added capacity in the clearance interval.

The effect of increasing left-turn *v/c* ratio is also rational, in the sense that the increased ratio results in reduced cycle length, albeit at a smaller rate than that observed for the threshold *v/c* ratios for through movements. Case 5 represents an interesting situation in which through and left-turn movements are required to operate at $X \leq 0.85$. In this case, a 4-phase signal is needed at the maximum cycle length of 150 sec. This result points to the critical nature of the left-turn movements in the example problem as well as to the limitation of optimal signal-setting methods in producing adequate capacity within the operational constraints set by the traffic engineer.

By far, the most surprising results were evident in Cases 8 through 13 in which the effects of *z* and *l* were studied. Although the trend generated by the optimization model was expected,

TABLE 4 SENSITIVITY ANALYSIS TO SELECTED MODEL PARAMETERS

Case	Max v/c (Thru)	Max v/c (Left)	z/ Cycle	l sec	Inter* Capc.	Copt sec.	# Phases
1 Bs.	0.85	0.90	1	3	6051	85	3
2	0.90	0.90	1	3	5858	70	3 **
3	0.95	0.90	1	3	5634	60	3
4	1.00	0.90	1	3	5398	50	3
5	0.85	0.85	1	3	5928	150	4
6	0.85	0.95	1	3	6017	80	3
7	0.85	1.00	1	3	5912	75	3
8	0.85	0.90	0.5	3	5876	150	4
9	0.85	0.90	1.5	3	6567	40	2
10	0.85	0.90	2	3	6567	40	2
11	0.85	0.90	1	3.25	6047	150	4
12	0.85	0.90	1	2.50	6007	70	3
13	0.85	0.90	1	2.00	6009	60	3

* Total effective movement capacity, including left turns in the clearance interval

** All 3 phase patterns include a protected left turn phase for movements 3 and 7 (ϕ3 in Fig. 1).

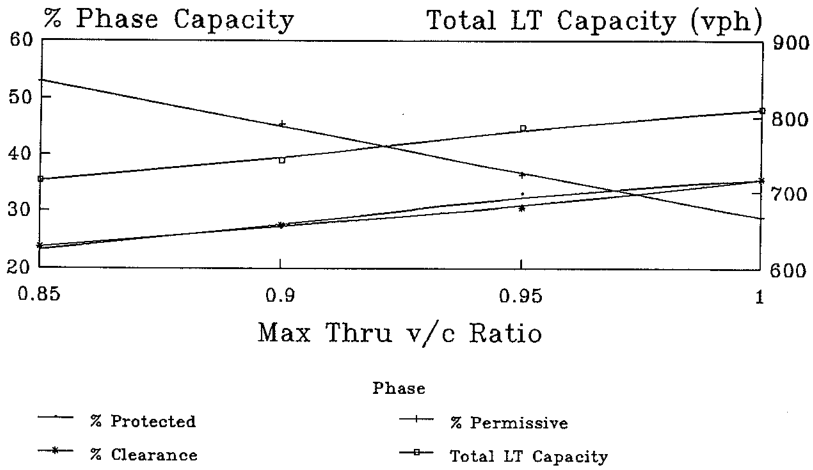

FIGURE 3 Apportionment of left-turn capacity by phase type.

the magnitude of the effects was rather substantial. Consider Cases 8, 9, and 10 in Table 4. Assuming that, on average, one left-turning vehicle uses the yellow interval every two cycles (Case 8), the model recommends a 4-phase signal at the maximum cycle. By increasing this value to three vehicles every two cycles (Case 9), then no protected phasing for any left-turn movement would be needed. In essence, this result gives credence to existing guidelines for protected phasing that stipulate the presence of a minimum number of left-turning vehicles per cycle before protected phasing is to be considered.

A similar pattern emerges on the effect of lost time (Cases 11 through 13), with cycle length and number of phases increasing with an increase in l. Attempts to optimize the traffic flows in Figure 2 with $l = 3.5$ sec per phase were unsuccessful, indicating that a lost time $\geq$3.25 sec per phase cannot sustain the prespecified threshold v/c ratios for at least some movements.

Total intersection capacity (including left turns in the clearance interval) is presented in Table 4. In general, capacity increased with the cycle length for fixed values of z, l, and number of phases. The 4-phase patterns did not compare favorably with either 2-phase or 3-phase patterns, because of the added lost time associated with the protected phases. A plot of intersection capacity versus cycle length for the 3-phase configurations is shown in Figure 4. The relationship confirms previous findings by Webster (*1*) regarding the reduced effect of increasing the cycle length on overall intersection capacity.

A final test of the derived settings for the base case (Case 1) in Table 4 was performed with the SIDRA 3.1 program developed by Akcelik (unpublished) at the Australian Road Research Board (ARRB). This model carries out the computational procedures for cycle length, splits, and delays as described in ARRB Report 123 (*3*) with some modifications. The model was run for the base case with 2-, 3-, and 4-phase options (Phases 2 through 4 in Figure 1) at cycle lengths varying from 40 to 150 sec. Figure 5 shows the resulting delays. As shown, the 3-phase pattern produced consistently lower delays than the 4-phase pattern, whereas the two-phase pattern had somewhat lower delays than the 3-phase pattern. The derived optimum cycle length of 85 sec had an average intersection delay of 21.2 sec/veh, only 0.50 sec/veh higher than the minimum delay solution of 20.7 sec/veh attained at a 75-sec cycle. However, the best 2-phase solution yielded a global minimum delay of 13.3 sec at a 35-sec cycle. Figure 6 shows a plot of the practical (or critical movement) reserve capacity versus cycle length. Reserve capacity is defined as

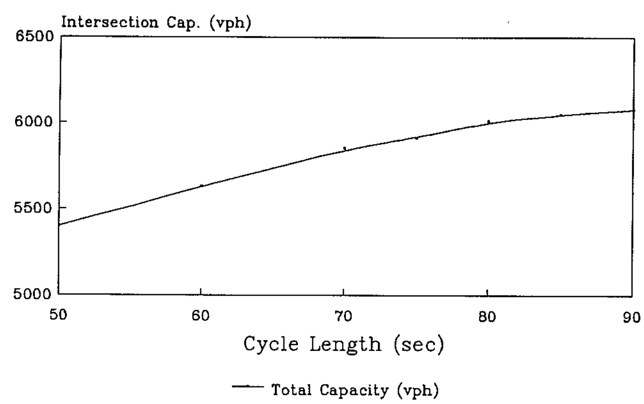

FIGURE 4 Capacity versus cycle length for l = 3, z = 1, and 3-phase setting.

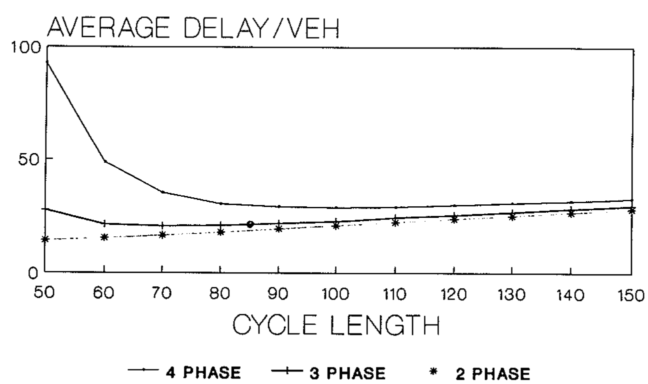

FIGURE 5 Delay versus cycle length by number of phases.

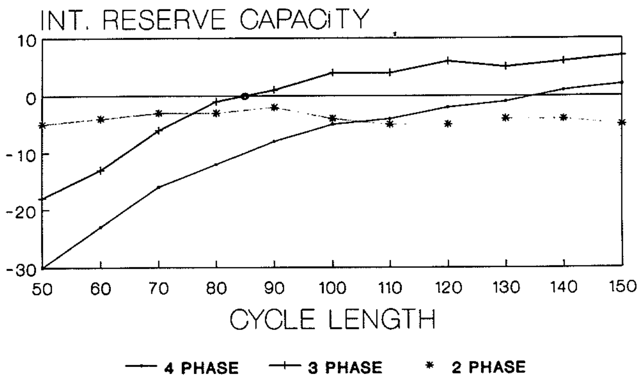

FIGURE 6 Reserve capacity by number of phases.

the difference between the practical capacity of the critical movements (i.e., when operating at the threshold v/c ratio) and the critical movements flow rate. In this case, none of the 2-phase cycle alternatives guarantees that each critical movement will operate below its specified threshold. On the other hand, the 4-phase pattern requires long cycle lengths (over 130 sec) to satisfy that constraint. The optimum solution for the 3-phase pattern is represented by the ordinate value 0 in Figure 6, indicating that the critical movements operate exactly at their practical capacity at the optimal relation. This solution would typically be the rule except in the event the minimum green constraints would govern the split allocation. Although the results in SIDRA indicate that the model solution does not produce optimal delays, these results nevertheless confirmed the model's ability to produce unique signal settings that are consistent with its objective and constraints.

CONCLUSION

A methodology was presented for integrating left-turn phasing treatment into the overall signal optimization scheme. Two apparent deficiencies in present signal-setting methods were addressed, namely their inability to determine the optimal number of phases and the alternative phase treatments for left-turn movements in each of these phases. The method described determines the optimal cycle length, splits, and left-turn phasing (permissive or protected-permissive) that satisfy a maximum v/c-ratio constraint for each movement. Global optimality is ensured as a result of formulating the problem as a binary-mixed-integer linear program. Results of the model appear to be consistent with signal-setting theory and empirical guidelines for left-turn treatment. From the limited sensitivity analysis conducted, two design parameters—lost time per cycle and assumed number of left-turning vehicles per cycle discharging the intersection during the clearance interval—appear to have a significant effect on the ultimate number of phases and on the magnitude of the cycle length—more so than the design v/c ratios stipulated in the formulation. The model was further tested with the SIDRA program for analysis of signalized intersections in Australia and found to produce acceptable delays although the formulation does not specifically account for delays. More important, the model delivered what it was designed to do—to produce a rational signal-setting method that guarantees no overflow for its critical movements using the shortest possible green time. A coherent framework was also provided in which trade-offs between the type of left-turn treatments are better understood and their effect on the ultimate capacity of the intersection evaluated. Two key limitations of the model are absence of an explicit delay minimization algorithm and the possibility of generating unfeasible solutions, both of which will be addressed in future versions of the model. Nevertheless, the proposed method concepts are useful for testing existing guidelines for left-turn phasing and suggesting means for improving their effectiveness.

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