Estimating Average Cycle Lengths and Green Intervals of Semiactuated Signal Operations for Level-of-Service Analysis

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A delay model is recommended in the 1985 *Highway Capacity Manual* (HCM) for the level-of-service analysis of signalized intersections. The use of this model for the evaluation of traffic-actuated signal operations requires the knowledge of the average cycle lengths and green intervals associated with the signal operation being analyzed. A method has been suggested in the HCM to estimate such operating characteristics of traffic-actuated signal operations. The method, however, is not reliable. An improved method for estimating average cycle lengths and green intervals, limited to semiactuated signal operations, is described. The method, which is developed through analytical modeling and computer simulation, is sufficiently simple and reliable. Realistic examples are used to illustrate the application of the method.

The evaluation of the performances of signalized intersections for signal timing and geometric design improvements is an important undertaking in traffic engineering. The performance evaluation of signalized intersections may be carried out in a number of ways. For existing intersections, field studies can yield the most reliable information concerning the relationships among traffic movements, signal operation, and geometric design; but they are time-consuming and expensive. Computer simulation is an attractive alternative to field studies. However, the use of simulation models for the analysis of signalized intersections has not become widespread because of various resource constraints imposed on traffic agencies. Under the circumstances, a method that can both be applied manually and with the aid of a personal computer for the performance evaluation of signalized intersections would be a useful tool to traffic engineers. An example of such a method is the level-of-service analysis methodology presented in Chapter 9 of the 1985 *Highway Capacity Manual* (HCM) (*1*).

The HCM (*1*) methodology provides a procedure for estimating stopped delays associated with a given signal operation. The delay estimates are then used to determine the level of service of a signalized intersection. For the evaluation of alternative traffic-actuated signal operations, this methodology requires the knowledge of the average cycle length and the corresponding average effective green intervals. The 1985 HCM (*1*) suggests that the following equations be used for estimating such averages:

$$C = \frac{LX_c}{X_c - \sum_i (V/S)_{ci}} \tag{1}$$

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and

$$g_i = (V/S)_{ci} C/X_i \tag{2}$$

where

- C = average cycle length,
- L = lost time per cycle,
- X_c = critical volume-to-capacity ratio for the intersection,
- X_i = volume-to-capacity ratio for Lane Group i,
- V = volume,
- S = saturation flow,
- $(V/S)_{ci}$ = ratio of critical volume to saturation flow for Lane Group i, and
- g_i = average effective green interval for Lane Group i.

Because the values estimated from Equations 1 and 2 affect the values of the estimated delays and the estimated levels of service, it is important that Equations 1 and 2 give reasonably accurate estimates. The adequacy of the 1985 HCM (*1*) method as represented by Equations 1 and 2 is discussed and an alternative means of estimating the average cycle lengths and green intervals is found. The discussions are limited to semiactuated signal operations that do not accommodate pedestrian timing.

SEMIACTUATED SIGNAL CONTROL

Semiactuated signals are frequently used at intersections where a major street meets a lightly traveled side street. A semiactuated signal operation normally contains a nonactuated signal phase and one or two vehicle-actuated phases. It may also allow either exclusive or concurrent pedestrian phasing. The nonactuated phase serves several or all of the vehicular movements on the major street. The vehicle-actuated phases are either for the side-street traffic or for several movements on the major street.

For a given traffic flow pattern, a semiactuated signal operation is governed by its detector configuration and timing settings. The detection of vehicles for the actuated phases may rely either on motion detectors or on presence detectors (*2*). The primary timing variable for the nonactuated phase is minimum green. For the actuated phases, the primary timing variables include minimum green, extension interval, and maximum green. When pedestrian-related phases exist, the pedestrian Walk and Don't Walk intervals also become part of the timing variables. The minimum green of a phase ensures that the green intervals of that phase will last for a duration not shorter than the minimum green setting. The extension interval of an actuated phase provides a mechanism for vehicles to extend a green interval through actuation of the detectors. The maximum greens limit the longest lengths of the green intervals of the actuated phases.

In a semiactuated signal operation, the nonactuated phase is given the green light as long as no vehicles actuate one of the detectors. If there are vehicles actuating the detectors during the minimum green of the nonactuated phase and such actuations result in a call for service, the green interval of the nonactuated phase would be terminated at the end of the minimum green. Otherwise, the green interval would be extended automatically until the time a vehicle actuates one of the detectors.

The green intervals of a vehicle-actuated phase depend on the settings for minimum green, extension interval, and maximum green. They also depend on the detector configuration and the manner in which the arriving vehicles actuate the detectors. If presence detectors are used, the green interval of an actuated phase can be extended beyond the minimum green as long as one of the detection areas associated with that phase is occupied by vehicles. The resulting green interval, however, cannot exceed the maximum green setting. Once the detection areas are left unoccupied, the green interval is extended automatically by one extension interval. In order to extend the green interval further, another vehicle must enter one of the detection areas before this extension interval expires.

If motion detectors are used for an actuated phase, the arriving vehicles must actuate the detectors at intervals not longer than the extension interval to extend the green intervals beyond the minimum green chosen for that phase. Each of such actuations of the detectors would cause a green interval to be extended by one extension interval from the moment of the actuation. Again, the resulting green interval cannot exceed the maximum green for that phase.

WEAKNESS OF THE 1985 HCM (*1*) METHOD

As described previously, semiactuated signal operations can be affected by a number of factors. In light of this operating characteristic, the HCM (*1*) method for estimating the average cycle length and green interval for this type of signal operation has left much to be desired. One critical weakness of the HCM (*1*) method is that this method does not address the impact of timing settings and detector configuration. Equation 1 demonstrates that the HCM (*1*) method does not consider any variable concerning timing settings and detector configuration. Instead, the HCM (*1*) method assumes that the average cycle length is only a function of the traffic volumes associated with both nonactuated and actuated phases. The use of the traffic volume associated with the nonactuated phase for estimating the average cycle length is especially troublesome. The reason is that, once the minimum green for the nonactuated phase is chosen, the traffic volume of the nonactuated phase has no effect on the signal operation. Therefore, the ratio of volume to saturation flow for the nonactuated phase should not be used for estimating the average cycle length.

Another major weakness of the HCM (*1*) method involves the assumption that the critical volume-to-capacity ratio X_c for any intersection being analyzed is about 0.85. In reality, this ratio can vary substantially from one intersection to another even under peak flow conditions; and it can be far smaller than 0.85. Consequently, the use of Equation 1 with X_c = 0.85 can aggravate estimation errors.

In order to examine the extent of estimation errors as produced by the HCM (*1*) method, four sets of data were collected at four intersections. A comparison of the average observed cycle lengths and the averages obtained by the HCM (*1*) method is presented in Table 1. The HCM (*1*) estimates are well below the observed values. Such discrepancies can yield significantly different estimates of delays.

AN ALTERNATIVE METHOD

Semiactuated operations are governed by complex interactions between the signal controls and the arriving vehicles. Attempting to account for such interactions accurately and in detail for estimating the resulting average cycle lengths and green intervals would invariably lead to a cumbersome method. Hence a method was developed through analytical modeling and computer simulation. The analytical modeling makes use of logical assumptions that simplify the estimation problem.

The simulation model used in this study is one developed at Clarkson University. The Clarkson University model is a microscopic model that simulates the signal operations at isolated intersections. The model has two major components, both of which contain a number of subroutines. One component is a flow processor that generates vehicles and moves them downstream through the intersection according to the prevailing flow and signal control conditions. The other component is a signal processor that is essentially a collection of various signal control logics.

In the flow processor, the location and speed of each simulated vehicle are updated once per second. Each simulated vehicle is assigned probabilistically a set of attributes related to vehicle length, maximum desired speed, directional movement, desired space headway from the vehicle ahead in a stationary queue, desired stopped location with respect to the stop line, driver reaction time, and driver sensitivity in a car-following situation. The model allows the simulation of the traffic movements at intersections where the following fea-

TABLE 1 AVERAGE OBSERVED CYCLE LENGTHS AND ESTIMATES OBTAINED BY HCM (*1*) METHOD

Intersection	V/S Ratio for Phase: Non-Actuated	Actuated 1	Actuated 2	Cycle Length, sec: Observed	Estimated
1	0.28	0.10	-	47.5	18.1
2	0.37	0.10	-	103.4	22.4
3	0.13	0.06	0.08	60.6	21.9
4	0.19	0.10	-	46.9	15.2

1. Assumed lost time per phase = 5 sec
2. Intersection
   1 : Elm and Union, Potsdam, NY
   2 : Central and Vly, Albany, NY
   3 : Taft Road and a Shopping Mall Entrance, Liverpool, NY
   4 : Sisson and Market, Potsdam, NY

tures may or may not exist: right-turn-on-red, auxiliary turning bay, mixed directional movements from a given lane, and opposed left turns.

The signal processor determines when the signal indications should be changed, according to the control logic being analyzed. For traffic-actuated signal operations, this processor can accept inputs from a variety of detectors. Each traffic lane may have a combination of several motion detectors and presence detectors. Such detectors may have call-delay or call-extension features.

The vehicular movements as simulated by the Clarkson University model agree reasonably well with the observed characteristics related to right-turn-on-red (*3*), opposed left turns (*4*), and queue dissipation (*5*). Because the model is developed for the purpose of comparative analysis of alternative signal control, special care has been taken to ensure that the model can realistically duplicate the interactions between vehicles and detectors. The validation of a simulation model for traffic-actuated signal operations is difficult to perform and is usually limited in scope. Nevertheless, the semi-actuated and full-actuated signal operations at four intersections as simulated by the Clarkson University model match observed operations rather well. The flow patterns at these intersections are shown in Figure 1. A comparison of the observed and the simulated operations is presented in Table 2.

The method for estimating the average green intervals and cycle lengths for semiactuated operations is described in the following section. This method is based on random vehicle arrivals.

ESTIMATION OF NONACTUATED AVERAGE GREEN

The green intervals of a nonactuated phase can be either equal to or longer than the minimum green chosen for that phase. Their average duration is governed primarily by the minimum green of the nonactuated phase and the traffic volume in the actuated phases than can call for the termination of the green intervals of the nonactuated phase.

Let m be the total number of lanes in the actuated phases; also let $\lambda_1, \lambda_2, \ldots, \lambda_m$ be the respective flow rates of such lanes. Assuming that the vehicles in an actuated phase arrive randomly at the stop line in the absence of interference by the signal control, then the headways between the vehicles in

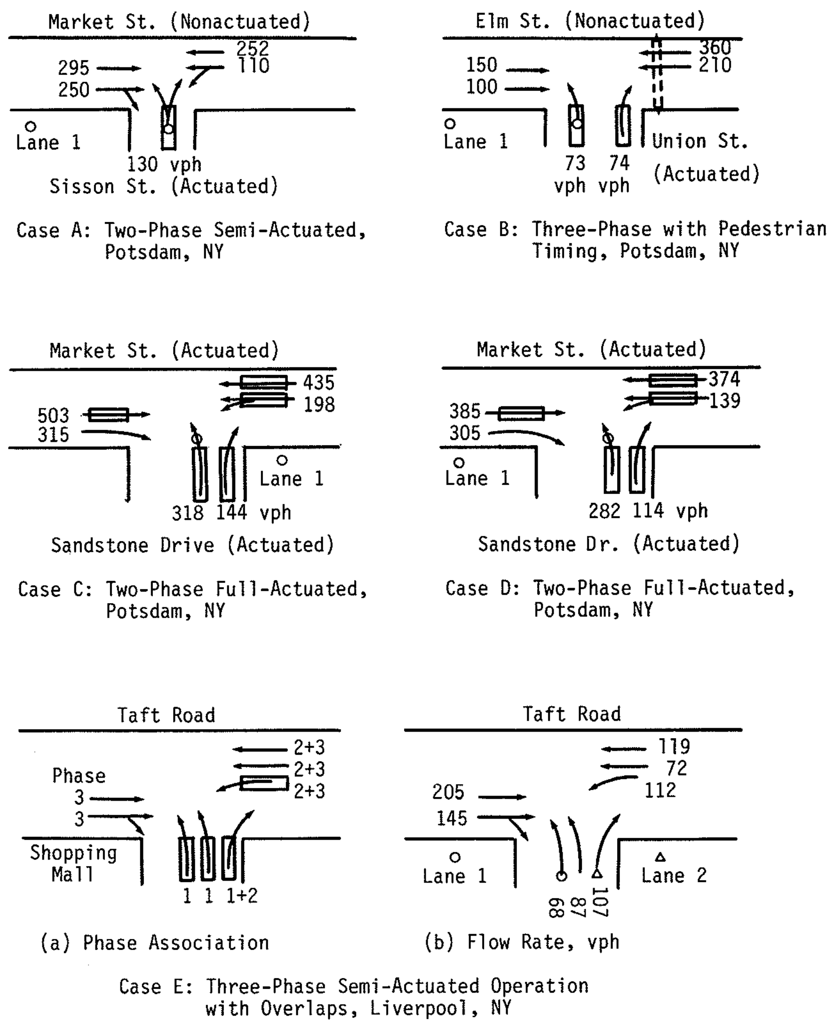

FIGURE 1 Five cases of signal operations observed at four intersections.

TABLE 2 OBSERVED CHARACTERISTICS OF FIVE SIGNAL OPERATIONS AND ESTIMATES OBTAINED FROM CLARKSON MODEL

		Green, sec				Average Stopped Delay, sec/veh	
		Observed		Simulated			
Case	Phase	Mean	S.D.[1]	Mean	S.D.	Observed[2]	Simulated
A	1	6.5	2.6	5.1	2.1	7.7 (Lane 1)	6.6
	2	32.4	25.2	30.3	24.6		
B	1	33.8	18.2	31.9	17.6	11.1 (Lane 1)	10.7
	2	5.4	2.1	5.1	2.3		
	3	24.0	0.0	24.0	0.0		
C	1	27.8	12.8	27.6	12.1	14.7 (Lane 1)	14.9
	2	12.4	6.3	13.0	6.3		
D	1	18.8	9.2	17.5	8.1	not available	
	2	10.7	5.8	9.5	5.2		
E	1	12.9	3.9	12.6	3.5	14.0 (Lane 1)	13.5
	2	9.2	4.1	9.4	4.0	3.1 (Lane 2)	2.7
	3	33.6	7.3	32.1	6.7		

[1] S.D. = Standard deviation
[2] Refer to Fig. 1 for lane designation

the combined flow that includes the flows in m lanes can be represented by the following probability function (*6*):

$$f(h \geq t) = e^{-(\lambda_1+\lambda_2+\ldots+\lambda_m)t} = e^{-\lambda t} \tag{3}$$

where $\lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_m$ and $f(h \geq t)$ is the probability that headway h is equal to or longer than a specified duration t.

A green interval of the nonactuated phase would be terminated at the end of its minimum green under several situations. First, a vehicle in an actuated phase that is expected to arrive at the stop line 2 or 3 sec before a green interval for the nonactuated phase begins may be forced to stop and wait for the next green interval. A call for service by such a vehicle would limit a green interval of the nonactuated phase to its minimum green. Second, a vehicle may have an expected arrival time at the stop line between the green onset of the nonactuated phase and the end of the corresponding minimum green. Such a vehicle can also limit a green interval of the nonactuated phase to its minimum green. Finally, the same signal operation would result if a vehicle, having an expected arrival time at the stop line several seconds after the minimum green of the nonactuated expires, actuates a detector. Generally, the nonactuated phase is assumed to receive a green interval equal to its minimum green if there are vehicles that have expected arrival times in a period equal to $G_{\min,n} + \beta$, where $G_{\min,n}$ is the minimum green of the nonactuated phase and β represents the sum of several seconds before and after the minimum green. Thus, the probability H_1 that the nonactuated phase would have a green interval equal to $G_{\min,n}$ is

$$H_1 = 1 - e^{-\lambda(G_{\min,n}+\beta)} \tag{4}$$

The probability that the nonactuated phase would have a green interval longer than its minimum green $G_{\min,n}$ is $1 - H_1$. The expected (i.e., average) length of a green interval, given that the green interval is longer than $G_{\min,n}$, can be estimated as

$$e^{\lambda G_{\min,n}} \int_{G_{\min,n}}^{\infty} \lambda t e^{-\lambda t} dt = G_{\min,n} + 1/\lambda \tag{5}$$

Therefore, the expected length G_n of the green intervals of the nonactuated phase is

$$\begin{aligned} G_n &= H_1 G_{\min,n} + (1 - H_1)(G_{\min,n} + 1/\lambda) \\ &= G_{\min,n} + (1/\lambda)e^{-\lambda(G_{\min,n}+\beta)} \end{aligned} \tag{6}$$

In order to test this model and determine an appropriate value for β, the Clarkson University model is used to determine the average nonactuated green intervals for a variety of operating conditions. For typical detector configurations, the value of β is found to be 4 sec. The simulated values of G_n and estimates obtained from Equation 6 for $G_{\min,n} = 15$ and 30 sec are shown in Figure 2.

AVERAGE GREENS OF VEHICLE-ACTUATED PHASES

The green intervals of an actuated phase can be significantly affected by the type of detectors used for vehicle detection. Therefore, the two types of detector deployment must be

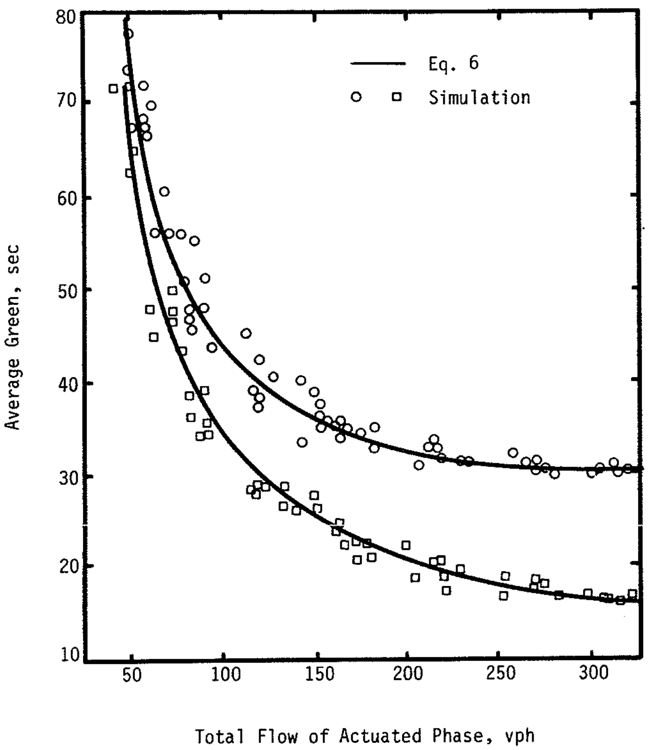

FIGURE 2 Average greens of nonactuated phase for minimum greens of 15 and 30 sec.

considered separately in the estimation of the average green interval of an actuated phase.

Motion Detection

When motion detectors are used, the minimum green of an actuated phase should be sufficient to allow all vehicles stored between the stop line and a detector to move into the intersection, after the green light is given to that phase. Such a minimum green would usually be about 10 sec or slightly longer. In general, a green interval can be extended beyond the minimum green of an actuated phase if there are vehicles actuating the detectors in succession at headways not longer than the extension interval. Because the traffic volume associated with an actuated phase is usually light, the chance for queueing vehicles to be present upstream of the detectors, after the minimum green of the actuated phase expires, is negligibly small. In other words, the extension of a green interval can be assumed to be done by randomly arriving vehicles that do not join the queues.

Let

$G_{\min,i}$ = minimum green of the ith actuated phase;
Q_i = total traffic flow of the ith actuated phase; and
E_i = extension interval of the ith actuated phase.

The probability that the green interval will not be extended beyond $G_{\min,i}$ equals the probability that there are no actuations of the detectors within E_i before the minimum green expires. With random arrivals, this probability, denoted H_2, can be estimated as

$$H_2 = e^{-Q_iE_i} \tag{7}$$

The probability that a green interval will be extended beyond $G_{\min,i}$ by at least one vehicle is $1 - H_2$. To extend a green interval n times ($n = 1, 2, \ldots$), the arriving vehicles should successively actuate the detectors n times at headways not longer than the extension interval E_i, and a headway longer than E_i must follow the last extension. Thus, given that there is at least one extension of a green interval, the expected number of repeated extensions N can be estimated as

$$N = \frac{\sum_{n=1}^{\infty} n(1 - e^{-Q_iE_i})^n e^{-Q_iE_i}}{(1 - e^{-Q_iE_i})} = e^{Q_iE_i} \tag{8}$$

Each of these N repeated extensions has an expected length that can be determined as

$$\frac{\int_0^{E_i} Q_i t e^{-Q_i t} dt}{(1 - e^{-Q_iE_i})} = \frac{(-E_i - 1/Q_i + e^{Q_iE_i}/Q_i)}{(e^{Q_iE_i} - 1)} \tag{9}$$

This expected length of individual extensions can be approximated by one-half of the extension interval, that is, $E_i/2$.

Given that there are an average of N repeated extensions, the first extension can be assumed to occur at $G_{\min,i} - E_i/2$ after the green onset. The first $N - 1$ extensions have an average length of $E_i/2$, and the last extension would result in an additional green equal to E_i. Therefore, N repeated extensions would yield a green interval equal to $G_{\min,i} - E_i/2 + (N - 1)E_i/2 + E_i$. When the possibility of having a green interval equal to $G_{\min,i}$ and that of having a green interval exceeding $G_{\min,i}$ are taken into account, the average green interval G_{ai} for the ith actuated phase becomes

$$G_{ai} = H_2G_{\min,i} + (1 - H_2)[G_{\min,i} - E_i/2 + (N - 1)E_i/2 + E_i] = G_{\min,i} + E_i(e^{Q_iE_i} - 1)/2 \tag{10}$$

In Equation 10, all the queueing vehicles present at the green onset of an actuated phase are assumed to be able to move into the intersection during the green interval. This assumption is reasonable unless a semiactuated control has a poor timing design and has to deal with heavy flows in actuated phases.

Presence Detection

The average green interval of an actuated phase that relies on presence detectors is more difficult to estimate. Such an average green interval can be significantly affected by the minimum green, extension interval, and number of lanes associated with an actuated phase. These characteristics are shown in Figures 3 and 4 on the basis of simulation.

The problem of estimating the average green interval for an actuated phase can be simplified by first transforming all the flows in the lanes associated with that phase into an equiv-

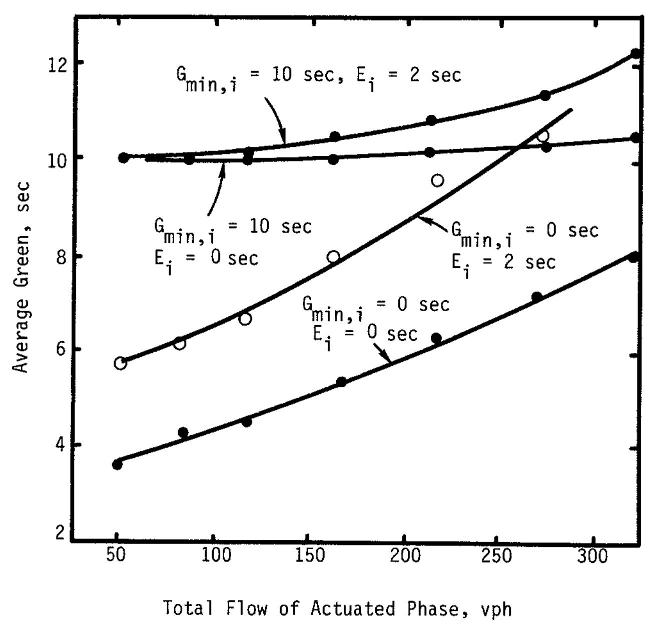

FIGURE 3 Variation of average greens of actuated phase with minimum green and extension interval (presence detection).

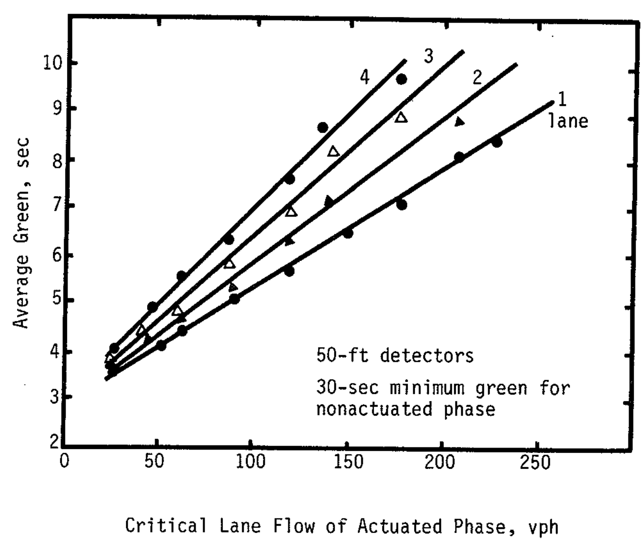

FIGURE 4 Variation of average greens of actuated phase with number of lane flows (presence detection).

alent single-lane flow. Simulation data, such as those shown in Figure 4, reveal that the transformation can be based on the following relationship:

$$Q_{ei} = q_{ci} + 0.3\sum_j q_{ji} \tag{11}$$

where

Q_{ei} = equivalent single-lane flow of the ith actuated phase,
q_{ci} = critical lane flow of the ith actuated phase, and
q_{ji} = the jth noncritical lane flow of the ith actuated phase.

Let X be the number of queueing vehicles in the equivalent signal lane at the beginning of a green interval. The average time required to discharge these and other vehicles that join the queue later can be estimated as

$$T = L + \frac{X}{S - Q_{ei}} \tag{12}$$

where

T = time needed to discharge queueing vehicles given that the initial queue length is X vehicles,
L = lost time caused by starting delay, taken to be 1.0 sec if $X = 1$, 1.5 sec if $X = 2$, and 2.0 sec if $X \geq 3$; and
S = saturation flow rate of the critical lane.

Assuming that the queueing vehicles in a lane can hold the green light and the maximum green is sufficiently long, then the resulting average green interval would be at least equal to T plus one extension interval. After queueing vehicles are discharged, those vehicles not in the queue would be able to extend the green interval if they move into the detection areas at headways not longer than the effective extension intervals they encounter. On average, such effective extension intervals for the vehicles in the ith actuated phase can be defined as

$$E_{ei} = E_i + (D + U)/V \tag{13}$$

where

E_{ei} = average effective extension interval for the ith actuated phase,
E_i = extension interval of the ith actuated phase,
D = detector length,
U = representative vehicle length, and
V = average speed over the detection area for those vehicles extending the green interval.

The probability that a green interval will not be extended by those vehicles not in a queue, after all queueing vehicles are discharged, can be estimated as

$$H_3 = e^{-Q_{ei}E_{ei}} \tag{14}$$

The probability of having at least one additional extension by those vehicles not in a queue is $1 - H_3$. The expected number of repeated extensions after all queueing vehicles are discharged can be estimated from Equation 8 by replacing E_i with E_{ei}. The average length of each extension is about $E_{ei}/2$. The first extension is expected to occur at $T + E_{ei}/2$ after all queueing vehicles are discharged. Therefore, given that a green interval is extended at least once by those vehicles not in a queue, the repeated extensions would have an expected length equal to

$$T_e = E_{ei}/2 + E_{ei}(e^{Q_{ei}E_{ei}} - 1)/2 + E_{ei} \tag{15}$$

Therefore, after all queueing vehicles are discharged, there is a probability of H_3 that a green interval would be extended by E_{ei}, and a probability of $1 - H_3$ that a green interval can be extended by T_e. With these probabilities, the expected length of the extensions by those vehicles not in a queue is

$$\begin{aligned} \Delta G &= E_i H_3 + T_e(1 - H_3) \\ &= \tfrac{1}{2}E_{ei}(1 + e^{Q_{ei}E_{ei}}) - (E_{ei} - E_i)e^{-Q_{ei}E_{ei}} \end{aligned} \tag{16}$$

Let $G_{\min,i}$ be the minimum green of the ith actuated phase. Then a green interval would equal $G_{\min,i}$ if the following condition is satisfied:

$$L + \frac{X}{S - Q_{ei}} + \Delta G \leq G_{\min,i} \tag{17}$$

The largest X that satisfies this condition, denoted X_m, is

$$X_m = (G_{\min,i} - L - \Delta G)(S - Q_{ei}) \tag{18}$$

For small values of $G_{\min,i}$, the value of X_m may become negative. In such case, X_m should be set equal to 0. Round off X_m to its smaller integer and let the resulting integer be denoted X_s. The probability that a green interval would equal $G_{\min,i}$ is the same as the probability that an initial queue length is no more than X_s vehicles. When $X_s = 1$ is involved, this probability can vary by as much as 0.15, depending on the phase plan of a semiactuated operation. Such a variation, however, does not have a significant effect on the estimates of average green intervals. For this reason, the probability F that a green interval equals $G_{\min,i}$ may be estimated as follows, regardless of the phase plan involved:

$$F = \sum_{x=1}^{X_s} P(x)/[1 - P(0)] \tag{19}$$

and

$$P(x) = (Q_{ei}R_i)^x e^{-Q_{ei}R_i}/x! \tag{20}$$

where $P(x)$ = the probability of having $x = 0, 1, 2, \ldots$ arrivals during the red interval R_i faced by the vehicles in the ith actuated phase. The values of F for X_s ranging from 0 to 5 veh are shown in Figure 5.

For each initial queue length that exceeds X_s, the resulting green interval has an average value equal to the sum of T (Equation 12) and ΔG (Equation 16). Therefore, the expected green interval resulting from $x \geq X_s$ can be estimated as

$$h = \left[\sum_{x=X_s+1}^{\infty} (T + \Delta G)P(x)\right] \Big/ \left[1 - \sum_{x=0}^{X_s} P(x)\right] \tag{21}$$

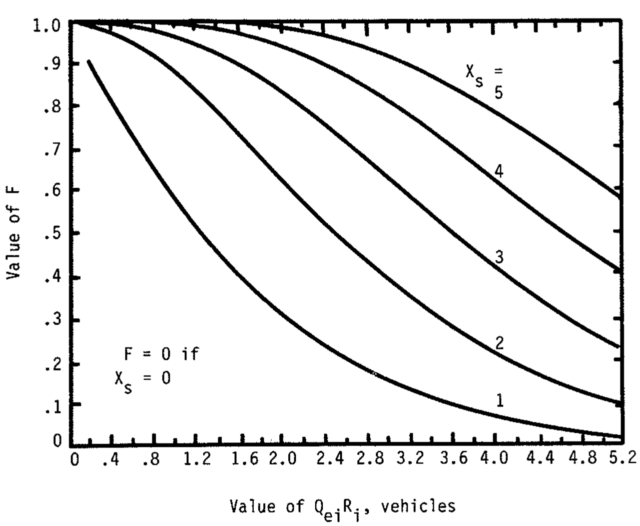

FIGURE 5 Values of F as a function of $Q_{ei}R_i$ and X_s.

Let

$$B = \left[\sum_{x=X_s+1}^{\infty} xP(x)\right] \Big/ \left[1 - \sum_{x=0}^{X_s} P(x)\right] \tag{22}$$

Then, Equation 21 can be rewritten as

$$h = L + \Delta G + \frac{B}{S - Q_{ei}} \tag{23}$$

Combining the case involving $x \leq X_s$ and that involving $x > X_s$, the expected green interval of an actuated phase can be estimated as

$$G_{ai} = FG_{\min,i} + (1 - F)\left(L + \Delta G + \frac{B}{S - Q_{ei}}\right) \tag{24}$$

where G_{ai} = average green interval of the ith actuated phase.

Let Z be the average time a vehicle in the first queueing position will remain in the detection area after the green onset (about 3 to 4 sec) and $G_{\max,i}$ = maximum green of the ith actuated phase. Then, the value of G_{ai} should be subject to the following constraint: $Z + E_i \leq G_{ai} \leq G_{\max,i}$. The sum of A and E_i represents the average green if only one vehicle receives the green in an actuated phase.

To avoid undue complications, neither Z nor $G_{\max,i}$ is explicitly taken into account in deriving G_{ai}. As a result, Equation 24 may underestimate the average green by about 2 sec when the flow rate of an actuated phase is small. This equation may also slightly overestimate the average green if there is a high probability for the vehicles in an actuated phase to extend the green to $G_{\max,i}$.

The value of B in Equation 22 represents the average queue length at the green onset of the ith actuated phase, given that the queue length exceeds X_s vehicles. Figure 6 may be used to obtain a quick estimate of B.

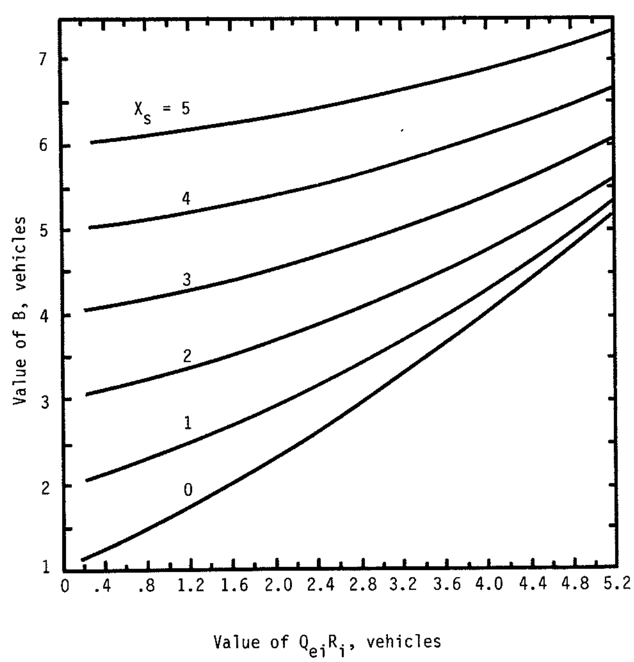

FIGURE 6 Values of B as a function of $Q_{ei}R_i$ and X_s.

Examples of estimates obtained from Equation 24 compared with estimates obtained from computer simulation are shown in Figure 7.

ESTIMATION OF $Q_{ei}R_i$ AND CYCLE LENGTH

In order to use Equation 24 to estimate the average green interval of an actuated phase, one must first estimate the product $Q_{ei}R_i$ (Equation 20). $Q_{ei}R_i$ represents the average number of queueing vehicles associated with Q_{ei} at the green onset of the ith actuated phase. Given the estimated green intervals of the nonactuated and the actuated phases, the average cycle length can be estimated. The value of $Q_{ei}R_i$ and the average cycle length C encountered by Actuated Phase i are affected by the phase plan involved. Three phase plans are considered. Each plan has a nonactuated phase and up to two actuated phases.

Two-Phase Operation

For a two-phase operation, $Q_{ei}R_i$ can be estimated as follows:

$$Q_{ei}R_i = Q_{ei}(G_n + Y_n) \tag{25}$$

where G_n is determined from Equation 6 and Y_n is the change interval of the nonactuated phase.

The average cycle length for a two-phase operation is

$$C = G_n + Y_n + G_{ai} + Y_i \tag{26}$$

where Y_i = change interval of Actuated Phase i. If the effective cycle length is to be used for evaluation purposes, G_n may be replaced by $G_{\min,n}$.

Three-Phase Operation Without Overlaps

Without phase overlaps, the vehicles in a given lane are allowed to move into the intersection only during the green interval of one designated phase. Let Phase i be the actuated phase of which the average green interval is to be determined, and let Phase j be the other actuated phase. Then, the $Q_{ei}R_i$ for Phase i can be approximated as

$$Q_{ei}R_i = Q_{ei}[G_n + Y_n + w_j(G_{aj} + Y_j)] \tag{27}$$

where

$$w_j = 1 - e^{-(G_{\min,n} + Y_n + G_{ai} + \beta)Q_j} \tag{28}$$

In these equations, w_j represents the probability that the other actuated phase exists in a cycle. As in Equation 6, the value of β may be taken to be 4 sec; Q_j is the total flow rate associated with Phase j.

It can be seen from Equations 27 and 28 that the estimation of G_{ai} requires the knowledge of G_{ai} and G_{aj}. Because both G_{ai} and G_{aj} may be unknown, the use of trial values becomes a necessity. The resulting estimates can be improved through an iteration process. Fortunately, G_{ai} and G_{aj} are usually short. A trial value either of 8 sec or the minimum green of the actuated phases, whichever is greater, would often yield good estimates even without iterations.

The average cycle length depends on how often an actuated phase exists in a cycle. The probability that a cycle includes Phase 1 and the nonactuated phase is $w_1(1 - w_2)/(w_1 + w_2)$, where w_1 and w_2 are the values obtained from Equation 28 for $j = 1$ ($i = 2$) and $j = 2$ ($i = 1$), respectively. The probability that a cycle includes Phase 2 and the nonactuated phase is $w_2(1 - w_1)/(w_1 + w_2)$. The probability that a cycle includes all the three phases is $1 - w_1(1 - w_2)/(w_1 + w_2) - w_2(1 - w_1)/(w_1 + w_2) = 2w_1w_2/(w_1 + w_2)$. Therefore, the average cycle length has an estimated value equal to

$$\begin{aligned} C = {} & G_n + Y_n + (G_{a1} + Y_1)w_1(1 - w_2)/(w_1 + w_2) \\ & + (G_{a2} + Y_2)w_2(1 - w_1)/(w_1 + w_2) \\ & + 2(G_{a1} + Y_1 + G_{a2} + Y_2)w_1w_2/(w_1 + w_2) \end{aligned} \tag{29}$$

Three-Phase Operation With Overlaps

The three-phase operation to be analyzed below refers to the one shown as Case E in Figure 1. In such an operation, a cycle may not include Phase 2, but it always has Phase 1 and the nonactuated phase. The $Q_{ei}R_i$ for Phase 1 can be estimated as

$$\begin{aligned} Q_{e1}R_1 = Q_{e1}\{ & G_n + Y_n \\ & + (G_{a2} + Y_2)[1 - e^{-Q_2(G_{a1} + \beta)}]\} \end{aligned} \tag{30}$$

where Q_2 = total flow of Phase 2. Because G_{a1} is unknown, a trial value must be used.

For Phase 2, the $Q_{ei}R_i$ can be estimated as

$$Q_{e2}R_2 = Q_{e2}Y_1 + Q_{e2}G_{a1}/[1 - e^{-Q_2(G_{a1} + \beta)}] \tag{31}$$

The average cycle length can be estimated as

$$\begin{aligned} C = {} & G_n + Y_n + G_{a1} + Y_1 \\ & + (G_{a2} + Y_2)[1 - e^{-Q_2(G_{a1} + \beta)}] \end{aligned} \tag{32}$$

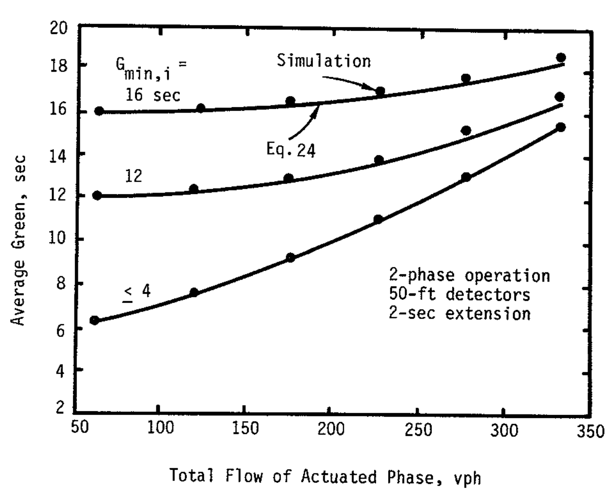

FIGURE 7 Computer-simulated average greens of actuated phase compared with estimates obtained from Equation 24.

For the evaluation of a three-phase operation with overlaps, the average green interval received by individual movements in an average cycle must be determined. Referring to Case *E* in Figure 1, the average green received by the left-turn vehicles in Phase 1 is the same as the average green of Phase 1. The average green received by the eastbound flows in Phase 3 is the same as the average green of Phase 3. The average green received by the right-turn flow in Phase 1 includes the average green interval and the change interval of Phase 1, plus the average green of Phase 2 if Phase 2 exists in a cycle. Therefore, the average green interval received by this right-turn flow (G_R) can be estimated as

$$G_R = G_{a1} + Y_1 + (G_{a2} + Y_2)[1 - e^{-Q_2(G_{a1}+\beta)}] \quad (33)$$

The left-turn flow in Phase 2 receives an average green interval that includes the average green of the nonactuated phase and, if Phase 2 exists in a cycle, the average green of Phase 2. This received average green (G_L) can be estimated as

$$G_L = G_n + G_{a2}[1 - e^{-Q_2(G_{a1}+\beta)}] \quad (34)$$

The westbound straight-through flows do not encounter a change interval when the signal operation moves from Phase 2 to Phase 3. Therefore, they receive an average green interval equal to

$$G_s = G_n + (G_{a2} + Y_2)[1 - e^{-Q_2(G_{a1}+\beta)}] \quad (35)$$

EXAMPLE APPLICATIONS

Example 1

This example is the two-phase operation shown as Case *A* in Figure 1. This operation has the following features relevant to the estimation of average cycle length and green interval:

- 15-sec minimum green for the nonactuated phase,
- 4-sec minimum green for the actuated phase,
- 0-sec extension interval,
- 2-sec effective extension interval,
- 60-ft presence detector with a 5-sec call delay,
- 3-sec yellow interval and 1.0-sec all-red interval for both phases,
- 43 percent right turns on Sisson Street,
- right-turn-on-red allowed, and
- Saturation flow rate of 1,400 vehicles per hour of effective green (vphg) for the actuated phase.

In addition, the actuated phase allows concurrent pedestrian timing, but the signal was never actuated by pedestrians during the course of a field study. For the flow pattern shown in Figure 1, the average observed green interval for the nonactuated phase is 32.4 sec and that for the actuated phase is 6.5 sec.

The permission to make right turns on red complicates the estimation of the average cycle length and green intervals. A model can be used to estimate the proportion of vehicles that can make right turns on red without actuating the detector on Sisson Street. As an approximation, all the vehicles on Sisson Street are assumed to have actuated the detector and called for service. On the basis of this assumption, the estimation of the average cycle length and green intervals follows:

1. Average Green of Nonactuated Phase G_n (Equation 6).
Inputs: $G_{\min,n}$ = 15 sec, λ = 130 vph, β = 4 sec
Output: G_n = 29.0 sec
2. Estimation of ΔG (Equation 16).
Inputs: E_{ei} = 2 sec, Q_{ei} = 130 vph, E_i = 0
Output: ΔG = 0.2 sec
3. Estimation of X_m and X_s (Equation 18).
Inputs: ΔG = 0.2 sec, $G_{\min,i}$ = 4 sec, S = 1,400 vphg, Q_{ei} = 130 vph, L = 2 sec (assumed)
Outputs: X_m = −0.63 veh, X_s = 0
4. Estimation of $Q_{ei}R_i$ (Equation 25).
Inputs: G_n = 29 sec, Y_n = 4 sec, Q_{ei} = 130 vph
Output: $Q_{ei}R_i$ = 1.19 veh
5. Estimation of F (Figure 5) and B (Figure 6).
Inputs: X_s = 0, $Q_{ci}R_i$ = 1.19 veh
Outputs: F = 0.0, B = 1.7 veh
6. Average Green of Actuated Phase G_{ai} (Equation 24).
Inputs: L = 1 sec (assumed for B = 1.7 veh) and other given data
Output: G_{ai} = 6.0 sec

Example 2

This example is the three-phase operation shown as Case *E* in Figure 1. The signal operation has the following features:

- 30-sec minimum green for the nonactuated phase,
- 10-sec minimum green for Phase 1,
- 7.5-sec minimum green for Phase 2,
- 3.5-sec extension interval for Phases 1 and 2,
- 50-ft detector lengths,
- Yellow interval and all-red interval for Phase 1 of 3.5 and 1.2 sec, respectively,
- Yellow interval and all-red interval for Phases 2 and 3 of 4 and 0 sec, respectively,
- Effective extension interval for Phase 1 of 6 sec,
- Effective extension interval for Phase 2 of 5.5 sec,
- Saturation flow for Phase 1 of 1,400 vphg, and
- Saturation flow for Phase 2 of 1,500 vphg.

The observed average green intervals for this signal operation are 33.6 sec for the nonactuated phase, 12.9 sec for Phase 1, and 9.2 sec for Phase 2. The average observed cycle length is 60.6 sec.

1. Average Green of Nonactuated Phase G_n (Equation 6).
Assumption: 50 percent of the 107-veh/hr right-turn flow of Phase 1 can call for the termination of the nonactuated phase
Inputs: $G_{\min,n}$ = 30 sec, λ = 68 + 87 + 107/2 = 208 vph, β = 4 sec
Output: G_n = 32.4 sec
2. Estimation of ΔG (Equation 16) for Phase 1.
Assumption: critical lane flow = 87 vph (not the 107-vph right-turn-on-red flow)
Inputs: Q_{e1} = 87 + 0.3(68 + 107) = 140 vph (Equation 11), E_{e1} = 6 sec, E_1 = 3.5 sec
Output: ΔG = 4.8 sec

3. Estimation of X_m and X_s (Equation 18) for Phase 1.
Inputs: $\Delta G = 4.8$ sec, $G_{\min,1} = 10$ sec, $S = 1{,}400$ vphg, $L = 2$ sec (assumed), $Q_{e1} = 140$ vph
Outputs: $X_m = 1.1$ veh, $X_s = 1$ veh
4. Estimation of $Q_{e1}R_1$ for Phase 1 (Equation 30).
Inputs: $G_n = 32.4$ sec, $Y_n = 4$ sec, $G_{a2} = 8$ sec (larger of 7.5 and 8 sec), $Y_2 = 4$ sec, $Q_2 = 112$ vph, $G_{a1} = 10$ sec (larger of 10 and 8 sec), $\beta = 4$ sec, $Q_{e1} = 140$ vph
Output: $Q_{e1}R_1 = 1.58$ veh
5. Estimation of F (Figure 5) and B (Figure 6) for Phase 1.
Inputs: $X_s = 1$ veh, $Q_{e1}R_1 = 1.58$ veh
Outputs: $F = 0.41$, $B = 2.7$ veh
6. Average Green of Phase 1, G_{a1} (Equation 24).
Inputs: $L = 1.5$ sec (assumed) and other relevant data
Output: $G_{a1} = 12.4$ sec
7. Estimation of ΔG (Equation 16) for Phase 2.
Inputs: $Q_{e2} = 112$ vph, $E_{e2} = 5.5$ sec, $E_2 = 3.5$ sec
Output: $\Delta G = 4.3$ sec
8. Estimation of X_m and X_s (Equation 18) for Phase 2.
Inputs: $\Delta G = 4.3$ sec, $G_{\min,2} = 7.5$ sec, $L = 2$ sec (assumed), $S = 1{,}500$ vphg, $Q_{e2} = 112$ vph
Outputs: $X_m = 0.46$ veh, $X_s = 0$
9. Estimation of $Q_{e2}R_2$ for Phase 2 (Equation 31).
Inputs: $G_{a1} = 12.4$ sec, $Y_1 = 3.5 + 1.2 = 4.7$ sec, $Q_2 = 112$ veh, $\beta = 4$ sec, $Q_{e2} = 112$ vph
Output: $Q_{e2}R_2 = 1.11$ veh
10. Estimation of F (Figure 5) and B (Figure 6) for Phase 2.
Inputs: $X_s = 0$, $Q_{e2}R_2 = 1.11$ veh
Outputs: $F = 0.0$, $B = 1.6$ veh
11. Average Green of Phase 2, G_{a2} (Equation 24).
Inputs: $L = 1$ sec (assumed) and other relevant data
Output: $G_{a2} = 9.4$ sec
12. Average Cycle Length (Equation 32).
Inputs: $G_n = 32.4$ sec, $Y_n = 4$ sec, $G_{a1} = 12.4$ sec, $Y_1 = 4.7$ sec, $G_{a2} = 9.4$ sec, $Y_2 = 4$ sec, $Q_2 = 112$ vph, $\beta = 4$ sec
Output: $C = 58.9$ sec

CONCLUSIONS

When the 1985 HCM (*1*) is used for the evaluation of traffic-actuated signal operations, the average cycle lengths and green intervals related to such operations must be estimated. A method is suggested in the manual to obtain the needed estimates. However, the suggested method is not reliable.

In order to provide a better alternative, a method is developed through analytical reasoning and computer simulation for semiactuated signal operations. The analytical reasoning, which attempts to identify the average operating characteristics of semiactuated operations, avoids elaborated modeling of the complex interactions between the vehicles and the signal control. The resulting method is reasonably simple and reliable. This method can be structured in a tabular form for manual applications and computerized for applications by a personal computer.

The method described can be expanded to deal with those signal operations that involve pedestrian phasing. Right-turn-on-red movements and their impact should be properly modeled to enhance the accuracy in the estimation of the average cycle length and green interval. Opposed left turns are usually not a significant problem for semiactuated operations, because the level of conflicts is low and the saturation flow of the left-turn lane can be readily estimated. Nevertheless, opposed left turns can also be modeled in the development of a generalized method.

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