# Kevlar Reinforced Prestressing for Bridge Decks

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Kevlar reinforced composites show considerable promise for use as prestressing tendons. Their high strength, low relaxation properties and resistance to chloride induced corrosion make them attractive for the design of prestressed concrete bridge decks and for rehabilitation or strengthening of existing bridge decks. Experimental work using Kevlar reinforced tendons for slab structures is presented and the design guidelines for using brittle materials in post-tensioning tendons are discussed. Structural issues of anchorage and reliance on resin socketed terminations and research necessary to develop composites to construction grade standards are addressed.

Substantial research has been conducted examining the use of composite materials for structural applications. Among the research has been an examination of both glass reinforced plastics and aramid reinforced plastics for prestressing applications (*1*). The primary motivation for the examination of these composites is their inherent resistance to corrosive conditions found in many structures. While both glass and aramids have some reactivity with alkali environments, they remain relatively impervious to sodium or calcium chloride solutions. The ability to reliably prestress bridge decks with these composites could enhance the durability of the bridge.

The issue of the cost of composites is of some concern. Glass fiber composites will cost in excess of $2.00 per pound. Aramid fibers, Kevlar being the most common in the United States, can cost in excess of $12.00 per pound. Even when corrected for their lower specific gravities and for their higher strengths, these materials will not compete with traditional prestressing steel on a first cost basis. If, however, the materials can extend the service life of a structure, or if they prevent the need to reconstruct a bridge deck, then they may become cost competitive. Current research and prototype installations are providing long term performance data which will assist engineers to determine the effectiveness of these materials.

Three synthetic materials are currently available for use as prestressing tendons. They are glass, aramid, and carbon. Typical stress strain curves for these fibers are shown in Figure 1 along with the stress strain curve for ASTM 416 - 270 ksi (1.20 GPa) seven wire prestressing strand. In addition to their high tensile capacity, the fibers are nearly linearly elastic to failure. The lack of a yield region creates significant concerns about the ductility of the structure and the method of design.

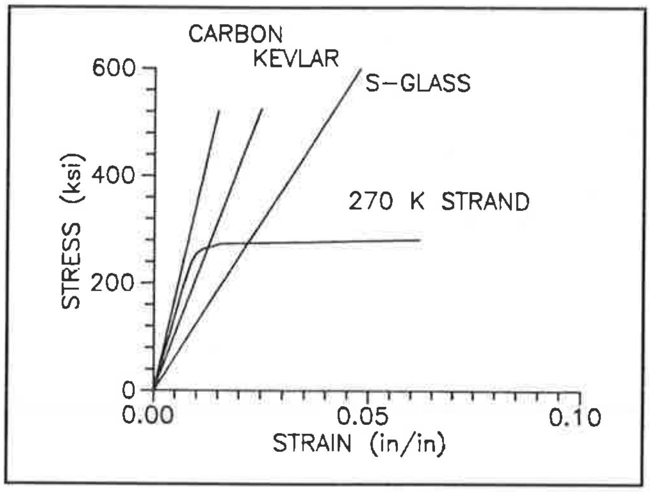

FIGURE 1 Sample Stress Strain Curves

This paper focus on the use of aramid fibers as reinforcing for prestressing tendons. The rationale for examining aramids is twofold. First, carbon fibers, while having a very high strength, also have a low strain to failure. When allocating the strain capacity of the fibers, as will be discussed later, there is little ductility reserve in carbon tendons used for prestressing applications. Glass fibers have the initial advantage of lower cost. E-Glass has been the focus of most development work in glass tendons. E-Glass has a lower strain to failure than S-Glass. E-Glass resistance to fatigue conditions which may be encountered in bridge decks suggests that aramids may be a superior choice for tendons.

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Aramid is the generic name for polyparaphenylene-terepthalamine. It is most commonly recognized by the trade name Kevlar, an E.I. duPont product. Other commercial aramid fibers are Twaron, manufactured by ENKA in Europe and Technora, made by Teijin Industries in Japan. Three grades of aramid fibers are available. Kevlar serves to differentiate these grades. Kevlar 29 has an elastic modulus of about 9.0 x $10^6$ psi (62 GPa) and a strain to failure of about 3.8 percent. Kevlar 49 has an elastic modulus of about 17 x $10^6$ psi (117 GPa) and a strain to failure of about 2.5 percent. Kevlar 149 has a modulus of 21.1 x $10^6$ psi (146 GPa) and a strain to failure of 1.4-1.5 percent (*2*). Kevlar 29 and Kevlar 49 both have tensile fiber strengths of 525 ksi (2.3 GPa) and Kevlar 149 has a slightly lower tensile stress. Twaron is similar to Kevlar 49 while Technora has properties between Kevlar 29 and Kevlar 49.

## CHARACTERISTICS OF ARAMID MATERIALS

Individual fiber strength, fiber strength capacity under sustained loads, durability in moist, alkali and chloride environments, fatigue resistance, and relaxation properties of aramid fibers are examined to determine the suitability of the fibers for prestressing applications.

### Tendon strength

The strength of a composite tendon must be examined from three viewpoints. First the strength of a single fiber must be considered, then the behavior of a fiber bundle assessed, and lastly the strength of a resin coated fiber bundle ascertained. The high strength of an individual fiber cannot be realized in a design application. Like a chain, a long fiber may be considered to be no stronger than its weakest segment. Probability theory indicates that the longer the fiber is, the lower its strength will be. This behavior has been demonstrated in laboratory experiments (*3*).

When many fibers are assembled into a parallel system, the reliability of the resulting tendon increases. The parallel fibers act as a redundant structural system, and the load carried by the weakest fiber is transferred to the stronger fibers when the weakest fibers fail. Providing that the strength of the remaining fibers is greater than the applied load, the tendon will continue to function, however the apparent strength of the fibers will be less than single fiber strength. This scenario assumes that the load is free to search out the weakest fiber. In actual bundles of twisted or resin coated fibers, once a fiber breaks, the load is transferred by friction, or by the matrix resin, to the adjacent fibers. Thus, the load does not have the opportunity to search out the next weakest fiber. Thus friction or resins increase the strength and the reliability of the tendon.

Extensive probability analysis by Phoenix and others (*4*) provide some guidance to the behavior of fiber bundles. Applying this theory to a Kevlar reinforced tendon, and using a probability of failure of 1 in 1000 for prestressing tendons, the theoretical tensile capacity of the tendon is about 315 ksi (2.2 GPa) based on the total fiber area. Glass and carbon fiber bundles undergo similar strength reductions. Glass is slightly more sensitive to strength loss with length than is Kevlar or Carbon.

If the fibers in the tendon are bound together with a resin, then the average strength of the tendon is determined by the volume fraction of the rod which is Kevlar. A typical Kevlar reinforced tendon will be 60 - 70 percent fiber and the remaining cross section will be resin. The resin strength contributes little to the total strength. Thus, based on the stresses predicted from probability theory and a volume fraction of 60 - 70 percent, the expected tendon strength is 190 - 214 ksi (1.3 to 1.5 GPa) using the total composite cross section. The DuPont Data Manual for Kevlar 49 (*5*) gives a tensile strength of 200 ksi (1.4 GPa) for a unidirectional Kevlar composite with a volume fraction of 60 percent fiber. This is consistent with the predictions based on probability theory. Since prestressing represents a proof load condition, stressing the tendons to 70 percent of their tensile capacity assures that the short term strength of the tendon is adequate. Using the DuPont guideline of 200 ksi (1.4 GPa), a jacking stress of 140 ksi (965 MPa) may be considered for prestressing applications.

### Sustained Load Capacity

Aramid fibers have a lower long term strength than the short term strength determined by static testing. This behavior is similar to concrete. A concrete cylinder will fail at 75 - 85 percent of its short term strength if left under a sustained high load. Test data indicates that Kevlar fibers have a 50 percent probability of failure after 100,000 hours if they are stressed above 70 percent of their short term fiber strength. S-Glass in comparison has a 50 percent probability of failure after 10,000 hours if stressed above 50 percent of its short term tensile strength (*5*). Thus Kevlar is superior to S-Glass for sustained load capacity and may be initially stressed to a slightly higher prestress level.

Seventy percent of the short term Kevlar strength is about 220 ksi (1.5 GPa) in a resin impregnated rod. The long term strength mechanics of sustained load capacity of resin reinforced rods with many fibers is not fully understood. However, allowing a jacking stress of 140 ksi (965 MPa) appears to be within the allowable range for long term strength retention. In order to increase the sustained strength capacity of the tendon a slightly lower jacking stress may be considered.

### Environmental Durability

Environmental durability addresses the performance of aramids in their working state. Specifically, three conditions are examined. They are normal and elevated temperature properties, moist environments, and alkali and chloride environments. The behavior of Kevlar 49 under static load at normal room temperature and at 250 °F (121 °C) are given in Table 1.

TABLE 1 STATIC LOAD DATA FOR KEVLAR 49

| Properties at Room Temperature | U.S. Units | S.I. |
|---|---|---|
| Axial Modulus | 18 x $10^6$ psi | 124 GPa |
| Axial Tensile Strength | 525 x $10^3$ psi | 3.6 GPa |
| Coefficient of Thermal Expansion | -2.9 x $10^{-6}$ in/in/°F | -5.2 x $10^{-6}$m/m/°C |
| Axial Strain at Break | 2.9 % | 2.9% |
| **Properties at 250 °F (121 °C)** | | |
| Axial Modulus | 15.4 x $10^6$ psi | 114 GPa |
| Axial Tensile Strength | 460 x $10^3$ psi | 3.2 GPa |
| Coefficient of Thermal Expansion | -2.9 x $10^{-6}$ in/in/°F | -5.2 x $10^{-6}$m/m/°C |
| Axial Strain at Break | 2.8% | 2.8% |

Data from Reference 5.

The loss of properties at elevated temperature is a concern, however, this magnitude of temperature increase is unlikely to occur in a bridge deck. The larger concern is the loss of strength associated with the resins used to bind the fibers. These resins are typically epoxies or vinylesters, and will have strength losses greater than the aramid fibers at elevated temperatures. The effects of strength loss in the resins is quite similar to the concerns about strength loss in epoxy coated strands and glass fiber reinforced tendons. The resin behavior suggest that the primary mode of failure at elevated temperature is likely to be debonding of the tendon at the resin interface.

The behavior of aramids in moist and alkali environments is a concern. The primary tensile strength of aramid fibers is generated by Van der Waals forces. Moisture can penetrate these bonds and cause a strength and modulus loss. Exposure tests of single fibers to water and to 5% salt water solutions have indicated no loss of strength and about an 8% loss of modulus in a 24 hour test at room temperature (*5*). Tests of Kevlar fibers in a cement matrix and increasing the temperature to 80 °C (180 °F) indicated a 30 percent strength loss after 30 days exposure (*6*). This loss is a combination of moisture, alkali, and elevated temperature effects. The surface to volume ratio of the individual fibers is very high, thereby maximizing the chemical reactivity. Glass fibers also display deterioration when exposed to the alkali environments found in concrete. Encasing the fibers in a resin may substantially change the response to this moisture. Glass reinforced tendons in prototype bridge structures have used an epoxy mortar grout in the tendon to assure bond and further protect the tendon (*7*). No long term durability tests of resin encased aramids were found in the literature.

### Fatigue Resistance

Fatigue tests were performed in England on tendons made of 64 small diameter composite rods (*8*). Tests were performed on composite rods made of fiber and epoxy resin, anchored in a steel socket using and epoxy. These tests indicated that the fatigue strength of aramids and carbons was superior to steel and that glass was more susceptible to fatigue damage than steel. The fatigue tests are summarized in Table 2.

TABLE 2 FATIGUE RESULTS OF COMPOSITE TENDONS

| Material | Number of Samples | Base Load % break strength | Load Range % strength | Average number cycles | Number of rods broken |
|---|---|---|---|---|---|
| E-Glass | 3 | 15-31 | 6-8 | .17 x$10^6$ | 15 |
| Kevlar 49 | 3 | 50-55 | 7-10 | 2.08 x$10^6$ | 0 |
| Carbon | 7 | 45-55 | 7-10 | 2.24 x$10^6$ | 0 |
| HS Steel | 2 | 35 | 5-7.5 | 2.04 x$10^6$ | 0-5 |

Data based on results given in Reference 8. The actual test programs contained additional tests outside the load ranges summarized in this table. The composite rods had a volume fraction of about 63 percent.

### Relaxation

Tests at Delft Technical University in the Netherlands and by Enka examined the long term relaxation characteristics of aramid tendons. They concluded that the relaxation losses for 100 years were about 20 percent (*9,10*). An independent relaxation test was undertaken to confirm these claims for a Kevlar aramid.

A four foot long (1.22 m) tendon comprised of 4 - 1/8 inch (3.2 mm) rods was secured in an epoxy socket anchor. The tendon was stressed to 70 percent of the tensile capacity in a double extra strong steel pipe. A load cell was installed under the dead end anchor, Figure 2. The resulting loads, normalized to the jacking stress equal to 1.0, are given in

Figure 3. Two phenomenon occur concurrently. First, there is movement of the epoxy plugs in the anchors. This accounts for much of the initial stress loss and resulted in the tendon being restressed shortly after the initiation of the tests. After restressing, dial gages were placed on the epoxy plugs to allow separation of the anchor movement from the tendon relaxation. Creep in the epoxy of the anchor continued during the test. Secondly, the tendon continued to relax, but at a lower rate for the test duration. The top line corresponds to the relaxation losses if effects of the anchor movement are removed from the test results. Test data was recorded for over 5000 hours. The corrected relaxation data at 5000 hours was about 11 percent. This agreed well with the european data.

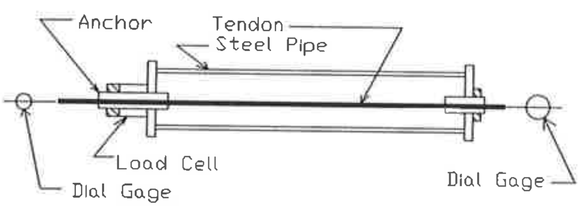
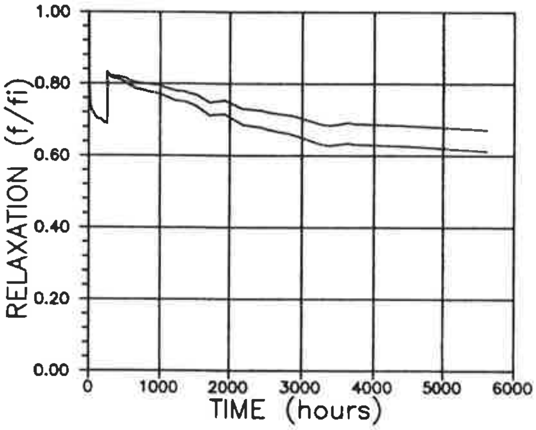


FIGURE 2 Relaxation Test Assembly



Unity of the graph represents 143 ksi (20.7 MPa) and corresponds to a load of 7.0 kips (31 kN).

FIGURE 3. Tendon Stress Versus Time

## STRUCTURAL BEHAVIOR

To investigate the short term behavior of concrete prestressed with Kevlar reinforced tendons, six prototype slab elements were constructed. Four tests examined the behavior of unbonded tendons and two tests were conducted using bonded tendons. The slab test was a 6 inch wide by 4 inch deep (152 x 100 mm) segment reinforced with a single tendon, Figure 4. The tendon consisted of a series of smooth 1/8 inch (3.2 mm) diameter rods. The rods were 68 percent Kevlar 49 in a vinylester resin. Tendons consisted of either 4 or 6 rods. The tendons were stressed to 120 ksi (830 MPa).

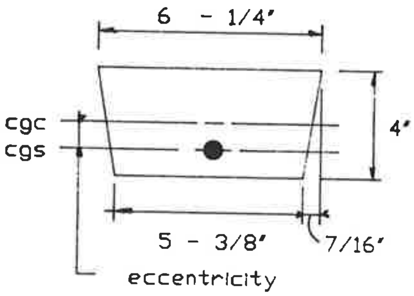


FIGURE 4 Test Cross Section

The beams were tests were conducted with a two point loading system. Beams were tested montonically to failure using a hydraulic jacking system. Data was recorded either with dial gages or with an automatic data acquisition system, Figure 5.

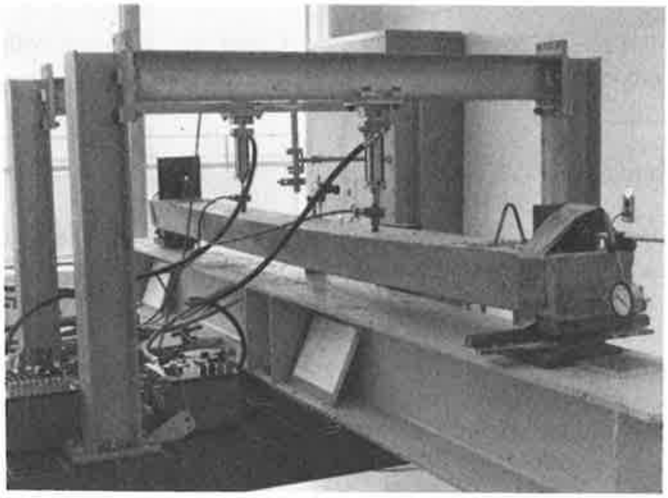


FIGURE 5 Test Arrangement

## Unbonded Tendon Results

The unbonded tendons specimens failed when the concrete exceeded its ultimate strain capacity. After initial crushing of the concrete, the sections continued to carry load, although the initial moment capacities were never regained. The test frame limited the total beam deflection, therefore it was not possible to strain the tendons to failure. Figure 6 shows a typical moment deflection curve for a six rod unbonded tendon system. The section was cycled several times after the initial failure. Deterioration of the section is evident by the gradual loss of deflection and strength capacity.

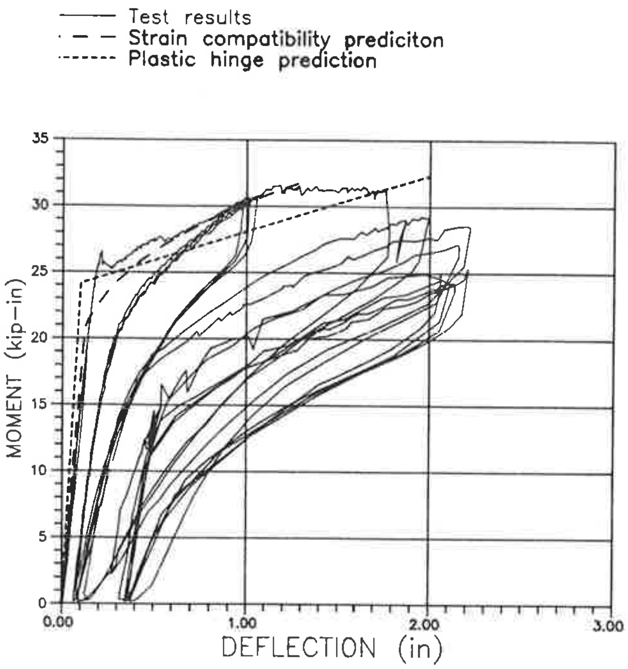
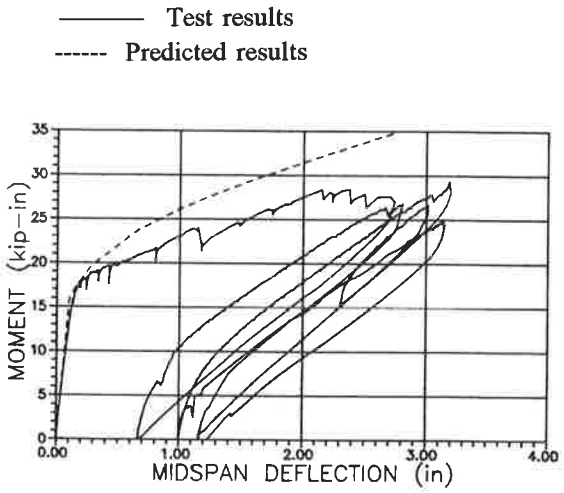


FIGURE 6 Moment Deflection Curve for a Beam with Unbonded Tendons

## Bonded Tendon Results

The bonded tendons failed by pullout of the tendon. The rods used in this research were originally intended for post-tensioning applications. As a consequence, the surfaces were not roughened to improve the bond with the concrete. The surface texture has the smoothness of a fiberglass fishing pole, and loss of bond was not unexpected. Furthermore, the test setup allowed only 27 inches (.69 m) between the end of the beam and the point of application of the load. For a 6 rod tendon, this is about 60 tendon diameters. Steel tendons have similar difficulties developing their capacity in that length. Figure 7 provides the moment deflection results for a six rod pretensioned beam. The sharp drop in moment capacity, followed by a quick recovery, represents the formation of cracks in the concrete. The jump in recovered deflection following the first maximum load cycle is a direct result of the tendon slipping. When the moment was controlled so that the tendon was not allowed to slip, the beam showed virtually complete deflection recovery. A uniform pattern of flexural cracks formed at about 6 inches (150 mm) on center.



The permanent deflection of about 0.7 inches (18 mm) is associated with the slip of the tendon.

FIGURE 7 Moment Deflection Results for a Beam with Bonded Tendons

## Test Implications

These tests lead to several short term conclusions. First, Kevlar reinforced tendons did provide the strength required for load carrying capacity on a short term basis. Second, the structures displayed substantial ductility during loading. The failure to rupture a tendon is a function of the shallow cross section and the inability to develop large tensile strains in the tendon. A deeper beam would may less ductility depending upon the prestressing reinforcement ratio.

One post-tensioned beam failed when one of the rods pulled out of the anchor. This confirms the concern regarding the long term reliance on epoxy bonded anchor systems. Tendon grouting is preferable to assure long term strength development.

## DESIGN IMPLICATIONS

The slab tests indicated that aramid prestressing materials are satisfactory for short term applications. Several questions arise on the actual application of Kevlar reinforced tendons in a design situation. These include the combined effects of using resin reinforced composites in wet, chloride environments, the long term strength degradation, anchorage of the tendons, and the reliability of brittle materials. These concerns lead to consideration of the tendon strain condition as a basis for strength and serviceability design.

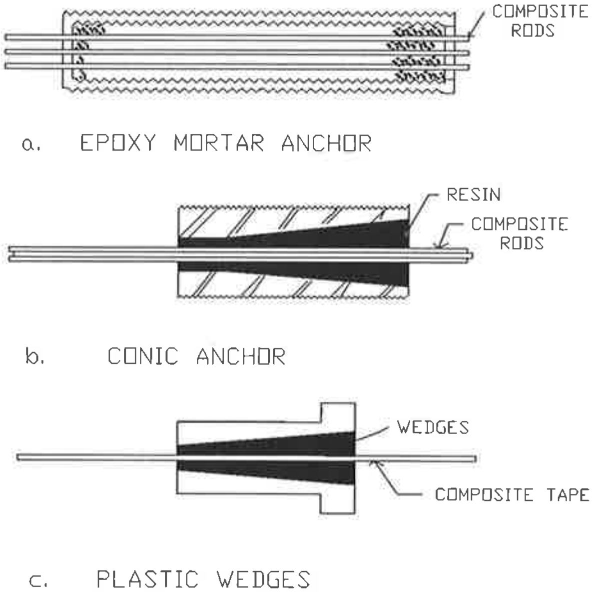
### Combined Material Effects

While the individual components performed satisfactorily in short term tests, the combined effects of moisture on resin coated fibers, fatigue and sustained high load are not yet defined. Tests at the University of Delaware are currently investigating the long term effects of Kevlar reinforced tendons in pretensioned concrete. Two tests of importance are underway. The first test is examining flexural fatigue of a thin plank pretensioned with Kevlar reinforced tendons. The plank is being fatigue tested in a salt water bath. The second test is examining the long term strength, relaxation and bond development length of small bars pretensioned with Kevlar reinforced tendons. For these tests, the surface texture of the rods are modified to assess the bond effects. Improved bond will assist not only the behavior in pretensioned concrete, but may also improve the resin socketed anchor strength.

One of the most important issues to be addressed is the mechanism of strength deterioration under sustained load. Is the strength reduction a stress phenomenon, or is it accompanied by a reduction of strain capacity? A stress reduction may be accompanied by reduction in apparent modulus of elasticity. This will lead to a softening of the structure and increased deflections. A loss of strain capacity can lead directly to rupture of the tendon and reduced ductility in the structure. Tests on concrete sections which have been cured for several months or years will provide some illumination of this behavior.

### Anchorage Issues

The tests reported above have depended upon epoxy socketed anchors for post-tensioning applications and for initially anchoring the pretensioned tendons. A mechanical anchor is desirable for pretensioning operations and for single strand post-tensioning systems. A reliable mechanical anchor could reduce the need to grout single strand tendons. Figure 9 shows several of the anchors commonly used with composite tendons. The Enka anchor uses plastic wedges, however, their system is primarily intended to hold the aramid reinforced tape during pretensioning operations and are not relied on to carry the service loads. The Japanese are conducting tests using a modified steel strand vice, but the details are not available.



a. Polystal anchor for glass reinforced composite tendons from reference 1.

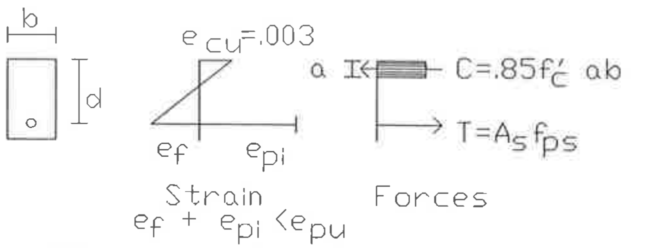
b. Conical anchor used for beam tests described in the preceding sections

c. Plastic wedges used in the Enka pretensioning systems from reference 10.

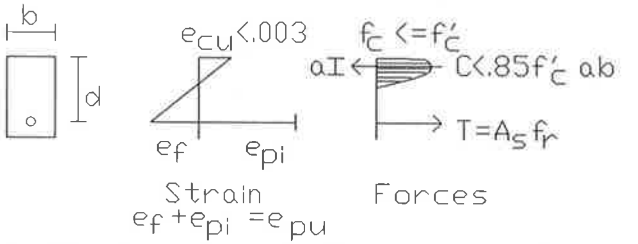
FIGURE 8 Representative Composite Tendon Anchor Systems

### Strain Compatible Design

The concept of strain compatibility has been prevalent in concrete design for decades. The brittle nature of composite tendons suggests that strain compatibility must be extended to the design of the tension reinforcement in addition to the limitations imposed on the concrete compressive strain. This is a direct result of the lack of an extended yield zone to provide a large tensile strain capacity. The highly uniform strain to failure behavior of composites suggests that a strain approach to strength prediction is desirable. Depending upon the location of the neutral axis, the depth of the structure and the structural configuration, the tendon may remain elastic, or it may exceed its ultimate strain capacity. Figure 9 indicates the possible stress and strain conditions which may occur in a concrete section reinforced with a composite tendon.



a. Strain condition for ductile behavior



b. Strain condition for brittle behavior

b = width of compression block
d = depth to prestressing tendon from compressive face
a = depth of equivalent compression
$\epsilon_{cu}$ = ultimate concrete strain
f'c = concrete compressive strength
$\epsilon_f$ = strain due to flexure
$\epsilon_{ps}$ = strain due to prestress and decompression
$\epsilon_r$ = rupture strain of the tendon
$A_s$ = area of prestressing strands
$f_{ps}$ = stress of prestressing strands
$f_r$ = stress of strands at rupture

FIGURE 9 Schematic representation of the strain compatibility of composite tendons in prestressed flexural member design.

The possibility of a brittle failure of the tendon suggests that sections designed with composite tendons should be treated differently than those with steel prestressing materials. Design modifications could address the capacity reduction factor, $\phi$, the load factor, or both. One option may be to substantially reduce the $\phi$ factor. Since the $\phi$ factor is supposed to account for variability of material property and construction tolerances, adjusting the $\phi$ factor does not directly address the issue of ductility. $\phi$ factors have yet to be developed for composite tendons, but their development should be consistent with other materials.

An alternative approach could be to define a supplemental load factor which is used in conjunction with the nominal capacity of the section based on the tensile strain capacity of the tendon. An increased load factor would then be applied to those conditions which may result in a tendon failure. The result is analogous to designing a prestressed concrete section which has a cracking moment close to its strength capacity. An additional margin of safety against failure is provided to assure that cracking does not lead to a brittle failure.

Appendix A develops a brittle reinforcement ratio for composite prestressing tendons. The brittle ratio uses the initial strain resulting from prestressing instead of attempting to combine the strain changes due to losses and to decompression of the tendon during flexural loading. Use of the prestressing strain is expedient because it represents a well defined condition while the losses are generally estimated values.

An important difference between composite tendons and steel tendons is that the rupture condition may be solved for directly. The resulting brittle reinforcement ratio is then dependent upon the initial prestressing strain and the flexural strain. The need to apportion the strain between prestressing and flexure indicates why the low strain to failure of carbon fibers makes them less likely candidates for prestressing applications.

Examining the brittle reinforcement ratio provides an interpretation of the difference between steel and composite tendons. Structures with actual reinforcement ratios less than the brittle reinforcement ratio will require tensile strains in excess of the tendon capacity. These tendons will rupture before the concrete compressive capacity is attained. Prestressing reinforcement ratios in excess of the brittle ratio will develop the concrete capacity before the tendon capacity is reached.

The brittle ratio is different than the balanced ratio associated with reinforced concrete. The balanced ratio defines the simultaneous yielding for the reinforcement and the compressive failure of the concrete. Sections with reinforcement ratios in excess of the balance ratio may fail suddenly since the reinforcement remains elastic when the concrete compressive strain is reached. The tensile strain for a balanced condition is generally less than 0.2 %. Composite tendons will strain to 1 to 1.5 % prior to rupture. This is in addition to the 1% strain used during prestressing. Consequently, while the tendons remain elastic, substantial deflection and cracking precedes failure. This ductility has been observed in the beam tests described earlier.

## Design Recommendations

Until more complete data is available, sections with prestress reinforcement ratios less than the brittle reinforcement ratio should be designed to have nominal capacity of 20 percent greater than the required ultimate capacity. The 20 percent recommendation follows from section 18 of the ACI building code provisions for prestressed concrete sections having cracking moments near to the strength requirements of the section (*11*). These sections require a strength capacity 20 percent greater than the cracking load to assure proper behavior. The strain reserve generated from this condition will provide assurance that the tendon does not rupture at less than the required strength.

## CONCLUSIONS

The body of knowledge regarding the behavior of composite prestressing tendons is growing rapidly. Glass fiber tendons have been used in prototypical bridges and aramid prestressing has been demonstrated in europe. The flexural strength of bonded tendons may be predicted using the same principles now available for prestressed concrete.

Short term tests have indicated that the aramid reinforced tendons are satisfactory for prestressing applications. The long term strength characteristics of aramid tendons appear satisfactory based on limited testing. Long term durability tests are being conducted. Shear performance of members prestressed with composite tendons has not been addressed. Mechanical anchors need to be developed to support commercial pretensioning operations for reliable post-tensioning applications.

Attention must be paid to the mode of failure of the tendon and supplemental load factors applied when tendon rupture is a possibility.

## ACKNOWLEDGEMENTS

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## REFERENCES

1. Dolan, C. W., "Developments in Non-Metallic Prestressing Tendons", Journal of the Precast/Prestressed Concrete Institute, Vol 35, No. 5 September/October 1990.

2. Riewald, P.G., Dingra, A.K., and Chern, T.S., "Recent Advances in Aramid Fiber Technology", Proceedings of the 6th International Conference in Composite Materials, Mathews, F.L., Buskell, N.C.R., Hodgkinson, J.M., and Morton, J., eds., Elseviner Applied Sciences, London, 1987.

3. Wagner, H.D., Schwartz, P., and Phoenix, S.L., "Lifetime Statistics for Single Kevlar 49 Filaments in Creep-Rupture", Journal of Material Science (21), 1986, pages 1868-1878.

4. Phoenix, S. L., "Statistical Theory for the Strength of Twisted Fiber Bundles with Applications to Yarns and Cables", Textile Research Journal, Vol. 49, No. 7, July 1979, pages 407-423.

5. DATA MANUAL FOR KEVLAR 49 ARAMID, E.I. DuPont Company, Chestnut Run Building 702, Wilmington, DE, May 1986.

6. Gale, D.M., Riewald, P.G. and Champion, A.R., "Cement Reinforcement with Man-Made Fibers", International Man-Made Fibers Congress, Dornbirn, Austria, 24-26 Sept, 1986.

7. Wolff, R. and Miesser, H-J. "New Materials for Prestressing and Monitoring Heavy Structures", Concrete International, Detroit, MI., vol 11, no. 9, September 1989, pages 86-89.

8. Walton, J.M., and Yeung, Y.T.C., "The Fatigue Performance of Structural Strands of Pultruded Composite Rods", Journal of the Institute of Mechanical Engineers, London, C286/86, 1986, pages 315-320.

9. "Relaxation of Aramid Tendons", Research Activities, Faculty of Civil Engineering, Delft University of Technology, Delft, TUDelft, 1985-86, page 71.

10. Gerritse, A., Schurhoff, H.J., and Maatjes, E., "Prestressed Concrete Structures with Arapree; Relaxation", contribution to IABSE Symposium, Paris, 1987. (Available from Hollandsche Beton Groep nv, R&D Department, Rijswijk, The Netherlands)

11. Building Code Requirements for Reinforced Concrete (ACI -318-86), American Concrete Institute, Detroit, MI, 1986.

## APPENDIX A
### Derivation of Brittle Reinforcement Condition for Bonded Composite Tendons

A brittle condition exists when the tensile capacity of the tendon occurs simultaneously with the compression failure of the concrete. In the case of concrete reinforced with a composite prestressing tendon, the tensile capacity of the tendon is also the rupture strength of the tendon. Figure A.1 shows the strain and stress state at the simultaneous occurrence of brittle failure of the tendon and compressive failure of the concrete.

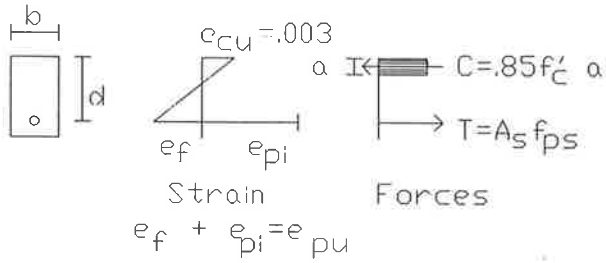

Figure A.1 Stress and Strain at Brittle Conditions

Examining Horizontal equilibrium

$$A_p f_{pu} = .85 f'_c \beta_1 c b \qquad (1)$$

Define

$$\rho_p = \frac{A_p}{b d} \qquad (2)$$

Then

$$\rho_p f_{pu} = .85 \beta_1 \frac{c}{d} \frac{f'_c}{f_{pu}} \qquad (3)$$

$$\rho_{pbr} = .85 \beta_1 \frac{c}{d} \frac{f'_c}{f_{pu}} \qquad (4)$$

By strain compatibility

$$\frac{c}{d} = \frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_f} \qquad (5)$$

And the constitutive behavior of composite tendons

$$\epsilon_f + \epsilon_{pi} = \epsilon_{pu} \qquad (6)$$

Therefore

$$\rho_{pbr} = .85 \beta_1 \frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_{pu} - \epsilon_{pi}} \frac{f'_c}{f_{pu}} \qquad (7)$$

**Definition of Terms**

| | |
|---|---|
| $A_p$ | Area of prestressed reinforcement |
| b | Width of compression face of the member |
| c | Depth from compression face to the neutral axis |
| d | Depth from the compression face to the prestressing centroid |
| $f'_c$ | Specified strength of concrete |
| $f_{pu}$ | Tensile capacity of the composite tendon |
| $\beta_1$ | Concrete strength reduction factor equal to 0.85 for $f'_c \leq 4000$ psi (27 MPa) and reduced .05 for every 1000 psi (6.9 MPa), but not less than 0.65. |
| $\epsilon_{cu}$ | Concrete compressive strain at failure, generally taken as 0.003 |
| $\epsilon_f$ | Strain in tendon due to flexure |
| $\epsilon_{pi}$ | Strain due to initial prestress |
| $\rho_p$ | Ratio of prestressed tension reinforcement $= A_p/bd$ |
| $\rho_{pbr}$ | Brittle reinforcement ratio when concrete reached $\epsilon_{cu}$ as the tendon reached $e_{pu}$ |