Finite Element Modeling, Analysis, and Design of Highly Skewed Post-Tensioned Concrete Bridges

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Skewed bridges have been designed for many years using approximate methods. Most of these methods do not account for the high torsional moments inherent in a skewed structure. More exact methods of analysis are time consuming and the stringent budgets do not permit the design consultant the luxury of using these more elaborate and exact methods. Recently, several highly skewed post-tensioned box girder bridges have been studied and designed by Parsons Brinckerhoff Quade & Douglas, Inc. (PBQ&D) in Tempe, Arizona. The high skew angle of these highway and railroad bridges with very heavy Live Loads was caused by the restricted right-of-way limitations and necessitated the use of more exact methods and/or dependable methods of analysis for these structures. Five skewed bridges were analyzed, by conventional plane frame analysis and by two different programs, using a finite element method of analysis. Two of these bridges were compared to verify the designs based on three dimensional grid analysis. In the preliminary stage the structures were sized and preliminary forces obtained by using CaltranS "Frame System" (5) and then the finite element models were generated in order to gain a better understanding of the effect of skew bending on the deformations, stresses, support reactions, and torsional moments.

Traditionally skewed bridges were analyzed and designed considering the longest length between the supports as the actual span length. Conventional wisdom assumed that such a conservative approach would result in a very safe design. The means or the methods to do a more precise analysis and design did not exist. The advent of computers and the implementation of finite element methods of analysis, have given the means to take a closer look at the behavior of the skewed bridges.

THE BASIC DIFFERENCE - SKEW BENDING EFFECT

What makes the behavior of skewed bridges so much different than from that of non-skew bridges? To explain this, it is necessary to take a closer look at the basic geometry and framing of a skewed concrete box girder bridge.

In any bridge, the principal bending occurs along the shortest axis between the supports, which happens to be perpendicular to the axis of supports. The longitudinal axis of a bridge is defined as the axis which is directed through the centriod of the cross section, and it is usually parallel to the center of the roadway or railroad.

In a non-skewed bridge the supports are along the transverse axis which is perpendicular to the longitudinal axis of the bridge. The principal bending of the girders and the whole structure occurs about the transverse axis parallel to the supports, along the longitudinal axis. This behavior has always dictated the direction of the girders in a concrete box girder bridge to be parallel to the longitudinal axis.

In a skewed bridge, the supports are not along the transverse axis. They are located along an axis skewed to the transverse axis of the bridge. The longitudinal axis of the bridge remains the same. The principal bending occurs about an axis parallel to supports along a new longitudinal bending axis which is perpendicular to the axis of the supports. This new longitudinal bending axis is skewed to the longitudinal axis and is almost parallel to the shortest distance between the supports. Depending on the length to width ratios and the location of piers and abutment supports, this longitudinal bending axis comes close to the line joining the supports at the two obtuse corners. The bending effects caused in a skewed bridge due to the principal bending along this new longitudinal bending axis is called "skew bending" and its effects are referred to as skew bending effects. This principal bending along the new longitudinal bending axis must be carried by the girders or webs, by resolving it along the longitudinal and transverse axis of the structure.

For a proper understanding of the design requirements, it is essential that the principal bending moment along the modified longitudinal axis be resolved to a bending moment along the longitudinal axis and a torsional moment about the longitudinal axis. The torsional moments caused by skew bending effects, in turn, cause an uneven distribution of horizontal and vertical shears at a section normal to the longitudinal axis of the bridge. This uneven distribution of horizontal and vertical shears, affects the location of maximum moments, produces large variations in support reactions and the post-tensioning forces which are normally negligible in a non-skewed bridge result in uplifts at certain supports

This study indicates that besides uplift reactions at supports, torsional moments are the principal concern in the design of skewed concrete box girders because they cause diagonal tension in exterior faces of the box and uneven variation of shear and support reactions. There are large deviations in magnitude and location of maximum positive moments, which complicate the location of the post-tensioning paths. The large variations in support reactions bring out the need to pay more attention to design of bearings.

Five highly skewed bridges included in the three bridge sites listed below were part of the Aviation Project in Tucson Arizona and are the subject of this paper:

1. Southern Pacific railroad overpass at Broadway (SPRR bridge); a two span structure, with skew angles of 37 degree at the pier, 43 degrees and 51 degrees at abutment supports. Figures 5 thru 11.

2. Council/Toole Avenue bridges over Broadway. Twin two span, variable depth structures with skew angles of 53 degrees at pier supports and 66 degrees at abutment supports. Figures 12 thru 18.

3. Euclid/Park Avenue overpass structures at SR 210. Twin two span structures with skew angles of 60 degrees at pier supports and 70 degrees at abutment supports. Figures 19 thru 20.

TOOLS AVAILABLE FOR ANALYSIS AND DESIGN

The following tools were selected for a closer evaluation from the scores of material available in the professional design field.

Parsons Brinckerhoff Quade & Douglas, Engineers, Architects and Planners, Tempe, Arizona

CALTRAN Bridge Memo for Designers 15-1

This internal memo to designers (1) by California Department of Transportation, basically gives charts for increasing the shears in the girders of bridges up to 45 degree skew, beyond which it recommends a more exact analysis, by using finite element programs like CELL4 or STRUDL. This memo addresses basically increasing the shear in girders which is only one of the many problems involved in the design of skewed bridges.

Cell4 Program

It was developed at University of California, Berkeley with the cooperation of California Department of Transportation, under the supervision of Professor A.C. Scordelis (2). CELL4, based upon the finite element method, is particularly suited for the analysis of post-tensioned box girder bridges of constant depth. The finite elements used are a combination of plane stress element (Q8D11) and plate bending element (Q19). The combination of these elements with appropriate transformations can take into account curved geometry and sloping of the exterior webs. The resulting element has 5 degree of freedom (DOF) per node in the local coordination system which, when transformed to global coordinate system, becomes 6 DOF.

MDC STRUDL Program

MDC STRUDL program (3), one of the commercially available programs that is widely used across the country, was used by PBQ&D for analysis and design of the hourglass shaped urban interchange structures for 7th Street and 7th Avenue over I-10 in Phoenix.

The structural element used to model these bridges is called "PBSQ2", a four noded element with 6 degrees of freedom developed by McDonnell Douglas Architectural Engineering of Construction Systems Company. This element is capable of carrying in plane loads (plane stress) and bending loads (plate bending) and was found suitable to model the top slab and bottom slab. To model the webs, a combination of "PBSQ2" elements and structural bending members were used. This combination was needed in order to simulate the behavior of the box girder. The curved shell element "SIPQ" with five DOF could have been used in place of the combined elements for the girder. But it was not used at the time, because of prohibitive computer costs.

The 7th Street bridge model consisted of 1800 joints, 1143 elements and 337 members.

3-Dimensional Grid Analysis Program

In his book "Bridge Deck Behavior", Edmund C. Hambly (4) explains the procedure for developing a grillage grid model. The methodology seems to have limitations in simulating the torsional rigidity of the box girder. In view of this, grid analysis was not favored. However there was an opportunity to compare the results of the finite element model with those of the grid analysis, as part of a design review on Euclid/Park Structures.

COMPARATIVE TESTS

The CELL4 program was selected for use in view of the savings offered in modelling, computer time, ease of obtaining sectional forces, moments and automatic generation of equivalent loads for post-tensioning forces. In order to ascertain and validate the results of CELL4 program two identical models were tested using CELL4 and STRUDL programs. They were both skewed structures, one a single span structure and the other a two span structure. Figure 1 and 3 represent the CELL4 and STRUDL models respectively for the single span bridge. Figures 2 and 4 represent CELL4 and STRUDL models respectively for the two span bridge. Figures 1 to 4 also illustrate the Dead Load (DL) reactions.

The differences between results of the two models though not exactly the same were within reasonable limits of accuracy required for design. The maximum stresses, moments and deflections were within 5% of each other. The model in Figure 2 has a thicker pier diaphragm than the model in Figure 4, which accounts for some of the differences in the reactions. The only results that have a wide variation are the two uplift reactions at the acute corners (see Figures 1 and 3) and needs further evaluation.

The closeness of results from both the programs gave us sufficient confidence to use CELL4 on all bridges except the Council/Toole Avenue bridge, which had a variable depth. This required STRUDL program to model the Council/Toole Avenue bridges.

The CALTRANS Frame System (5) Program was used to obtain order of magnitude of the post-tensioning forces to be applied and the cable path. Various versions of this program with different names are being widely used by design consultants in the west coast.

SOUTHERN PACIFIC RAILROAD (SPRR) BRIDGE OVER BROADWAY

The SPRR bridge is a two span structure with 3 railroad tracks and one highway maintenance road. Figure 5 shows the general geometry of the bridge and the framing of the girders and centerline of the tracks.

Modeling

Having gained confidence in the results of CELL4 program, the model for the SPRR bridge was created. The finite element discretization mesh is shown in Figure 6. In order to increase the accuracy of the results in the finite element analysis, the aspect ratio of the elements was kept to 1 in critical areas and to 2 in noncritical areas. The finite element model for this structure has 1428 joints and 1818 elements.

Discussion of Results

Dead Load Reactions

Figure 7 shows the reactions due to DL at abutment supports and piers. Note the variations in reactions from one end of the abutment to the other. The variations in support reactions, location of maximum moments and the magnitude of these moments and stresses was surprising compared to what one normally expects from a conventional approach. For bridges with this span range, the Live Load (LL) from the railroad are about twice the Deal Load (DL), where as in the case of highway LL effects are about a third of the DL. Figure 8 shows the points of maximum moment. Note the fluctuations in points of maximum moment.

Maximum Moments

In a two span structure, with roller supports at each end, the points of maximum moment would usually be located at 40% of the span from the exterior roller supports. In span 1, the location of the points of maximum moments vary from 32 percent in girder 1 and to 54 percent in girder 8 (Figure 8). This effect is reversed in span 2. The location of the points at which maximum moment occurs vary from 48 percent in girder 1 to 32 percent in girder 8. These variations clearly demonstrate the skew bending behavior and the inherent torsional moment and its effect on the behavior of the structure. The situation was the same in span 2 except it was in reverse order. Note also the fluctuations in the values of maximum moments in different girders.

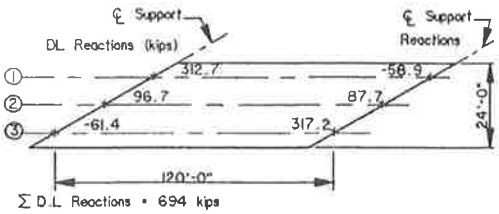


Fig. 1 Single Span CELL4 Test Model

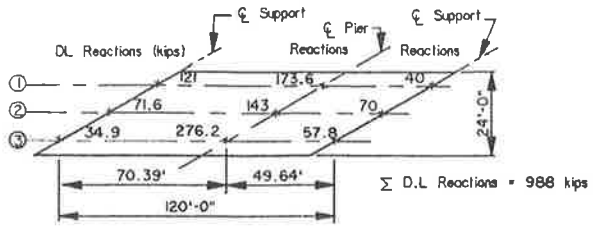


Fig. 2 Two Span CELL4 Test Model

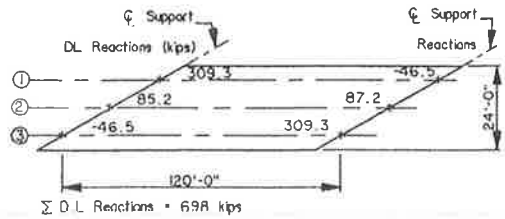


Fig. 3 Single Span STRUDL Test Model

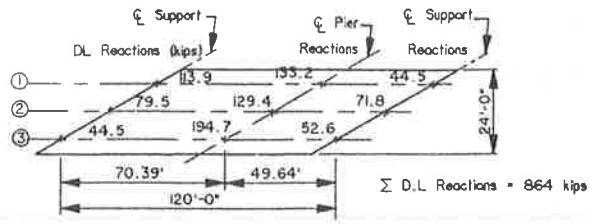


Fig. 4 Two Span STRUDL Test Model

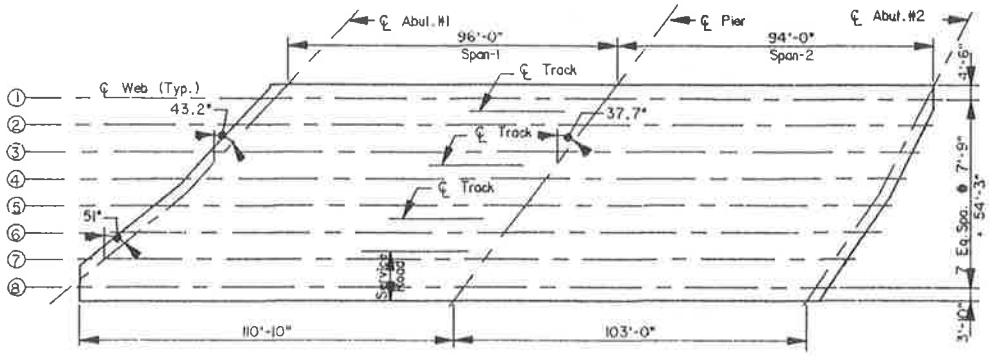


Fig. 5 Sectional Framing Plan - SPRR Bridge

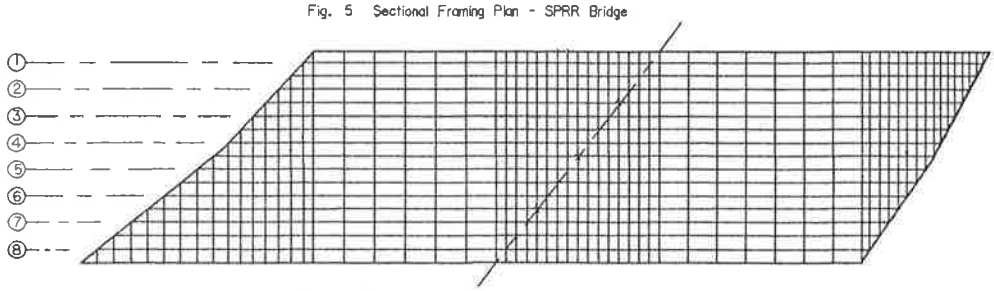


Fig. 6 Finite Element Discretization Mesh - SPRR Bridge

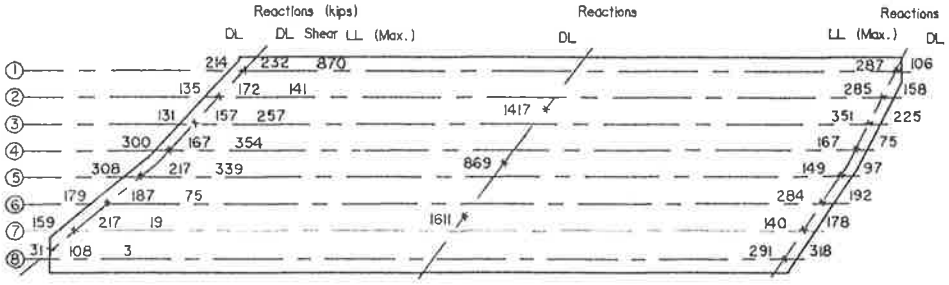


Fig. 7 Dead Load Reactions and Shears - SPRR Bridge

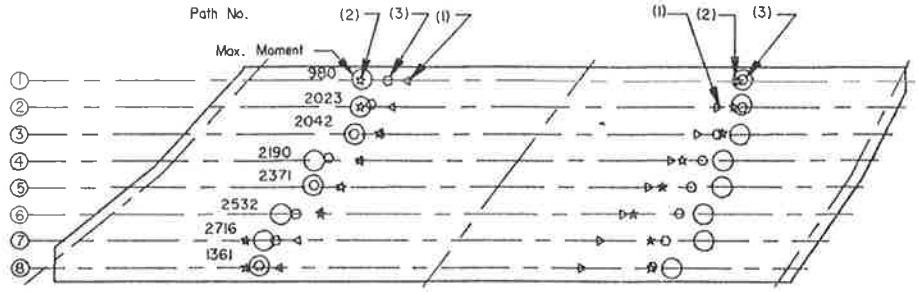


Fig. 8 Points of Max. Moments and Sags in Tendon Paths - SPRR Bridge
Max. D.L. Moments (Ft.Kips) in Span (1)

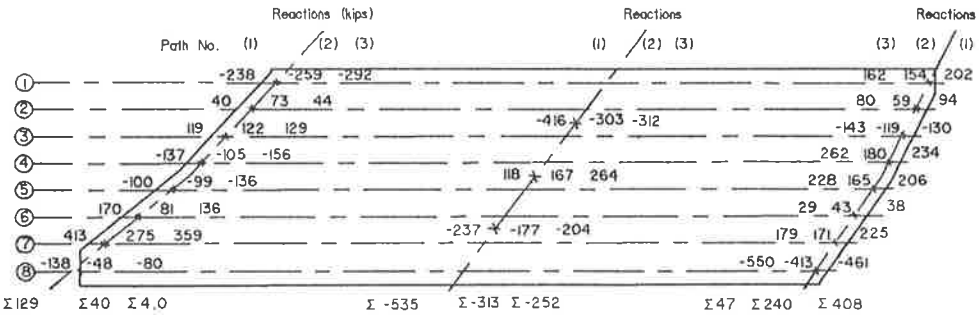


Fig. 9 Reactions Due to Alternative Post Tension (P/T) Paths - SPRR Bridge

UNBALANCED P/T REACTIONS

Σ = 2.0 (1)

Σ = -33.0 (2)

Σ = -1.0 (3)

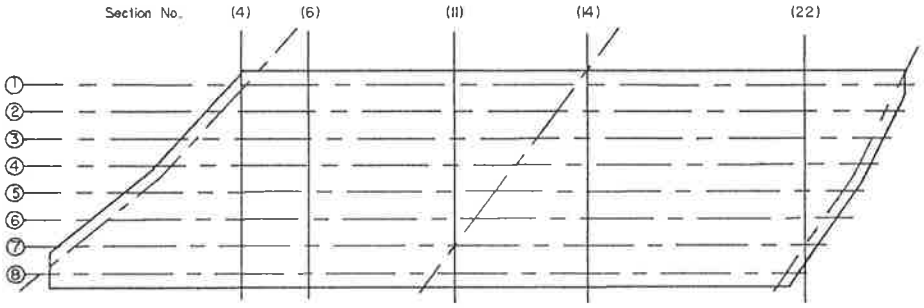


Fig. 10 Torsional Section Plan - SPRR Bridge

Table 1 Section Torsional Moment and Shear for SPRR Bridge

SECTION	4		6		11		14		22	
	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear
DL	-12236	-250	-12063	151	235	739	-9215	-997	-7763	538
DL+SDL	-18729	-414	-17537	168	1171	1088	-14958	-1486	-9623	756
DL+SDL+LL	-38788	-1020	-31298	-22	3768	1674	-12305	-1604	-7176	664
P_1 =3910 Path 1	17327	774	21967	147	-10517	-1630	25256	2071	12488	-1011
P_1 =3520 Path 2	22371	735	26315	-107	-8033	-1709	13162	1924	14948	-1084
P_1 =3520 Path 3	19975	651	24566	-60	-10241	-1635	15708	1950	14945	-1010

Units Are Foot-kips and kips

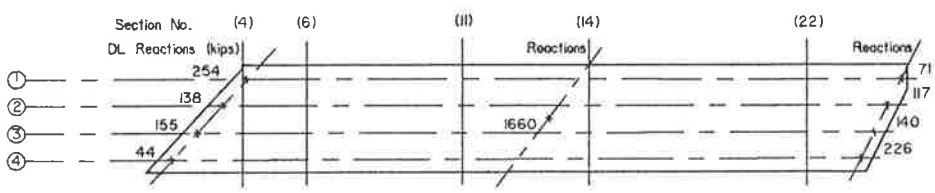


Fig. 11 Sectional Framing Plan - Half of SPRR Bridge

Table 2 Section Torsional Moment and Shear for Half of SPRR Bridge

SECTION	4		6		11		14		22	
	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear	Torsion	Shear
DL	-2199	-296	-2257	-108	-2273	416	-1599	-586	-1585	169
DL+SDL	-3883	-483	-3845	-179	-3485	662	-2853	-948	-2270	266
DL+SDL+LL	-9679	-1161	-9900	-440	-8749	1280	-2281	-1043	-1709	173

Units Are Foot-kips and kips

Initially the thought was to reverse this trend by applying the equivalent upward post-tensioning forces over a longer length of girders, by varying the location of sag points. The supplemental design criteria by SPRR (6) required on having 50 psi to 200 psi compression under any loading condition. This made the location of sag points and inflection points in the tendon path more difficult, since the secondary moments due to post-tensioning are largely influenced by their locations. The skew effect made it even harder to estimate the effect of the sag and inflection points on secondary moments. Three post-tension paths were investigated, varying the sag points and are illustrated in Figure 8.

Post Tensioning Cable Path

Path No. 1: Initially path No. 1 was tried, to determine the effect of post-tensioning. The sag point was at 50 percent in Girder No. 1 and varied linearly to 60 percent at Girder No. 8. We anticipated this path to help lift girders 1 and 2 in span 1. It did not give the anticipated results. Instead it resulted in large tension stresses in the bottom slab near the piers.

Path No. 2: The tendon paths were moved to coincide with the location of points of maximum moments in girders 1, 2, 7, and 8 and was kept the same in the remaining girders. The results of path No. 2 showed an improvement over path No. 1, with some small compressive stresses (150 psi) in the bottom slab near the pier support.

Path No. 3: Finally path No. 3 was tried with a view to obtaining higher compressive stresses near the pier supports. Note the location of sag points vary from 41 percent at girder 1 to 54 percent at girder 8. This path gave the required results with compressive stresses in the bottom slab near the pier exceeding 250 psi.

Figure 9, shows the support reactions induced by the three paths. Note the reversals in support reactions at certain locations. This did not happen with the DL of the structure. This in a way indicates that the concept of load balancing method may not eliminate the problems of torsion and the large variations in support reactions induced by the DL and LL on the structure.

Note the post-tensioning reactions (Figure 9) exceed the DL reactions (Figure 7) at the exterior supports and result in uplift. The bearings at these locations were designed for uplift capacities. The critical section for torsion is section 4 Figure 10. The post-tensioning force does have the beneficial effect of reducing the torsion due to DL and LL as evidenced by figures in Table 1. Note also the reversal in torsion for certain LL conditions at other non-critical sections.

Torsional Moments and Effect of Width/Length Ratio

Figure 10 shows the location of the five sections used for computing torsional moments and shears. The torsional moments obtained due to DL, LL, and the three different post-tensioning paths are shown in Table 1. Note that the DL torsion moments are huge and are almost constant between the closest edges of the abutment and pier supports. It was thought that one of the reasons for this huge torsional moment might be the large width of the structure, causing skew bending effects. So the structure was cut by half and a finite element model was run using CELL4 program. The results of this model are shown in Figure 11 and Table 2. Comparison of results in Table 1 and Table 2 indicate that the total torsion was reduced to 35 percent of the full bridge. The corresponding section resisting torsion was reduced to 38 percent of the full bridge. Also the torsion capacity of the structure was reduced to 32 percent of the full bridge.

The steel required for torsion based on Thomas T. C HSU (7), ACI building code (8), (9) and ACI - analysis of structural systems for torsion (10) was found to be the same for both bridges. This indicates that the length to width ratios have practically very little effect on the amount of torsion steel required in a skewed box girder bridge. Since there was no apparent gain in cutting the structure by half, it was decided not to break the structure and to use the full model for verifying the LL and post-tensioning loads.

COUNCIL/TOOLE AVENUE BRIDGES

The extremely tight vertical clearances imposed by the roadway profiles, dictated the use a two span structure with a variable depth. The skews at the abutment and pier supports for the east bound (EB) and west bound (WB) lanes varied by a large amount and introduced a kink in the pier diaphragm. As a result it was decided to have two separate structures, one for east bound and the other for the west bound. Both the structures were analyzed separately and the results of the east bound (EB) structure are presented here.

Figure 12 shows the geometry and framing of girders for the EB structure. Note the extremely high skews varying from 45 degrees to 66 degrees. Figure 13 shows the typical longitudinal section, with the parabolic haunch.

Modelling

Since CELL4 program did not have the capability of analyzing variable depth box girders, it was decided to model this structure using the MDC STRUDL program. The finite element called "SIPQ", available in McAuto STRUDL library, was used in this model. The "SIPQ" is a quadrilateral curved element with four corner nodes and four midside nodes with five degrees of freedom (DOF), three of which are linear DOF and two are rotational DOF about a nodal local axis. This model had a total of 1482 elements and 4119 joints. The aspect ratio was kept to 1 in critical areas and to 2 in other areas. The design of SPRR bridge was slightly ahead of this bridge, which enabled to use the experience gained and avoid some of the iterations we went through earlier on.

Discussion of Results

Reactions and Moments

In Figure 14 are shown the locations of points of maximum moment and the DL support reactions. The support reactions are more than 100 percent different at certain locations. Note the heavy reactions at the top left hand corner and the bottom right hand corner supports. The line joining these points, almost coincides with one of pier supports. Though it is not perpendicular to the lines of support, it seems to act like the principal longitudinal bending axis of the structure. The reaction at this pier is more than twice the reaction at the other pier and greatly affects the design of the cantilever pier diaphragm. The effects of skew bending and torsion are again very evident, as illustrated in Table 3. As in the SPRR bridge, Figure 14 illustrates the enormous fluctuations in the location of points of maximum moments. The distance from the center of abutment to these points varies from 25 percent of span for girder 1 to 55 percent for girder 5, which is very different than the usual 40% of span for a two span structure with roller supports at the ends.

Cable Path

Two post-tensioning paths were tried. The first one was with sag points of cable path coinciding with the location of maximum moments. This path did not help in balancing the stresses due to DL and LL. Also the required upward deflection in span 1 was not obtained.

The second post-tensioning path was modified as shown in Figure 15, locating all the sag points at 40 percent of the span from the abutment in span 1 and at 30 percent of the span from abutment in span 2. The span 2 being shorter, the amount of sag in span 2 was smaller than in

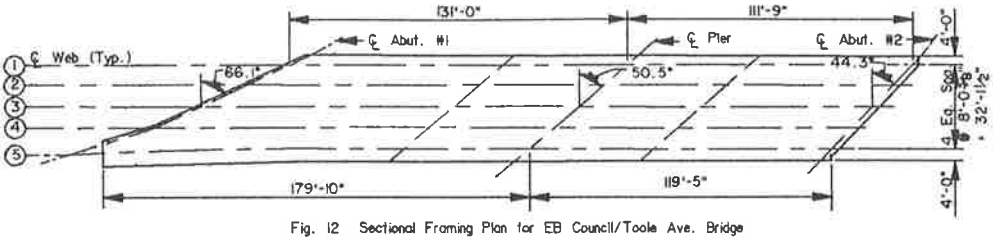


Fig. 12 Sectional Framing Plan for EB Council/Toole Ave. Bridge

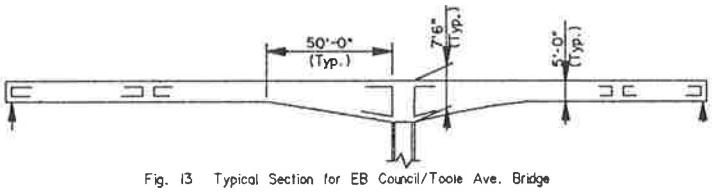


Fig. 13 Typical Section for EB Council/Toole Ave. Bridge

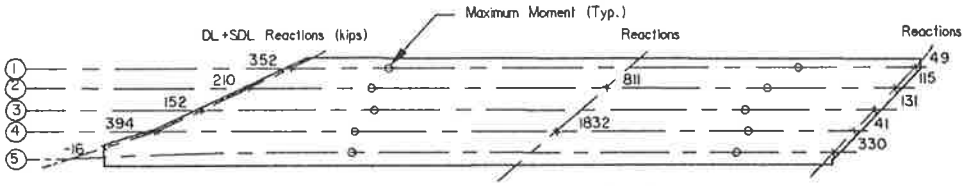


Fig. 14 Dead Load Reactions and Points of Max. Moments for EB Council/Toole Ave. Bridge

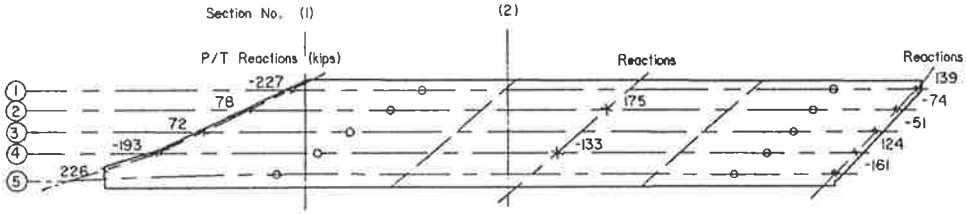


Fig. 15 Points of Max. Sags in Tendon Paths for EB Council/Toole Ave. Bridge

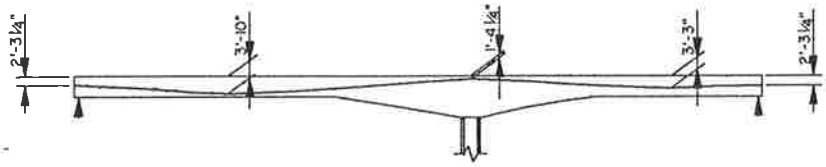
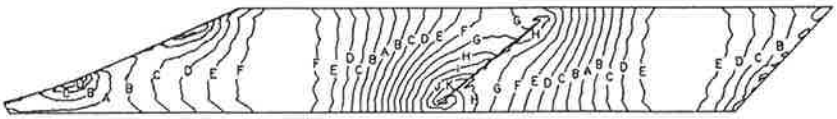


Fig. 16 Typical Tendon Path for EB Council/Toole Ave. Bridge

Table 3 Section Torsional Moments for EB Council/Toole Ave. Bridge

SECTION	1	2
DL	-7370	-7585
DL+SDL	-9167	-9435
DL+SDL+LL	-11563	-11893
P/T	6210	4994

Units Are Foot-kips



Units Lbs/in^2

A 0.000E+00
B 7.806E+01
C 1.561E+02
D 2.342E+02
E 3.122E+02
F 3.903E+02
G 4.683E+02
H 5.464E+02
I 6.245E+02
J 7.025E+02
K 7.806E+02
L 8.586E+02
M 9.367E+02

Fig. 17 Stress Contours Top Slab (DL) - EB Council/Toole Ave. Bridge

Units Lbs/in^2

A 0.000E+00
B 1.174E+02
C 2.348E+02
D 3.522E+02
E 4.696E+02
F 5.870E+02
G 7.044E+02
H 8.218E+02
I 9.392E+02
J 1.057E+03
K 1.174E+03
L 1.291E+03
M 1.409E+03

Fig. 18 Stress Contours Bottom Slab (DL) - EB Council/Toole Ave. Bridge

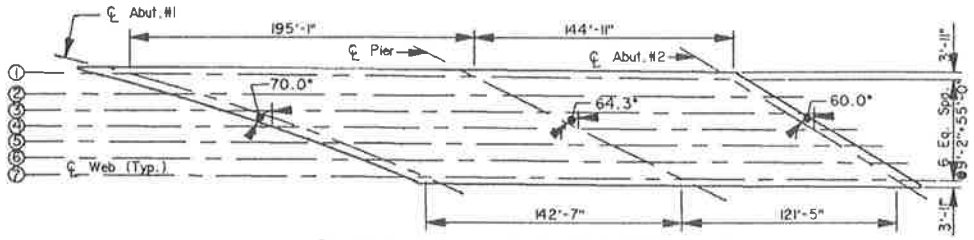


Fig. 19 Sectional Framing Plan for WB Euclid/Park Ave. Bridge

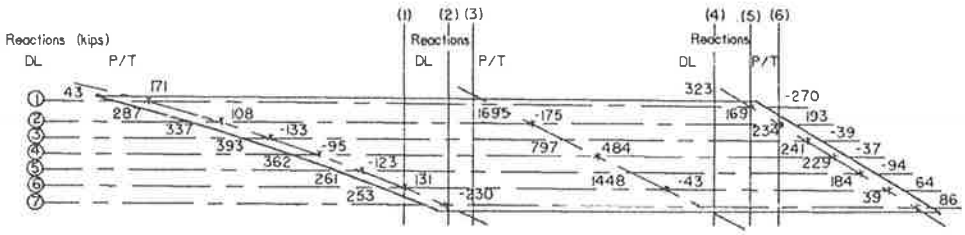


Fig. 20 Sectional Framing Plan for WB Euclid/Park Ave. Bridge

Table 4 Section Torsional Moments for WB Euclid/Park Ave. Bridge

SECTION	1	2	3	4	5	6
	Torsion	Torsion	Torsion	Torsion	Torsion	Torsion
DL	10640	21828	21971	16780	16653	7085
DL+SDL	12690	26058	26209	19900	19757	8323
DL+SDL+LL	15396	31694	31632	24403	24397	10324
P/T	-11273	-23392	-17097	-13341	-18965	-8340

span 1. Figure 16, illustrates the cable path with reduced eccentricity in span 2, to help lift off span 1. This was done with a view to help the upward deflection in span 1. The reactions due to post-tensioning force are shown in Figure 15 and the torsional moments are listed in Table 3. These results have a pattern similar to the ones obtained for the SPRR bridge using CELL4.

Stress Contours

Stress contours were plotted for several cases of loadings, DL, LL and post-tensioning forces. Figure 17 shows the stress contours in the top slab and Figure 18, the stress contours in the bottom slab for the DL of the structure. Note the concentration of stress contours at the bottom left hand edge and top right hand edge of pier support line. These stress contours really helped us to evaluate the critical points for checking the stresses and to arrive at details of the post-tensioning path for each girder.

EUCLID/PARK STRUCTURES

These are two highway bridges with large skews varying from 60 degrees to 70 degrees, similar to Council/Toole Avenue bridges. Parsons Brinckerhoff was involved in the review of final design for these two structures. The design was based on a 3-dimensional grid analysis. CELL4 program was used for the design verification. The basic geometry of one of the structures is shown in Figure 19. The finite element model consisted of 1,116 joints; and 1,370 elements. The aspect ratio of elements was 1 in critical areas and 2 in less critical areas.

The DL reactions and post-tensioning reactions are shown in Figure 20.

Load Balancing Method

The load balancing method is explained in design of prestressed concrete structures by T.Y. Lin (11). By load balancing, the moments due to post-tensioning are made equal the maximum moment due to DL and LL. In load balancing the axial stress effect of the post-tension force is neglected. As such, this method yields uneconomical design resulting in excessive post-tensioning force.

The design of this structure was based on the assumption that by using the load balancing method for post-tensioning, the skew bending effects of DL and LL will be reversed, leaving the structure without any torsional moments. Table 4 gives the sectional torsional moments from CELL4 model for DL, (DL + SDL), (DL + SDL + LL) and the post-tensioning force used for load balancing. For location of these sections see Figure 20.

By inspection of results in Table 4, it can be seen that the post-tensioning force does not offset the torsional moments due to (DL + LL + SDL). In view of this it would be erroneous to assume load balancing as a cure for eliminating torsional moments and to assume that there is no need for a more exact method of analysis.

Dependability of Grid Model

In the initial stages, there were wide variations of reactions and moments between the results of the 3-dimensional grid and finite element models. With some iterations in the development of properties for the grid model, the gap was narrowed down and the final reactions from grid model were close to the ones from finite element model. However the torsional moments as furnished for the grid were much lower than those given by the finite element model. This is a major drawback for dependability of the grid methodology for the analysis of highly skewed bridges.

INTERMEDIATE DIAPHRAGMS

The model for the SPRR bridge was tested with and without intermediate diaphragms. There was negligible difference in moments,

stresses and support reactions between the two models. No transfer of loads to other girders were observed. The differences noticed were attributable to the additional DL added by the introduction of the intermediate diaphragm.

CONCLUSIONS AND RECOMMENDATIONS

• Analysis and design of highly skewed bridges requires careful evaluation of skew-bending effects.

• Finite element models, with proper aspect ratios, provide the best means to evaluate the behavior of highly skewed bridges.

• Support reactions are unpredictable by conventional procedures.

• Torsional moments induced by the high skews need to be addressed in the design. The torsion capacity of the post-tensioned concrete box girders was not adequate to resist the total torsions induced on the structures. Torsion steel was required in all the five bridges.

• The load balancing procedure does not offset all the skew bending effects in a post-tensioned concrete structure. The post-tensioning force required for load balancing is much higher than what is required for balancing the stresses.

• Three dimensional grid analysis does not give a true account of the behavior of the highly skewed structure. The results are very sensitive to the torsional rigidity of the members assumed in a grid model and the comfort level for dependability of results is low.

• Intermediate diaphragms do not have any noticeable effect on the Behavior of the box girder structure or the re-distribution of loads.

• Structures with skews greater than 25 degrees should be avoided if an accurate method of analysis like finite element methodology is not used for analysis. Otherwise it seems imperative to use a more exact method of analysis like finite element methodology to assure the structural design integrity.

Based on the numerical results of the models, there is a need for further study of the effects of skew bending of concrete box girder bridges. The study should include second order analysis and inclusion of time and creep factors and non linear effects in concrete. Also, the numerical results should be correlated with experimental data for future recommendations of design guidelines.

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