Heavy Loads on Prestressed Girders: The Probability of Flexural Cracking

VERNE A. GEIDL

Bridge engineers often need to determine the maximum safe load that may be allowed to pass over a prestressed concrete bridge. Frequently the need for the load to pass is urgent, requiring a swift but accurate evaluation of safety. Many times the maximum load is governed by flexural cracking, and the bridge must be analyzed to find the cracking moment capacity of the girders. A conventional bending analysis uses deterministic strength properties even though properties such as concrete strength and prestress force are random variables. Because of this, a conventional load capacity analysis cannot reveal the probability of failure (i.e., by flexural cracking) associated with the passage of an extreme load. Therefore, bridge engineers have been forced to try to account for the probability of failure by a combination of experience and engineering judgement. This approach is sometimes disquieting for those engineers required to set load limits, issue overload permits, or approve large temporary construction loads. A computer program was written to use a very accurate analytical model and probabilistic methods to calculate a population of cracking moment capacities, rather than a single, deterministic value. Using this data and information about the extreme load moment, the engineer can calculate an associated probability of cracking failure. This result along with other information can be used to help the engineer in making a final decision about the safety of the passage of an extreme load.

PROBABILISTIC CAPACITY ANALYSIS TOOL

Introduction

A research study (1), for the U.S. Department of Transportation through the TRANSNOW Northwest Transportation Center and in cooperation with the Idaho Transportation Department, produced a computer program to do probabilistic bending analysis of prestressed girders. The program is called Probabilistic Capacity Analysis Tool (PROCAT) and produces a population of cracking moment capacities (See Figure 1). The results of the program can be used to find the probability that a heavy load will cause flexural cracking. PROCAT can accept several standard cross-sections or any section made up of a combination of rectangular and triangular areas. The probabilistic analysis uses a very exact, analytical model that employs the non-linear stress-strain relationships for concrete. The model also accounts for the fact that live-load strains in the prestressing steel cause an
increase in the prestressing force (and moment capacity). In addition to PROCAT, a program called STEP1 was written to prepare input for the analysis.

Department of Civil Engineering, University of Idaho, Moscow, Idaho 83843

The Input Program

STEP1 is an interactive program that prepares the necessary input files for the analysis program. The entire input process usually takes only a few minutes. The user has the option of selecting one of several standard shapes or a nonstandard shape. The standard shapes include bulb-tees and AASHTO I-girders. To fit the needs of a particular agency, more standard shapes can be easily added to the program by creating the appropriate data files.

For nonstandard shapes, the user subdivides half of the cross

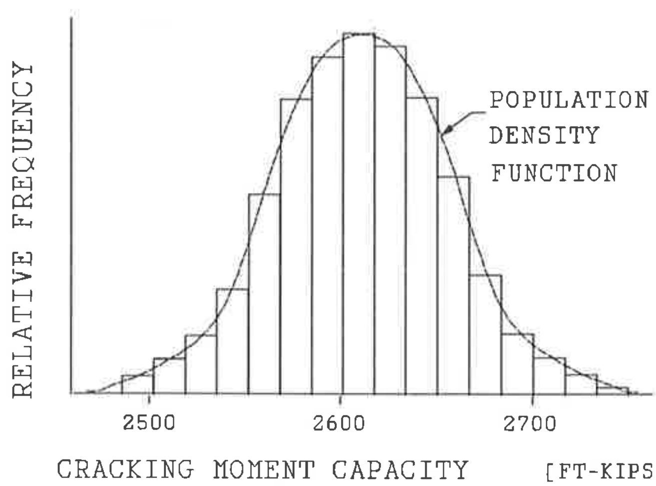

FIGURE 1 Sample population.

section into rectangular and triangular areas; the program assumes the shape has a vertical axis of symmetry (Figure 2). The maximum number of areas allowed in the half-section is 50. Each area has seven attributes: width; height; type (i.e., rectangle or triangle); vertical position (i.e., distance from the top of the girder to the top of the area); cast-in-place indicator (1 for cast-in-place, 0 for precast); class (i.e., whether the area is above or below the clear web (See Figure 2)); and 28-day concrete compressive strength. The "class" attribute is needed to deal with the random variation in girder dimensions during Monte Carlo simulation.

The user enters the specific weight of the concrete, not including the weight of any imbedded reinforcing steel or other dead load. Because the girder dimensions are random values, the analysis program automatically calculates the girder dead load, including all longitudinal reinforcing steel (both prestressed and non-prestressed). The program also requests the value for the additional dead load that acts on the precast section, not including the dead load of the precast concrete and any imbedded longitudinal mild steel or prestressing steel. The user also enters the number, and locations of mild steel reinforcing bars and prestressing steel. One-half-inch, seven-wire, 270 ksi strand and Grade 60 mild steel reinforcing are assumed. The mild steel and prestressing steel are used in the calculation of the cracking moment capacity as explained later.

Analysis Options and Features

Probabilistic Analysis

PROCAT, will perform either a probabilistic analysis or a deterministic, mean-value analysis; both solve for the flexural cracking moment. The probabilistic analysis is accomplished by simulating a random sample of prestressed girders. The user selects the size of the sample (i.e. the number of girders to be simulated). The program uses Monte Carlo simulation to produce the sample; the Monte Carlo simulation procedure is discussed in detail later in the paper. The total cracking moment capacity of each girder in the sample is found using a very exact analytical model. The calculation of these moment capacities results in a corresponding sample (or population) of cracking moment capacities.

Important Features

Some important features of the analysis program are as follows:

- The maximum sample size that can be specified is 1000 girders.
- For a probabilistic analysis, the user must enter two numbers to be used as the seed values for the random number generator. This assures that all probabilistic analyses will be done with different series of random numbers.
- Because prestress losses are often a point of disagreement, the program offers maximum flexibility in this area. The user has the choice of specifying either a deterministic loss value (even though all other variables are randomly simulated) or the mean and standard deviation of the losses. For both options, default values are available (4).
- For the probabilistic analysis, the output consists of the mean and standard deviation of the total cracking moment capacity. In addition, the entire sample of cracking moment capacities is saved for statistical evaluation by the user. The input data are also included for user verification.
- For the mean-value analysis, the output consists of the mean cracking moment capacity of the girder and all of the input values for the girder (for user verification).

Mean-Value Analysis

As the name implies, the mean-value anaylsis finds the cracking moment capacity of the girder using the mean values for all properties and dimensions. Since, in this case, all properties and dimensions are single-valued (i.e., deterministic), this analysis produces only a single-valued result, rather than a population. However, there may be situations where the user simply wants to do a mean-value analysis, since this represents the expected value of the moment capacity. The expected value represents the location of the centroid of the area under the population curve. For symmetrical distributions, such as the normal distribution, the expected value is also the most likely value (i.e, the mode). For unsymmetrical distributions, such as the Weibull, the expected value does not coincide with the mode.

Software Standards and Portability

Both PROCAT and STEP1 are very portable. Both programs have been compiled and run on IBM PC/AT and compatible microcomputers. The analysis software also accommodates batch or interactive runs and has been

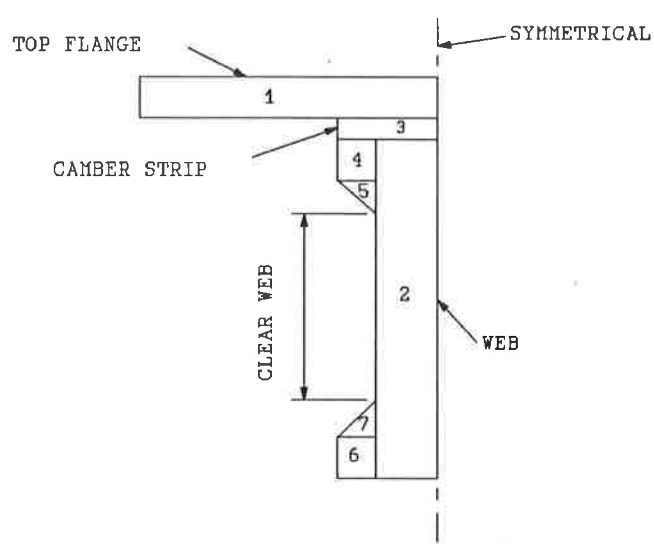

FIGURE 2 Typical cross section for input.

TABLE 1 RANDOM VARIABLE RESISTANCE PARAMETERS

RANDOM VARIABLE	DISTRIBUTION TYPE	MEAN	STANDARD DEVIATION	REFERENCE
Top Flange Width	Normal	+5/32* in.	1/4 in.	[7]
Top Flange Depth	Normal	0.0* in.	3/16 in.	[7]
Web Thickness	Normal	0.0* in.	3/16 in.	[7]
Girder Depth	Normal	+1/8* in.	5/32 in.	[7]
Effective Depth to Mild Steel	Normal	+1/8* in.	11/32 in.	[7]
E of Mild Steel	Normal	29000 ksi	957 ksi	[6]
F_y of Mild Steel	Log Normal	66.8 ksi	5.5 ksi	[6]
Initial Prestress	Normal	189 ksi	2.8 ksi	[8]
Prestress Loss	Normal	36 ksi	5.75 ksi	[8]
Prestress Ultimate Stress	Normal	281 ksi	7.0 ksi	[8]
Prestress Ultimate Strain	Normal	0.05	0.0035	[8]
Prestress E	Normal	28400 ksi	568 ksi	[8]
Depth to Prestressing	Normal	0.0* in.	1/16 in.	[8]

*Deviation from nominal dimension.

compiled and run on IBM 4300 series computers under VM/CMS and MVS/XA2.2 operating systems. The original programs, which were the basis for this software, have also been run on VAX, Prime, and Cray computers. Both PROCAT and STEP1 are written in ANSI Fortran 77

MONTE CARLO PROCEDURE

Simulation of Properties and Dimensions

For a probabilistic analysis, the random properties (6)(7)(8) (Table 1) of a sample girder are produced by Monte Carlo simulation. The statistics of the random concrete properties (i.e., compressive strength, modulus of elasticity, and flexural tensile strength) depend on the specified 28-day compressive strength and are calculated as outlined in reference (2). The user can easily change or customize the statistical parameters if desired.

Random values for the material properties and dimensions are produced in the following way. I use the term property to represent either a material property or a dimension. For each property, a random number generator produces a random number from the appropriate distribution type. For example, if the property is normally distributed, the random number generator produces random values that are normally distributed. Also, for a normal distribution, these random values have a mean of zero and a standard deviation of one and are between negative and positive infinity.

The simulated random property is computed by algebraically adding the product of the random number and the standard deviation of the property to the mean value of the property. The random number represents a random number of standard deviations from the mean. In this example, since the random numbers have a mean value of zero, as the sample size is increased, the mean value of the simulated property will approach the mean value of the property. That is, the mean plus zero equals the mean. In other words, the simulated property accurately represents the actual property. Each sample girder has a unique set of random values, simulated in this way, for its properties.

Finding the Cracking Moment Capacity

Selecting a Trial Curvature

The analytical model used in PROCAT was first developed by Hognestad(2), was later used by Mirza et al.(4) and Ellingwood et al.(5), and recently was applied to prestressed bulb-tee girders (3). The process of using the analytical model to find the cracking moment can be summarized as follows. Once the properties of a girder are determined, an arbitrary curvature, ϕ, is imposed on the girder. That is, we specify that the girder is deflected downward. The curvature represents the slope of the strain distribution on the girder cross section, however we do not know the moment that caused the curvature. The equilibrium moment that corresponds to that curvature must be found.

Selecting a Trial Top-of-Girder Strain

Since the curvature only represents the slope of the strain distribution, we need to assume the strain at some point in the section to completely define a unique strain distribution. Therefore, a trial value for the top-of-girder strain (Figure 3) is

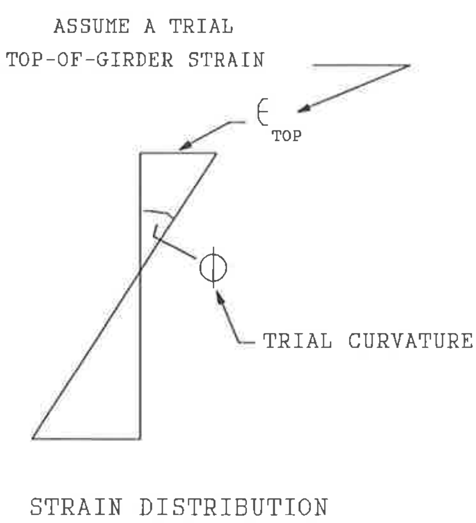

FIGURE 3 Strain distribution on cross section (precast deck).

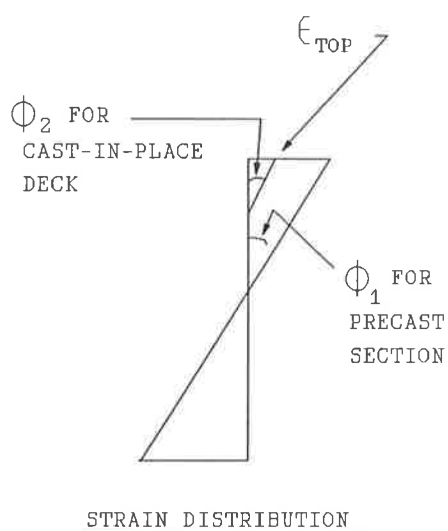

FIGURE 4 Strain distribution on cross section (CIP deck)

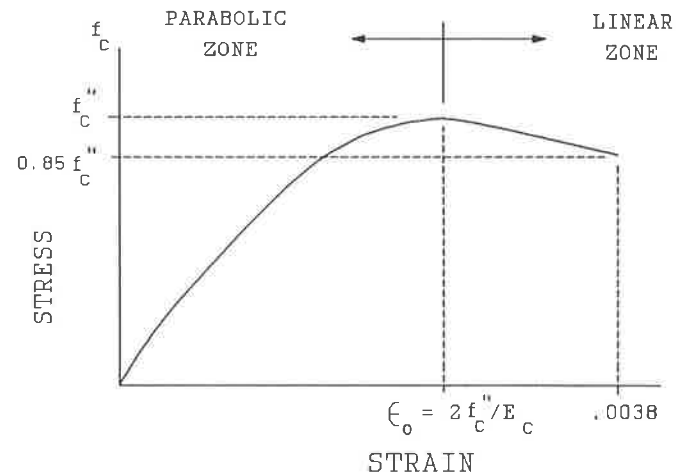

FIGURE 5 Concrete stress vs. strain.

assumed in order to establish a trial strain distribution. For a girder with a cast-in-place deck, the strain distribution is slightly more complicated (Figure 4). However, there is a fixed difference between the curvature of the precast girder and the curvature of the cast-in-place deck. There is also a fixed difference between the strain at the top of the precast girder and the strain at the bottom of the cast-in-place deck.

The Equilibrium Strain Distribution

Using the trial strain distribution and the stress-strain relationships (Figures 5 and 6) for the materials (2)(4), the total tensile and compressive forces, T and C, on the cross section are found. For the mild steel forces, the program uses an elastic-perfectly-plastic stress-strain curve using the mild steel

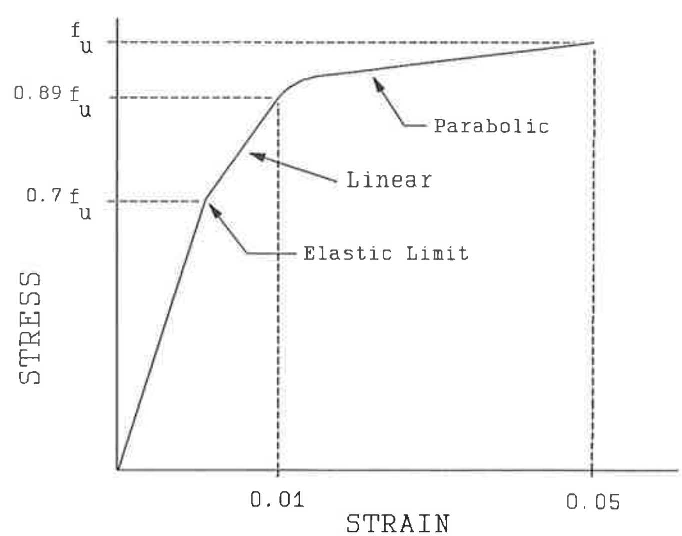

FIGURE 6 Prestressing steel stress vs. strain.

properties listed in Table 1. The current model assumes planes remain plane under bending (i.e., a linear strain distribution). Creep and shrinkage effects are only included by way of the prestress losses. If, after the force calculation, the net force (T - C) is not zero, a new trial value for the top-of-girder strain is selected and the process repeated. This process of varying the top-of-girder strain shifts the strain distribution (Figure 7). For example, if T is greater than C, then the strain distribution should be shifted to the right to increase C and decrease T. If T is less than C, the strain distribution should be shifted to the left. If T is equal to C (i.e., the net force is zero), the equilbrium value for the top-of-girder strain has been found (Figure 8). The corresponding equilibrium moment for the trial value of ϕ is then calculated using the tensile and compressive forces on the section.

The Cracking Moment

Once equilibrium is found, PROCAT checks to see if the equilibrium moment is also the cracking moment. The equilibrium stress at the bottom of the girder is compared to the flexural cracking strength of the concrete. If the equilibrium stress is not equal to the cracking stress, a new trial curvature is selected and the process is repeated. For example, if the equilibrium stress is less than the cracking strength, a larger curvature would be tried. If the two stresses are equal, the equilibrium moment is the cracking moment capacity of the sample girder (Figure 9).

This process is repeated for each sample girder. Because the properties for each sample girder have unique values, the cracking moment capacities will also be unique. If the sample consists of, say, 100 girders, this Monte Carlo procedure computationally represents the building and testing of 100 individual girders.

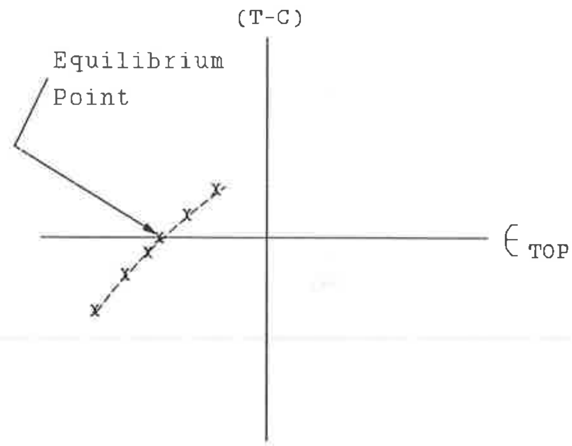

Figure 8 Schematic of search for equilibrium moment.

USING PROCAT OUTPUT

The Parent Distribution

The probabilistic analysis produces a sample population of cracking moment capacities for the subject girder. It is desirable to be able to describe that population mathematically. There are many known theoretical distribution types (for example, the normal distribution and the Weibull distribution) that can be considered to represent the sample population. The distribution that best represents the sample is known as the parent distribution and can be found using standard statistical methods. Once a parent distribution type is chosen, this implies that we believe our sample came from a larger population distributed according to the parent distribution. Previous studies (9) found that the population of moment resistances for

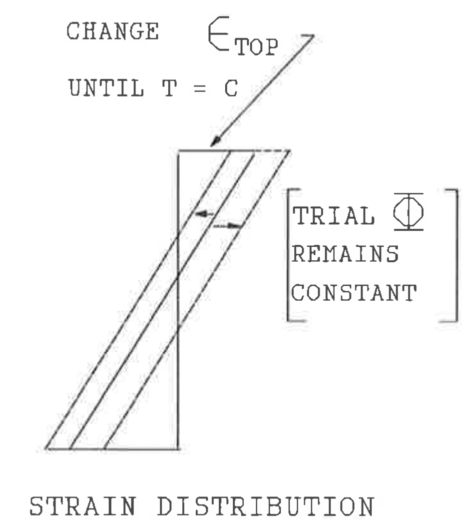

FIGURE 7 Shifting strain distribution for equilibrium.

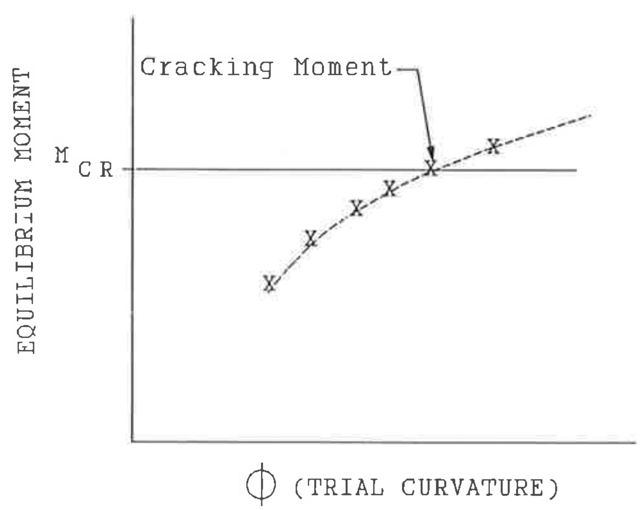

FIGURE 9 Schematic of search for cracking moment.

bulb-tee girders could usually be represented by a normal distribution. No attempt is made here to recommend a parent distribution type for cracking moment resistances of prestressed bridge girders. Since nearly all of the random properties are normally distributed, it is reasonable to assume that the moment capacity might also be normally distributed. However, for a given girder, the sample population produced by PROCAT should be statistically evaluated to determine a parent distribution.

Finding the Probability of Failure

Given adequate information about the load moment, the probability of failure can be calculated using the cumulative distribution function for the parent distribution. For example, suppose the population of moment resistances for a bridge girder turns out to be normally distributed (Figure 10). Further, suppose an extreme load needs to pass over the bridge and that the load will result in a total moment of 2500 ft-kips in the girder. Assume the extreme load is carefully controlled so that the 2500 ft-kip moment can be taken to be deterministic (i.e., not random). Using the mean, $\mu=2610$, and the standard deviation, $\sigma=35.6$, of the cracking moment capacity, we first calculate the standard normal variable, z, and get

$$z = (2500 - \mu)/\sigma = -3.09 \qquad (1)$$

where

z = number of deviations from the mean,
μ = mean cracking moment capacity, and
σ = standard deviation of cracking moment capacity.

The probability that the girder will have a moment capacity, M, less than 2500 ft-kips is

$$P[M \leq 2500 \text{ ft-kips}] = \Phi(z) = .001 \qquad (2)$$

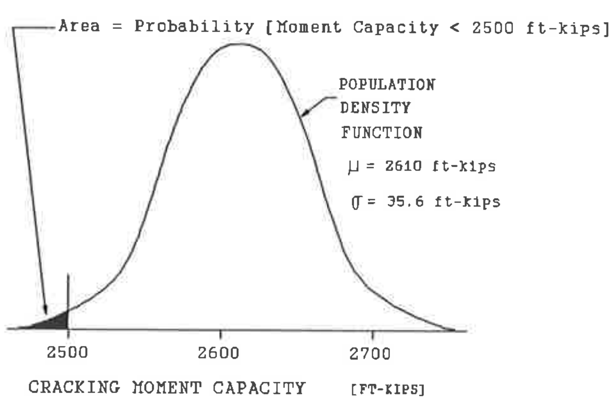

FIGURE 10 Population of cracking moment capacities.

where

Φ = the standard normal cumulative distribution function.

Values for the standard normal cumulative distribution are available in tables and on many hand-held calculators. Physically, this probability represents the area in the tail of the population curve below a value of 2500 ft-kips (See Figure 10). This result indicates that there is a 1/1000 chance that the load would cause flexural cracking in the girder.

An alternative to the use of a parent distribution, such as Φ above, is to use the actual, discrete cumulative distribution of the sample. Other methods are also available as outlined in reference (10). In any case, using normal methods of reliability theory, a probability of failure can be calculated for either a deterministic extreme load or a probabilistic extreme load.

Realistic Application of Results

Some discussion of how to apply the results obtained by PROCAT is in order. It is important to note that PROCAT only produces information about the cracking capacity of a girder. Information about the load moment must come from other sources. As suggested in the example, the load moment is also a random variable. Its variability must be either minimized or quantified in some way in order to calculate a meaningful probability of failure. Often engineers choose to minimize that variability by weighing each axle of the load, carefully controlling the location of the wheel lines on the bridge, limiting the speed of the load (e.g., 5 mile-per-hour maximum), prohibiting shifting of gears, and maintaining a constant speed (i.e., no braking or acceleration). The engineer may also use finite element analysis or methods such as those presented by Bakht and Jaeger (11) to calculate a more accurate load moment. In any case, one should attempt to minimize the uncertainty associated with the load moment analysis. By carefully controlling the load and the moment calculation, it is often reasonable to establish a deterministic value for the load moment. This may affect the theoretical purity of the resulting probability of failure, but not necessarily its practical usefulness. Individual agencies may well choose not to calculate a probability of failure, but they can still use the knowledge about the variability of the cracking moment capacity to aid the process of dealing with extreme loads.

The statistical results that PROCAT provides are a valuable source of **additional information.** The engineer can use this additional information to **help** make a decision about the passage of the extreme load. That is, **the information provided by using PROCAT should not be seen as a substitute for engineering judgement, but rather as an additional aid to that judgement.** The situation is similar to the way finite element techniques are used to help in this same area. The intention is that PROCAT will help the engineer to make better decisions about the safety of the passage of extreme loads.

CONCLUSIONS

Probabilistic analysis is a powerful tool to aid the engineering decision-making process. The computer program PROCAT, introduced in this paper, is written specifically for finding the probabilistic, flexural cracking strength of prestressed concrete girders; it does not find the statistical parameters of the applied load moment. The cracking strength statistics provided by the software is important, additional information to help in assessing the safety of extreme loads on prestressed concrete structures. By providing the strength information needed to calculate a probability of failure, PROCAT contributes to a more rational process for dealing with load limits, overload permits, and construction loads.

ACKNOWLEDGEMENTS

The author appreciates the funding provided by the TRANS-NOW Northwest Transportation Center in cooperation with the U.S. Department of Transportation and the significant funding provided by the Idaho Transportation Department.

REFERENCES

1. V. A. Geidl and S. Saunders. Calculation of Reliability for Time-Varying Loads and Resistances. In *Structural Safety*, Elsevier, Amsterdam, Vol. 4, 1987, pp.285-292.
2. E. Hognestad. *A Study of Combined Bending and Axial Load in Reinforced Concrete Members*. Bulletin No. 399, Engineering Experiment Station, University of Illinois, Urbana, 1951, 128 pp.
3. B. Khafagi and V. A. Geidl. Reliability of Cracked, Prestressed Girders. In *Proceedings: ASCE Conference on Probabilistic Mechanics*, Blacksburg, VA., 1988, pp. 309-312.
4. S. Mirza, D. Kikuchi and J. MacGregor. Flexural Strength Reduction Factor of Bonded Prestressed Concrete Beams. In *Journal of the American Concrete Institute*, July-Aug., 1980, pp. 237-246.
5. B. Ellingwood, T. V. Galambos, J. G. MacGregor and C. A. Cornell. *Development of a Probability Based Load Criterion for American National Standard A58*. NBS Special Publication 577, National Bureau of Standards, June 1980.
6. S. A. Mirza and J. G. MacGregor. Variability of Mechanical Properties of Reinforcing Bars. In *Journal of the Structural Division*, ASCE, Vol. 105, No. ST5, Proc. Paper 14590, May, 1979, pp. 921-937.
7. S. A. Mirza and J. G. MacGregor. Variations in Dimensions of Reinforced Concrete Members. In *Journal of the Structural Division*, ASCE, Vol. 105, No. ST4, Proc. Paper 14495, April, 1979, pp. 751-766.
8. S. A. Mirza, M. Hatzinikolas and J. G. MacGregor. Statistical Descriptions of Strength of Concrete. In *Journal of the Structural Division*, ASCE, Vol. 105, No. ST6, Proc. Paper 14628, June, 1979, pp. 1021-1037.
9. V. A. Geidl. *Structural Reliability Assessment of AASHTO Limit States Design for Prestressed Concrete Bulb-Tee Bridge Girders*, Department of Civil Engineering, University of Idaho, Moscow, Idaho, May, 1985.
10. P. Thoft-Christensen and M. J. Baker. *Structural Reliability Theory and Its Applications*. Springer-Verlag, New York, 1982.
11. B. Bakht and L. G. Jaeger. *Bridge Analysis Simplified*. McGraw-Hill, New York, 1985.