Upper Confidence Limit of Local Pier-Scour Predictions

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Eighty-three on-site measurements of local scour at bridge piers were gathered from both published and unpublished sources and analyzed using multiple regression analysis to obtain local scour prediction equations. This investigation differs from most previous studies of scour at bridge piers because only on-site measurements were used. The regression analysis provides a probability statement about the distribution of the prediction, from which an upper confidence limit of the depth of local scour can be determined. An upper confidence limit having an appropriate exceedance probability can be used for design of bridge pier foundations to provide a desired factor of safety.

INTRODUCTION

Exposure or undermining of bridge-pier foundations from the erosive action of flowing water can result in structural failure of a bridge, requiring a major expenditure for repair or replacement (see Davis, 1984 for case histories of scour problems at several bridges). Therefore, bridge-pier foundations need to be designed for the maximum depth of scour that is likely to occur for a specified design condition. Spread footings, drilled pier foundations, and caisson foundations need to be placed well below the estimated depth of scour; and piles piers need to be long enough to support the structure after scour has occurred.

The term "scour" is used here to mean a lowering by erosion of a stream-bed below an assumed natural level or other appropriate datum. "Depth of scour", or "scour depth", refers to the depth of material removed below the selected datum. Scour is of concern primarily at bridge crossings on alluvial streams, but stream-bed erosion occurs in nearly all waterways.

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Scour at a bridge waterway is usually separated into three components: (1) General scour, (2) constriction scour, and (3) local scour. General scour is the progressive aggradation or degradation of the stream channel caused by changes in channel controls (such as a dam construction or gravel mining), changes in sediment supply, or changes in stream form (such as a change from a meandering to a braided stream). Constriction scour is caused by the reduction of waterway area by bridge embankments, piers, and other obstructions, that increase flow velocities within and near a stream channel generally for a short distance up-stream of and downstream of the bridge crossing. Local scour is caused by flow disturbances created by piers and abutments, resulting in an abrupt decrease in stream-bed elevation in the immediate vicinity of these structure.

The phenomena of local scour at a bridge pier has been studied often, and numerous mathematical and graphical relations for predicting depth of local scour at a bridge pier have been developed. Most of these relations are based entirely on experimental data collected in small-scale laboratory flumes and provide widely varying predictions for a given set of pier, approach flow, and bed material characteristics (see Jones, 1984, for a comparison of several local pier scour prediction methods).

Departing from the experimental approach, 83 on-site measurements of local scour at bridge piers were assembled from published sources, and local pier-scour prediction equations for live-bed scour conditions were developed from multiple regression analysis of these data. The regression analysis also provides a probability statement about the distribution of the prediction, from which an upper confidence limit of the predicted depth of local scour can be determined. An upper confidence limit having an appropriate exceedance probability can be used for design of bridge pier foundations to provide a desired factor of safety.

MECHANISM OF LOCAL SCOUR

Local scour is classified as clear-water or live-bed depending on whether or not sediment is supplied to the scour hole by the approaching flow. Clear-water scour occurs when the approach flow is not capable of transporting sediment to the scour hole. The maximum depth of local scour for clear-water flow is attained when the capacity for transport of material out of the scour hole is reduced to zero (that is, when an equilibrium condition is reached). Live-bed scour occurs when material is continuously supplied to a scour hole. Because of the movement of bed forms in a stream channel when live-bed flow exists, the lowest point in a scour hole will oscillate periodically about an average value. The equilibrium scour depth for live-bed conditions is defined as the distance between the temporal averages of the ambient stream-bed elevation and the elevation of the lowest point in the scour hole.

Piers are classified as blunt-nosed or sharp-nosed depending on their geometry. Blunt-nosed piers have square or rounded upstream ends; sharp-nosed piers have a tapered upstream end that terminates at a sharp edge. Scour around a blunt-nosed pier is initiated by a downward flow along the upstream face of the pier caused by a vertical pressure gradient that develops as approaching flow impinges on the nose of the pier. Weaker downward flows will occur along the tapered sides of a sharp-nosed pier. As flow is deflected downward and around a pier, velocities near the bed are increased and large-scale eddy structures (vortices) are formed (Laursen and Toch, 1956; Roper and others, 1967; Melville, 1975; and Dargahi, 1987). Local scour at a pier is caused by the combined action of vortices and accelerated flow around the pier.

ON-SITE MEASUREMENTS

On-site measurements of local scour at bridge piers were assembled from available published and unpublished sources and are summarized in Tables 1 and 2. The depth of local scour, d_s, is defined as the distance from the lowest point in the scour hole to the estimated elevation that the stream-bed would have assumed if the pier was removed. Scour holes were measured using a variety of devices including sonic depth meters, sounding weights, and sounding rods. Each measurement was made during a sustained high flow, and is assumed to represent an equilibrium condition.

Variables governing local scour at a pier consist of those that describe the pier, the approach flow, and the streambed material. Pier data consist of a pier shape code (1 = a square-nosed pier, 2 = a round-nosed pier, and 3 = a sharp-nosed pier); the pier width, b; and the pier length, ℓ. Approach flow data consist of the flow depth, y_a; the depth-averaged flow velocity, V_a; and the angle at which the flow approaches the pier, α (α equals zero if the pier is aligned with the approaching flow). Bed material variables consist of the median particle diameter, d_{50}; and the geometric standard deviation, $\sigma_g = \sqrt{d_{84}/d_{16}}$; where d_i is the intermediate diameter of a particle for which i percent (by weight) of the material is finer. Reported bed material characteristics are assumed to represent the material forming the bed of the approaching channel and the bed surrounding the pier.

ANALYSIS OF DATA

Using dimensional analysis, the variables governing local scour at a bridge pier were combined to form the following functional relation for relative depth of local scour:

$$\frac{d_s}{b} = f\left\{\phi, \frac{b'}{b}, \frac{y_a}{b}, F_a, R_p, S_s, \frac{b}{d_{50}}, \sigma_g\right\} \quad (1)$$

where ϕ is a dimensionless pier-shape factor, $b' = b \cos \alpha + \ell \sin \alpha$ is the pier width projected normal to the approach flow, $F_a = V_a/\sqrt{gy_a}$ is the Froude number of the approach flow, $R_p = V_a b/\nu$ is the Reynolds number based on approach-flow velocity and pier width, and $S_s = \rho_s/\rho$ is the specific gravity of the bed material.

Because viscosity (or water temperature), specific gravity of bed material, and bed material gradation were rarely reported for the on-site measurements, the variables R_p, S_s, and σ_g were eliminated from the analysis. However, the influence of water viscosity, bed material density, or bed material gradation on local scour at a bridge pier may be significant. Shen and others (1969)

TABLE 1. SUMMARY OF ON-SITE MEASUREMENTS OF LOCAL SCOUR AT BRIDGE PIERS: REFERENCES, LOCATION, MEASUREMENT NUMBER, DATE, AND COMMENTS

Reference(s) and Location of Measurement	Measurement Number	Date of Measurement[a]	Comments
Bata and Todorovic (1960):			
Danube River Bridge at	1	--/--/26[b]	
Novi Sad, Yugoslavia	2	--/--/58	
Neill (1965):			
Beaver River LaCorey	3	16/19/62	center pier
Bridge, Alberta, Canada			
Breusers (1970), and			
Breusers and others (1977):			
Niger River Onitsha Bridge	4	--/--/--	
Melville (1975):			
Waikato River Tuakau	5	08/15/58	
Bridge, New Zealand			
Norman (1975):			
Susitna River	6	07/02/71	pier 1
Anchorage-Fairbanks	7	"	pier 2
Hwy. Bridge near	8	"	pier 3
Sunshine, Alaska	9	08/11/71	pier 1
	10	"	pier 2
	11	"	pier 3
Knik River Bridge	12	07/11/65	pier 5
near Palmer, Alaska			
Knik River Bridge	13	06/17/66	pier 3
near Eklutna, Alaska	14	"	pier 4
	15	"	pier 5
	16	"	pier 6
	17	"	pier 7
	18	06/24/66	pier 1
	19	"	pier 2
	20	"	pier 3
	21	"	pier 4, exposed foundation
	22	"	pier 5
	23	"	pier 6
	24	"	pier 7
	25	06/28/66	pier 1
	26	"	pier 2
	27	"	pier 3
	28	"	pier 4, exposed foundation
	29	"	pier 5
	30	"	pier 6
	31	"	pier 7

TABLE 1. SUMMARY OF ON-SITE MEASUREMENTS OF LOCAL SCOUR AT BRIDGE PIERS: REFERENCES, LOCATION, MEASUREMENT NUMBER, DATE, AND COMMENTS (continued)

Reference(s) and Location of Measurement	Measurement Number	Date of Measurement[a]	Comments
Tazlina River	32	04/22/69	
Richardson Hwy. Bridge	33	09/02/71	
near Glennallen, Alaska	34	09/04/71	
	35	10/01/71	
Tanana River	36	07/16/71	pier 1
Richardson Hwy. Bridge	37	"	pier 2
at Big Delta, Alaska	38	"	pier 3
	39	"	pier 4
Tanana River Anchorage-Fairbanks Hwy. Bridge at Nenana, Alaska	40	07/30/67	
Snow River Seward Hwy. Bridge near Seward, Alaska	41	09/22/70	
Chang (1980):			
Red River Hwy. 6 Bridge	42	12/27/77	
near Grand Encore, La.	43	06/16/78	
Red River Hwy. 8 Bridge	44	06/28/77	
near Boyce, La.	45	06/06/78	
Red River Hwy. 3026 Bridge	46	06/06/77	
near Alexandria, La.	47	11/21/77	
	48	06/19/78	
Atchafalaya River	49	07/14/77	
Hwy. 190 Bridge near	50	"	
Krotz Springs, La.	51	02/29/77	
	52	"	
	53	07/12/78	
	54	"	
Mississippi River Hwy. 65 Bridge at Natchez, Miss.	55	09/07/77	
Mississippi River Hwy. 190 Bridge at Baton Rouge, La.	56	06/14/77	
Hopkins and others (1980):			
Red River Texas St. Bridge Bridge at Shreveport, La.	57	12/18/72	

TABLE 1. SUMMARY OF ON-SITE MEASUREMENTS OF LOCAL SCOUR AT BRIDGE PIERS: REFERENCES, LOCATION, MEASUREMENT NUMBER, DATE, AND COMMENTS (continued)

Reference(s) and Location of Measurement	Measurement Number	Date of Measurement[a]	Comments
Davoren (1985):			
Oahu River near	58	09/20/82	Cylindrical pier
Twizel, New Zealand	59	09/22/82	constructed specifically
	60	06/10/82	to study local scour
	61	01/19/82	
	62	01/20/82	
	63	08/09/82	
	64	07/04/82	
	65	07/05/82	
	66	08/11/82	
	67	07/15/82	
	68	04/28/82	
Jarrett and Boyle (1986), and Jarrett (written commun., 1986):			
South Platte River	69	10/02/84	pier 5
County Rd. 87 Bridge	70	"	pier 6
at Masters, Colo.	71	"	pier 7
	72	06/25/84	pier 5
	73	"	pier 6
	74	"	pier 7
	75	05/18/84	pier 5
Arkansas River	76	05/23/84	pier 1
County Rd. 613 Bridge	77	"	pier 3
near Nepasta, Colo.	78	06/05/84	pier 1
	79	07/27/84	pier 1
Rio Grande Hwy. 285 Bridge	80	05/22/84	
at Monte Vista, Colo.	81	06/01/84	
South Platte River Hwy. 37	82	05/21/84	pier 1
Bridge near Kersey, Colo.	83	06/26/84	pier 1

[a] Date given as mm/dd/yy.
[b] Information not available.

TABLE 2. SUMMARY OF ON-SITE MEASUREMENTS OF LOCAL SCOUR AT BRIDGE PIERS: PIER DATA, APPROACH FLOW DATA, AND BED MATERIAL DATA

Mea-sure-ment num-ber	Depth of local scour, in meters	Pier data			Approach flow data			Bed material data	
		Shape code[a]	Width, in meters	Length, in meters	Depth, in meters	Veloc-ity, meters per second	Angle, in degrees	Median diameter, in milli-meters	Geometric standard deviation
1	4.30	2	4.50	14.00	18.80	1.84	0	0.25	2.24
2	3.00	2	4.50	14.00	17.40	2.28	0	0.25	2.24
3	1.74	2	1.92	17.37	5.39	1.80	5	0.50	--[b]
4	7.80	2	8.50	8.50	9.00	0.65	12	0.67	--
5	2.75	1	2.40	8.85	3.45	0.96	10	0.78	2.30
6	0.76	3	1.52	6.10	5.80	1.98	0	70	--
7	0.76	3	1.52	6.10	4.10	2.59	0	70	--
8	0.61	3	1.52	6.10	3.40	2.13	0	70	--
9	0.61	3	1.52	6.10	5.30	3.05	0	70	--
10	0.61	3	1.52	6.10	6.60	2.90	0	70	--
11	0.61	3	1.52	6.10	5.20	3.51	0	70	--
12	0.82	3	1.80	9.60	5.50	3.67	0	1.5	--
13	0.30	2	1.52	11.58	1.20	0.49	0	0.5	--
14	0.30	2	1.52	11.58	1.50	0.76	0	0.5	--
15	0.30	2	1.52	11.58	1.20	0.88	0	0.5	--
16	0.76	2	1.52	11.58	0.50	0.27	0	0.5	--
17	1.22	2	1.52	11.58	0.60	0.15	0	0.5	--
18	0.61	2	1.52	11.58	2.10	1.52	0	1.6	--
19	0.61	2	1.52	11.58	2.00	1.55	0	1.6	--
20	0.91	2	1.52	11.58	3.00	1.58	0	1.6	--
21	1.22	2	1.52	11.58	3.20	1.98	0	1.6	--
22	1.37	2	1.52	11.58	3.00	1.80	0	1.6	--
23	1.07	2	1.52	11.58	2.60	2.07	0	1.6	--
24	1.83	2	1.52	11.58	3.00	1.83	0	1.6	--
25	0.46	2	1.52	11.58	0.90	0.94	0	1.6	--
26	0.61	2	1.52	11.58	0.90	0.98	0	1.6	--
27	0.46	2	1.52	11.58	1.80	1.10	0	1.6	--
28	0.61	2	1.52	11.58	2.40	1.16	0	1.6	--
29	0.76	2	1.52	11.58	2.30	1.13	0	1.6	--
30	0.46	2	1.52	11.58	1.50	1.13	0	1.6	--
31	0.76	2	1.52	11.58	2.00	0.98	0	1.6	--
32	0.60	3	4.60	--	0.60	--	0	90	--
33	1.50	3	4.60	--	3.70	2.90	0	90	--
34	1.70	3	4.60	--	4.60	3.51	0	90	--
35	0.90	3	4.60	--	1.50	0.61	0	90	--
36	1.80	2	1.52	10.36	3.70	2.16	35	14	--
37	2.10	2	1.52	10.36	3.70	2.22	35	14	--
38	1.80	2	1.52	10.36	4.60	2.07	35	14	--
39	2.40	2	1.52	13.56	4.30	1.74	35	14	--
40	1.80	2	3.05	17.60	6.70	2.59	0	15	--
41	0.90	2	0.98	0.98	1.70	1.61	0	8.0	--
42	4.00	2	4.90	12.80	1.80	--	0	0.053	--
43	4.60	2	4.90	12.80	4.60	--	0	0.053	--
44	3.70	2	8.20	8.20	4.90	0.46	0	0.060	--
45	4.30	2	8.20	8.20	4.30	0.61	0	0.060	--

TABLE 2. SUMMARY OF ON-SITE MEASUREMENTS OF LOCAL SCOUR AT BRIDGE PIERS: PIER DATA, APPROACH FLOW DATA, AND BED MATERIAL DATA (continued)

Mea-sure-ment num-ber	Depth of local scour, in meters	Pier data			Approach flow data			Bed material data	
		Shape code[a]	Width, in meters	Length, in meters	Depth, in meters	Veloc-ity, meters per second	Angle, in degrees	Median diameter, in milli-meters	Geometric standard deviation
46	7.30	2	13.00	38.00	4.10	0.55	5	0.027	--
47	6.80	2	13.00	38.00	3.40	0.66	15	0.027	--
48	8.50	2	13.00	38.00	5.40	1.16	20	0.027	--
49	4.30	3	9.80	12.50	11.00	0.73	5	0.008	--
50	8.20	3	9.80	12.50	12.80	0.81	30	0.008	--
51	4.60	3	9.80	12.50	13.60	1.08	15	0.008	--
52	7.90	3	9.80	12.50	16.30	1.22	25	0.008	--
53	4.00	3	9.80	12.50	11.60	0.82	15	0.008	--
54	7.60	3	9.80	12.50	13.40	0.91	25	0.008	--
55	6.10	1	9.40	19.50	19.50	1.80	0	0.036	--
56	10.40	2	19.50	38.00	11.30	0.66	15	0.036	--
57	2.80	2	3.66	17.30	3.60	0.64	0	0.100	--
58	1.30	2	1.50	1.50	3.10	2.38	0	20	--
59	1.30	2	1.50	1.50	3.00	2.69	0	20	--
60	0.80	2	1.50	1.50	2.50	2.54	0	20	--
61	0.90	2	1.50	1.50	1.40	2.65	0	20	--
62	0.90	2	1.50	1.50	1.30	2.43	0	20	--
63	0.40	2	1.50	1.50	1.30	2.68	0	20	--
64	0.40	2	1.50	1.50	1.00	2.39	0	20	--
65	0.50	2	1.50	1.50	0.90	2.33	0	20	--
66	0.40	2	1.50	1.50	0.90	2.56	0	20	--
67	0.40	2	1.50	1.50	0.70	2.24	0	20	--
68	0.40	2	1.50	1.50	0.60	--	0	20	--
69	0.61	1	0.29	3.66	0.76	1.04	15	1.5	--
70	0.61	1	0.29	3.66	0.61	1.36	15	1.5	--
71	0.52	1	0.29	3.66	0.73	1.17	15	1.5	--
72	0.58	1	0.29	3.66	0.43	1.13	10	2.3	--
73	0.46	1	0.29	3.66	0.58	1.02	10	2.3	--
74	0.49	1	0.29	3.66	0.70	1.12	10	2.3	--
75	0.66	1	0.29	3.66	1.81	1.22	15	2.3	--
76	0.64	2	1.22	6.40	2.13	1.17	0	0.6	--
77	0.40	2	1.22	6.40	0.55	0.69	0	0.6	--
78	1.22	2	1.22	6.40	2.32	1.70	0	0.6	--
79	0.61	2	1.22	6.40	0.70	0.66	0	0.6	--
80	0.37	3	0.94	27.43	1.40	1.54	0	7.9	--
81	0.15	3	0.94	27.43	1.22	1.35	0	4.3	--
82	0.98	3	0.52	8.29	3.21	1.68	10	1.2	--
83	0.65	3	0.52	8.29	2.14	1.17	10	1.8	--

[a]Pier shape code defined as follows: 1 = square-nosed pier, 2 = round-nosed pier, 3 = sharp-nosed pier.
[b]Information not available.

present a prediction equation that contains R_p, and conclude that the influence of viscosity on relative depth of local scour is significant. Raudkivi (1986) and Chiew (1989), on the basis of analyses of experimental data, conclude that bed material gradation has a significant influence on local scour depth because of the ability of a graded sediment to form an armour layer in the scour hole and on the surrounding stream-bed. Future on-site measurements of local scour at bridge piers need to include both temperature and bed-material gradation data.

Multiple linear regression was used to develop prediction equations for relative depth of local scour, based on the proposed functional relation in equation 1. A multiple regression model is written in matrix notation as

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \tag{2}$$

where $\mathbf{y}$ is an (n×1) vector of the observations, $\mathbf{X}$ is an (n×p) matrix of (p - 1) independent variables augmented by a column of ones, β is a (p×1) vector of the regression coefficients to be determined, and ε is an (n×1) vector of random errors. The least-squares estimator of β is

$$\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}. \tag{3}$$

The vector of fitted values corresponding to a vector of observed values is computed as

$$\begin{aligned}\hat{\mathbf{y}} &= \mathbf{X}\hat{\beta} \\ &= \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \\ &= \mathbf{H}\mathbf{y}\end{aligned} \tag{4}$$

where the (n×n) matrix $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is called the "hat" matrix. The hat matrix and its properties play a central role in regression analysis.

The effect of pier shape was evaluated by defining two indicator variables (Draper and Smith, 1981, p. 241), Z_1 and Z_2, as follows:

$$Z_1 = \begin{cases} 1, & \text{if the pier has a square-nose} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1, & \text{if the pier has a sharp-nose} \\ 0, & \text{otherwise} \end{cases}$$

Hence, $(Z_1, Z_2) = (1, 0)$ for a square-nosed pier; $(Z_1, Z_2) = (0, 1)$ for a sharp-nosed pier; or $(Z_1, Z_2) = (0, 0)$ for a round-nosed pier.

Only live-bed scour measurements were considered in the analysis. Measurements were classified as live-bed scour if the approach flow velocity was greater than a critical velocity, V_c, calculated using the relation developed by Neill (1968) for coarse uniform bed material,

$$V_c = 1.58\sqrt{(S_s - 1)gd_{50}}\left(\frac{y_a}{d_{50}}\right)^{1/6}, \tag{5}$$

where S_s was assumed to equal 2.65 for all measurements. Sixty-six measurements were classified as live-bed scour.

Logarithmic transformation of all variables except the indicator variables was found to provide the best linear relations. The logarithm of relative depth of local scour was regressed against every possible combination of the seven independent variables. The best two, three, four, five, and six-variable regression equations, chosen on the basis of Mallow's C_p statistic, are summarized in Table 3. The two indicator variables, Z_1 and Z_2, were retained in all of the models because experiments have shown a definite dependence of local scour depth on pier shape (Breusers and others, 1977). During preliminary analyses, measurement number 55 was found to exert a large influence on the regression models and was determined to be an "outlier". It was deleted from the data set for final analyses. The best six-variable equation differs slightly from an equation presented in a previous analysis of the data set (Froehlich, 1988) because of the deletion of the outlier measurement. The six-variable equation presented here is preferred.

The five-variable regression equation minimizes the standard error of estimate of the logarithm of relative depth of local scour and contains parameters that are all significant at the 0.10 confidence level. The prediction equation written in terms of untransformed values is

$$\frac{\hat{d}_s}{b} = 0.40\,\phi\left(\frac{b'}{b}\right)^{0.53}\left(\frac{y_a}{b}\right)^{0.46}\left(\frac{b}{d_{50}}\right)^{0.052} \tag{6}$$

where

$$\phi = \begin{cases} 1.4, & \text{for square-nosed piers} \\ 1.0, & \text{for round-nosed piers} \\ 0.5, & \text{for sharp-nosed piers} \end{cases}$$

PREDICTING NEW OBSERVATIONS

Relative depth of local scour predicted by equation 6 is the expected response to a specified set of regressors, and is not known with certainty. However, a local scour depth that has a small probability of being exceeded needs to be used for designing pier foundations to insure against the failure of a bridge. A probability statement about the distribution of $\log(d_s/b)$ provides a means of evaluating the accuracy of the estimate and of determining an appropriate design value. Letting

$$x_0' = \left[1,\ Z_1,\ Z_2,\ \log\left(\frac{b'}{b}\right),\ \log\left(\frac{y_a}{b}\right),\ \log\left(\frac{b}{d_{50}}\right)\right] \tag{7}$$

be the transpose of the vector of predictor variables at a particular site, a $100(1 - \alpha)$ percent upper confidence limit for $\log(d_s/b)$ is computed as

$$\log(\hat{d}_s/b)_0 + t_{\alpha,n-p}\sqrt{\text{var}(\mathbf{x}_0\hat{\beta})} \tag{8}$$

(Montgomery and Peck, 1982, p. 141), where

$$\log(\hat{d}_s/b)_0 = \mathbf{x}_0'\hat{\beta}() \tag{9}$$

is the predicted value of $\log_e(d_s/b)$,

$$\hat{\beta}' =$$

$$\left[-1.150,\ 0.412,\ -0.422,\ 0.534,\ 0.464,\ 0.052\right] \tag{10}$$

is the transpose of the vector of regression model parameters, $t_{\alpha,n-p}$ is the one-sided t-distribution statistic corresponding to an exceedance probability α and n-p degrees of freedom, n is the number of observations used in the regression, p is the number of regression model parameters,

$$\text{var}(\mathbf{x}_0\hat{\beta}) = \hat{\sigma}^2\left(1 + \mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0\right) \tag{11}$$

is the estimated variance of prediction, and $\hat{\sigma}^2$ is the residual mean square error (estimated model error). For the 5-variable regression model, n = 65, p = 6, $\hat{\sigma}^2 = 0.1056$, and

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 0.1433 & -0.0196 & 0.0375 & -0.0056 & -0.0360 & -0.0160 \\ -0.0196 & 0.2566 & 0.0410 & -0.0962 & -0.0059 & 0.0018 \\ 0.0375 & 0.0410 & 0.1436 & -0.0077 & -0.0340 & -0.0075 \\ -0.0056 & -0.0962 & -0.0077 & 0.1039 & -0.0220 & -0.0011 \\ -0.0360 & -0.0059 & -0.0340 & -0.0220 & 0.0523 & 0.0045 \\ -0.0160 & 0.0018 & -0.0075 & -0.0011 & 0.0045 & 0.0022 \end{bmatrix} \tag{12}$$

One-sided t-distribution statistics for n - p = 59 degrees of freedom and several confidence levels are given in Table 4. An upper estimate of $\log(d_s/b)$ is obtained from equation 8 by selecting an appropriate confidence level, say 99 percent, and obtaining the corresponding t statistic from Table 4. Use of an upper limit of scour depth for design of pier foundations will provide a factor of safety needed to insure against failure of the structure.

Hidden Extrapolation

Extrapolation beyond the region defined by the data used to develop a regression equation may provide an unreliable prediction. It is possible that an equation that predicts accurately in the region spanned by the data will perform poorly outside that region. Because the levels of the regressor variables jointly define the region containing the data, it is easy to extrapolate inadvertently. Although each member of a given set of regressors may lie within the range of that particular variable, it is possible that the point defined by the set lies outside the region defined by the original data.

To detect a hidden extrapolation, the smallest convex set containing all of the original data points is considered and

TABLE 3. SUMMARY OF LOCAL PIER-SCOUR REGRESSION EQUATIONS

Number of vari-ables	Constant	Coefficient of Z_1	Z_2	$\log \frac{b'}{b}$	$\log \frac{y_a}{b}$	$\log F_a$	$\log \frac{b}{d_{50}}$	Coeffi-cient of determ-ination	Stan-dard error of estimate
2	-0.625	1.214	0.064[a]	---[b]	---	---	---	0.388	0.508
3	-0.739	0.447	-0.089[a]	0.724	---	---	---	0.601	0.414
4	-0.761	0.367	-0.241	0.560	0.355	---	---	0.713	0.353
5	-1.150	0.412	-0.422	0.534	0.464	---	0.052	0.762	0.325
6	-1.220	0.396	-0.450	0.549	0.497	0.190[a]	0.094	0.767	0.325

[a]The hypothesis that the coefficient equals zero cannot be rejected at the 0.10 significance level.

[b]Variable is not included in the regression model.

TABLE 4. ONE-SIDED t STATISTICS FOR SEVERAL CONFIDENCE LEVELS AND 59 DEGREES OF FREEDOM

Confidence level, in percent	One-sided t-distribution statistic
50.0	0.000
60.0	0.254
70.0	0.527
80.0	0.848
90.0	1.30
95.0	1.67
97.5	2.00
99.0	2.39
99.5	2.66

referred to as the regressor variable hull (RVH). If a point defined by a set of regressors lies inside or on the boundary of the RVH the prediction involves interpolation, while extrapolation is needed if the point lies outside the boundary. The diagonal elements h_{ii} of the hat matrix **H** can be used to determine the position of a point $\mathbf{x}_i$ (Montgomery and Peck, 1982, p. 143). The values of h_{ii} are dependent on the Euclidean distance of a point $\mathbf{x}_i$ from the centroid of the RVH, and on the density of the points in the RVH. Usually the largest value of h_{ii}, say h_{max}, corresponds to a point that lies on the boundary of the RVH in a region of the x-space where the density of observations is relatively low.

The location of a point $\mathbf{x}_0$ relative to the RVH is reflected by the quantity

$$h_{00} = \mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0. \tag{13}$$

A point $\mathbf{x}_0$ for which $h_{00} \geq h_{max}$ lies outside an ellipsoid enclosing the RVH, and is a point for which extrapolation would be needed. If $h_{00} \leq h_{max}$, the point $\mathbf{x}_0$ is inside the ellipsoid and possibly within the RVH, and would be considered an interpolation point. Generally, the smaller the value of h_{00}, the closer the point $\mathbf{x}_0$ lies to the centroid of the RVH. For the 5-variable prediction equation listed in Table 3, $h_{max} = 0.214$. When using equation 5 to predict a local scour depth, h_{00} needs to be calculated using the $(\mathbf{X}'\mathbf{X})^{-1}$matrix (equation 12) and the $\mathbf{x}_0$ vector of regressor variables, then compared to h_{max} to determine whether or not the prediction is an extrapolation.

EXAMPLE APPLICATION

To demonstrate use of the local scour prediction equation and calculation of an upper confidence limit to be used for design, consider the following values for a hypothetical square-nosed bridge pier:

$$b = 1.0 \text{ m}$$
$$\ell = 6.0 \text{ m}$$
$$\theta = 5 \text{ deg}$$
$$y_a = 4.0 \text{ m}$$
$$V_a = 2.0 \text{ m/s}$$
$$d_{50} = 2 \text{ mm}$$
$$S_s = 2.65$$

Dimensionless variables for the example are $\frac{b'}{b} = 1.52$, $\frac{y_a}{b} = 4.0$, $\frac{b}{d_{50}} = 500$; and $V_c = 1.0$ m/s (from equation 5). Critical velocity exceeds the approach flow velocity; hence, live-bed conditions exist and equation 6 is applicable. The transpose of the vector of predictor variables is

$$\mathbf{x}_0' = [1.0,\ 1,\ 0,\ 0.418,\ 1.386,\ 6.215]$$

and $h_{00} = \mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0 = 0.232$. The predicted relative depth of local scour $\hat{d}_s/b = 1.58$ from equation 6, and the 99 percent upper confidence limit of $\log(d_s/b)$ yields $d_s/b = 3.73$. Because $h_{00} \geq h_{max}$, the point $\mathbf{x}_0$ is outside the region defined by the variables used to fit the regression model parameters. Hence, the prediction is an extrapolation and needs to be evaluated carefully before being accepted.

SUMMARY AND CONCLUSIONS

On-site measurements of local scour at bridge piers obtained from both published and unpublished sources were assembled, and prediction equations for relative depth of local live-bed scour were developed using multiple regression analysis. The best prediction equation obtained was a 5-variable model that requires a description of pier shape, and values of pier width, pier length, angle at which flow approaches a pier, approach flow depth, and the median size of bed material. In addition, an estimate of approach flow velocity is needed to determine if live-bed scour conditions exist at a site. The regression analysis provides a probability statement about the accuracy of a prediction, based on particular values of the regressor variables, which allows an upper confidence limit of relative depth of local scour to be calculated. The upper confidence limit provides a factor of safety for design that is based on sound statistical theory.

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