Managing Low-Volume Road Systems for Intermittent Use

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In some areas of the United States, particularly in gentle topography, closely spaced road systems have been developed that, although low cost per mile to construct, are costly in total to maintain. These low-volume roads are often characterized by short periods of use separated by long periods of little or no use. During periods of little or no use, periodic maintenance must be done both to maintain public safety and to prevent degradation. Closing a road requires an investment to prevent unplanned entry and also often requires modification of drainage facilities. Opening a closed road involves an investment. Managers must decide which roads should be left open and maintained, which roads should be closed, which roads should be reopened, and in what period. Managing roads for intermittent use differs from classic transportation development where the emphasis is on construction and transport costs and does not include intermittent road management options and costs. An efficient mathematical formulation for intermittent road management and a numerical example are given.

As low-volume road systems for natural resource development areas become more extensive, a problem becomes how to manage these systems. Typically, in timber harvesting regions even during periods of use, natural resource development roads may average less than 100 vehicles per day, and then for periods of several years or more, they may have no planned use. In some areas, more roads have been developed than will be needed during any one period of activity. If all roads are left open, for purposes of public safety and to prevent damage to the road, some level of periodic road maintenance must be done. Alternatively, roads could be closed through uses of gates, temporary earth embankments, or other barriers, and drainage structures could be modified to prevent damage from water.

Managers must decide which roads should be open in each time period and which roads should be closed. Considerations include the cost of opening roads, closing roads, maintaining roads while they are open, maintaining roads while they are closed, and transportation costs over the alternative routes between entry points in the road network and timber destinations during each period of activity. Although for some roads, the decisions can be made independently of other roads, often, they cannot. In general, a transportation schedule may involve several entry points into a road network during each period, and alternative routes to destinations may exist. Therefore, the planned activities may have to be considered simultaneously.

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The intermittent road management problem differs from the usual road development problem in that in the latter problem only costs of development, maintenance during periods of use in proportion to the level of the activity, and transportation costs are considered. Road openings, closings, and costs that occur during periods of nonuse are not considered.

One way in which to analyze the intermittent road management problem is to use mixed integer linear programming (MIP). In the following sections, the problem is formulated and a numerical example is provided.

FORMULATION

From a known schedule of entries into an existing road system, the problem is to determine which roads are to remain open and which are to be closed and for how long. The objective is to minimize the discounted cost of transportation costs, road opening costs, road closing costs, and road maintenance costs. By minimizing discounted costs is meant to minimize the sum of truck transportation, road opening, road closing, and road maintenance costs over a specified time horizon taking into account the time value of money.

The objective function becomes

$$\text{Minimize} \sum c_{it}x_{it} + \sum u_{it}B_{it} + \sum v_{it}G_{it} + \sum w_{it}D_{it} \qquad (1)$$

where

c_{it} = discounted unit cost of transport and variable road maintenance for Road i in Period t;
u_{it} = discounted cost of fixed road maintenance for Road i in Period t if it is open;
v_{it} = discounted cost of closing Road i in Period t;
w_{it} = discounted cost of opening Road i in Period t;
x_{it} = volume of traffic units using Road i in Period t;
B_{it} = indicator of period Road i is open, where B_{it} = 1 in all periods Road i is open, and B_{it} = 0 at all other times;
G_{it} = indicator of period Road i is to be closed, where G_{it} = 1 in period Road i is to be closed, and G_{it} = 0 at all other times; and
D_{it} = indicator of period Road i is to be opened, where D_{it} = 1 in period Road i is to be opened, and D_{it} = 0 at all other times.

There are four kinds of relationships (constraints) in the problem—relationships to (a) conserve flow at each node in the road network, (b) to signal which period the road is open, (c) to signal which period the road is to be closed, and (d) to signal which period the road is to be opened. It is necessary to declare that only one type of variable, B_{it}, is an integer variable, with a value of 0 or 1. The variables G_{it} and D_{it} are continuous variables that will be forced to act as integer variables using the relationships discussed next.

Conservation of Flow

Because all traffic that enters a road node (junction) in Period t must exit the node, it is required that

$$\sum x_{kt} = \sum x_{jt} \qquad \text{for all nodes and time periods} \qquad (2)$$

where x_{kt} = traffic entering node in Period t and x_{jt} = traffic leaving node in Period t.

There will be one equation for each node in the road network for each period of the analysis.

Signal When Road Is Open

To do this, an integer variable, B_{it}, is assigned that must be equal to 1 in all periods the road is open, and 0 otherwise. This is done by requiring that B_{it} must be 1 if there is traffic over the road segment in Period t by the relationship

$$MB_{it} \geq x_{it} + x_{i't} \qquad \text{for all } i \text{ and } t \qquad (3)$$

where M is a number sufficiently large to guarantee that if there is traffic over a road segment (B_{it} = 1) that M will be larger than the total flow over the segment in both directions. $X_{i't}$ indicates the possibility of flow in either direction over the link. Equation 3 also permits the possibility that the road could be open with no traffic.

Signal When Road Was Open, But Now It Is Time To Close It

For the first period, because the roads were all assumed to be open at the beginning of Period 1, it is required that

$$B_{i1} + G_{i1} \geq 1 \qquad \text{for all } i \qquad (4)$$

If the road is open, B_{i1} will be 1 and the objective function will force G_{i1} to be 0 to minimize costs. By this is meant, that if B_{i1} is 1, then Equation 4 is satisfied regardless of what positive value G_{i1} takes. Because G_{i1} is not necessary to satisfy Equation 4, the linear programming algorithm will never include it as it only adds cost to the objective function. If B_{i1} is 0, then G_{i1} will be 1 because the sum of B_{i1} and G_{i1} must be greater than or equal to 1. It is not possible to have the road open and apply closing costs at the same time. For subsequent periods, it is required that

$$B_{i,t+1} + G_{i,t+1} \geq B_{it} \qquad \text{for all } i \text{ and } t \qquad (5)$$

This expression means that if the road was open in the previous period, then it must either be kept open in the next period or the closing costs must be applied. Because B_{it} and $B_{i,t+1}$ must be 1 or 0, the objective function will force $G_{i,t+1}$ to be 0 when it can be, or at most equal to 1 to satisfy Equation 5.

Signal When Road Was Closed, But Now Is Time To Open It

If a road will be open in this time period, but it was closed in the previous time period, then the road must be opened. This is done by requiring that

$$D_{it} + B_{i,t-1} \geq B_{it} \qquad \text{for all } i \text{ and } t \geq 2 \qquad (6)$$

If the road was open in the previous period ($B_{i,t-1}$ = 1), and is to remain open in this period (B_{it} = 1), then the objective function will force D_{it} to be 0. On the other hand, if the road was not open in the previous period ($B_{i,t-1}$ = 0) and is to be open in this period, then the objective function will force D_{it} to be equal to 1.

These complete the relationships necessary to ensure that the roads are open when they must carry traffic and that roads can be closed when there is no traffic. The objective function of Equation 1 is the criterion that is used to determine which roads are open, which are closed, and for how long.

EXAMPLE

A 15-road segment, 10-node, 3-period problem is used to demonstrate what has been discussed (Figure 1). All roads are initially assumed to be open. Closing costs, opening costs, and periodic (fixed) road maintenance costs required when the road is open are presented in Table 1. To simplify the presentation, these costs are assumed equal for all road segments (links). The costs have already been discounted to the present from the middle of the period at which they occur. Nodes where traffic enters the network and the period of entry are presented in Table 2. All traffic exits the network at Node 10. The discounted transport costs to transport one truckload over each segment during each period are presented in Table 3. In this example, there is no difference in cost for a truck to go in either direction over a road segment. Maintenance costs proportional to traffic have been included in the transport rate.

The equations corresponding to Equations 1–6 were formulated and the problem solved using the linear programming software LINDO (*1*). The optimal value of the objective function was $273,580. The open-road segments for each time period using the optimal strategy are shown in Figure 2. The road segment from Node 1 to Node 2 is left open in Time Period 2, whereas the road segment from Node 4 to Node 5 is closed. If road opening and closing costs had not been considered simultaneously with the choice of transport routes, the solution could have differed. For example, if road opening and closing costs are ignored, the problem re-solved, and then closing and opening costs are manually added as indicated by the solution of the revised problem, the solution value is

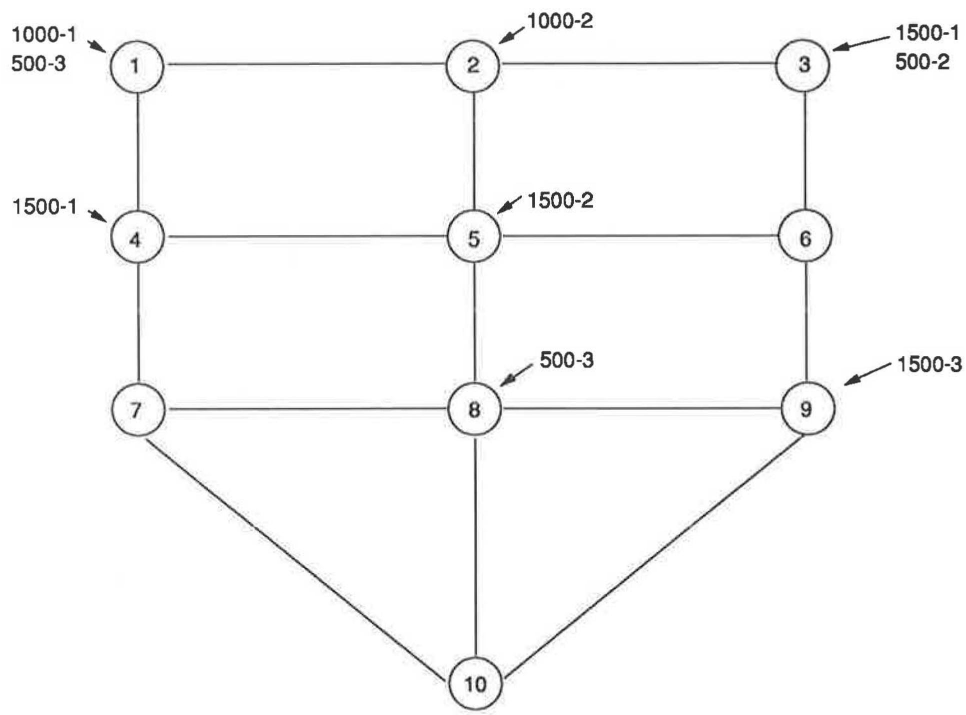

FIGURE 1 Existing road network—the first number indicates the number of truck loads entering the network; the second number is the time period.

TABLE 1 FIXED COSTS FOR OPENING, CLOSING, AND MAINTAINING ROAD LINKS

Period	Link	Opening Cost $	Closing Cost $	Fixed Main. Cost $
1	each	9000	2000	7500
2	each	7397	1644	6164
3	each	6080	1351	5067

TABLE 2 TRUCK LOADS ENTERING ROAD NETWORK IN EACH TIME PERIOD

Truck Loads	Node	Period
1000	1	1
1500	4	1
1500	3	1
1000	2	2
500	3	2
1500	5	2
500	8	3
1500	9	3
500	1	3

$310,161, for an increase of approximately 13 percent. The increase of $36,581 is caused by solving the transportation problem without simultaneous consideration of opening and closing costs. Naturally, the solution is then to minimize transport costs. The open road segments for the revised problem are shown in Figure 3.

PRACTICAL CONSIDERATIONS

There are several considerations an organization will have to weigh before using this technique to solve road closure problems. As with any numerical technique, good cost data are required. Computing facilities are also a consideration. Computing time goes up rapidly when mixed-integer problems exceed 50 integer variables. Depending on the complexity of the problem and the speed of the computer, computing time may be prohibitive. For problems larger than 50 integer variables, at least a 25-Mhz 80386 or equivalent microcomputer with a math coprocessor is recommended.

Several techniques can be used to reduce solution time:

1. Reduce the number of integer variables. Each 0,1 integer variable can double the possible number of solutions, therefore doubling the solution time required. However, not all road segments need to be integer variables. For example, if a road segment from Node 1 to Node 2 (Link 1–2) in Period 3 will be open regardless of other road management decisions, then add the constraint equation $B_{123} = 1$ and do not declare B_{123} to be an integer variable. Record all road segments that will definitely be open or closed in this manner.
2. Prepare a tight formulation. MIP solution time can be reduced by preparing a tight formulation. Integer program-

TABLE 3 DISCOUNTED SUM OF TRANSPORT PLUS VARIABLE MAINTENANCE COSTS PER TRUCK LOAD PER LINK

Link	Period 1 $/Load/Link	Period 2 $/Load/Link	Period 3 $/Load/Link
1-2	8.00	6.58	5.40
1-4	7.00	5.75	4.73
2-3	10.00	8.22	6.76
2-5	4.00	3.29	2.70
3-6	7.00	5.75	4.73
4-5	5.00	4.11	3.38
4-7	9.00	7.40	6.08
5-6	8.00	6.58	5.40
5-8	6.00	4.93	4.05
6-9	6.00	4.93	4.05
7-8	10.00	8.22	6.76
7-10	5.00	4.11	3.38
8-9	9.00	7.40	6.08
8-10	6.00	4.93	4.05
9-10	8.00	6.58	5.40

(a)

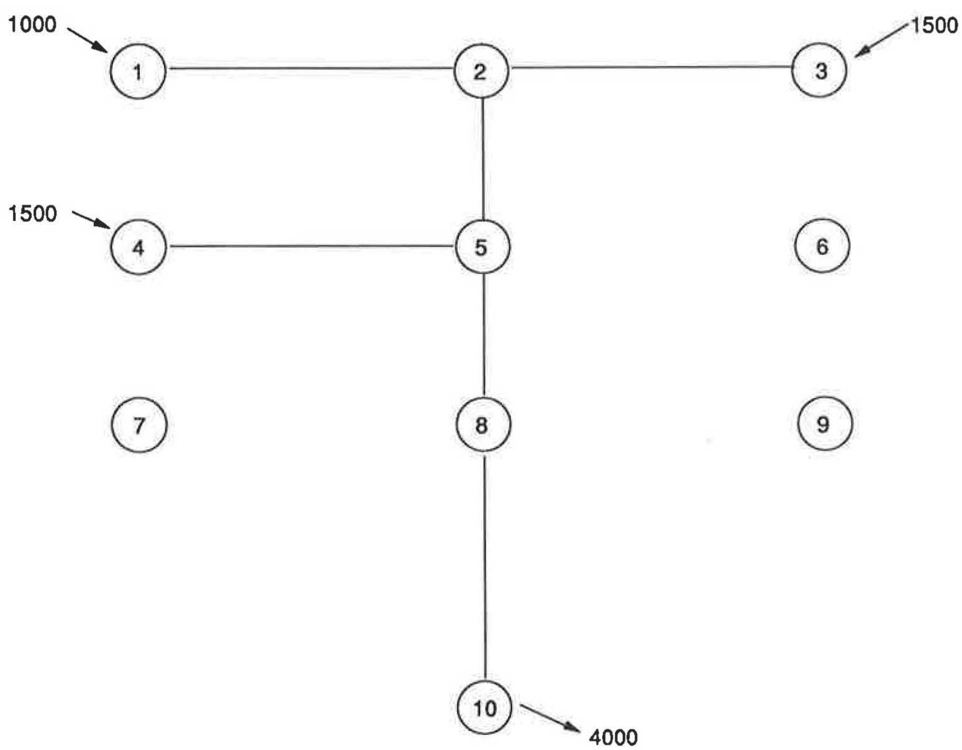

FIGURE 2 Open road segments resulting from simultaneous consideration of transport, road maintenance, and opening and closing costs for (*a*) Period 1, (*b*) Period 2, and (*c*) Period 3. Total cost of transport, road maintenance, and opening and closing road links is $273,580. **(*continued on next page*)**

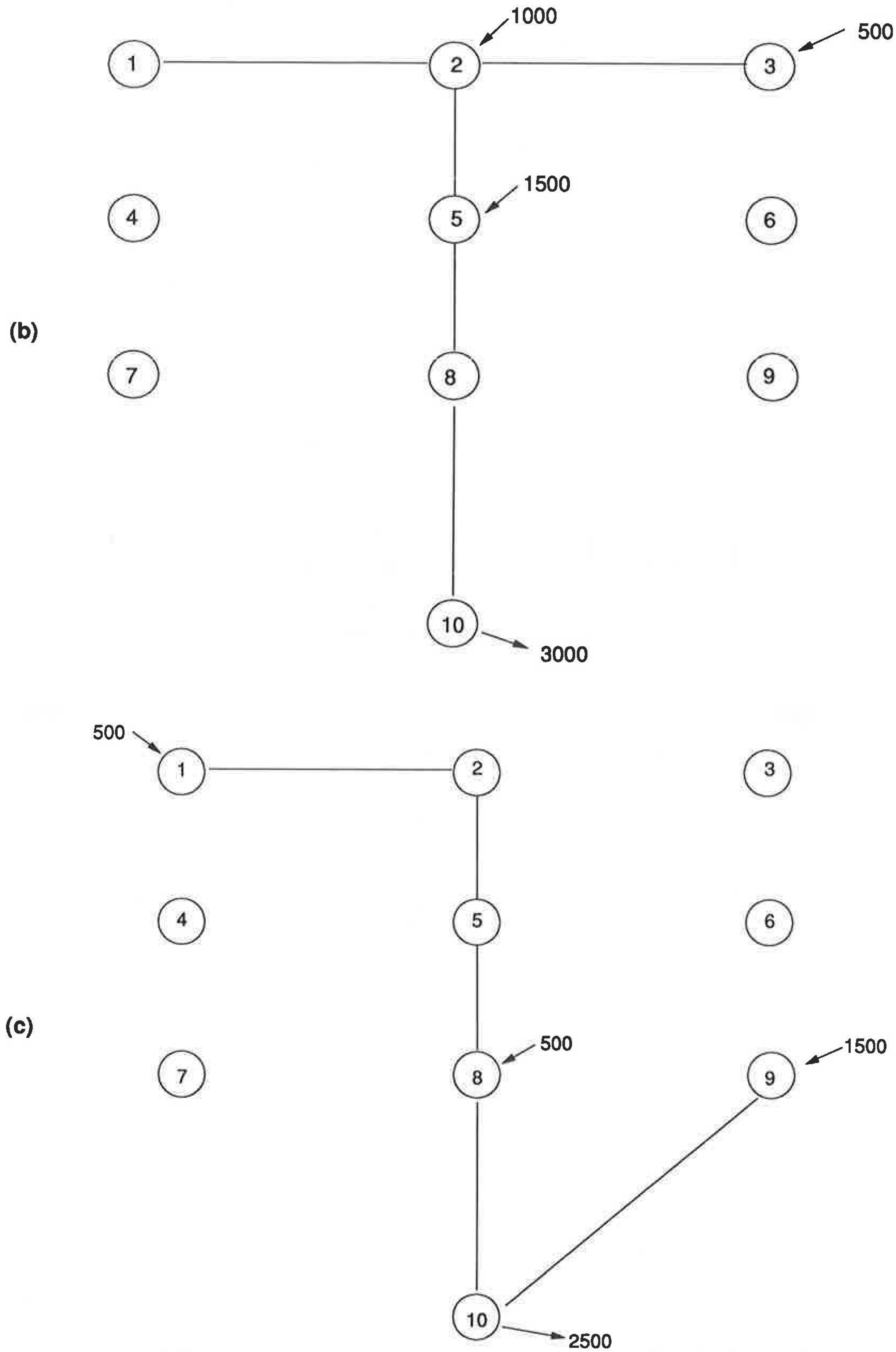

FIGURE 2 *(continued)*

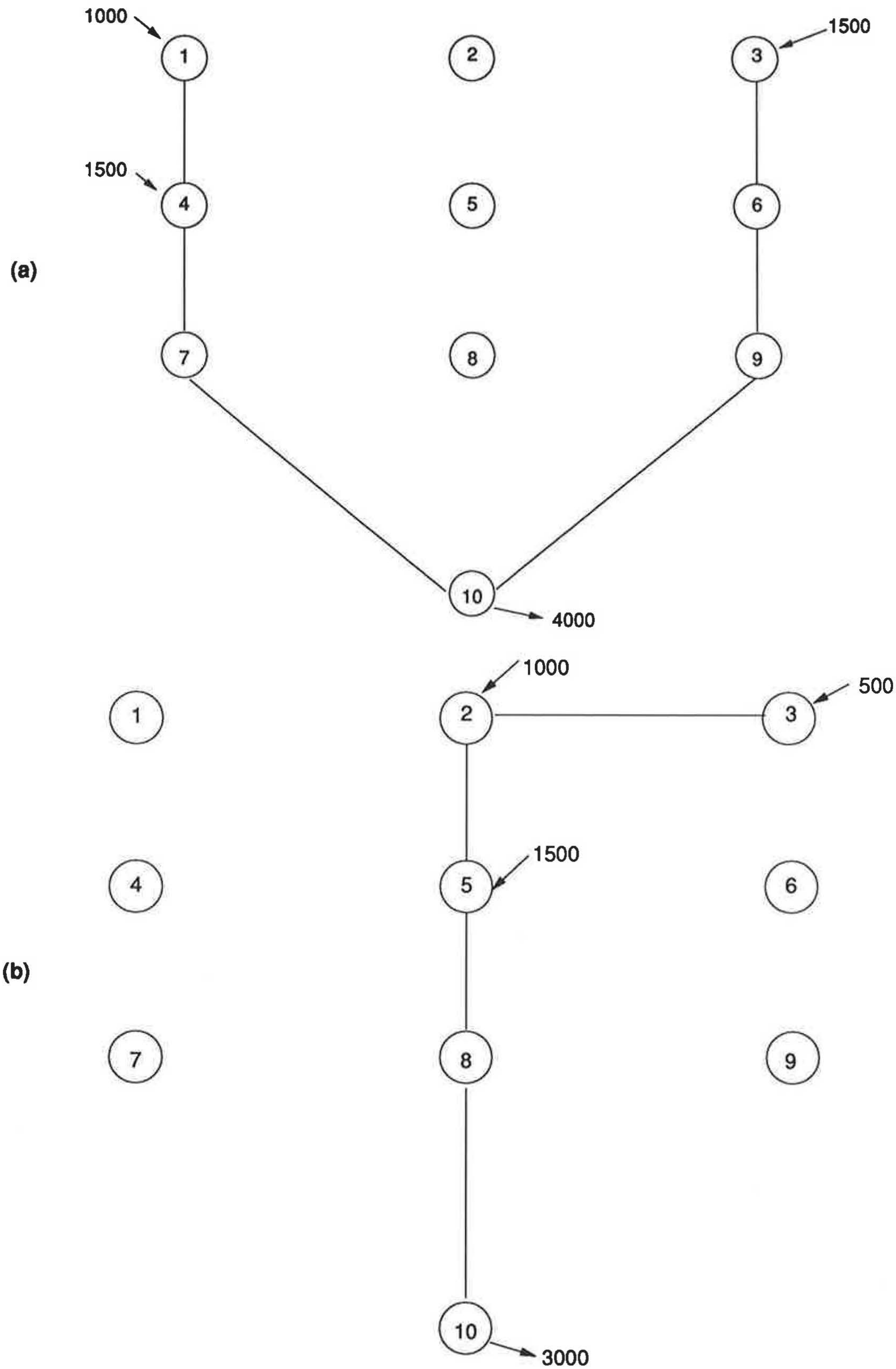

FIGURE 3 Open road segments if opening and closing costs were not considered simultaneously with transport and road maintenance costs for (*a*) Period 1, (*b*) Period 2, and (*c*) Period 3. Total cost of transport, road maintenance, and opening and closing road links is $310,161, a 13 percent increase over the optimum solution. **(*continued on next page*)**

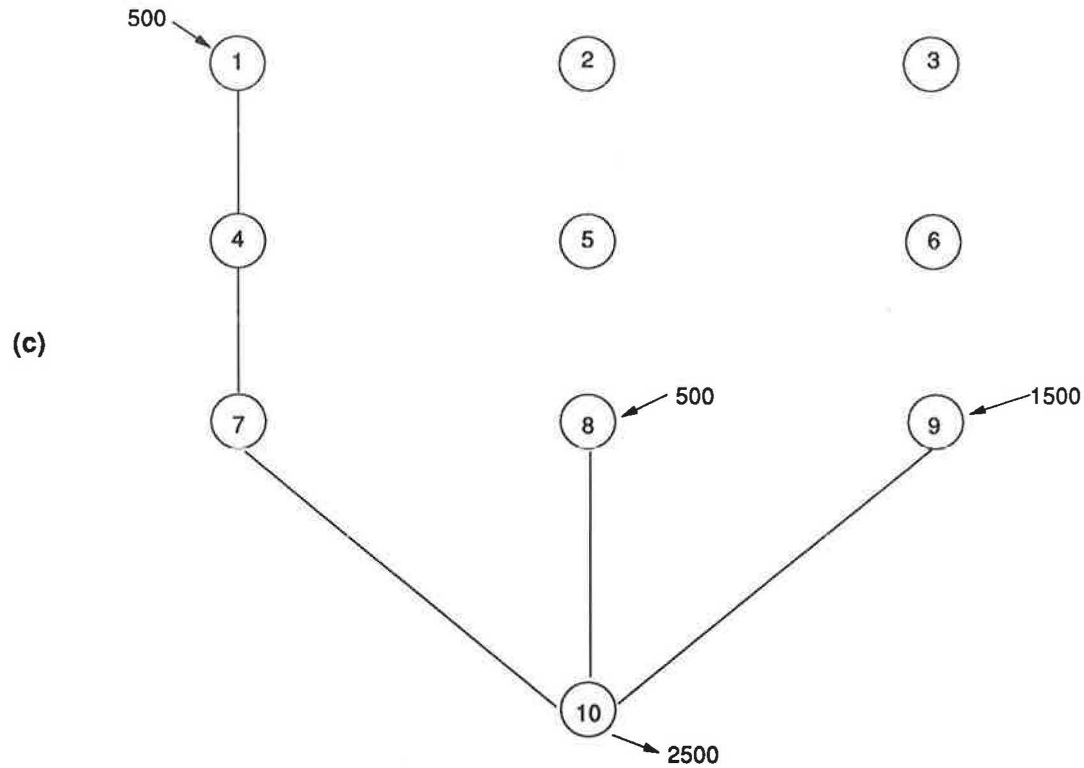

FIGURE 3 *(continued)*

ming packages solve MIPs by first solving the problem as a linear problem (LP), then by iterating the integer variables to solve the MIP. Without a discussion about integer programming algorithms, it suffices that if the initial LP solution is close to the final MIP solution, the formulation is said to be tight. The formulation can be tightened by using a logical large number M in Constraint Equation 3. A number that is too small will make the formulation infeasible when the appropriate B_{ijt} variables are set to 1. However, a number that is extremely large will allow the B_{ijt} variables to take on very small values during the LP solution. This causes the LP solution to be further from the MIP solution, creating a loose formulation and causing a longer solution time. The smallest possible number to use and still ensure feasibility is to set M equal to the total volume entering the road system during that period.

3. Place bounds on the solution. The number of possible solutions to an MIP formulation is 2 to the Nth power, where N is the number of integer variables. To avoid testing all possible solutions in full, an integer programming algorithm using "branch and bound" techniques will examine a solution set until it is clear this set, or branch, is not the optimal one. When the algorithm reaches this point, it will pursue a different branch. Usually, a bound can be placed on the solution value so the MIP will not be pursued beyond this value. A reasonably good bound can save considerable computing time. Three methods for determining good bounds are (a) past experience; (b) run the program, look at the first feasible solution, estimate how much could be saved from this solution, then rerun the program with a revised bound; and (c) use a heuristic procedure to find a good feasible solution and use this as the lower bound.

One final consideration is the type and size of road system being analyzed. This formulation assumes the road system in question is large enough that optimum management strategies are not obvious. Some systems managed by small landowners may have few alternatives where such a complex formulation is not required. Other road systems may have characteristics that do not fit in well with this type of formulation. An example would be extremely broken ownership in which the landowner may not have control over opening and closing the majority of roads.

CONCLUSION

The preceding formulation is one technique to create management strategies for intermittent road use. It may be modified to account for other operational characteristics such as capacity constraints. For applications in which more than 50 integer variables are involved, a 25-Mhz 80386 microcomputer or equivalent with math coprocessor is recommended. All attempts should be made to reduce the number of integer variables. Remember that each integer variable can double the number of possible solutions and double the computing time. Having a tight formulation and placing reasonable bounds on the solution can also reduce computing time considerably. Large areas should be divided into logical road system boundaries where no more than 500 integer variables exist. As faster computers become available, larger problems can be solved with the formulation presented here.

REFERENCE

1. L. Schrage. *Linear, Integer, and Quadratic Programming with LINDO.* Scientific Press, Palo Alto, Calif., 1984, 274 pp.