Comparison of Dynamic and Static Backcalculation Moduli for Three-Layer Pavements

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Deflection data collected from the falling weight deflectometer (FWD) have mostly been analyzed by using the static layered elastic analysis method. Analysis might be improved by including dynamic effects, such as inertia, damping, and resonance. Results from a finite element backcalculation program developed to account for these factors are discussed. The program can perform both static and dynamic backcalculation analyses. Dashpots or dampers are installed at the boundary nodes (bottom and lateral) to simulate half-space conditions, thus avoiding the need to specify a rigid base (say at 20 ft) at the bottom of the subgrade. Such dashpots absorb propagating waves in the dynamic analysis, thus preventing wave reflection off the rigid boundary. Backcalculation results computed from two existing methods and the proposed method using FWD data obtained at four sites in Nevada are compared. The results indicate that the moduli of the asphalt layers are not affected by the type of analysis (static or dynamic) for any of the sites. Lower base and higher subgrade moduli were consistently computed in the dynamic analysis compared with the static analysis.

Backcalculation of layer moduli from dynamic deflection measurements is becoming an accepted means of estimating in situ material properties (*1*). This is commonly done by matching deflections measured under a known dynamic load with theoretical deflections generated in an analytical model of the pavement by varying the elastic moduli of the layers. Most backcalculation procedures use linear elastic layered models. Though such models are simple and useful, they have limitations. One major limitation is that the applied load is assumed to be static. This is not the case in modern deflection testing where an impulse load is used.

According to previous research (*2,3*), dynamic instead of static analysis should be performed on nondestructive testing data obtained through dynamic loading. Pavement deflection under a static load is different from that under a dynamic or impulse load because of viscoelastic pavement properties and dynamic effects such as inertia, damping, and resonance. Dynamic analysis would therefore provide a more accurate estimate of the pavement modulus from backcalculation.

PAST RESEARCH USING DYNAMIC ANALYSIS

One of the more recent and well-known research efforts using dynamic analysis was performed by Mamlouk (*2,4*). The elastodynamic method was used to calculate the deflection of pavement subjected to cyclic (road rater) loading. That model has a rock or rigid layer at some depth (about 15 ft) in the subgrade. Roesset and Shao (*5*) determined that the rigid layer has to be located at least 70 ft from the surface to prevent boundary effects for a dynamic analysis. Mamlouk's research also included material and radiation damping in its elastodynamic analysis. (The characteristics of radiation damping will be discussed later.) The viscoelastic theory determines the change in strain of a material under load with time.

Roesset and Shao (*5*) also studied pavement response using dynamic analysis and determined that calculated deflections are different from those in static analysis. However, theirs was not a backcalculation model and did not provide modulus values for the pavement layers.

OBJECTIVE OF RESEARCH

The major objective of this paper is to show the difference in backcalculated moduli between dynamic and static analyses using falling weight deflectometer (FWD) data. A dynamic backcalculation program that can closely simulate the FWD impulse load and perform analysis for a three-layer pavement has been developed. The program, called FEDPAN, uses the finite element method and can simulate the behavior of the pavement under the FWD load. It includes both the effect of pavement inertia and damping in the dynamic analysis and can perform static backcalculation analysis. One of the major advantages of a finite element–based analysis is that the nonlinear material property characterization can be easily incorporated in the study. Such a study is currently under way.

APPROACH TO PROGRAM DEVELOPMENT

The Structural Analysis Program IV (SAP IV) (*6*), a widely used and accepted linear finite element program, was used as the base program in FEDPAN to calculate theoretical deflections. SAP IV can perform axisymmetric and other types of analyses. The axisymmetric analysis was chosen for FEDPAN because of the symmetrical deflection basin due to the FWD load. The response of an axisymmetric problem can be obtained by analyzing only a radial section. This type of analysis also greatly reduces memory requirements and computing time. To further reduce the mesh size, dashpots or dampers

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were added to the model to absorb radiating waves created by the FWD loading.

The purpose of installing dashpots at boundary nodes is to reduce the size of the finite element domain that should be discretized. A smaller domain reduces the number of elements, which in turn reduces the memory storage and computation time required. The dashpots located at the boundary absorb the radiating waves [P waves (compression waves) and S waves (shear waves)] caused by the impulse loading. This viscous boundary simulates the presence of similar pavement layer materials beyond the boundary and gives the effect of a continuum layer for the waves to propagate away from the source, even though the actual discretized domain is small. Without the dashpots, the waves reflect off the rigid boundary and back into the domain. The boundary dashpot characteristics depend on wave velocities, material properties, and the area of the boundary elements. The relationships are as follows (*7*):

$$\text{Dashpot forces} = \text{stress (compression or shear)} * \text{area of element}$$

Here,

$$\text{stress} = \rho V_{c/s} \dot{w}_{n/t} \tag{1}$$

where

ρ = mass density of element,
$V_{c/s}$ = compression or shear wave velocity,
$\dot{w}_{n/t}$ = normal or tangential wave velocity,
$V_c = (G/\rho)^{1/2}$,
$V_s = (1/S)V_c$,
G = shear modulus,
$S = [(1 - 2\nu)/2(1 - \nu)]^{1/2}$, and
ν = Poisson's ratio.

The dashpots are used to simulate a continuum condition, but they can be removed to simulate the presence of a rigid or rock layer.

The dynamic analysis is performed in the time domain incrementally. This method is suitable for impact loading problems in which the time of loading is short. In the dynamic analysis with dashpots, the equation of motion is

$$[M]\ddot{x} + ([C] + \text{dashpots})\dot{x} + [K]x = F(t) \tag{2}$$

where

$[M]$ = mass matrix,
$\ddot{x}$ = acceleration,
$[C]$ = Rayleigh damping matrix,
$\dot{x}$ = velocity,
$[K]$ = stiffness matrix,
x = displacement, and
$F(t)$ = applied force as a function of time.

The Rayleigh damping matrix used in Equation 2 will be explained in the next section. The closure algorithm used in this program is the CHEVDEF algorithm (*8*), which is used in many other backcalculation programs.

CHARACTERISTICS OF PHYSICAL MODEL

In this study, the finite element pavement mesh is made up of four node rectangular elements, as shown in Figure 1 (top). The nodes at the boundaries of the mesh are either fixed (hinged) or on rollers. None of the fixed nodes at the bottom boundary can move, either vertically or laterally, which simulates a rigid boundary. The right and the left boundary nodes are on rollers and can move vertically but not laterally. Figure 1 (bottom) indicates that dashpots are installed at these boundary nodes except for the left boundary, because of the axisymmetric loading. For clarity, only the dashpots that absorb compressional waves are shown in the figure. A boundary with dashpots installed at the nodes will be referred to as a viscous boundary, and fixed nodes at the bottom nodes, which simulate the presence of a rock or rigid layer, will be referred to as a rigid boundary. The zone of influence at the surface caused by a load is typically 10 to 12 times the radius of the footing (the radius of the FWD loading plate is approximately 6 in.), and, therefore, the right boundary of the mesh was located approximately 15 ft (30 times the radius) to the right of the load. Deflections were recorded on the surface of the pavement at 0.0, 7.9, 11.8, 23.6, and 39.4 in. from the center of the load, which correspond to the locations of geophones in the field test.

Though FEDPAN can estimate moduli of a three-layer pavement, the pavement layers can be subdivided into thinner layers, up to a total of eight. Memory allocation in the program can be increased to accommodate more sublayering, if required.

The FWD impulse loading curve is simulated in the model by using a Haversine equation. With this equation, the nodes

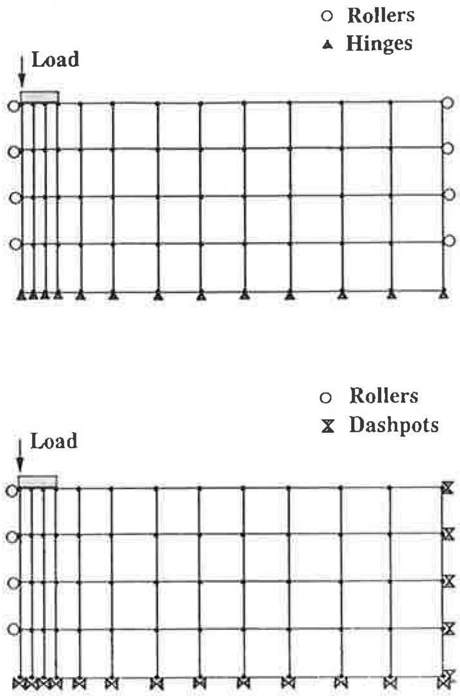

FIGURE 1 Finite element method representation: *top*, four node elements; *bottom*, dashpots installed at boundary nodes.

under the FWD loading plate will experience a change in load with time. The FWD load is spread over the plate, and the nodal forces are computed on the assumption of uniform stress distribution under the plate. This type of stress distribution is commonly used in foundation engineering. The duration of the simulated impulse loading is 30 msec, with peak load at 15 msec. The response of the pavement is observed for 90 msec. Any peak in deflections during the 90-msec period is recorded. This ensures that delays in peak deflections at the nodes due to damping or other dynamic effects of the pavement are accommodated. The material damping used in the model is characterized by the Rayleigh damping equation, which gives the damping matrix as a sum of mass and stiffness proportional components:

$$[C] = \alpha[M] + \beta[K] \tag{3}$$

where

$[C]$ = Rayleigh damping matrix,
α = mass proportional coefficient, and
β = stiffness proportional coefficient.

In FEDPAN, the mass proportional coefficient is set to zero. When α is zero, undesirable high-frequency components of the response will be filtered out [see Figure 2 (*9*)].

VERIFICATION OF COMPUTER PROGRAM—FEDPAN

To test the FEDPAN program, pavement moduli values obtained after a few iterations with FEDPAN were used in the original SAP IV. The programs produced the same deflection results, as expected. The tests were performed for both the static and dynamic analyses. The computed static stress and deformation results were checked against classical solutions available in the literature (*10*), and the responses observed were similar.

The values of the dashpot coefficients generated by the program were also verified by hand calculations. To check whether the dashpots were working correctly, the dashpot coefficients of the bottom boundary were gradually increased. The surface deflections of the pavement model decreased and approached the deflections obtained using a rigid layer, as shown in Figure 3. This indicates that the dashpots were functioning correctly. Researchers also concluded that these types of dashpots are good absorbers for both harmonic and nonharmonic waves (*6*).

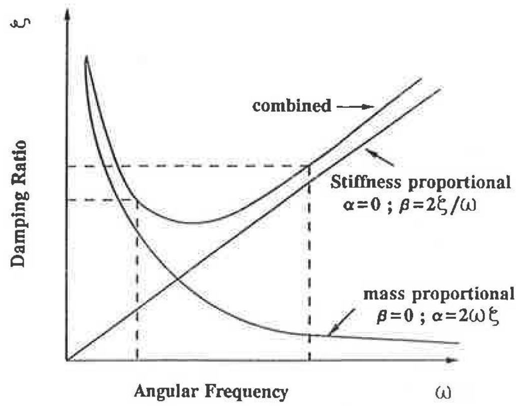

FIGURE 2 Rayleigh damping (*9*).

FEDAT, an input data generator program, was written to help create the data file for FEDPAN. FEDAT generates the nodal coordinates and other data required in the backcalculation. It is interactive and prompts for layer thicknesses and number of sublayers in each layer. It also prompts for the type of analysis (static or dynamic) and generates dashpots for the dynamic analysis if so desired. The β value used in material damping and other material properties are specified by the user. Stress values in any element or in rows of elements can be determined if they are requested by the user. Hundreds of input items required to set up any finite element–based analysis are therefore reduced to a minimum by FEDAT. This substantially reduces extensive and, often, time-consuming data preparation.

LIMITATIONS AND ASSUMPTIONS

In FEDPAN, the pavement materials are assumed to be homogeneous and isotropic in each layer. FEDPAN performs only a linear analysis, even though behavior in the unbound pavement materials may be nonlinear. An effort to incorporate nonlinear analysis into FEDPAN is currently being made. The material damping ratio (ζ) in this study was assumed to be 5 percent, a value commonly used in pavement response analysis (*2*). This may be achieved by selecting the β value used in Rayleigh damping by the following equation:

$$\beta = 2\zeta/\omega_n \tag{4}$$

where

ζ = critical damping ratio,
ω_n = natural angular frequency = $2\pi f$, and
f = fundamental frequency of the pavement (Hz).

The fundamental frequency of the pavement was assumed to be 14 Hz. The fundamental frequency of the pavement

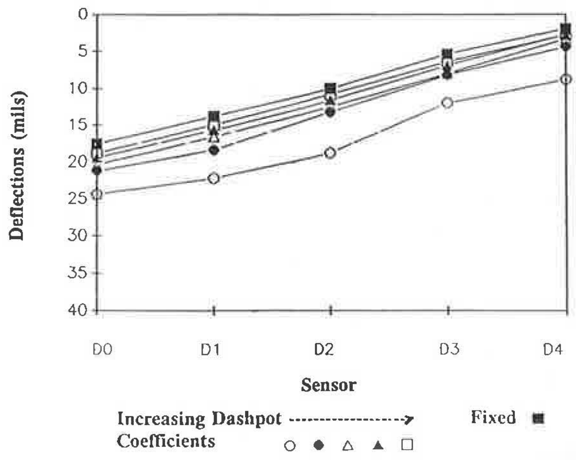

FIGURE 3 Results from dashpot tests.

structure is a function of structural mass and stiffness. Researchers showed that typical pavement sections with 20 ft of subgrade have fundamental frequencies in the vicinity of 14 Hz (*2,3*). On the other hand, if the fundamental frequencies for the test sections are known, they can be used in Equation 4.

Other assumptions used in this study, such as unit weights and initial guess moduli (seed moduli), are given in Table 1. The procedure adopted uses an overall damping ratio of 5 percent for the pavement section. On the other hand, if evidence exists that damping in the asphalt layer is substantially different from that in the bottom layers, the damping matrix can be constructed element by element using the steps outlined by Idriss et al. (*11*). They generated a damping matrix for seismic soil response studies that takes into account the variable damping in soil elements.

Observations of trial tests showed that the CHEVDEF (*8*) closure algorithm used in this program is not very sensitive to the seed moduli or the maximum and minimum moduli range, as long as the moduli calculated fall within the range. Lee (*12*) reported that the backcalculation program EVERCALC (layered elastic program), which he developed, showed the same lack of sensitivity. The upper limit of the layer moduli values has been selected to be quite high, but this does not affect the computed results. The algorithm will converge as long as the field data are good. If the percent summation of the absolute differences between the calculated and the measured deflections is less than or equal to 6 percent (tolerance), convergence in the backcalculation procedure is achieved. When this happens, the assumed moduli values are considered the corresponding pavement layer modulus.

TEST SITE DESCRIPTION

The sites used in the study are located in Nevada. FWD tests were performed by the Nevada Department of Transportation (NDOT) using a Dynatest 8000 FWD. Initially, five sites were chosen for the study, but one site was dropped when layer thickness data from the construction record were found to be questionable. The remaining four were Sites 11, 12, 16, and 31. The thickness profiles for these sites are shown in Table 2. Deflections used in the backcalculation were obtained on the same marked spots during different seasons.

RESULTS AND ANALYSIS

The results obtained are subject to the limitations and assumptions described earlier. The results also do not reflect the stress sensitivity of the unbound materials. Three backcalculation analyses were performed using FEDPAN: static analysis with rigid boundary, dynamic analysis with rigid boundary, and dynamic analysis with viscous boundary.

Parametric Tests

Two parametric tests were conducted using FEDPAN with data from Site 12. The first was to determine the thickness of elements to be used in the subgrade. The moduli values calculated for four equal sublayers in the 240-in. subgrade were compared with the moduli values calculated for six equal sublayers using the same 240-in. subgrade for all three methods of analysis. The results are presented as ratios of moduli

TABLE 1 MATERIAL PROPERTIES USED IN STUDY

	Layer 1	Layer 2	Layer 3
Maximum (psi)	5,000,000	500,000	250,000
Minimum (psi)	100,000	2,000	2,000
Poisson's Ratio	0.35	0.35	0.40
Seed Moduli (psi)	500,000	25,000	20,000
Unit Weight (lb/ft^3)	144	125	115

TABLE 2 PAVEMENT PROFILES FOR TEST SITES

Site	Thickness (inches)			No. of Layers		
	AC	Base	Subgrade	AC	Base	Subgrade
11	4.25	11.00	240.00	1	1	4
12	8.25	16.00	240.00	1	1	4
16	9.75	11.00	240.00	1	1	4
31	16.25	13.00	240.00	2	1	4

in Figure 4. The ratios were obtained by dividing the pavement layer moduli by the corresponding moduli computed with the six-layer characterization for the subgrade.

Figure 4 indicates only a small difference in moduli given by the static analysis in both the four- and six-layer sublayering of the subgrade, and the difference in moduli using dynamic analysis with dashpots was even smaller. The dynamic analysis with rigid bottom yielded the greatest difference, up to 12 percent in the base.

This test showed that the 240 in. of subgrade can be divided into four sublayers and still provide a good estimate compared with division into six sublayers, which requires more computation time.

The second parametric test determined the influence of the location of the bottom boundary. Moduli values computed for all three layers by FEDPAN for subgrade thicknesses (D_s's) of 240 and 120 in. (both with four equal subgrade sublayers) were compared. Figure 5 shows the ratio of moduli values normalized using the moduli values computed with 120 in. of subgrade. Figure 5 indicates that the modulus of asphalt concrete (AC) is not affected by either the location of the rigid bottom or the type of analysis. However, the moduli values of base and subgrade were substantially affected.

The static analysis with rigid boundary at 120 in. in the subgrade indicated a stronger base and a weaker subgrade. The dynamic analysis with rigid boundary showed the same trend. On the other hand, results from the dynamic analysis with dashpots (viscous boundary) indicated the opposite trend. After the parametric tests, the location of the bottom boundary was set at 240 in. in the subgrade with four equal sublayers in all subsequent analyses.

Backcalculation of Field Test Results

The backcalculation was performed for the static analysis, dynamic analysis with viscous boundary, and dynamic analysis with rigid boundary using the FWD data collected by NDOT for the selected sites. Results obtained were averaged for Sites 11 (two seasons), 12 (four seasons), 16 (three seasons), and 31 (two seasons). Comparisons between the different analyses are shown in Figures 6 through 10. The ratio method of comparison was used in order to evaluate the effects of the different analyses without addressing seasonal effects, even though the strength of the pavement layers varies with the season. For all three analyses, an examination of the backcalculated moduli results obtained for different seasons indicated that

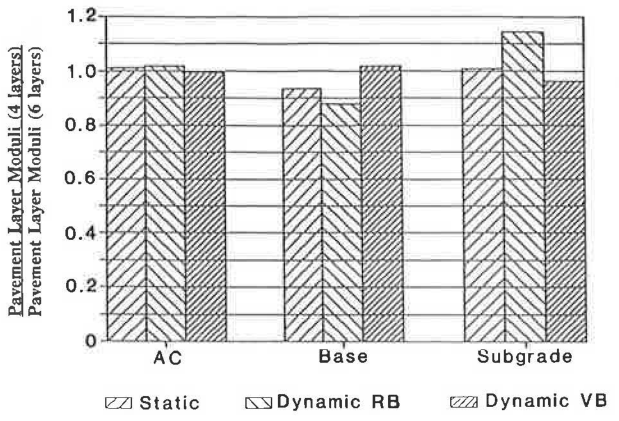

FIGURE 4 Sensitivity of backcalculated moduli to sublayering of subgrade (RB indicates rigid boundary; VB indicates viscous boundary).

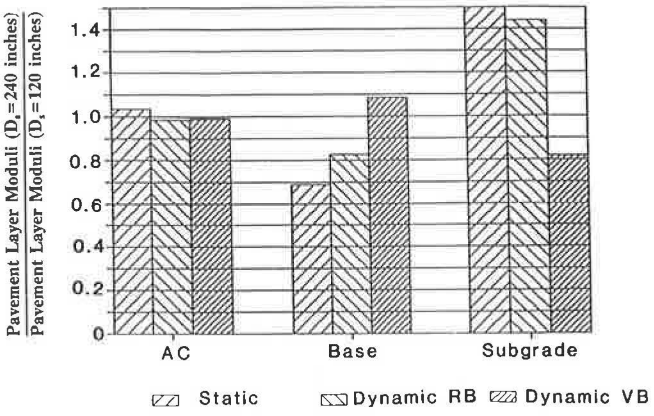

FIGURE 5 Sensitivity of backcalculated moduli to subgrade thickness (RB indicates rigid boundary; VB indicates viscous boundary).

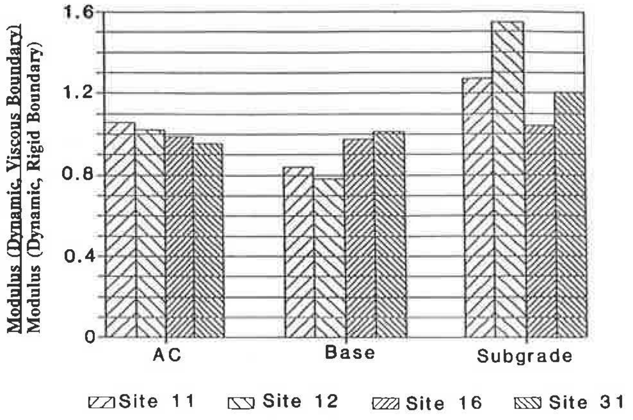

FIGURE 6 Ratio of moduli for dynamic viscous boundary to moduli for dynamic rigid boundary.

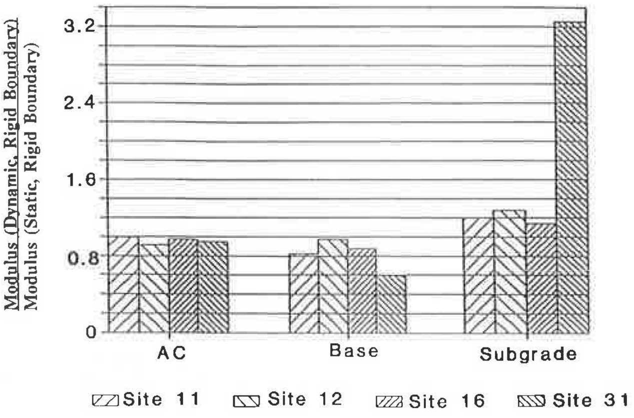

FIGURE 7 Ratio of moduli for dynamic rigid boundary to moduli for static analysis using FEDPAN.

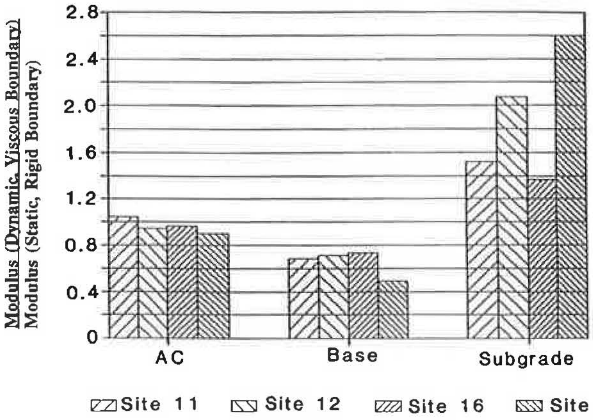

FIGURE 8 Ratio of moduli for dynamic viscous boundary to moduli for static analysis using FEDPAN.

the same trend existed, so the results were averaged and reported. The average percent difference for the backcalculated results (last iteration) and the average number of iterations used to reach convergence are given in Table 3.

Comparison of Results for Dynamic Viscous Boundary and Dynamic Rigid Boundary

Figure 6 compares dynamic backcalculated moduli values for viscous and rigid boundary models. The figure indicates that the AC moduli were not affected by the inclusion of dashpots at the boundaries at any of the sites. In particular, the asphalt moduli at Sites 16 and 31 (with thick AC layers of 9.75 and 16.2 in., respectively) were not significantly affected by the presence of dashpots. But for base and subgrade layers, the backcalculated moduli values were substantially affected by the backcalculation procedure, except at Site 16.

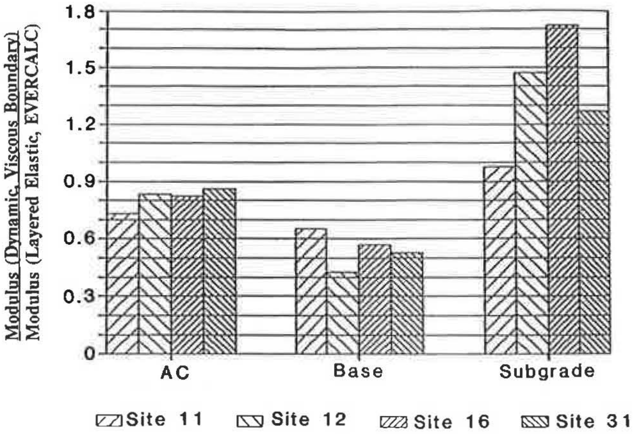

FIGURE 9 Ratio of moduli for FEDPAN dynamic viscous boundary to moduli for EVERCALC.

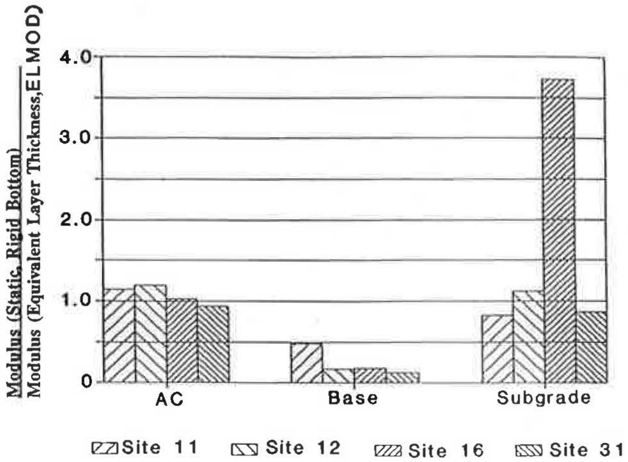

FIGURE 10 Ratio of moduli for FEDPAN (static) to moduli for ELMOD.

Comparison of Dynamic Results with Static Results

Comparisons of results obtained using FEDPAN for the two dynamic analyses and the static analysis are presented in Figures 7 and 8. Again, the static and dynamic analyses yield very similar results in the estimate of AC modulus. However, the subgrade and base moduli values can be substantially affected by the type of analysis. These results indicate that the static backcalculation procedure, which neglects inertia and other dynamic effects, may lead to an underestimation of the subgrade moduli. The base moduli, on the other hand, may be overestimated by the static analysis.

TABLE 3 AVERAGE CONVERGENCE RESULTS FOR 11 DEFLECTION SITES

	Iteration	Difference (%)
EVERCALC	3.0	3.47
FEDPAN		
Static	3.0	4.17
Dynamic Rigid Boundary	3.3	4.12
Dynamic Viscous Boundary	3.2	3.26

Comparison of FEDPAN Results with EVERCALC and ELMOD Results

Figure 9 compares the results obtained from dynamic analysis with viscous boundary and EVERCALC (*12*). Moduli ratios of FEDPAN (static) to ELMOD (which uses the Equivalent Thickness Method) are shown Figure 10. In general, the two figures indicate that the backcalculated moduli are quite different. The base modulus is overestimated by both EVERCALC and ELMOD when compared with FEDPAN, and the subgrade modulus estimates vary. The main reasons are the differences in the methods of analysis and the assumptions used by the backcalculation programs. In the case of FEDPAN, when the rigid bottom boundary option was used, the rigid bottom was located at a depth of 20 ft into the subgrade, whereas EVERCALC can consider only a semi-infinite subgrade.

CONCLUSIONS AND RECOMMENDATIONS

This paper describes a dynamic model (FEDPAN) for backcalculation of FWD deflection data to obtain pavement layer moduli values. Dashpots introduced at the bottom and lateral right boundaries allow the size of the finite element domain to be reduced, providing for efficient analysis. The dashpots also reduced the effects of reflection waves by simulating half-space conditions without having to use a rigid boundary at a distance from the load (say at 20 ft in the subgrade). The conclusions to be presented are subject to the limitations and assumptions discussed throughout this paper. The conclusions are as follows:

1. If bedrock (or a stiff layer) exists near the surface, the bottom boundary of the mesh can be located there when using FEDPAN. If the stiff layer does not exist, the rigid bottom boundary may be located at some distance (at least 20 ft in the subgrade) with dashpots connected to the nodes. When dashpots are provided, the waves that reach the viscous boundary are absorbed, and wave reflection does not take place. This case represents a semi-infinite subgrade.
2. The study indicates that four sublayers are sufficient in the 20 ft of subgrade when a static analysis or a dynamic analysis with dashpots is used. A subgrade with six sublayers can improve the accuracy of the results, especially in the dynamic analysis when a rigid boundary at approximately 20 ft is required.
3. AC layer moduli were not affected by either the static or the dynamic analyses. The sites considered represent different combinations of asphalt and base layer thicknesses. The thickness of the asphalt layers used in this study varied from 4.25 to 16.2 in.
4. The dynamic analysis produced higher subgrade moduli and lower base moduli than the static analysis. For pavements with thick asphalt layers, the dynamic analysis produced much higher subgrade moduli. However, because of the thick asphalt layers at Sites 16 and 31, further research is recommended to determine whether the reading from the outermost sensor can adequately represent the stress experienced by the subgrade only, and not a combination of stresses from other layers.
5. When the dynamic backcalculation analysis with viscous boundary was performed, the computed subgrade moduli values were higher and the base moduli values were lower than those computed by the dynamic analysis with rigid boundary or the static analysis.
6. The results obtained from two static analysis programs, one with FEDPAN and another with EVERCALC, confirmed that the location of a rigid boundary is extremely important. Comparison of the two analysis programs ELMOD and EVERCALC indicated that different programs and methods of analysis can produce quite different results.
7. Limitations of the proposed method include the inability to model nonlinear soil properties and lateral variation in moduli values, which should be overcome in future models.
8. Comparisons of the backcalculated moduli from FEDPAN with the pavement moduli obtained from laboratory tests should also be made.

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