Advanced Backcalculation Using a Nonlinear Least Squares Optimization Technique

N. SIVANESWARAN, STEVEN L. KRAMER, AND JOE P. MAHONEY

In recent years the analysis of pavement structures has relied increasingly on characterizing material properties (such as resilient modulus) by use of nondestructive deflection testing and backcalculation procedures. An important element common to all backcalculation procedures—the technique used to achieve a "convergence" of the measured and calculated deflection basins—will be described. A convergence method based on the use of nonlinear least squares is described. The method was adapted to a layered elastic program (CHEVRON N-layer). This convergence approach improves moduli estimates over prior procedures; however, the most important element is the ability to efficiently backcalculate not only layer moduli but also layer thicknesses. This ability is illustrated by using hypothetical two- and three-layer pavement sections and by using real data for a three-layer section.

In recent years the design and rehabilitation of pavement structures has relied increasingly on accurate characterization of the mechanical properties of the materials that compose them. A number of nondestructive testing techniques have been developed to evaluate some of these mechanical properties. The falling weight deflectometer (FWD) method has seen widespread use, in large part because of its ability to impose dynamic loading on a pavement structure similar to the loading imposed by truck traffic.

Conventional interpretation of the results of an FWD test generally involves backcalculation of estimates of the elastic moduli of the various layers of the pavement section. Backcalculation procedures seek to define a set of elastic moduli that best describe the pavement deflections observed from the FWD test in the framework of a particular pavement model. A number of pavement models and backcalculation procedures have been employed to interpret FWD tests. Some of the issues involved in accurate and reliable FWD test interpretation are discussed, and a versatile backcalculation procedure that has exhibited improved performance characteristics relative to many previously used procedures is presented. This procedure is then extended to allow backcalculation of other parameters, namely layer thicknesses, in addition to layer moduli.

In the FWD test, a transient impulse load is applied to a pavement surface by a cushioned falling weight. The response of the pavement surface is measured at a number of points at different distances from the weight. The response is generally measured by velocity transducers, with the velocity time history integrated to provide a time history of pavement deflection. The test may be repeated several times at a particular location and the results averaged to reduce random errors, or the test may be repeated with different loads to evaluate stress dependence of layer moduli.

For current methods of interpretation of the results of FWD tests, the maximum displacement at each velocity transducer is used to define a deflection basin, which is interpreted as having resulted from a statically applied load. This approach discards a great deal of potentially useful information contained in the load and displacement signals. Using a solution for a layered system of linear, elastic materials and assuming that layer thicknesses and Poisson's ratios are accurately known, the moduli of the individual layers providing the best agreement with the observed deflection basin are considered to represent the stiffness of the various materials.

BACKCALCULATION

Problems of backcalculation, sometimes referred to as parameter identification or system identification, are common in many areas of science and engineering. Basically, they involve situations where an input is transformed by some process to an output. An analytical or numerical model is used to describe the process. Usually, the input and the output are known, and the backcalculation problem becomes one of identifying the model parameters. Such methods were formalized by electrical engineers who identified model parameters for various electrical components by matching a known input signal to the component with the measured output signal.

Backcalculation of FWD Test Results

In FWD test interpretation, the input is the impulse load applied to the pavement surface by the falling weight, the output is the measured deflection basin, and the process is the mechanical transfer of the kinetic energy of the falling weight at the point of impact to the work done in deforming the pavement. The input, related to the particular FWD apparatus and weight being used, is generally well known. The output, expressed in terms of the deflection basin, is measured and therefore also known. The process, which is typically described by a layered elastic mechanical model, depends on model parameters, which include the modulus, thickness, and Poisson's ratio of each layer. For most current FWD test

Department of Civil Engineering, FX-10, University of Washington, Seattle, Wash. 98195.

interpretation procedures, the layer thicknesses and Poisson's ratios are assumed to be accurately known, leaving the layer moduli as the only unknown variables. The FWD backcalculation problem is shown schematically in Figure 1, where the layer moduli are contained in the vector **E**. The problem is to find the model parameters that best describe the known deflection basin produced by the known FWD load, using the layered elastic model.

Requirements of a Backcalculation Method

A variety of different methods have been used for backcalculation of layer moduli in the interpretation of FWD test results. In this paper, the term backcalculation will refer to the numerical process by which the layer moduli are calculated rather than the more broadly defined problem of pavement analysis. These methods have ranged from simple, largely manual methods to more sophisticated numerical methods. To be useful to practicing pavement engineers, a good backcalculation method must possess certain characteristics.

First, and most obvious, it must be accurate. Satisfactory performance of pavement overlays or other rehabilitative measures designed by a mechanistic-empirical process depends on accurate characterization of layer moduli. Consequently, a suitable backcalculation method must be able to recognize and correct even small errors in layer moduli in order to develop an accurate solution.

Second, it must converge rapidly. Under production conditions the interpretation of large numbers of FWD tests is usually required. Under other conditions it is desirable to interpret the results of an FWD test in the field immediately after its completion. In either case, important decisions often must be made quickly on the basis of the backcalculated layer moduli. It is therefore important that the backcalculation procedure allow processing of large amounts of data in the shortest possible time. However, in the future, rapid increases in computational capabilities resulting from advances in computer hardware may allow some sacrifice of computational efficiency for robustness or versatility.

Third, the backcalculation method must be robust—it must converge to a correct solution, even under difficult circumstances. Such circumstances arise from errors and uncertainty in the measured test results, pavement structures that cause the backcalculation problem to be ill-conditioned, and poor initial estimates of the layer moduli.

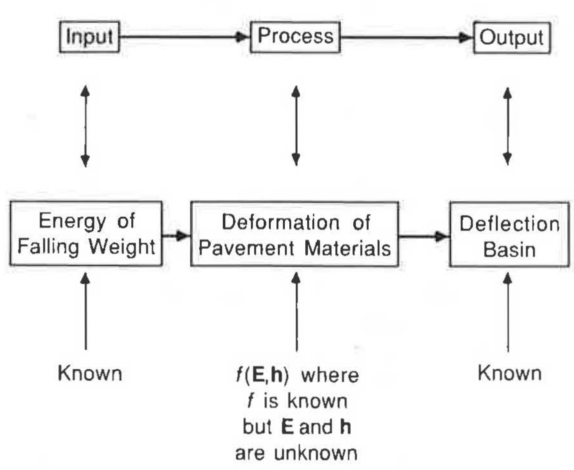

FIGURE 1 Schematic view of backcalculation process.

Fourth, the backcalculation method should be versatile. In looking toward the future, it is desirable to base FWD backcalculation on methods that can account for parameters other than layer moduli alone. Future FWD backcalculation methods may, for example, use the entire time history of motion at each sensor with dynamic modeling to backcalculate dynamic moduli and damping, as described in a frequency domain approach by Lytton (*1*). The advanced procedure described in this paper can backcalculate both layer thickness and layer moduli simultaneously and is being adapted to include other parameters as well.

Current Backcalculation Methods

A number of computer programs have been developed for analysis of FWD test results. Many of these programs have been patterned after the __ DEF series of programs (CHEVDEF, BISDEF) developed by the U.S. Army Corps of Engineers (*2*). These programs employ a gradient search technique for iteration toward the correct set of layer moduli. In the formulation used in these programs, a successive linear least squares approach is used, taking advantage of empirically linearized model parameters. The linearization of model parameters allows a system of simultaneous equations to be solved for the layer moduli at each iteration. If the parameters were truly linear, only one iteration would be required; however, the linearization is only approximate, so successive iterations are required to approach the correct solution. This approximate linearity simplifies the FWD interpretation problem to the point where satisfactory accuracy and efficiency can be obtained for many data sets by a limited optimization method.

A different approach is used by the program MODULUS (*3*) developed at Texas A&M University. Before the actual backcalculation process, MODULUS computes a series of normalized deflection basins using the BISAR program with layer moduli that cover the range of moduli anticipated in the field. The number of normalized deflection basins increases rapidly with the number of unknown parameters in the backcalculation problem. The deflection basins are stored in a data base for subsequent comparison with measured deflection basins. By using this data base, a Hooke-Jeeves pattern search algorithm, and three-point Lagrangian interpolation, a reasonable set of moduli can be attained quickly. The Hooke-Jeeves algorithm is a direct search technique that relies only on function values, neglecting first- and second-derivative information. It can handle nonlinear problems, but it often requires significantly more iterations to reach a solution than more recently developed nonlinear optimization techniques. The approach taken by MODULUS is distinctly faster than other approaches for production cases in which many deflection basins in the same pavement geometry are to be evaluated. When pavement conditions change, however, the time-consuming task of generating normalized deflection basins must be repeated.

In summary, currently available backcalculation procedures seek only to evaluate pavement layer moduli. The accuracy and efficiency of most rely on the empirically observed linear dependence of pavement deflection on the logarithm of layer modulus. These methods generally perform satisfactorily within the limited framework of conventional FWD test interpretation. In order to broaden the scope and capabilities of the FWD test, however, improved backcalculation procedures must be used.

NONLINEAR LEAST SQUARES OPTIMIZATION

A nonlinear least squares optimization approach for FWD backcalculation problems is proposed in this paper. The nonlinear least squares approach has certain advantages over current procedures for conventional FWD backcalculation, but it has many more advantages when viewed in the light of more advanced FWD backcalculation procedures. The advantages will be illustrated by using the proposed approach to backcalculate layer moduli and layer thickness at the same time.

Criterion Function

After selection of a model to represent the system and the quantities to be measured, the backcalculation problem can be expressed as an optimization problem in which the objective is to estimate a set of model parameters that best describes the measured quantities. How well the model describes the measured quantities can be evaluated by defining a criterion function as a function of the differences between measured and model-predicted quantities. The optimization process then seeks to minimize the value of the criterion function. Selection of the criterion function can strongly influence the accuracy and efficiency of the optimization process.

In backcalculating layer moduli from FWD data, the measured quantities are the pavement deflections at the various sensor locations. Hence the criterion function should represent the discrepancy between the measured deflections and those predicted by the model. Several criterion functions can be defined. For the FWD backcalculation problem involving *n* deflection measurements on a pavement section of *M* layers with unknown modulus and thickness, the most common criterion functions can be expressed as follows:

- Sum of absolute differences:

$$f(\mathbf{E},\mathbf{h}) = \frac{1}{n}\sum_{i=1}^{n} |d_i^c(\mathbf{E},\mathbf{h}) - d_i^m| \quad (1)$$

- Sum of absolute relative differences:

$$f(\mathbf{E},\mathbf{h}) = \frac{1}{n}\sum_{i=1}^{n} \left|\frac{d_i^c(\mathbf{E},\mathbf{h}) - d_i^m}{d_i^m}\right| \quad (2)$$

- Sum of squared differences:

$$f(\mathbf{E},\mathbf{h}) = \frac{1}{n}\sum_{i=1}^{n} [d_i^c(\mathbf{E},\mathbf{h}) - d_i^m]^2 \quad (3)$$

- Sum of squared relative differences:

$$f(\mathbf{E},\mathbf{h}) = \frac{1}{n}\sum_{i=1}^{n} \left[\frac{d_i^c(\mathbf{E},\mathbf{h}) - d_i^m}{d_i^m}\right]^2 \quad (4)$$

where

$d_i^c(\mathbf{E},\mathbf{h})$ = calculated deflection at Location *i* based on **E** and **h**,
$\mathbf{E}$ = $\{E_1, E_2, E_3, \ldots, E_M\}$ (unknown moduli of the layers),
$\mathbf{h}$ = $\{h_1, h_2, h_3, \ldots, h_{M-1}\}$ (unknown layer thicknesses), and
d_i^m = measured deflection at Location *i*.

Each criterion function defined above has its own advantages and disadvantages, and the quality of its performance is problem dependent. The first two functions are nonsmooth, meaning that their slopes are not necessarily continuous, and consequently optimization techniques that use first-derivative (either analytical or numerical) information cannot be used for estimation. This is a major disadvantage, because optimization methods that use first-derivative information often perform much better than methods that use only function values. From a statistical standpoint, the preferred form of the criterion function depends on the nature of the random error of the measurements, as summarized in Table 1.

The measured deflections contain errors arising from the accuracy of the deflection-measuring system. The specified accuracy of most available FWD devices is on the order of ±2 percent of the measured deflection for commonly used ranges of loading (*4*). The random error can therefore be approximated, for criterion function selection, as normally distributed with zero mean and a constant coefficient of variation. Table 1 indicates that the sum of squared relative differences (Equation 4) is the preferred criterion function for use in backcalculating layer moduli from FWD data.

TABLE 1 SELECTION OF PREFERRED CRITERION FUNCTION FORM

Random Error Characteristics				
Distribution	Mean	Standard Deviation	Coefficient Variation	Preferred Criterion Function
Laplace	Zero	Constant	-	Sum of Absolute Differences
Laplace	Zero	-	Constant	Sum of Relative Differences
Normal	Zero	Constant	-	Sum of Squared Differences
Normal	Zero	-	Constant	Sum of Squared Relative Differences

Nonlinear Least Squares Optimization Method

A number of optimization methods are available to minimize the sum of squared relative differences, that is, to solve the following problem:

$$\text{minimize } f(\mathbf{E},\mathbf{h}) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{d_i^c(\mathbf{E},\mathbf{h}) - d_i^m}{d_i^m}\right]^2 \tag{5}$$

Optimization methods developed for general minimization problems, including direct search and quasi-Newton methods, can be used to solve this problem. However, methods that take into account the special structure of the sum of squared relative differences converge substantially faster than general minimization methods. If the relative error at Location i is represented by

$$r_i(\mathbf{E},\mathbf{h}) = \frac{d_i^c(\mathbf{E},\mathbf{h}) - d_i^m}{d_i^m} \tag{6}$$

the criterion function can be expressed, after multiplying by the constant n for convenience, as

$$f(\mathbf{E},\mathbf{h}) = \sum_{i=1}^{n} [r_i(\mathbf{E},\mathbf{h})]^2 = \mathbf{r}^T\mathbf{r} \tag{7}$$

where $\mathbf{r} = \{r_1, r_2, r_3, \ldots, r_n\}$, the relative error (residual).

Then the gradient of the criterion function is

$$\nabla f = 2\mathbf{A}\mathbf{r} \tag{8}$$

where $\mathbf{A} = \{\nabla r_1, \nabla r_2, \ldots, \nabla r_n\}$, and the Hessian can be written as

$$\mathbf{H} = \nabla^2 f = 2\mathbf{A}\mathbf{A}^T + 2\sum_{i=1}^{n} r_i \nabla^2 r_i \tag{9}$$

The gradient and Hessian are the respective multidimensional equivalents of the slope and curvature of a one-dimensional function. In this formulation, the first part of the Hessian is known as soon as the gradient ∇f has been evaluated. Because $\mathbf{r}^T\mathbf{r}$ is being minimized, the relative errors are often small. Consequently, the second part of the Hessian may be negligibly small, so that a good approximation to the Hessian may be made by neglecting the second part, which gives

$$\mathbf{H} \approx 2\mathbf{A}\mathbf{A}^T \tag{10}$$

A solution can then be obtained iteratively by incorporating the approximated Hessian into the Levenberg-Marquardt algorithm (*5,6*).

Numerical studies have indicated that the residuals vary approximately linearly with the logarithm of layer moduli. This behavior can be exploited to speed convergence by choosing $\log E_i$ as the parameter to be determined instead of E_i. As previously noted, many existing backcalculation methods rely on this behavior; they would not perform satisfactorily without linearization of this aspect of the problem. Because of this linearity, the ∇r_i^2 terms are very small (zero if the relationship is linear) when layer moduli are the only unknowns, and the second part of the Hessian becomes smaller yet, further improving the Hessian approximation used above. With the nonlinear least squares approach, the structure of the selected criterion function allows the Hessian to be easily and rapidly approximated, resulting in a more efficient search for the parameters that minimize the criterion function.

The nonlinear least squares approach has two main advantages in addition to its ability to handle nonlinear problems. First, an optimization method that uses the above approximation to the Hessian will converge much faster than other methods. An approximation to the Hessian is obtained as soon as the gradient is calculated in each iteration; other nonlinear optimization methods may require several iterations to obtain a satisfactory estimate of the Hessian. Second, because $2\mathbf{A}\mathbf{A}^T$ is always positive definite, the criterion function is convex and will have a unique minimum. It should be noted, however, that the criterion function is not proven to be convex. The claim that the neglected term in the Hessian is very small compared with the term retained in the approximation is only supported by numerical experiments, and no theoretical evaluation of the claim is possible because of the complex nature of the problem.

VERIFICATION OF NONLINEAR LEAST SQUARES OPTIMIZATION APPROACH

The nonlinear least squares optimization approach meets all the requirements of a good backcalculation method and has a number of advantages over other methods. The advantages were explored by incorporating the approach into an existing backcalculation program. The LMDIF routine, available in the MINPACK collection of FORTRAN programs developed at the Argonne National Laboratory (*7*), is a well-tested nonlinear least squares routine that takes advantage of the least squares problem structure. This optimization routine was used for the studies described in this paper and has been incorporated into EVERCALC, a layered elastic pavement analysis program developed for the Washington State Department of Transportation. The revised version of the program, EVERCALC 3.0, and the previous version, EVERCALC 2.0, will be compared in terms of the requirements of a good backcalculation method.

Accuracy and Efficiency

The nonlinear least squares optimization approach leads to accurate solutions. It can recognize small differences between computed and measured deflection basins and make the layer modulus adjustments necessary to reduce the differences to an acceptable value. However, most other backcalculation methods also lead to accurate solutions when error tolerances are set at low values, at least when the problem is well conditioned and the seed moduli are reasonably close to the actual moduli.

In order to achieve accurate solutions, however, many iterations may be required. Because of the ease with which the conventional backcalculation problem (i.e., evaluation of layer moduli alone) can be linearized, nearly all backcalculation methods are reasonably efficient when circumstances are not difficult—when errors and uncertainty are minimal, the backcalculation problem is well conditioned, and seed moduli are close to the actual moduli. In the absence of these conditions the efficiency of backcalculation methods is closely related to their robustness.

Robustness

The principal advantage of the nonlinear least squares optimization approach for conventional FWD test interpretation is in robustness. The approach offers distinct improvements in accuracy and speed when the backcalculation problem is complicated by errors and uncertainty in deflection measurements, very sharp or very small modulus contrasts, or poor seed moduli. Because these complications are not uncommon in practice, the benefits of using the nonlinear least squares optimization approach can be considerable.

Several sources contribute to uncertainty in the measured deflections and thus to uncertainty in the backcalculated layer moduli. One of the most common is the limited accuracy of the deflection-measuring system of available FWD devices. To evaluate the effects of such errors on backcalculated layer moduli, a simulation was carried out with the hypothetical four-layer pavement system used in a study of measurement error effects by Irwin et al. (*4*) and shown in Figure 2.

In the simulations by Irwin et al., the instrument error was assumed to be normally distributed with a standard deviation of 1.95 μm. The true deflection basin, characterized by six deflections at offsets of 0, 8.86, 11.8, 20.7, 29.5, and 53.2 in., was first computed for the pavement shown in Figure 2 for a load of 10,000 lb and a plate of radius 5.9 in. using the CHEVRON N-layer program. Thirty deflection basins were then created by adding randomly generated errors to the computed deflections. The layer moduli for each of the 30 deflection basins were backcalculated with EVERCALC 3.0. The results are presented in Table 2, along with those obtained with EVERCALC 2.0 and MODCOMP 2 (*4*). The mean EVERCALC 3.0 moduli were generally somewhat closer to the true moduli, but the variability in backcalculated moduli, reflected in the coefficient of variation of the modulus for each layer, was significantly lower for EVERCALC 3.0. The improvement in the performance of EVERCALC 3.0 (small in this case) is largely attributed to the robust nonlinear least squares optimization routine, though some may be due to

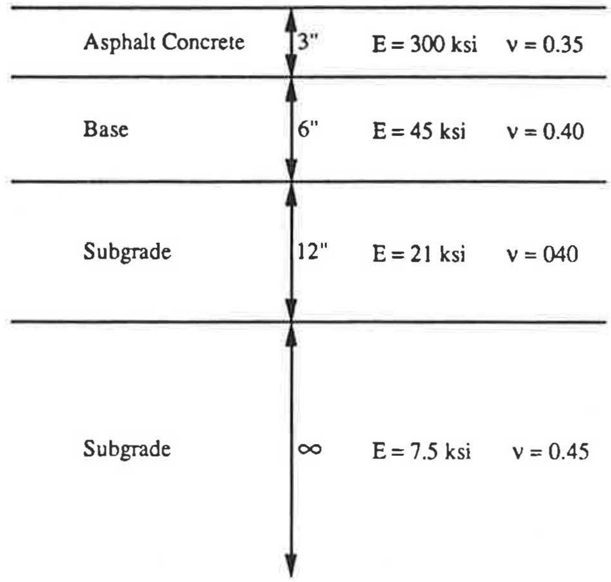

FIGURE 2 Four-layer pavement structure analyzed by Irwin et al. (*4*).

differences in the modeling approaches used in the two programs.

Versatility

The nonlinear least squares optimization approach has capabilities beyond those of conventional backcalculation procedures. This versatility is described in the following section.

ADVANCED BACKCALCULATION BY NONLINEAR LEAST SQUARES OPTIMIZATION APPROACH

All currently available FWD interpretation programs require the thickness of each pavement layer to be specified before backcalculation. These layer thicknesses can be measured accurately from core samples but are usually obtained from design and construction records. However, as-built layer thicknesses can vary significantly from those described in the design and construction records. Measured asphalt thicknesses have been observed to differ from those in design and construction records by up to 128 percent (*8*). Because pave-

TABLE 2 COMPARISON OF EFFECTS OF MEASUREMENT ERRORS ON BACKCALCULATED LAYER MODULI

		MODCOMP 2 Moduli		EVERCALC 2.0 Moduli		EVERCALC 3.0 Moduli	
Layer	Actual (ksi)	Mean (ksi)	COV (%)	Mean (ksi)	COV (%)	Mean (ksi)	COV (%)
AC	300.0	306.0	16.3	290.5	18.6	295.2	12.2
Base	45.0	44.6	13.2	46.0	13.3	45.5	9.0
Subbase	21.0	21.3	6.6	20.5	7.3	21.0	4.8
Subgrade	7.5	7.5	1.2	7.5	1.2	7.5	0.8

ment deflections are sensitive to layer thicknesses, even modest errors in assumed layer thicknesses can lead to large errors in backcalculated layer moduli.

An advanced backcalculation procedure that uses the nonlinear least squares optimization approach has been developed to backcalculate both layer moduli and layer thickness from the same set of FWD data. Currently used optimization procedures may not be capable of solving such nonlinear problems accurately and efficiently.

Problem Characteristics

Some insight into the FWD moduli backcalculation problem can be obtained from examination of a simple, hypothetical two-layer backcalculation problem. Consider the pavement section of Figure 3, in which a 10-in. layer of asphalt concrete (AC) is underlain by an infinitely thick subgrade. The actual moduli of the AC and subgrade are 500 and 20 ksi, respectively. If the deflection basin predicted for this section by the CHEVRON N-layer program is considered the measured deflection basin, values of the sum of squared relative differences criterion function can be computed for other combinations of layer moduli. The values of the criterion function for other layer moduli will reflect how close they are to the actual moduli. Figure 4a shows contours of the sum of squared relative differences criterion function for this hypothetical problem with a 10-in. AC thickness. Figure 4b shows the sensitivity of the criterion function to the AC layer thickness at the actual layer moduli.

Certain characteristics of the FWD moduli backcalculation problem are apparent from Figure 4. The criterion function is relatively sensitive to subgrade modulus and relatively insensitive to AC modulus. These observations are not unexpected, because most of the pavement surface deflection results from deformation of the subgrade soils. The size of the 0.01 criterion function contour [which corresponds to 10 percent root-mean-square (RMS) of the relative error] is also of note. It encompasses an AC modulus range of 300 to 800 ksi and a subgrade modulus range of 18 to 24 ksi. Clearly, a criterion function tolerance of much less than 0.01 or an RMS relative error tolerance less than 10 percent is necessary to ensure reasonable accuracy of backcalculated moduli. As the number of layers in the pavement section increases while the number of deflection measurements remains the same, this range (i.e., the volume encompassed by a fixed value of the criterion function or the RMS relative error) also increases, requiring much tighter error tolerances. The criterion function is also sensitive to the thickness of the AC layer. Hence, accurate knowledge of layer thickness is necessary for accurate backcalculation of layer moduli. This sensitivity of layer moduli to layer thickness was well illustrated by Irwin et al. (*4*).

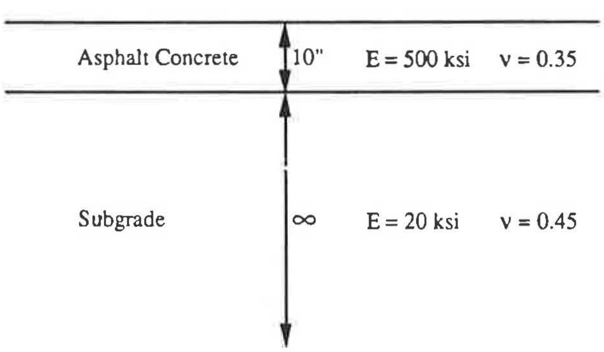

FIGURE 3 Hypothetical two-layer pavement structure.

Illustration of Advanced Backcalculation Procedure

When the assumed layer thicknesses specified in a conventional backcalculation problem are incorrect, the resulting backcalculated layer moduli will be incorrect. Whether the backcalculated layer moduli are too high or too low will depend on whether the layer thicknesses were underestimated or overestimated. The effect of such errors can be illustrated by introducing incorrect layer thicknesses into a conventional analysis of a pavement section of known properties. The benefits of the advanced backcalculation procedure can then be illustrated by considering the layer thicknesses as unknowns and iterating toward a set of layer moduli and layer thicknesses that best matches the measured deflection basins.

Two-Layer Case

Using the CHEVRON N-layer program, a deflection basin was generated for the two-layer pavement section of Figure 3, which consists of a 10-in.-thick AC layer with modulus E_1 = 500 ksi overlying a subgrade with modulus E_2 = 20 ksi. A 10 percent error in AC thickness could lead to the specification of a 9- or 11-in. thickness in a conventional backcalculation analysis. Assuming either of these incorrect AC thicknesses, a conventional backcalculation procedure would converge to an incorrect set of layer moduli. Using EVERCALC 3.0 with fixed values of AC thickness (conventional mode), a backcalculated AC modulus of 648.2 ksi was obtained from four different sets of seed moduli when the AC thickness was assumed to be 9 in. Thus a 10 percent underestimation of AC thickness resulted in a 29.6 percent overestimation of AC modulus. When the AC thickness was incorrectly assumed to be 11 in., the AC modulus was backcalculated to be 402.6 ksi, again from four different sets of seed moduli. The subgrade modulus was accurately predicted for all cases. The optimization process is shown graphically in Figure 5a. The actual solution is represented by the large point at the center of the cube, the initial conditions by the medium-size points along the edges of the cube, and the conditions at the end of each iteration by the small points at the kinks in the optimization "paths." As can be seen, the conventional mode of backcalculation requires that the optimization paths remain in the same AC thickness plane, so the actual solution cannot be reached.

The advanced mode of EVERCALC 3.0 was then used, starting from the same seed values of layer modulus and AC layer thickness, with the AC layer thickness also considered an independent variable. The results are shown graphically in Figure 5b. The optimization paths converge, in two or three iterations, to values close to the actual solution. The numerical results are presented in Table 3. In each case, the advanced

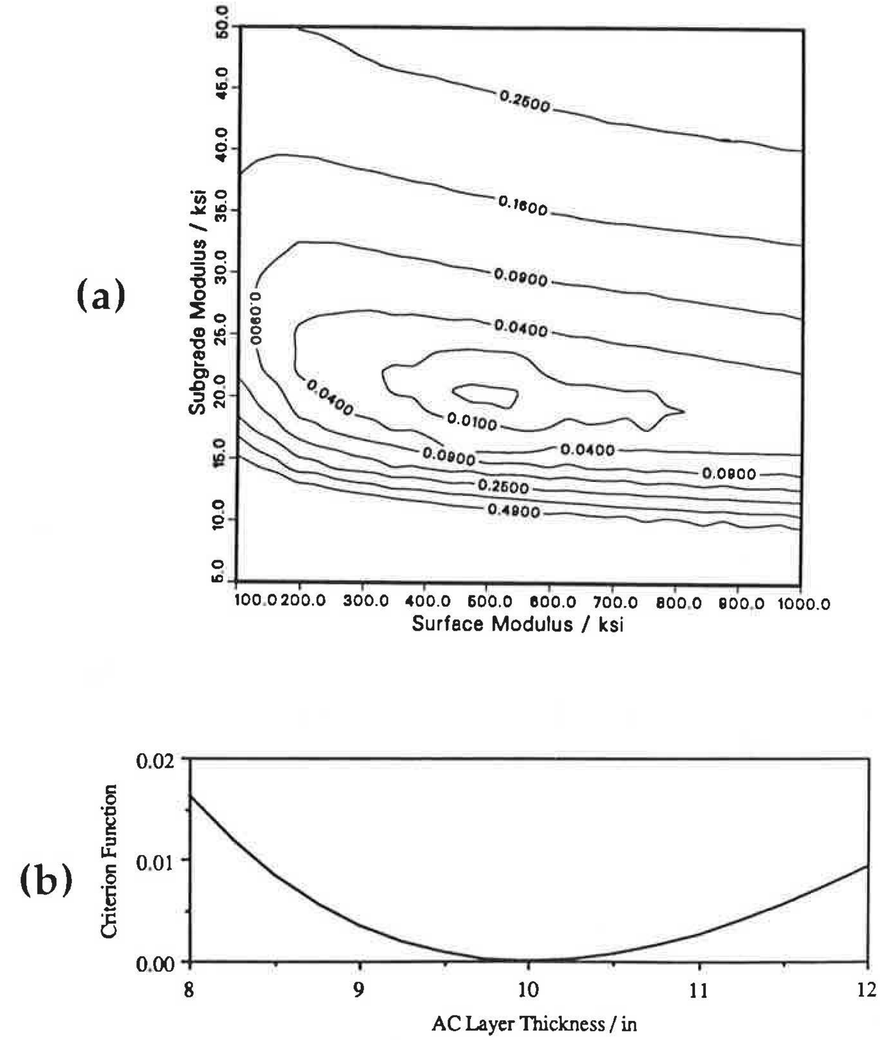

FIGURE 4 (a) Criterion function contours with AC thickness set at actual value; (b) variation of criterion function with AC thickness with moduli set at actual values. [Criterion function value = (RMS error)^2.]

backcalculation procedure of EVERCALC 3.0 converged to the correct moduli for the AC and subgrade layers and to the correct thickness of the AC layer. Averaging the modulus and thickness values from eight sets of seed values (seed moduli and seed thickness) gave differences between the actual and backcalculated values of less than 0.1 percent.

Three-Layer Case

A similar analysis was performed for the three-layer pavement structure shown in Figure 6. This pavement structure consisted of 5 in. of AC (E_1 = 500 ksi) over 10 in. base material (E_2 = 40 ksi) over subgrade (E_3 = 20 ksi). In the three-layer analyses, pavement thickness errors of $\pm$20 percent were assumed. Four sets of incorrect layer thickness assumptions were evaluated, corresponding to (h_1, h_2) values of (4, 8), (4, 12), (6, 8), and (6, 12), where h_1 and h_2 are the AC and base layer thicknesses in inches, respectively. Each set was evaluated once from seed moduli twice as large as the actual moduli and once from seed moduli half as large as the actual moduli. The sections were analyzed with EVERCALC 3.0, first in the conventional mode and then in the advanced mode starting from the ending point of the conventional analysis. The results for the eight cases are presented in Table 4. Again, it can be seen that errors in assumed pavement layer thickness resulted in significant errors in backcalculated layer moduli, with the exception of the subgrade modulus, when conventional backcalculation techniques were used. When the layer thicknesses were released as independent variables in the advanced mode of EVERCALC 3.0, the program again converged rapidly toward the correct modulus and thickness for all layers. Averaging the modulus and thickness values from each of the eight sets of seed values gave errors no larger than 0.3 percent.

During each iteration, both layer moduli and layer thicknesses are adjusted at the same time, as shown in Figure 5b. Consequently, the increase in the number of parameters required for the advanced approach does not lead to a proportional increase in the number of iterations or the total com-

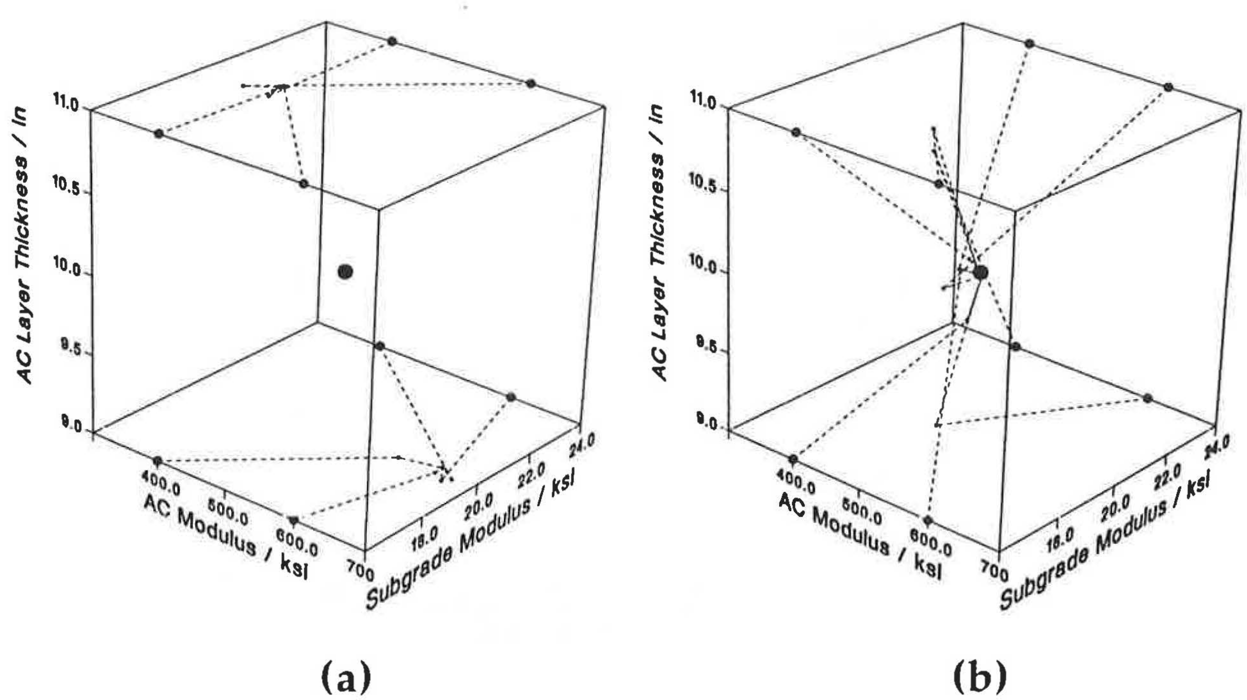

FIGURE 5 Optimization paths taken by EVERCALC 3.0 during backcalculation of hypothetical two-layer problem: a, conventional mode; b, advanced mode.

TABLE 3 COMPARISON OF BACKCALCULATED MODULI AND LAYER THICKNESS—TWO-LAYER CASE

	Seed Values			Calculated Values*			Calculated Values**			
Case	E1	E2	h	E1	E2	RMS Error (%)	E1	E2	h	RMS Error (%)
1	400	24	9.000	648.2	20.1	0.8	500.0	20.0	10.002	0.01
2	600	24	9.000	648.2	20.1	0.9	501.0	20.0	9.990	0.07
3	400	15	9.000	648.2	20.1	0.8	500.0	20.0	10.002	0.08
4	600	16	9.000	648.2	20.1	0.9	500.1	20.0	10.001	0.08
5	400	24	11.000	402.6	19.9	0.8	500.6	20.0	9.994	0.00
6	600	24	11.000	402.6	19.9	0.9	501.0	20.0	9.990	0.04
7	400	16	11.000	402.6	19.9	0.8	500.7	20.0	9.995	0.08
8	600	16	11.000	402.6	19.9	0.9	500.0	20.0	10.001	0.00
						Average Values	500.4	20.0	9.997	
						Error (%)	0.08	0.00	0.03	

*h fixed at seed value (conventional mode of backcalculation)
**h freed as independent variable (advanced mode of backcalculation)

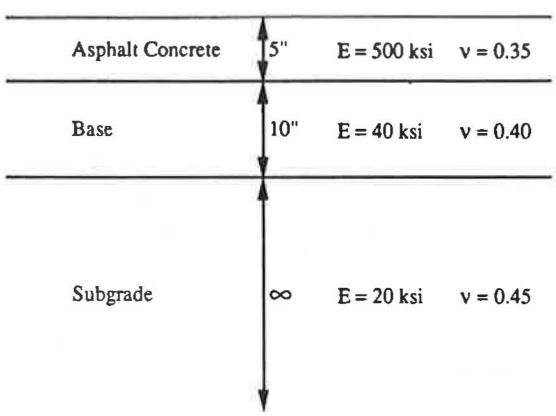

FIGURE 6 Hypothetical three-layer pavement structure.

puting time. In fact, the time required to reach a solution in the advanced mode was typically only 10 percent greater than in the conventional mode.

Application to Real Data

The advanced backcalculation procedure was applied to actual deflection basin data obtained by a Washington State Department of Transportation FWD (Dynatest Model 8000) at a test site (Milepost 20.85 on SR-11) in northwestern Washington State. AC and base thicknesses were measured at three locations in the 1,000-ft-long test section; average values were 5.2 in. and 28.8 in., respectively. The deflection basin and the results of conventional backcalculation by five different backcalculation programs were described by Mahoney et al.

TABLE 4 COMPARISON OF BACKCALCULATED MODULI AND LAYER THICKNESS—THREE-LAYER CASE

	Seed Values					Calculated Values*				Calculated Values**					
Case	E1	E2	E3	h1	h2	E1	E2	E3	RMS Error (%)	E1	E2	E3	h1	h2	RMS Error (%)
1	1000	80	40	4	8	696.2	58.7	20.0	0.8	500.7	40.0	20.0	4.995	9.992	0.01
2	1000	80	40	4	12	877.7	39.4	19.9	0.5	500.5	40.1	20.0	4.996	9.985	0.01
3	1000	80	40	6	8	347.1	39.6	20.1	0.5	501.0	40.4	20.0	4.995	9.990	0.01
4	1000	80	40	6	12	373.4	31.3	20.1	0.2	499.3	40.1	20.0	5.000	9.960	0.06
5	250	20	10	4	8	696.4	58.6	20.0	0.8	501.3	40.2	20.0	4.981	9.975	0.03
6	250	20	10	4	12	877.4	39.4	19.9	0.5	496.3	40.2	20.0	5.010	9.861	0.06
7	250	20	10	6	8	347.2	39.6	20.0	0.5	500.0	40.0	20.0	4.999	10.010	0.02
8	250	20	10	6	12	373.1	31.4	20.1	0.2	500.0	40.0	20.0	4.996	9.985	0.05
									Average Values	499.9	40.1	20.0	4.997	9.970	
									Error (%)	0.02	0.25	0.00	0.06	0.30	

*h fixed at seed value (conventional mode of backcalculation)
**h freed as independent variable (advanced mode of backcalculation)

(*9*). The conventional backcalculation programs used the measured AC and base thicknesses to backcalculate moduli for each of the three layers. The programs exhibited considerable variability in backcalculated AC modulus, ranging from 518 to 761 ksi with an average of 621 ksi, but much more consistent base and subgrade moduli (25.0 ksi and 26.4 ksi, respectively). The average RMS error at convergence for the five programs was 2.7 percent.

Advanced backcalculation was performed with gross errors in assumed layer thickness. Seed thicknesses of 4.0 in. for the AC layer (77 percent of measured thickness) and 32.8 in. for the base layer (114 percent of measured thickness) were used. The results are summarized in Table 5. The advanced procedure iterated to backcalculated thicknesses of 5.4 in. for the AC and 26.3 in. for the base and, simultaneously, to backcalculated layer moduli of 656, 22, and 27 ksi for the AC, base, and subgrade layers, respectively. The RMS error for the advanced backcalculation procedure was 0.8 percent. The backcalculated thicknesses are in good agreement with the measured thicknesses, and the backcalculated moduli are in good agreement with those backcalculated by the conventional backcalculation programs.

POTENTIAL LIMITATIONS

The verification of the advanced backcalculation procedure described in this paper is based on one real deflection basin and two hypothetical pavement sections with calculated deflection basins. The probability of the parameters backcalculated with the nonlinear least squares optimization approach being close to the actual parameters increases with increasing system overdeterminism (ratio of number of independent data points to number of unknown parameters). Therefore, increasing the number of unknowns by adding layer thicknesses to the list of unknown parameters implies that additional data points may be necessary, particularly for many-layered pavement structures. In the three-layer structures considered here, five unknowns (three moduli and two thicknesses) were accurately calculated from deflection basins

TABLE 5 COMPARISON OF BACKCALCULATED MODULI AND LAYER THICKNESS—SR-11 FIELD SITE

Method	AC	Base	Subgrade	RMS Error (%)
ELMOD	518	28	23	1.9
ELSDEF	550	27	25	5.8
EVERCALC 2.0	761	23	27	1.2
ISSEM4	592	25	28	n.a.
MODCOMP2	686	22	29	1.8
Average	**621**	**25.0**	**26.4**	**2.7**
EVERCALC 4.0	**656**	**22**	**27**	**0.8**

Layer	Measured Thickness (in)	Seed Thickness (in)	Backcalculated Thickness (in)
AC	5.2	4.0	5.4
Base	28.8	32.8	26.3

defined by six deflections. Whereas the advanced backcalculation procedure worked well in these cases, the effects of large measurement errors that may exist in real data on the confidence intervals of backcalculated parameters have not yet been fully investigated.

Increasing the overdeterminism of the FWD backcalculation problem (with consequent improvement in confidence intervals), however, can easily be accomplished. By representing the stress-dependent layers with more fundamental parameters—for example, k_1 and k_2 [from $E = k_1(\theta)^{k_2}$]—the data from multiple load drops can be combined in a single backcalculation analysis. As an example, consider a four-layer system subjected to four load drops. If two parameters are necessary to describe the load-dependent modulus for each layer except the AC, the total number of unknowns will be 10, including the thicknesses of the upper three layers. If the FWD apparatus is configured to make six deflection measurements, a total of 24 independent data points will be generated. With 10 unknowns calculated from 24 data points, the backcalculation problem is highly overdetermined and the advanced procedure is likely to converge to an accurate solution. Investigation of this approach is continuing.

SUMMARY

An improved optimization technique for backcalculating pavement layer moduli has been described. The optimization technique (nonlinear least squares) can converge to a solution more quickly using wide ranges of input data, such as seed moduli. Further, the technique can be used to backcalculate layer thicknesses.

Data were presented in Tables 3 and 4 illustrating the effect of incorrect layer thicknesses on backcalculated moduli. Clearly, the ability to determine both the layer moduli and thicknesses represents an improvement in backcalculation technique.

FUTURE ACTIVITIES

In an extension of the work described in this paper, an advanced version of EVERCALC, EVERCALC 4.0, will be tested on actual field data. The data will include FWD deflection basins on pavement sections that have been cored for thickness determination and for which the samples have been tested in the laboratory for resilient moduli (AC by ASTM D4123 and unstabilized base and subgrade materials by a modified triaxial test similar to AASHTO T274). Though laboratory resilient moduli do not necessarily represent the "true" moduli, this information will allow evaluation of the procedure. The ability of the proposed technique to estimate depths to rock (rigid) layers and material parameters, such as k_1 and k_2 [from $E = k_1(\theta)^{k_2}$], for unstabilized layers will also be evaluated. The nonlinear least squares optimization technique will also be used to examine the minimum number of deflection sensors, sensor spacings, and FWD load levels necessary to accurately backcalculate both layer moduli and layer thicknesses from surface deflections.

REFERENCES

1. R. L. Lytton. Backcalculation of Pavement Layer Properties. In *Nondestructive Testing of Pavements and Backcalculations of Moduli* (A. J. Bush III and G. Y. Baladi, eds.). ASTM STP 1026. American Society for Testing and Materials, Philadelphia, Pa., 1989, pp. 7–38.
2. A. J. Bush III. *Nondestructive Testing for Light Aircraft Pavements, Phase II: Development of the Nondestructive Evaluation Methodology*. Report FAA-RD-80-9-II. Federal Aviation Administration, U.S. Department of Transportation, 1980.
3. J. Uzan, R. L. Lytton, and F. P. Germann. General Procedure for Backcalculating Layer Moduli. In *Nondestructive Testing of Pavements and Backcalculations of Moduli* (A. J. Bush III and G. Y. Baladi, eds.). ASTM STP 1026. American Society for Testing and Materials, Philadelphia, Pa., 1989, pp. 217–228.
4. L. H. Irwin, W. S. Yang, and R. N. Stubstad. Deflection Reading Accuracy and Layer Thickness Accuracy in Backcalculation of Pavement Layer Moduli. In *Nondestructive Testing of Pavements and Backcalculations of Moduli* (A. J. Bush III and G. Y. Baladi, eds.). ASTM STP 1026. American Society for Testing and Materials, Philadelphia, Pa., 1989, pp. 229–244.
5. K. Levenberg. A Method for the Solution of Certain Nonlinear Problems in Least Squares. *Quarterly of Applied Mathematics*, Vol. 2, 1944, pp. 164–168.
6. D. W. Marquardt. An Algorithm for Least Squares Estimation of Nonlinear Parameters. *SIAM Journal*, Vol. 11, 1963, pp. 431–441.
7. J. J. More, B. S. Garbow, and K. E. Hillstrom. *User Guide for MINPACK-1*. Argonne National Laboratory, Argonne, Ill., 1980, 261 pp.
8. S. W. Lee. Backcalculation of Pavement Moduli by Use of Pavement Surface Deflections. Ph.D. dissertation. University of Washington, Seattle, 1988, 247 pp.
9. J. P. Mahoney, N. F. Coetzee, R. N. Stubstad, and S. W. Lee. A Performance Comparison of Selected Backcalculation Computer Programs. In *Nondestructive Testing of Pavements and Backcalculations of Moduli* (A. J. Bush III and G. Y. Baladi, eds.). ASTM STP 1026. American Society for Testing and Materials, Philadelphia, Pa., 1989, pp. 452–469.