Assessing Tiltrotor Technology: A Total Logistics Cost Approach

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The feasibility and competitive potential of tiltrotor aircraft technology in the short-haul scheduled passenger transportation market is studied through optimization of transport systems combining tiltrotor and turboprop service. A total logistics cost function is used, the arguments for which are tiltrotor frequency, turboprop frequency, tiltrotor market share, and in one case the number of vertiports. When the system involves one center city vertiport and one airport at the city periphery, total cost may be minimized when tiltrotor market share is either greater than 50 percent or 0. Conditions when tiltrotor is definitely feasible or definitely infeasible are represented by functional relationships between the difference in per seat aircraft operating cost and the ratio of per flight aircraft operating cost, the parameters of which reflect the size of the market, traveler valuations of the cost of airport-vertiport access, and traveler valuations of time spent waiting for flights. Curves relating optimal tiltrotor market share to tiltrotor per seat operating cost exhibit discontinuities as the optimum shifts from exclusive tiltrotor service, to mixed service, to exclusive turboprop service. When multiple vertiports are allowed, the optimum number is almost always 10 or fewer and, for most markets, fewer than 4. However, the total logistics cost is fairly insensitive to the number of vertiports, as long as frequencies at any given number are set to their optimum values. Even under extreme conditions when the optimum number of vertiports is 16, the multiple vertiport system market share is, in most cases, only marginally greater than that of a system involving just one vertiport. However, the multiple vertiport system is better able to compensate for constraints on tiltrotor aircraft size. This is particularly important because the sizes receiving the most consideration are smaller than optimum for markets of several hundred passengers or more. If at all possible, however, civil tiltrotor aircraft should be upsized to reduce total aircraft operating cost.

The ability to take off and land vertically, while attaining turboprop performance during cruise, makes tiltrotor aircraft a promising civil aircraft technology. The prospects for civil application depend on two sets of factors. The first set—not the subject of this paper—concerns how obstacles to technological change, particularly of large-scale public systems, can be surmounted. Issues of certification, industry and public confidence, and infrastructure provision are to be included here.

The second set of factors concerns the inherent capabilities of the aircraft and how these capabilities fit the service preferences of the traveling public. The purpose of this paper is to explore these. In particular, we ask whether and under what conditions tiltrotor technology would reduce the total logistics cost of air travel when introduced into scheduled passenger service. In order to do this we will optimize, with respect to total logistics cost, systems that include both tiltrotor and conventional turboprop technology as travel options.

The term "total logistics cost" refers to the total cost associated with air travel. For present purposes, total logistics cost is assumed to have four components: (a) the cost of owning and operating aircraft, (b) the cost to passengers of accessing airports and vertiports, (c) the cost to passengers of waiting for a scheduled flight, and (d) the cost to passengers of spending time in the aircraft during the flight. Excluded from consideration are various external costs, in particular those deriving from noise and air pollution. Also excluded are accident costs and costs associated with marginal delay increases that result from operating at congested airports.

The sole focus of this research, then, is on tiltrotor feasibility in the context of the cost-frequency-accessibility tradeoff in the short-haul scheduled air transportation market. Previous studies have explored the potential of tiltrotor technology in other markets, such as corporate flying (*1*), oil exploration (*2*), and as a direct substitute (that is, providing airport-to-airport service) for conventional takeoff and landing aircraft (*3,4*). The use of tiltrotor flying from vertiports in scheduled service has also received some attention (*2,3*), but in these studies frequency levels were assumed rather than considered as an inherent element of the trade-offs that must be considered. By ignoring the impact of frequency on total logistics cost, former analyses have overlooked a source of scale economies that tends to encourage service concentration. Additionally, key supply variables, the determination of which is heavily influenced by frequency considerations, such as aircraft size and the number of vertiports, have received scant attention. This research seeks to fill some of these deficiencies.

The "total" logistics cost model used here, in addition to excluding some cost items, makes simplifying assumptions about the structure of included costs. For example, we assume that the ownership and operation costs for a given aircraft technology can be represented by a linear function involving only two parameters. Similarly, passenger costs are linearized: access cost on a per unit distance basis and waiting cost on a per unit time basis. It is recognized that passenger costs may not be fully linear as depicted, and furthermore that the importance of access and waiting elements varies from passenger to passenger. At least at the initial stages of an inquiry, however, the assumption of linear, homogeneous waiting and access costs is appropriate.

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CENTER-CITY VERTIPORT

This model considers the idealized situation depicted in Figure 1. We assume a square urban area of constant density with a vertiport located in the center and an airport on the periphery. Conventional turboprop aircraft operate out of the airport and tiltrotor aircraft use the vertiport. We explore how these two services compete with each other in a hypothetical point-to-point market where the destination lacks a separate vertiport facility. Such a destination could be envisioned either as a smaller city or as one in which separate vertiport infrastructure has yet to be deployed.

The essential trade-off here is between accessibility and operating cost: how much additional operating cost is the central location of the vertiport worth? To approach this question, we imagine that the airport and the vertiport have respective market areas, as shown in Figure 1. Within a given market area, it is assumed that all trips will be made using the appropriate facility. The relative size of the two market areas thus determines the relative market shares of the two services. We then seek to minimize total logistics cost by choosing optimum values for three decision variables: the location of the boundary, or equivalently the market share of the tiltrotor service; the frequency of flights from the airport; and the frequency of flights from the vertiport. These variables will be chosen so as to minimize the total logistics cost of transporting a fixed number of passengers, whose true origins are assumed to be distributed uniformly about the city, to a single destination. Optimum aircraft size, a function of the three decision variables, will also be determined.

In analyzing this problem, we do not mean to imply that some government official is or should be authorized to determine values for the three decision variables. Instead, we might imagine some commuter airline wishing to optimize its mix of turboprop and tiltrotor services. We might even hope that the optimum could be achieved without any central decision maker, perhaps with a set of government interventions to eliminate market failures. Thus the optimization problem previously defined is relevant even in the context of a market economy.

It is also recognized that the concept of a market area in which all trips are made from the same facility is an oversimplification. There will be cases in which costs can be reduced by allowing some trips originating at a certain point to use one facility and some to use the other, depending on the specifics of flight schedules and desired travel times. Such possibilities are very difficult to treat analytically, and are better handled using computer simulation techniques. The philosophy here is to rely on simple analytic techniques that yield a clear, if approximate, representation of the system and the variables to which it is sensitive. The use of sharply delineated market area boundaries, which may compromise realism to some extent, greatly facilitates the analysis.

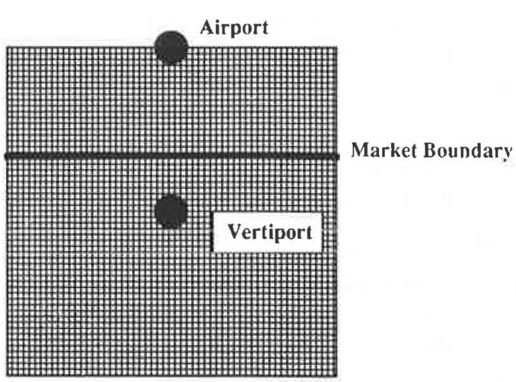

FIGURE 1 Square city.

Before proceeding further, it is useful to define a set of variables that influence the solution to the problem. We first define a full set of variables and then use them to define a reduced set that will be used in subsequent calculations.

Let

C_t = cost/flight for tiltrotor,
C_c = cost/flight for conventional (turboprop) aircraft,
S_t = cost/seat for tiltrotor,
S_c = cost/seat for conventional (turboprop) aircraft,
L = the length of a side of the city,
T_a = passenger access cost for a trip of length L,
T_w = average passenger waiting time cost when flight frequency is 2/day,
N = the number of passenger trips/day, and
LF = aircraft load factor.

This set of parameters can be reduced by observing that solutions depend on the relative magnitudes of the four cost parameters only. Also, because we treat load factor parametrically, it is useful to divide it into S_t and S_c to get a cost per passenger. We therefore define

$$C'_t = C_t/C_c$$

$$S'_c = S_c/LF \quad C_c$$

$$S'_t = S_t/LF \quad C_c$$

$$T'_a = T_a/C_c$$

$$T'_w = T_w/C_c$$

Our decision variables are

F_t = tiltrotor frequency (flights/day),

F_c = conventional aircraft frequency (flights/day), and

P = tiltrotor market share.

We now derive the logistics cost function for the problem. As already noted:

$$TLC(F_c,F_t,P) = AOC(F_c,F_t,P) + PAC(P) + PWC(F_c,F_t,P) + PTC(P) \quad (1)$$

where

TLC = total logistics cost,
AOC = aircraft operating cost,
PAC = passenger access cost,
PWC = passenger waiting cost, and
PTC = passenger in-aircraft time cost.

We will assume that the travel time for tiltrotor and conventional aircraft are equal. Boeing estimates that tiltrotor would have at most a 9-min travel time advantage for the 200 nautical mile (NM) stage length considered in this analysis (2). It is unlikely that passengers' valuation of this difference would be very high in relation to the other costs involved. Therefore, only the first three terms on the right-hand side of Equation 1 need be considered. We now turn to specifying each of these in terms of the variables already defined.

The cost to operate an aircraft for a given stage length depends on the size of the aircraft. The cost is approximated as a linear function of the number of seats, with a positive intercept. The intercept reflects economies of scale: one 40-seat flight costs less than two 20-seat flights. On the other hand, the linear coefficient reflects the fact that one 40-seat flight costs more than one 20-seat flight. Analysis of Boeing cost data for turboprops and tiltrotor shows that this linear function approximates operating costs of both aircraft quite well.

As previously noted, size is not an explicit decision variable in our problem. Rather, the size is obtained by assuming a particular load factor and then determining the number of seats necessary to accommodate the passenger traffic at the assumed load factor with the optimum service frequency. Thus,

$$\text{Tiltrotor seats/flight} = \frac{NP}{LF \cdot F_t}$$

$$\text{Turboprop seats/flight} = \frac{N \cdot (1 - P)}{LF \cdot F_c} \tag{2}$$

Therefore, the aircraft operating cost, in terms of the normalized parameters, in which the cost of a turboprop flight is set to unity, is given by

$$\begin{aligned} AOC(F_c, F_t, P) &= F_c + C_t' F_t \\ &\quad + N[PS_t' + (1 - P)S_c'] \end{aligned} \tag{3}$$

Passenger access cost (PAC), depends on the market boundary, and also how we measure distance. For analytical simplicity, we assume that the city has a street system consisting of an infinitely dense square grid oriented parallel to the city boundary. Distance between two points X and Y, with coordinates (X_1, X_2) and (Y_1, Y_2), is given by

$$D(X, Y) = |X_1 - Y_1| + |X_2 - Y_2| \tag{4}$$

The market area should be shaped so that at any point along the boundary the difference in access cost to the airport and the vertiport is the same. Under the geometry and distance metric assumed, this implies that the rectangular market areas are as depicted in Figure 1. Other distance metrics, such as the Euclidian, would yield curvilinear market area boundaries.

When the market area boundary is between the airport and the vertiport, total access can be derived by dividing the city into three regions: the airport market area, the portion of the vertiport market area between the vertiport and the airport, and the rest of the city (the portion below the vertiport in Figure 1). It is seen that the total access cost is

$$\begin{aligned} PAC(P) &= NT_a' \{(1 - P)[\tfrac{1}{4} + \tfrac{1}{2}(1 - P)] \\ &\quad + (P - \tfrac{1}{2})[\tfrac{1}{4} + \tfrac{1}{2}(P - \tfrac{1}{2})] + \tfrac{1}{4}\} \end{aligned} \tag{5}$$

in which the three terms included in the brackets correspond to the three regions. Simplifying this expression yields

$$PAC(P) = NT_a'\left(P^2 - \frac{3P}{2} + 1\right)\left(\frac{1}{2} \le P \le 1\right) \tag{6}$$

Note that PAC is minimized when $P = 3/4$, that $PAC(1/2) = NT_a'/2$, and that $PAC(1) = NT_a'/2$. All of these results can be easily verified from inspection of Figure 1. However, the expression is valid only when $P \geq 1/2$. It will presently be shown that all cost-minimizing solutions have P either in this range or equal to 0. For completeness, we note that when $P < 1/2$, the access cost is

$$PAC(P) = NT_a'\left(\frac{3}{4} - \frac{P}{2}\right)\left(0 \le P < \frac{1}{2}\right) \tag{7}$$

The evaluation of PWC requires an understanding of how waiting cost varies with the number of flights. We assume that for service to be viable, at least two flights/day—one at the beginning and one at the end—must be offered. Other flights would be scheduled throughout the day. If preferred travel times were uniformly distributed over time, the third flight would be scheduled in the middle of the day and would cut the average difference between a passenger's preferred travel time and the time when a flight is available by 50 percent. Likewise, if four flights were available, the optimum schedule would be to have one of the midday flights in the morning and one in the afternoon, and this would yield an average time difference of 1/3 that under the two-flights/day scenario. Thus,

$$PWC(F_c, F_t, P) = NT_w'\left(\frac{P}{F_t - 1} + \frac{1 - P}{F_c - 1}\right)(F_c, F_t \geq 2) \tag{8}$$

Equations 3, 5, and 8 define the total logistics cost, exclusive of in-plane travel time, given in Equation 1. Before turning to the question of optimization, it is useful to get a sense of the magnitudes of the different variables included in this function. These are summarized in Table 1.

The per flight costs of turboprop were obtained by regressing total costs—capital, ownership, and operating—for a 200-NM mission against the number of seats for a set of hypothetical aircraft of sizes ranging from 8 to 75 seats, based on the Boeing data. The estimates of the intercept and slope are \$300 and \$25, respectively. Boeing provides similar costs for a set of tiltrotor aircraft of varying sizes. Regression on these yields an intercept of \$700 and a slope of \$40. Civil tiltrotor cost estimates are highly uncertain, however, and more recent (to date unpublished) figures are reported to be substantially lower. In this analysis, we will assume as a baseline that tiltrotor per flight and per seat costs are 40 percent higher than those for turboprop technology.

The waiting cost parameter, T_w, is the cost a traveler would attribute to waiting if the service frequency were two/day. The average time difference between preferred and actual flight times depends on whether passengers opt for the flight scheduled nearest to their preferred time, or choose the first flight before (or after) that time. In the first case, a uniform distribution of preferred travel times spread over a 16-hr day yields an average time difference of 16/4 or 4 hr, whereas in

TABLE 1 ESTIMATED VALUES OF KEY VARIABLES

Variable (Symbol)	Dollar Range	Value in Baseline Scenario Dollar	Normalized
Turboprop Cost per Flight (C_c)	--	300	1.00
Turboprop Cost per Seat (S_c)	--	25	0.13
Tiltrotor Cost per Flight (C_t)	--	420	1.40
Tiltrotor Cost per Seat (S_t)	--	35	0.18
Waiting Cost @ 2 Flights per day (T_w)	30-120	60	0.20
Access Cost @ Distance = L (T_a)	20- 80	45	0.15

the second case it would be 8 hr. Assuming that this waiting time is valued at roughly half an average $20/hr wage rate, we obtain a T_w between $40 and $80. Recognizing that these estimates are quite "soft," we widen the range to be considered from $30 to $120, with $60 assumed for the baseline.

The access cost parameter, T_a, depends on the size of the urban area, average travel speeds, and unit costs for expenditures and travel time. We consider a large urban area, 30 mi across. Assuming an average travel speed of 30 mi/hr, we obtain a time to cross the city of 1 hr. Because access time tends to be unpleasant and, unlike waiting time, cannot be used for other activities, it is valued at the full wage rate. This results in an access time cost of $20. The expenditure depends on the mode of access. If this were a taxi, it would be roughly $1.25/mi, or $40, whereas for a private automobile it would be $10 to $20, depending on the specific vehicle and whether ownership costs are included. This implies a total access cost for a cross-city trip of $30 to $60. Again, widening the range to reflect the uncertainty of these estimates, we assume that T_a falls in the $20 to $80 range. For our baseline case, we let T_a be $45.

The market size, N, is highly variable. Most origin-destination (O-D) markets are very small—1 passenger/day or less—but the vast majority of total passenger traffic is in markets of 100 passengers/day or more. An upper limit—for a market such as New York to Boston—is about 10,000 passengers/day. The subsequent analysis will treat N parametrically, assuming 10,000 passengers/day as an upper limit. Finally, the load factor, LF, is set at 65 percent. This is representative of the load factors of major airlines but may be somewhat high for short-haul services. On the other hand, load factors throughout the industry have generally been increasing, so that by the time civil tiltrotor service becomes available, load factors of 65 percent may be quite realistic.

The minimization of the logistics cost function occurs in two steps. First, we find optimum frequency values as a function of P. The appropriate first order conditions are

$$\frac{\partial TLC}{\partial F_c} = 1 - \frac{NT'_w(1 - P)}{F_c^2} = 0 \rightarrow F_c^* = \left(NT'_w(1 - P)\right)^{\frac{1}{2}} + 1 \quad (9)$$

$$\frac{\partial TLC}{\partial F_t} = C'_t\,\frac{NT'_w P}{F_t^2} = 0 \rightarrow F_t^* = \left(\frac{NT'_w P}{C'_t}\right)^{\frac{1}{2}} + 1 \quad (10)$$

Note that these conditions are valid for $0 < P < 1$. When $P = 1$, then F_c^* is clearly 0, as is Ft^* when $P = 0$.

Substituting these expressions into the logistics cost function, we obtain a function involving only P as a decision variable:

$$\begin{aligned} TLC^*(P) = {} & 2(T'_w N)^{\frac{1}{2}}[(C'_t P)^{\frac{1}{2}} + (1 - P)^{\frac{1}{2}}] \\ & + N[PS'_t + (1 - P)S'_c] \\ & + T'_a N\left[D_1\left(P^2 - \frac{3P}{2} + 1\right) + D_2\left(\frac{3}{4} - \frac{P}{2}\right)\right] \\ & + D_3 C'_t + D_4 \end{aligned} \quad (11)$$

$D_1 = 1$ when $P \geq ½$ and 0 otherwise,
$D_2 = 1$ when $P < ½$ and 0 otherwise,
$D_3 = 1$ when $P > 0$ and 0 otherwise, and
$D_4 = 1$ when $P < 1$ and 0 otherwise.

We are now concerned with minimizing this function. Before deriving the first order conditions, it is useful to plot the function in Equation 10. This is done in Figures 2 and 3 for different values of the various parameters. This exercise demonstrates that the minimum of the function can occur at one of three locations: at $P = 0$, at $P = 1$, or at $½ < P < 1$. No minimum can occur at $0 < P \leq ½$ because the second derivative of TLC is always negative in this interval. Thus, any interior extremum less than ½ must be a maximum and not a minimum. The intuitive explanation for this is that for $P <$ ½, any boundary has the same accessibility differential.

A local minimum does not always exist in the region where $P >$ ½. Although the second derivative is positive over most of the region between 0.5 and 1.0 in all of the cases shown, it becomes negative as P approaches 0.1, and sometimes the inflection point occurs before a minimum is reached. In other cases, the local minimum of the variant of Equation 1 that applies for $½ \leq P < 1$ (that is, where $D_1 = D_3 = D_4 = 1$) occurs when $P <$ ½.

The first order condition on P is

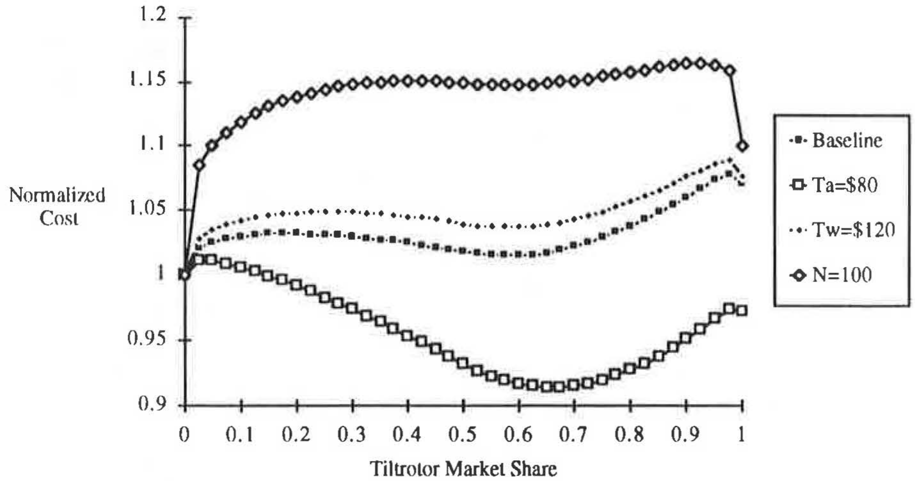

FIGURE 2 Total logistics cost versus tiltrotor market share under alternative demand scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

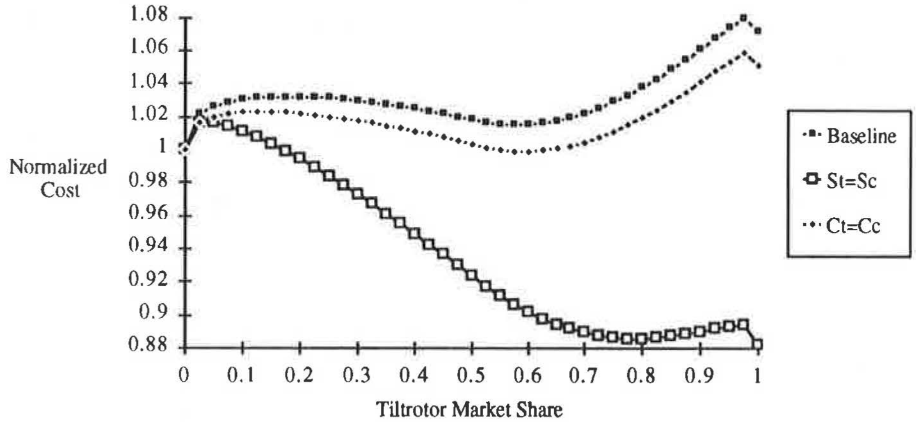

FIGURE 3 Total logistics cost versus tiltrotor market share under alternative tiltrotor cost scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

$$\begin{aligned}\frac{dTLC^*}{dP} &= N(S'_t - S'_c) - NT'_a\left(2P - \frac{3}{2}\right) \\ &\quad + (T'_wN)^{\frac{1}{2}}\left[\left(\frac{C'_t}{P}\right)^{\frac{1}{2}} - \left(\frac{1}{1-P}\right)^{\frac{1}{2}}\right] \\ &= 0 \rightarrow P* \\ &= \frac{3}{4} - K\left[\left(\frac{C'_2}{P^*}\right)^{\frac{1}{2}} - \left(\frac{1}{1-P^*}\right)^{\frac{1}{2}}\right] - \Delta S\end{aligned} \tag{12}$$

where

$$K = \left(\frac{T'_w}{4NT'^2_a}\right)^{\frac{1}{2}} \tag{13}$$

and

$$\Delta S = \frac{S'_t - S'_t}{2T'_a} \tag{14}$$

This condition is a necessary one for an interior minimum of TLC. As already observed, such an interior minimum need not exist, and if it does, it need not be the global minimum. For the tiltrotor to be economically feasible in the system being considered, it must have cost characteristics such that some $P^* > 0$ minimizes TLC. This may occur under one of two conditions: either $TLC^*(1) < TLC^*(0)$, or a local minimum of $TLC^*(\bullet)$ exists and that local minimum is less than $TLC^*(0)$. The first of these conditions implies that

$$\Delta S < \frac{1}{8} - 2K[(C'_t)^{\frac{1}{2}} - 1] - \frac{C'_t - 1}{2NT'_a} \tag{15}$$

The second condition is not so easily represented because the first order condition for P does not yield an explicit expression for P^*. It is, however, possible to define a condition under which the interior minimum does not exist. This occurs if the solution to Equation 12 occurs when $P \leq 0.5$. Substituting $P = 0.5$ into the first order condition, we obtain the inequality:

$$\Delta S > \frac{1}{4} - (2)^{\frac{1}{2}}K[(C'_t)^{\frac{1}{2}} - 1] \tag{16}$$

This defines a sufficient condition for there being no local minimum greater than 0.5; and hence a guarantee that such a minimum will not produce a lower cost solution than $TLC(0)$.

Comparing inequalities (Equations 15 and 16), note that for $C_t' > 1$, the right-hand side of Equation 16 is always greater. Thus, a sufficient condition for tiltrotor to be feasible (assuming $C_t' > 1$) is given by Equation 15. On the other hand, the inequality (Equation 16) defines a sufficient condition for the infeasibility of tiltrotor. The feasibility frontiers for the tiltrotor defined by inequalities Equation 15 and Equation 16 are plotted in Figure 4. Combinations of ΔS and C_t' to the inside of the inner curve are definitely feasible, whereas those to the northeast of the outer curve are definitely infeasible. Obviously, feasibility depends on the market conditions represented by T_a', T_w', and N. All three of these variables define the variable K (see Equation 13), which completely defines the outer curve. K and the quantity NT_a' define the inner curve. It can been seen in Figure 4 how a change in the value of T_a' causes the feasibility curves to shift.

Combinations of ΔS and C_t' between the two curves may or may not be feasible. There appears to be no simple representation of which will be the case. One could take Taylor series to arrive at a closed form approximation of the solution to Equation 12 and then use it to arrive at an inequality that precisely (subject to the Taylor approximation) defines the conditions under which the tiltrotor is feasible, but this appears to be more trouble than it is worth. It is far simpler to evaluate $TLC(0)$, $TLC(1)$, and $TLC(P^*)$ to determine which is the least. To do this, one can find P^* either by iterating (if P^* exists, the convergence is very rapid when one starts at $P = 3/4 - \Delta S$) or using a Taylor series.

The heuristic previously mentioned was used to trace the dependence of the cost-minimizing tiltrotor market share on different cost and demand factors, using the baseline scenario defined in Table 1. The minimizing tiltrotor market share was then plotted against tiltrotor cost per seat, S_t, for the baseline and for different scenarios in which one of the variables defining the baseline was set to an alternative value. The results are shown in Figures 5 and 6.

The baseline market share curve (which is plotted on both figures) indicates that when S_t is very close to the \$25 assumed for S_c, minimum cost is attained by having the tiltrotor serve 100 percent of the market. However, when S_t reaches \$27, the minimum cost solution becomes one in which the market

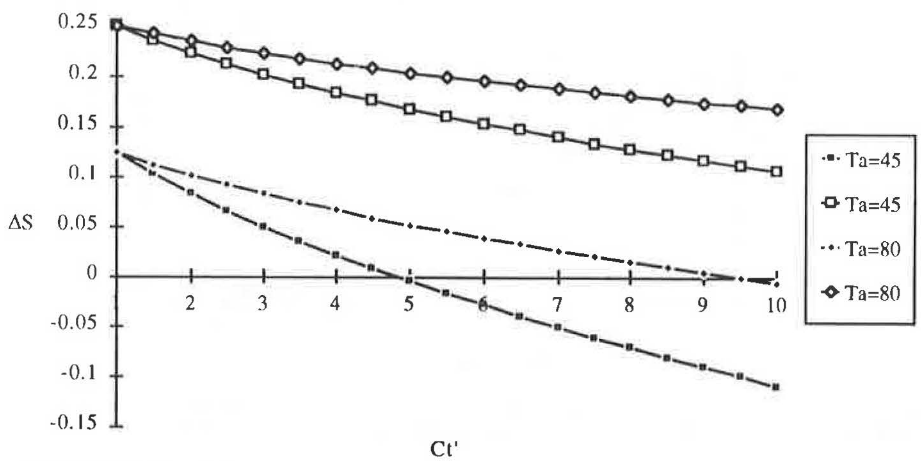

FIGURE 4 Feasibility boundaries under alternative access cost scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

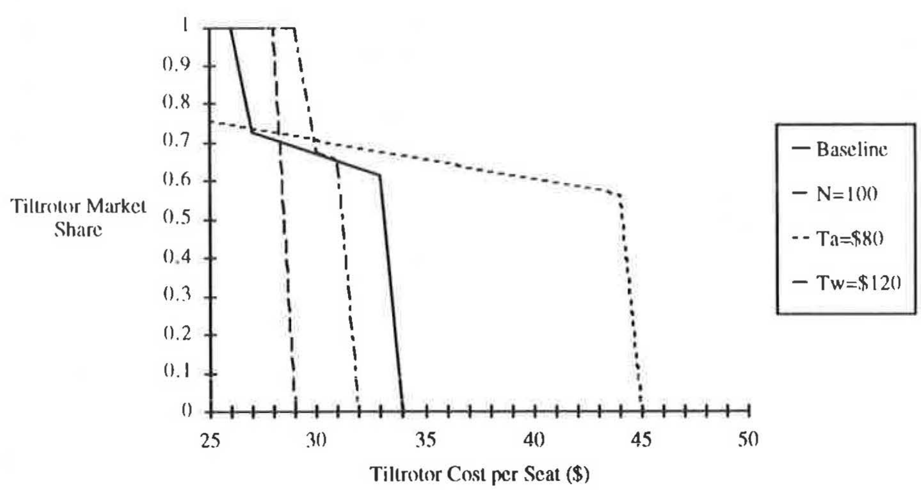

FIGURE 5 Market share versus cost per seat under alternative demand scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

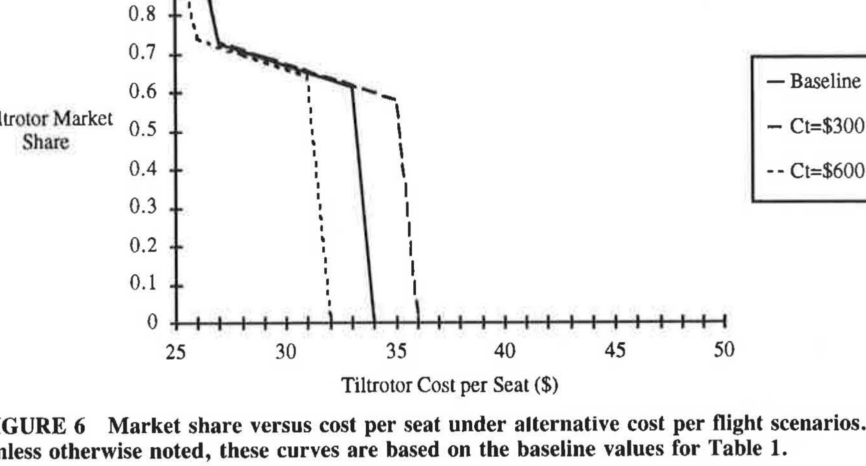

FIGURE 6 Market share versus cost per seat under alternative cost per flight scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

is shared. The optimum market share for tiltrotor then decreases roughly linearly, reflecting a case in which variation in ΔS drives the solution to Equation 12. When S_t reaches $34, 36 percent more than S_c, minimum cost is attained through exclusive use of turboprop aircraft.

The steeply sloped portions of the market share curves are not strictly accurate. Rather, the portions reflect discontinuities when the solution shifts from $P = 1$ to $P = P^*$ to $P = 0$. For the baseline case, these discontinuities occur between $26 and $27, and between $33 and $34, respectively.

The sensitivity of the market share curve to market size, access cost, and waiting cost is indicated in Figure 5. For a market of 100 passengers/day (as opposed to the 1,000/day assumed for the baseline), the cost-minimizing solution shifts directly from $P = 1$ to $P = 0$. For such a small market, competing services cannot be economically justified in the face of the waiting time reduction that could be attained from service consolidation. Changing the value of T_w from $60 to $120 also has the effect of favoring service monopoly, although in this case market sharing does minimize cost for a small range of tiltrotor cost per seat values. In contrast, a T_a value of $80 (compared with the $45 assumed for the baseline), extends the range over which market sharing is the minimum cost solution by increasing the benefit of assigning passengers to a nearby facility.

The sensitivity of the market share curve to tiltrotor cost per flight, C_t is shown in Figure 6. The major effect of changing this parameter is to shift the points where the solution shifts from $P = 1$ to $P = P^*$. The value of P^* itself is quite insensitive to variation in C_t. This again shows that, under the baseline assumptions, ΔS is the critical determinant of the solution to Equation 11.

DIAMOND-SHAPED CITY

We now alter the geometry of the problem so that the city, although still square, is now oriented diagonally to the rectangular street system so that the square looks like a diamond. The vertiport is still in the center of the city, but the airport is now located in a corner. The new geometry is indicated in Figure 7.

The new system has two advantages over the earlier one. First, if we draw an axis through the vertiport and the airport, it is clear that the width of the city diminishes as we move toward the latter. The effect is qualitatively similar to having the density of demand be higher near the center of the city, a condition we observe in the real world. Second, the new system is better suited to the analysis of multiple vertiports, discussed in the next section. The city-center vertiport in the diamond-shaped city thus provides a more suitable comparison with the multi-vertiport system.

The new problem differs from the old one only in how access cost varies with P. As before, the market area boundaries are straight lines perpendicular to a line connecting the airport and vertiport. In this case, one can show that the access cost is related to P by

$$PAC(P) = NT'_a\,(2)^{\frac{1}{2}}\left\{\frac{[8(1-P)^3]^{\frac{1}{2}}}{3} + \frac{P}{2} - \frac{1}{6}\right\} \qquad \left(\frac{1}{2} \leq P \leq 1\right)$$

$$PAC(P) = NT'_a(2)^{\frac{1}{2}}\left(\frac{2}{3} - \frac{P}{2}\right) \quad \left(0 \leq P < \frac{1}{2}\right) \tag{17}$$

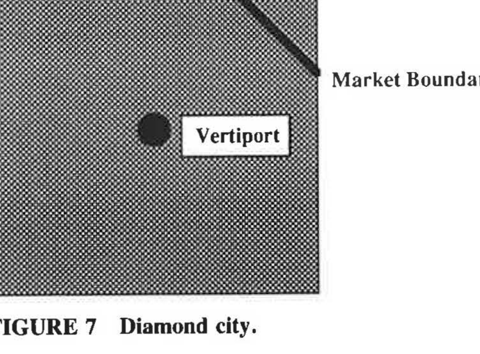

FIGURE 7 Diamond city.

As before, the solution proceeds by first optimizing the frequency variables and deriving a univariate function between *TLC* and *P*. In this case, the function is

$$TLC*(P) = 2(T'_wN)^{\frac{1}{2}}[(C'_tP)^{\frac{1}{2}} + (1-P)^{\frac{1}{2}}] + N[PS'_t + (1-P)S'_c] + NT'_a(2)^{\frac{1}{2}}\left\{D_1\left(\frac{[8(1-P)^3]^{\frac{1}{2}}}{3} + \frac{P}{2} - \frac{1}{6}\right) + D_2\left(\frac{2}{3} - \frac{P}{2}\right)\right\} + D_3C'_t + D_4 \qquad (18)$$

The relationship between *TLC* and *P* for the diamond-shaped city and the square city under the same assumed values for the demand and cost variables is indicated in Figure 8. The advantage of tiltrotor vertiport service is substantially greater under the diamond geometry, with the minimum cost occurring when *P* is roughly 0.8. This difference reflects the greater accessibility advantage afforded by a center-city location under the diamond geometry.

This function has the same possible minima as the one for the square city. We can again establish the conditions under which the first order and second order conditions on *P* are satisfied only for $P \leq 0.5$, as well as the conditions under which $TLC(0) \leq TLC(1)$.

In the case of the diamond shaped boundary, the first order condition on *P* is

$$N(S'_t - S'_c) - NT'_a(2)^{\frac{1}{2}}\left([2(1-P)]^{\frac{1}{2}} - \frac{1}{2}\right) + (T'_wN)^{\frac{1}{2}}\left[\left(\frac{C'_t}{P}\right)^{\frac{1}{2}} - \left(\frac{1}{1-P}\right)^{\frac{1}{2}}\right] = 0 \rightarrow$$

$$P = 1 - 2\left\{\frac{1}{4} + \Delta S + \frac{K}{(2)^{\frac{1}{2}}} \times \left[\left(\frac{C'_t}{P}\right)^{\frac{1}{2}} - \left(\frac{1}{1-P}\right)^{\frac{1}{2}}\right]\right\}^2 \qquad (19)$$

The outer feasibility boundary for the diamond-shaped city is thus the same as that for the square city, except that T_a is replaced with $T_a(2)^{\frac{1}{2}}$ and K with $K/(2)^{\frac{1}{2}}$. The inner feasibility boundary is given by

$$\Delta S < \frac{1}{6} - \frac{2K}{(2)^{\frac{1}{2}}}\left[(C'_t - 1)^{\frac{1}{2}}\right] - \frac{C'_t - 1}{2(2)^{\frac{1}{2}}NT'_a} \qquad (20)$$

These feasibility boundaries for both the square and diamond-shaped cities are plotted in Figure 9. Both boundaries are shifted outward for the latter, reflecting the greater values of a center-city location in this geometry.

Following the same procedure as for the square city, curves relating tiltrotor market share to various demand and cost parameter values were developed. The results, along with the comparable ones for the square city, are shown in Figure 10. Again, the greater accessibility advantage under the diamond geometry is evident.

MULTIPLE VERTIPORTS

The foregoing analyses assume systems consisting of one vertiport and one airport. In these systems, the advantage of the tiltrotor technology derives from the ability to centrally locate the tiltrotor terminal facility. A second potential advantage of the technology is that, because vertiports are relatively inexpensive, tiltrotor services could be offered from multiple

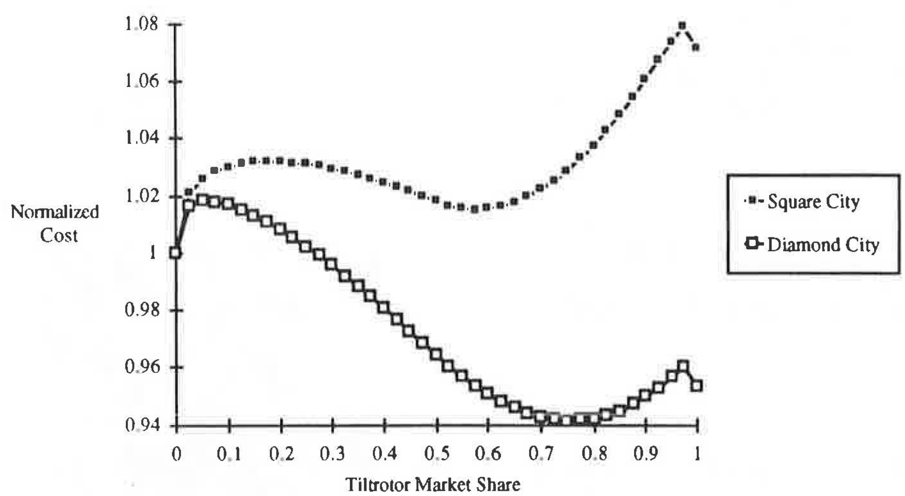

FIGURE 8 Total logistics cost versus tiltrotor market share, square- and diamond-shaped cities. Unless otherwise noted, these curves are based on the baseline values for Table 1.

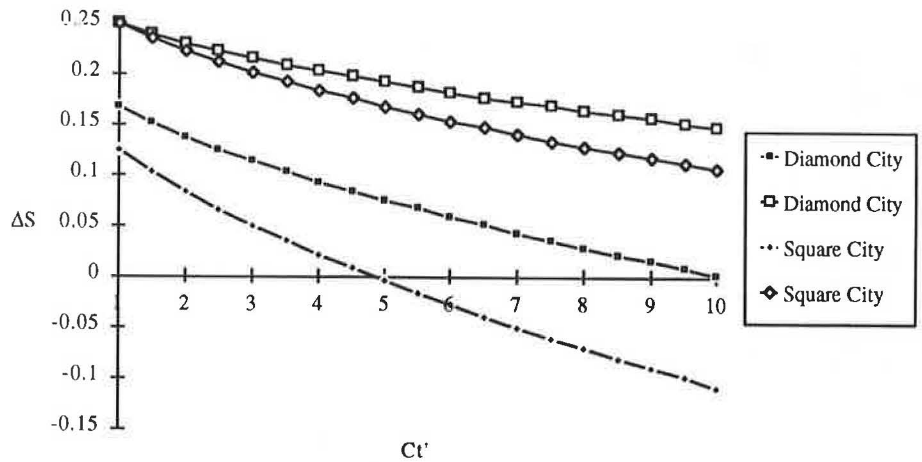

FIGURE 9 Feasibility boundaries for square- and diamond-shaped cities. Unless otherwise noted, these curves are based on the baseline values for Table 1.

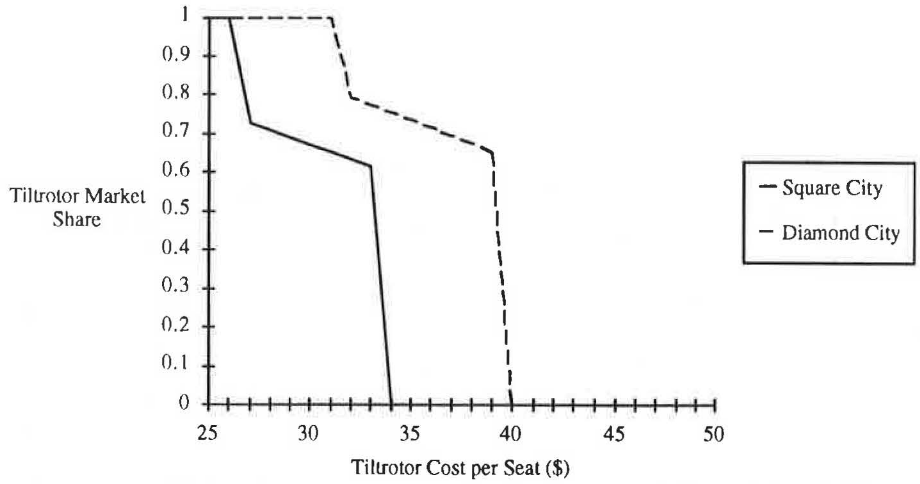

FIGURE 10 Market share versus cost per seat for square- and diamond-shaped cities. Unless otherwise noted, these curves are based on the baseline values for Table 1.

vertiports throughout an urban region. We now turn to the analysis of a system in which the number of vertiports is included as a decision variable.

The multiple vertiport system is depicted in Figure 11. The shape of the city, location of the airport, and geometry of the street network are the same as for the diamond city, single vertiport case discussed in the last section. In addition to having more than one vertiport, the system is different in that the market area of the airport is depicted as square rather than triangular. As will be elaborated later in this paper, the shape is not entirely realistic, but is assumed in order to arrive at an approximation of the total logistics cost.

To develop the total logistics cost function, imagine first that $P = 1$ so that the entire city is served by tiltrotor aircraft. If there were one vertiport, its optimal location would obviously be the center of the city. If there were four vertiports, it is equally clear that the optimal location pattern would be to locate the vertiports along each diagonal at the center of each of the city's four quadrants. The optimal locations for nine vertiports would similarly be at the center of the nine squares created by trisecting the city along each direction. Thus, when the number of vertiports, n, is a perfect square, the access cost (assuming $P = 1$) would be

$$PAC_t = \frac{N(2)^{\frac{1}{2}}T'_a}{3(n)^{\frac{1}{2}}} \tag{21}$$

When n is not a perfect square, Equation 21 is only an approximation. The actual access cost will be somewhat higher in these cases because market areas can no longer be perfect squares. For example, when $n = 2$, optimal location of vertiports yields an access distance of about 12 percent greater than that predicted by Equation 21. As n increases, the approximation becomes more accurate.

Waiting costs for the multiple vertiport system are the same as for the earlier ones, assuming that tiltrotor frequency is the same from each vertiport. Operating costs differ from the earlier cases only in that a given tiltrotor frequency F_t entails $n \cdot F_t$ flights. Thus, the total logistics cost function for the

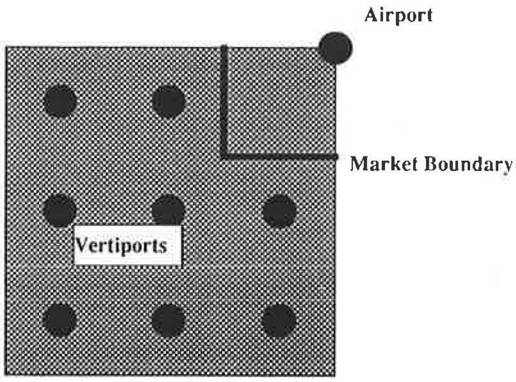

FIGURE 11 Diamond city with multiple vertiports.

multiple vertiport system, assuming $P = 1$, is

$$TLC_t(n, F_t) = nC'_tF_t + N\left(S'_t + \frac{T'_w}{F_t - 1} + \frac{(2)^{\frac{1}{2}}T'_a}{3(n)^{\frac{1}{2}}}\right) \quad (22)$$

Equation 22 excludes costs associated with the provision of vertiports. This is because our model is concerned with a specific O-D market. Vertiports, on the other hand, would serve many such markets. Vertiport costs thus represent a joint cost that cannot be allocated to a specific market. To incorporate these costs, one would have to model all of the potential tiltrotor markets involving the city simultaneously. In general, we expect unit vertiport costs to be small enough to justify excluding them, thereby allowing modeling of individual markets.

The first order condition on F_t for this system is

$$\frac{\partial TLC_t}{\partial F_t} - nC'_t - \frac{NT'_w}{(F_t - 1)^2} = 0 \rightarrow F_t = \left(\frac{NT'_w}{nC'_t}\right)^{\frac{1}{2}} + 1 \quad (23)$$

Plugging the optimum value for F_t into Equation 22, we obtain

$$TLC^*_t(n) = 2(nNT'_wC'_t)^{\frac{1}{2}} + nC'_t + N\left(S'_t + \frac{(2)^{\frac{1}{2}}T'_a}{3(n)^{\frac{1}{2}}}\right) \quad (24)$$

First order conditions on n require

$$\frac{dTLC^*_t}{dn} = \left(\frac{NT'_wC'_t}{n}\right)^{\frac{1}{2}} + C'_t - \frac{N(2)^{\frac{1}{2}}T'_a}{6(n^3)^{\frac{1}{2}}} = 0 \quad (25)$$

This is a cubic equation in the square root of $1/(n)^{\frac{1}{2}}$.

Solving Equation 25 for our baseline case, we obtain an optimal value for n of approximately 2. TLC^* and its various components against n are plotted in Figure 12. Observe that the cost curve is relatively flat, with $TLC^*(10)$ only about 10 percent greater than the minimum $TLC^*(2)$.

In Figure 13 the sensitivity of n^* to market size and to other demand and cost factors is indicated. n^* increases roughly with the square root of N, an approximation that improves as the second term of Equation 25 becomes small in relation to the other two terms. A similar approximation holds for the relationship between n^* and T'_a. On the other hand, n^* is approximately inversely related to the square roots and T'_w and C'_t, when the second term of Equation 25 is relatively small.

The results shown in Figure 13 suggest that under most circumstances, cities would require 10 or fewer vertiports. For many cities, the optimal number of vertiports is more likely to be 4 or fewer. The fact that vertiport facility costs are excluded from this analysis means that these estimates are, if anything, on the high side.

We now consider competition between the multiple vertiport system and turboprop service from a single airport, again located at the corner of the city. A closed form approximation of the total logistics cost for the system, consisting of the multiple vertiports and the single airport, can be derived on the basis of Figure 11. From this, we observe that when n is a perfect square and $(1 - P) = m/n$, where m is a perfect

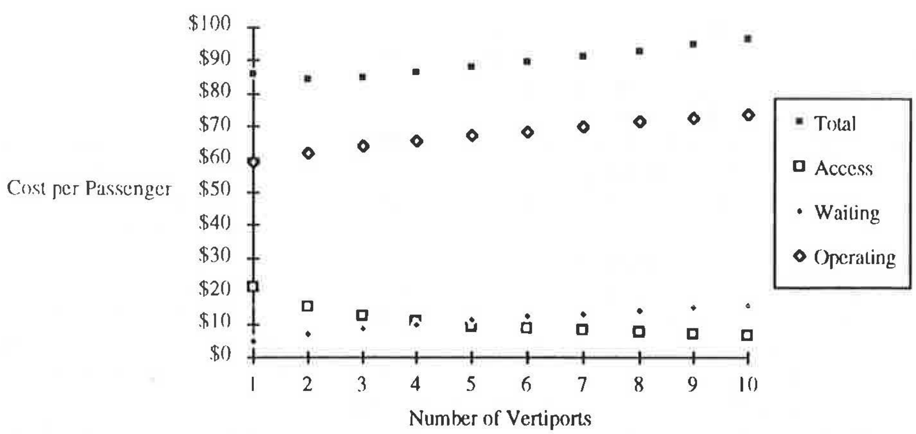

FIGURE 12 Cost per passenger versus number of vertiports. Unless otherwise noted, these curves are based on the baseline values for Table 1.

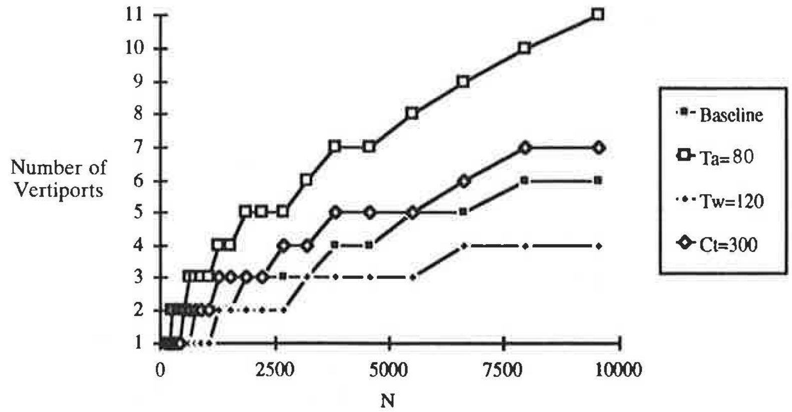

FIGURE 13 Optimum number of vertiports under alternative scenarios. Unless otherwise noted, these curves are based on the baseline values for Table 1.

square, the airport market area can be approximated as a square consisting of the m tiltrotor market areas closest to the airport. In effect, tiltrotor service from these m vertiports disappears, while the service and market areas of the remaining n - m vertiports are unaffected. (In fact, market areas will be affected, with those of the remaining vertiports closest to the airport elongated in the direction of the airport But in light of the approximate nature of this analysis, requiring all vertiports to have market areas of the same shape is a worthwhile simplification.)

It is therefore clear that, to a first approximation, the total logistics cost per passenger for the vertiport system is unaffected by the market share of that system. Let this cost be TLC_t^{**}, where the ** indicates that the cost assumes optimization of both frequency and the number of vertiports per area of the city. The total logistics cost of the vertiport-airport system then becomes

$$TLC(TLC_t^{**},F_c,P) = P \cdot TLC_t^{**} + \frac{2(2)^{\frac{1}{2}}T'_aN[(1-P)^3]^{\frac{1}{2}}}{3} + NS'_c(1-P) + \frac{T'_wN(1-P)}{F'_c - 1} + F'_c \qquad (26)$$

When m or n are not perfect squares, Equation 25 is clearly an approximation, the accuracy of which increases as do n and P.

As in the previous examples, minimization of Equation 21 proceeds by solving the first order condition on frequency and using the results to define a relationship between TLC and P. The relationship obtained is

$$TLC^*(TLC_t^{**},P) = P \cdot TLC_t^{**} + \frac{2(2)^{\frac{1}{2}}T'_aN[(1-P)^3]^{\frac{1}{2}}}{3} + NS'_c(1-P) + 2[N(1-P)T'_w]^{\frac{1}{2}} + 1 \qquad (27)$$

This resulting first order condition on P is

$$\frac{dC}{dP} = T'_a(2)^{\frac{1}{2}}N(1-P)^{\frac{1}{2}} - 2\left(\frac{NT'_w}{1-P}\right)^{\frac{1}{2}} + N(S'_c - C_t) = 0 \qquad (28)$$

We should also be concerned with the second order condition:

$$\frac{d^2C}{dP^2} = \frac{NT'_a(2)^{\frac{1}{2}}}{2(1-P)^{\frac{1}{2}}} - \left(\frac{NT'_w}{(1-P)^3}\right)^{\frac{1}{2}} > 0 \qquad (29)$$

For relatively large N (100 or more), the second order condition on P will be met unless P is quite close to 1. As in the previous examples, tiltrotor market share can be optimized by evaluating the logistics cost function at $P = 0$, $P - 1$ and $P = P^*$.

The feasibility condition for tiltrotor is obtained by substituting $P = 0$ into Equation 26. Note, however, that application of this condition requires prior determination of the value of TLC_t. Thus, while it is straightforward to use these results to evaluate the feasibility of a multiple vertiport tiltrotor system, it is not possible to construct the type of feasibility boundaries that were used in the single vertiport cases.

Furthermore, recall that under baseline conditions we found that the optimal number of vertiports per area of the city is 2. Unfortunately, the approximations we have used to estimate P^* are not very accurate when n is so small. As a practical matter, cases involving very small n (e.g., less than 4), should be handled individually rather than with the approximations used here. Optimal vertiport locations would have to be determined explicitly, and market areas would have more complicated shapes. Consequently, these problems would not yield meaningful closed form solutions, but would instead have to be handled numerically.

Such techniques are beyond the scope of the present research. Instead, we examine a case with very high demand (8,500/day), access cost at the upper limit of the plausible range, and waiting cost and per flight costs at the low ends of their sets of likely values. Under these conditions, the optimal number of vertiports is 16. Although this may well exceed the present requirements of any city, it does provide a situation in which the approximations derived as described will be accurate.

Tiltrotor market share against tiltrotor cost per seat for both the single and multiple vertiport cases under such conditions are plotted in Figure 14. Multiple vertiports clearly result in a higher market share for tiltrotor. However, the market share difference is less than 0.1 over most of the range of per seat costs. Only at very high costs per seat, when tiltrotor market share goes to 0 for the single vertiport case, is there a substantial difference. The result is somewhat surprising in light of the large number of facilities in the multiple vertiport system, and the extreme assumptions concerning access and waiting cost used to generate that large number.

As a final basis of comparison between the single and multiple vertiport cases, optimal aircraft market size under various traveler and operator cost conditions is plotted in Figures 15 and 16. Under baseline assumptions in the multiple vertiport case, optimal size ranges from just over 20 (for a 100-passenger/day market), to 140 for market with 10,000 passenger trips/day. In the case of the single vertiport system, optimal size is undefined for low values of N, because, under baseline assumptions, tiltrotor aircraft would have zero share of markets of this size range. When N reaches 500 or so, tiltrotor service becomes feasible with an optimal aircraft size of 60 to 70 seats. At 10,000 passengers/day, the optimal size reaches that of a widebody aircraft—between 250 and 300 seats.

Tiltrotor studies to date have focused on tiltrotor aircraft in the 20- to 40-seat size range, with 75 seats as an upper limit. Under our baseline assumptions, aircraft in the upper portion of this size range would be appropriate for markets of 1,000 passengers or fewer in a multiple vertiport system, or 500 passengers in a single vertiport system. In view of the relative insensitivity of total logistics cost to the number of vertiports (see Figure 10), larger markets could be served without great compromise in system performance by operating smaller than optimum aircraft out of more than the optimum number of vertiports. The single vertiport system lacks this flexibility, and its performance (competitiveness with turboprop service) would be greatly impaired if an optimally sized aircraft is unavailable.

We do not present optimum sizes for the turboprop aircraft. In general, sizes are smaller than those of tiltrotors under the single vertiport scenario, because cost per flight (C_c) is smaller,

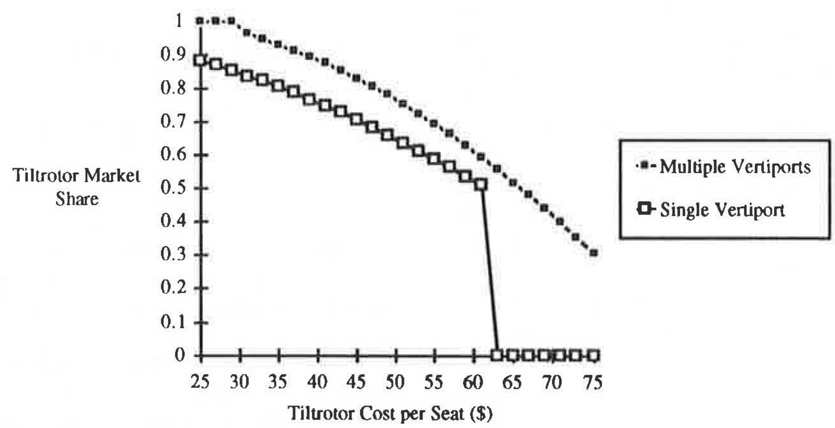

FIGURE 14 Market share versus cost per seat for single and multiple vertiports. Unless otherwise noted, these curves are based on the baseline values for Table 1.

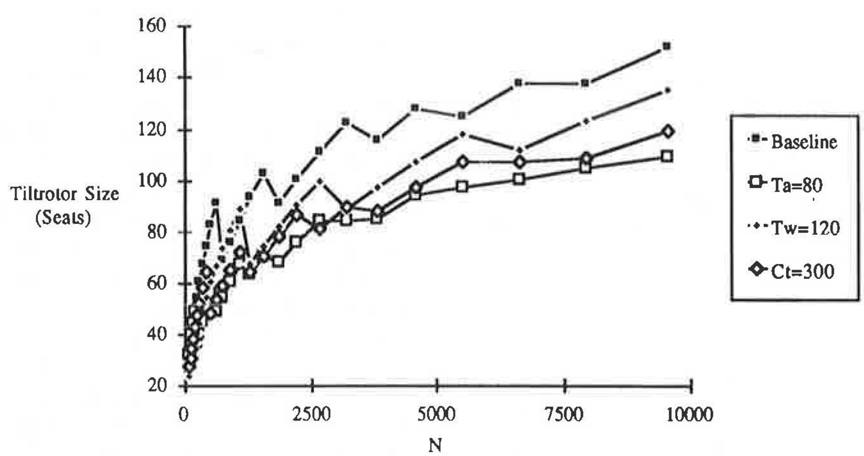

FIGURE 15 Optimum tiltrotor size, multiple vertiport case. Unless otherwise noted, these curves are based on the baseline values for Table 1.

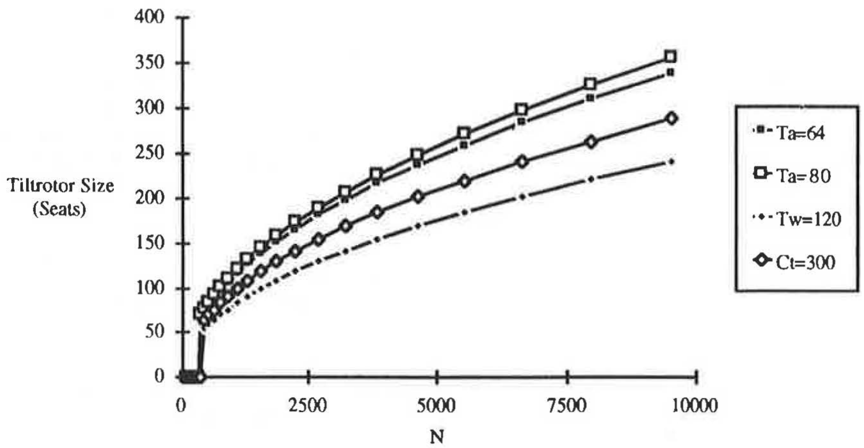

FIGURE 16 Optimum tiltrotor size, single vertiport case. Unless otherwise noted, these curves are based on the baseline values for Table 1.

and larger than tiltrotor sizes under the multiple vertiport scenario. The size constraints of turboprop technology are comparatively unimportant, however, because of the availability of jet aircraft for missions for which turboprops are too small.

CONCLUSION

We have explored the operating cost, passenger cost, and market size conditions under which tiltrotor technology would and would not be feasible in scheduled passenger service. For single vertiport systems, these conditions can be represented as functional relationships between the difference in operating cost per seat and the ratio of operating cost per flight for tiltrotor and turboprop aircraft. These functional relationships include the other relevant variables as parameters.

The accessibility advantage of tiltrotor technology is substantial, and enables the technology to be feasible even when its costs substantially exceed those of fixed-wing turboprop aircraft. A hypothetical tiltrotor technology, in which costs (both per seat and per flight) exceeded those of turboprops by 40 percent, would capture over 70 percent of the market under the diamond city scenario and "best guesses" concerning passenger access and waiting costs. If multiple vertiports are permitted, the tiltrotor technology becomes somewhat more competitive.

The analysis suggests that sizing of tiltrotor aircraft is an important issue. The 31-seat civil version favored by Boeing (*2*) is too small for markets of more than a couple of hundred daily passengers. Further, in such low-volume markets, it is unlikely that tiltrotor service would be viable unless it completely replaced conventional service. Under the multiple vertiport system, much of the operating cost disadvantage of these small aircraft can be made up by the access cost reduction of multiple vertiport service. Undersized tiltrotor aircraft would be a substantial detriment in a single vertiport system, however. In any case, the prospects for tiltrotor technology would be substantially improved if a larger version were developed.

No single size of tiltrotor aircraft can adequately serve all markets. The multiple vertiport system again helps resolve this dilemma, by introducing the number of vertiports from which service is offered as a degree of freedom that can respond to market size variation. Nonetheless, the ability to "stretch" tiltrotor aircraft could be an important advantage, particularly given that this ability exists for the competing technology.

There are several limitations to this analysis that require attention in subsequent research. The reliance on analytical models with closed form representations has, it is hoped, allowed insight to the trade-offs and sensitivities involved that numerical techniques frequently fail to provide. Yet it is clear that the latter could allow relaxation of the many assumptions and improve the accuracy of the many approximations that the analytical approach necessitates. This is particularly obvious in the case in which there are multiple but "few" vertiports. Other assumptions that could be eliminated under a numerical approach include the uniform distribution of preferred travel times throughout the day, and that all trips from a given point must use the same vertiport.

Our treatment has ignored issues related to market segmentation. Clearly, some travelers value waiting and travel time more than others. Just as travelers' origins and destinations are distributed throughout the physical space of the city, and their preferred travel times spread throughout the day, so are they also distributed in "time valuation space." A more detailed analysis would explicitly incorporate this variation, and might well extend the conditions under which tiltrotor service would be economically viable in scheduled service. In focusing on the average traveler, our analysis has been directed at the potential of tiltrotor technology to compete in the main market rather than its ability to occupy smaller market niches.

Finally, as noted at the outset, our analysis has focused on the "planner's solution" to the systems we have considered. In fact, passengers and airlines will make choices that optimize the system only when their individual objects are completely aligned with those of the system. This raises issues of pricing and profit that require further study.

ACKNOWLEDGMENTS

This research was carried out while the author was a participant in the National Aeronautics and Space Administration (NASA) Stanford American Society for Engineering Education Summer Faculty Fellowship Program. The author would like to thank Larry Alton and John Zuk, Civil Technology Office, NASA Ames Research Center, for their generous support and assistance.

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Publication of this paper sponsored by Committee on Aviation Economics and Forecasting.