Lateral Clearance to Vision Obstacles on Horizontal Curves

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Evaluation of the sight distance adequacy on highway horizontal curves with single or multiple obstacles requires determination of the minimum sight distance on the curve. The current method of establishing such a minimum is to plot the sight distance profile for the given curve and obstacle. Approximate relationships have been developed for establishing the sight distance profile. However, there is no explicit, exact solution available for determining the minimum sight distance on the curve. Exact formulas have been derived to relate the available sight distance to the circular curve parameters, lateral clearance of the obstacle, its location along the curve, and the locations of observer and object. These relationships are then used to derive closed-form solutions of the minimum sight distance, S_m. To facilitate practical use, values of S_m are established for typical ranges of the curve parameters, lateral clearance, and obstacle location. Values of the maximum lateral clearance, which is required in design, are also provided. The methodology and results should be valuable in the operational and cost-effectiveness analysis of highway locations with restricted sight distances.

The sight distance on highway horizontal curves may be restricted by such physical features as longitudinal barriers, cut slopes, foliage, and other structures. For safe operations, the available sight distance at any point on the traveled way must be greater than the sight distance needed for stopping, passing, or decision at complex locations. The available sight distance is a function of the horizontal curve parameters, locations of the observer and object, and the location of the vision-limiting obstacle inside the curve.

The stopping sight distance (SSD), presented by AASHTO (*1–4*), is one of the basic considerations in the design of highways. Design values for SSD applicable to all highways are presented by AASHTO. A new approach to SSD that considers the functional classifications of highways was recently presented by Neuman (*5*). Design values for passing sight distance (PSD) involving passenger cars on two-lane highways are presented in AASHTO's 1984 *Policy on Geometric Design of Highways and Streets* (Green Book) (*4*). Design values for PSD for all combinations of passing involving a passenger car and a truck have been developed by Harwood and Glennon (*6*). These design values are based on a model developed by Glennon (*7*) that logically accounts for the kinematic relationships among the passing, passed, and opposing vehicles and explicitly contains vehicle-length variables. Design values for decision sight distance (DSD) are presented by AASHTO (*4*) and Neuman (*5*), and their usefulness and application have been evaluated by McGee (*8*).

A number of models exist that relate the available sight distance and the lateral clearance on horizontal curves. AASHTO presents a model that relates the sight distance, S_m, and maximum lateral clearance, M, for $S_m \leq L$, where L is the length of the curve (*4*). The case $S_m > L$ is not presented by AASHTO. When either the vehicle or vision obstacle is situated near the ends of the curve, the less clearance is needed. The studies by Olson et al. (*9*), Neuman and Glennon (*10*), and Glennon (*11*) show that when the vehicle is on the tangent within a distance S from the point of curvature, PC, the maximum lateral clearance needed varies from 0 to M (when the vehicle is at PC). The required lateral clearance for all points within $S/2$ beyond PC is also less than M. For these situations, AASHTO recommends the use of a graphical procedure or the curves empirically developed by Raymond (*12*).

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To eliminate the need for the graphical procedure, Waissi and Cleveland (*13*), on the basis of the results of the NCHRP report by Olson et al. (*9*), derived approximate relationships that relate the available sight distance to the horizontal curve parameters, locations of observer and road object, location of obstacle, and lateral clearance to a single obstacle to vision located inside the curve. Relationships for determining the maximum lateral clearance for $S_m > L$ are also presented.

For a given obstacle on the curve, the available sight distance varies as the observer moves along the tangent and curve. Clearly, there is a minimum value of sight distance on the traveled path that determines the adequacy of sight distance on the curve. There is no explicit, exact solution available for determining this minimum sight distance. The purpose of this paper is threefold:

1. To derive exact relationships for determining the available sight distance for arbitrary locations of the observer and obstacle,
2. On the basis of the preceding, to develop relationships for determining the minimum sight distance, and
3. To establish evaluation and design values for practical use.

THEORETICAL DEVELOPMENT

Relationships for determining the minimum sight distance, S_m, are developed for a single obstacle located on the inside of a simple horizontal curve between PC and PT (point of tangency). Both the observer and object are assumed to be located on the centerline of the inside lane. Three cases are considered:

- Case 1: Observer before PC and object beyond PT,

- Case 2: Observer before PC and object on curve, and
- Case 3: Observer and object on curve.

Case 1: Observer Before PC and Object Beyond PT

The geometry of this case is shown in Figure 1. As the observer moves toward PC, the available sight distance decreases, reaches a minimum value, and then increases again. The lateral clearance requirements should be based on this minimum value. In Figure 1, x_1 is the distance from the observer to PC, and x_2 is the distance from the object to PT.

Available Sight Distance

The available sight distance is given by

$$S = L + x_1 + x_2 \tag{1}$$

where S equals available sight distance, and L equals curve length.

With the law of sines for triangles $abPT$ and $acPC$, x_1 and x_2 can be expressed in terms of the angles θ_1 and θ_2, shown in Figure 1, as

$$x_1 = m_1 \sin\theta_1/\sin(\theta_1 + \alpha) \tag{2}$$

$$x_2 = m_2 \sin\theta_2/\sin(\theta_2 + \beta) \tag{3}$$

where

m_1 = distance from the obstacle to PC,
m_2 = distance from the obstacle to PT,
α = angle at PC between the tangent and the line to the obstacle, and
β = angle at PT between the tangent and the line to the obstacle.

Note that in Equation 2, $\sin(\theta_1 + \alpha) = \sin(180 - \theta_1 - \alpha)$, and similarly for Equation 3. These four elements, which are constant for a given curve and obstacle, are computed using triangles $oaPC$ and $oaPT$ as follows:

$$m_1 = [R^2 + (R - m)^2 - 2R(R - m)\cos I_1]^{1/2} \tag{4}$$

$$m_2 = [R^2 + (R - m)^2 - 2R(R - m)\cos(I - I_1)]^{1/2} \tag{5}$$

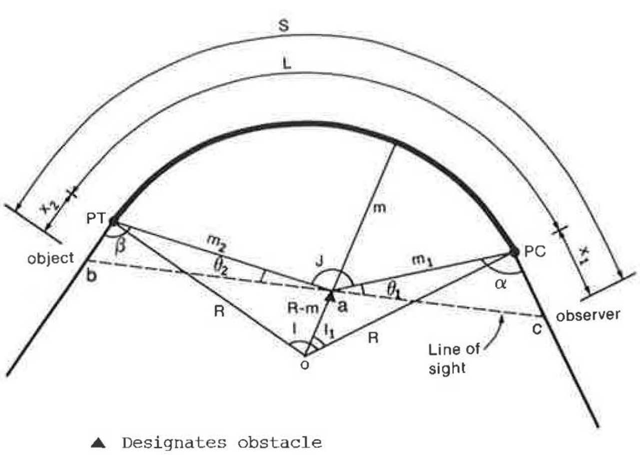

FIGURE 1 Case 1: Observer before PC and object beyond PT.

$$\alpha = 90° + \cos^{-1}\{[-(R - m)^2 + R^2 + m_1^2]/2Rm_1\} \tag{6}$$

$$\beta = 90° + \cos^{-1}\{[-(R - m)^2 + R^2 + m_2^2]/2Rm_2\} \tag{7}$$

where

R = curve radius,
m = lateral clearance between the centerline of the inside lane and obstacle,
I_1 = central angle from PC to the obstacle, and
I = central angle from PC to PT.

The angles θ_1 and θ_2 are related by

$$\theta_2 = 180° - \theta_1 - J \tag{8}$$

Since $\angle\, aPCo = \alpha - 90°$ and $\angle\, aPTo = \beta - 90°$, the angle J is obtained as follows:

$$J = \alpha + \beta + I - 180° \tag{9}$$

Substituting θ_2 of Equation 8 into Equation 3, the available sight distance of Equation 1 can be written as

$$S = L + [m_1 \sin\theta_1/\sin(\theta_1 + \alpha)] + [m_2 \sin(\theta_1 + J)/\sin(\theta_1 + J - \beta)] \tag{10}$$

in which the curve length L equals $R\pi I/180$.

Condition for S_m

Differentiating Equation 10 with respect to θ_1 and equating $dS/d\theta_1$ to zero gives

$$\sin(\theta_1^* + \alpha)/\sin(\theta_1^* + J - \beta) = (m_1 \sin\alpha/m_2 \sin\beta)^{1/2} \tag{11}$$

in which θ_1^* is the critical angle corresponding to the minimum sight distance, S_m. The derivation of Equation 11 is included in Appendix A. A successive approximation method for solving Equation 11 to determine θ_1^* is given in Appendix B. Note that Equation 11 implies that θ_1^* must be greater than $(\beta - J)$. For equal or smaller values, the line of sight from the observer to the obstacle does not intersect with the tangent beyond PT. After determining θ_1^*, S_m is computed by substituting θ_1^* into Equation 10.

If the obstacle lies at the midpoint of the curve ($I_1/I = 0.5$), then $m_1 = m_2$ and $\alpha = \beta$. With these values, Equation 11 yields $\theta_1^* + \alpha = 180 - (\theta_1^* + J - \beta)$ (note that $\alpha \neq J - \beta$ is based on Equation 9). Thus,

$$\theta_1^* = (180° - J)/2 \quad \text{for} \quad I_1/I = 0.5 \tag{12}$$

which implies that $x_1 = x_2$, as expected.

Case 2: Observer Before PC and Object on Curve

In this case, the observer is on the tangent at a distance x_1 from PC, and the object is on the curve at a distance L_1 from PC (Figure 2).

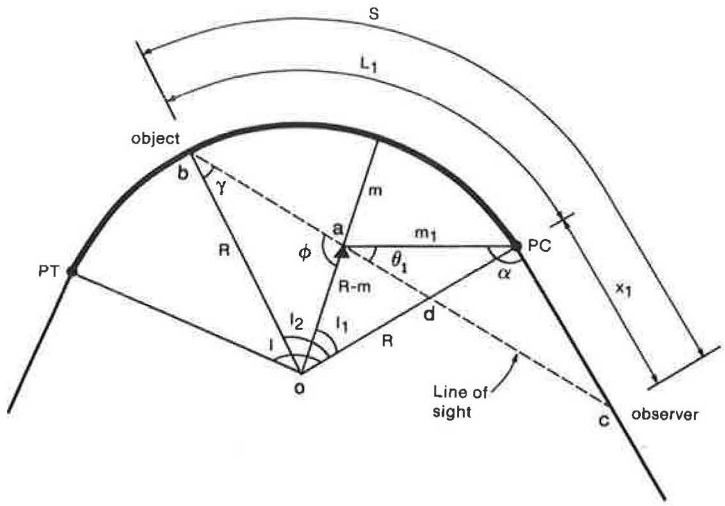

FIGURE 2 Case 2: Observer before PC and object on curve ($I_1/I \leq 0.5$).

Available Sight Distance

The available sight distance is given by

$$S = L_1 + x_1 \quad (13)$$

where L_1, the distance from PC to the object, is given by

$$L_1 = RI_2 \quad (14)$$

and x_1 is given by Equation 2 (m_1 and α are given by Equations 4 and 6). I_2 is the central angle between PC and the object (in radians). The angle $oda = \theta_1 + \alpha - 90°$. Therefore, using triangle obd, I_2 is obtained:

$$I_2 = 270° - \gamma - \alpha - \theta_1 \quad (15)$$

Using triangles oab and oad, respectively, γ and ϕ are obtained:

$$\gamma = \sin^{-1}[(R - m)\sin\phi/R] \quad (16)$$

$$\phi = I_1 + \alpha + \theta_1 - 90° \quad (17)$$

Now Equation 13 can be written as follows:

$$S = RI_2 + [m_1 \sin\theta_1/\sin(\theta_1 + \alpha)] \quad (18)$$

Condition for S_m

Differentiating Equation 18 with respect to θ_1 and equating $dS/d\theta_1$ to zero give (Appendix A)

$$\frac{m_1 \sin\alpha}{R \sin^2(\theta_1^* + \alpha)} - \frac{(R - m)\sin(\theta_1^* + \alpha + I_1)}{[R^2 - (R - m)^2 \cos^2(\theta_1^* + \alpha + I_1)]^{1/2}} = 1 \quad (19)$$

Solving Equation 19 by successive approximations gives the critical angle θ_1^* (Appendix B). S_m is then computed by substituting θ_1^* into Equation 18.

Because of the symmetry of the horizontal curve, Case 2 may occur only when I_1/I is less than 0.5. For I_1/I greater than 0.5, the minimum sight distance occurs when the observer is on the curve and the object is beyond PT. The solution of this situation is also given by Equations 13–19 after switching the positions of PC and PT, and of the observer and object.

Case 3: Observer and Object on Curve

In Case 3, both observer and object are on the curve. Figure 3 shows the geometry of this case. Let I_3 denote the central angle from the observer to the obstacle and I_4 the central angle from the obstacle to the perpendicular line od. Using triangle ace, $\tan I_4 = ae/ec$. But $ae = (R - m) - oe$. From triangle oec, $ec = R \sin I_3$ and $oe = R \cos I_3$. Thus,

$$I_4 = \tan^{-1}[(R - m - R\cos I_3)/(R \sin I_3)] \quad (20)$$

where I_3, which equals $I_1 - \angle coPC$, is given by

$$I_3 = I_1 - (180x_1/\pi R) \quad (21)$$

and x_1 is the distance along the curve between the observer and PC

Available Sight Distance

The available sight distance is given by (Figure 3)

$$S = 2R(I_3 - I_4) \quad (22)$$

where I_3 and I_4 are in radians. Substituting for I_4 from Equation 20 into Equation 22 gives

$$S = 2R\{I_3 - \tan^{-1}[(R - m - R\cos I_3)/(R \sin I_3)]\} \quad (23)$$

Condition for S_m

Differentiating Equation 23 with respect to I_3 and equating dS/dI_3 to zero gives (Appendix A)

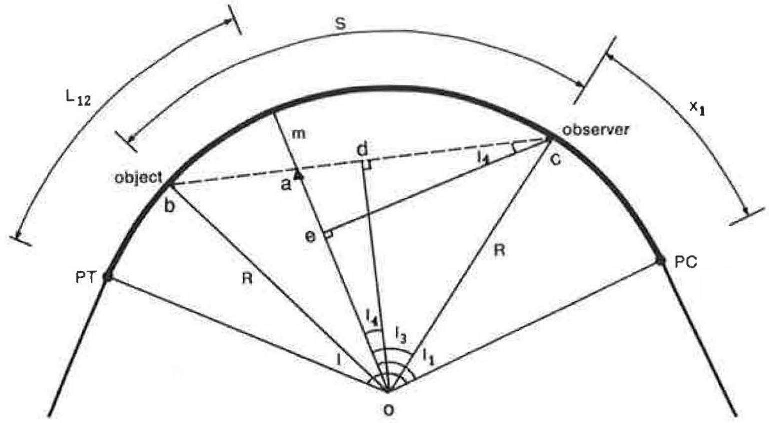

FIGURE 3 Case 3: Observer and object on curve.

$$[R(1 - \cos I_3^*) - m](R - m) = 0 \quad (24)$$

from which

$$m = R(1 - \cos I_3^*) \quad (25)$$

Substituting for m into Equation 20 gives $I_4 = 0$, which implies that S_m occurs when the observer and object are at equal distances from the obstacle. Thus, from Equation 22, $S_m = \pi R I_3^*/90$ or $I_3^* = 90S_m/\pi R$ (where I_3^* is in degrees). Substituting for I_3^* into Equation 25 gives

$$m = R[1 - \cos(90S_m/\pi R)] \quad (26)$$

which is the formula presented by AASHTO (*4*). It is clear that for Case 3, S_m is independent of the location of the obstacle for any given value of m.

Conditions for Case Determination

The geometry of the conditions for different cases is shown in Figure 4. Line *oc* is a radial line passing through the obstacle. The line from PC to *d* is perpendicular to this radial line. If m is less than *cb*, this is Case 3. If m is greater than *cb* but less than *ca*, this is Case 2. If m is equal to or greater than *ca*, this may be Case 1 or 2.

Given R, I, m, and I_1, the following steps are used to determine the respective case and the minimum sight distance:

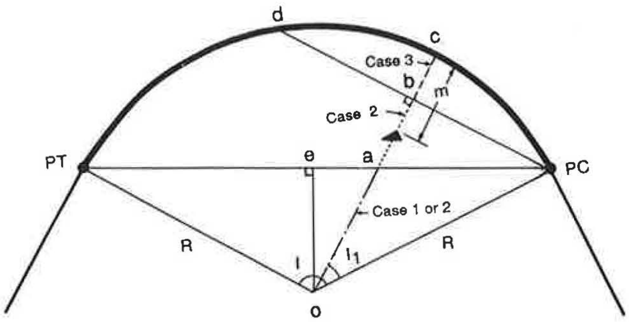

FIGURE 4 Geometry of the conditions for case determination.

1. S_m corresponds to Case 3 if the following condition is satisfied:

$$m \leq R[1 - \cos I'] \quad I' = \min\{I_1, I - I_1\} \quad (27)$$

The right-hand side equals *cb* in Figure 4. If this condition is not satisfied, go to the next step.

2. S_m corresponds to Case 2 if the following condition is satisfied:

$$m < R\{1 - [\cos(I/2)/\cos(I/2 - I_1]\} \quad (28)$$

The right-hand side of Equation 28 equals *ca* in Figure 4. This is the radial distance from the curve center to the line of sight when the observer is at PC and object is at PT. If this condition is not satisfied, S_m may correspond to Case 1 or 2. Go to the next step.

3. To determine whether S_m corresponds to Case 1 or 2, first calculate θ_1^*, x_1, and x_2 for Case 1. If x_1 and x_2 are positive or zero, then S_m corresponds to Case 1. Otherwise, S_m corresponds to Case 2.

The following numerical example illustrates these steps. Suppose that $R = 1{,}500$ ft, $I = 38.2°$, $m = 30$ ft, and $I_1 = 7.64°$. In Step 1, the right-hand side of Equation 27 equals 13.32 ft and Equation 27 is not satisfied. Therefore, this is not Case 3. In Step 2, the right-hand side of Equation 28 equals 53.74 ft. Therefore, Equation 28 is satisfied and S_m corresponds to Case 2. For Case 2, calculate $m_1 = 200.12$ ft (Equation 4) and $\alpha = 167.58°$ (Equation 6). Solving Equation 19 by successive approximations (Appendix B) gives $\theta_1^* = 3.25°$. Then, $\phi = 88.47°$ (Equation 17), $\gamma = 78.42°$ (Equation 16), $I_2 = 20.75°$ (Equation 15), and $S_m = 614$ ft (Equation 18).

PRACTICAL ASPECTS

The minimum sight distance on a horizontal curve must be determined to know whether the required sight distance (stopping, decision, or passing) is satisfied. The sight distance profile is a necessary input to the cost-effectiveness analysis of locations with restricted sight distances. In this section the sight distance profile, the application of the presented methodology to multiple obstacles, and a comparison with the NCHRP method are discussed.

Sight Distance Profile

The sight distance profile for a given obstacle for different values of the lateral clearance is shown in Figure 5. The obstacle is located at $I_1/I = 0.3$. The horizontal axis shows the location of the observer at various points of the tangent and curve. The PC is designated as the reference point, with the locations before it being negative and the locations beyond it being positive. The vertical axis shows the available sight distance for any given location of the observer. For example, for $m = 15$ ft, the minimum sight distance (435 ft) occurs when the observer is about 90 ft beyond PC. The sight distance profile is established using the developed relationships by computing the available sight distance for successive values of x_1. Unlike vertical curves, the minimum sight distance on a horizontal curve with a single obstacle occurs at a specific point on the traveled way rather than through a section of the traveled way.

For locations with restricted sight distances, the sight distance profile provides the length of the road within which the sight distance is restricted. This length is required for the operational and cost-effectiveness analysis developed by Neuman et al. (*14*). The probability that a critical event will occur at the location is directly proportional to the length of restricted sight distance. For example, if the required sight distance in Figure 5 is 600 ft and $m = 15$ ft, the SD profile shows that the length with a restricted sight distance is about 400 ft.

Application to Multiple Obstacles

For a horizontal curve with multiple obstacles, the sight distance profile of one obstacle interferes with the profiles of other obstacles. The actual sight distance profile is an envelope of the individual profiles, as shown in Figure 6. The horizontal curve has four obstacles with the indicated lateral clearances and locations on the curve. It is clear that Obstacle 2 is critical because it gives the least value of S_m (430 ft). The minimum sight distances can be determined using the developed relationships by considering each obstacle as a single obstacle (note that some obstacles may not have their minimum values on the sight distance envelope).

The interface among the profiles of various obstacles is an important element that should be considered in improving the sight distance on the curve. As noted in Figure 6, S_m on the curve can be improved by increasing the lateral clearance at Obstacle 2, but only to $S_m = 450$, which corresponds to Obstacle 4. Any further improvement in sight distance would require increasing the lateral clearances at both Obstacles 2 and 4, and so on. Thus, an obstacle that is currently not critical may become critical as other obstacles are displaced.

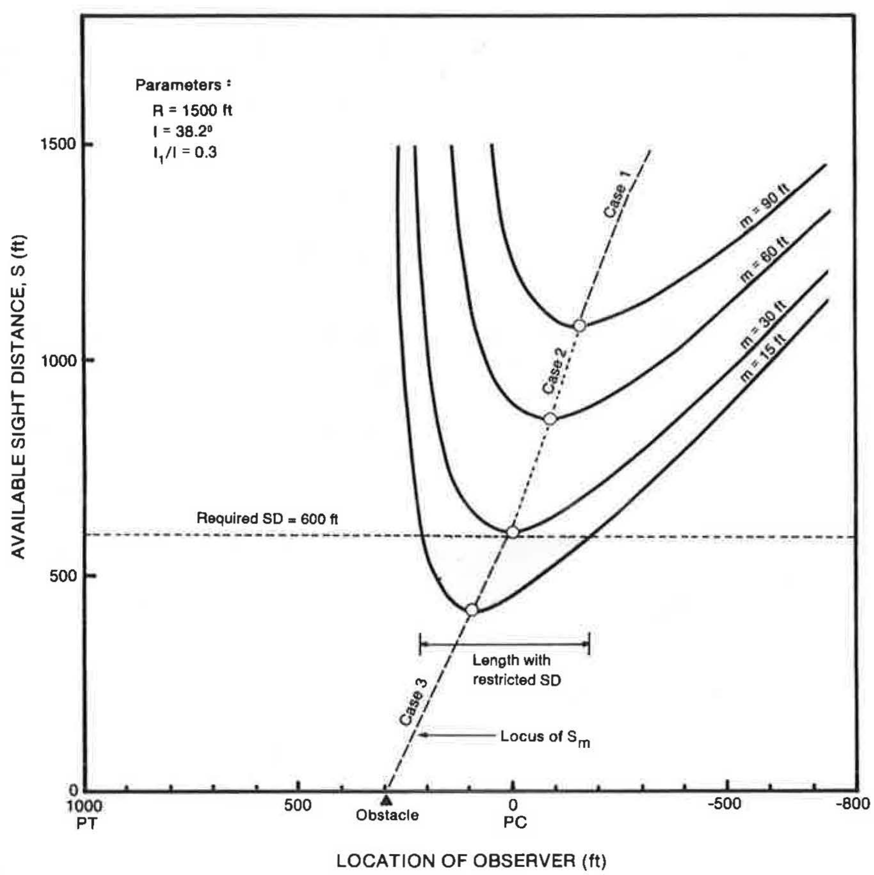

FIGURE 5 Sight distance profile on horizontal curve for different lateral clearances.

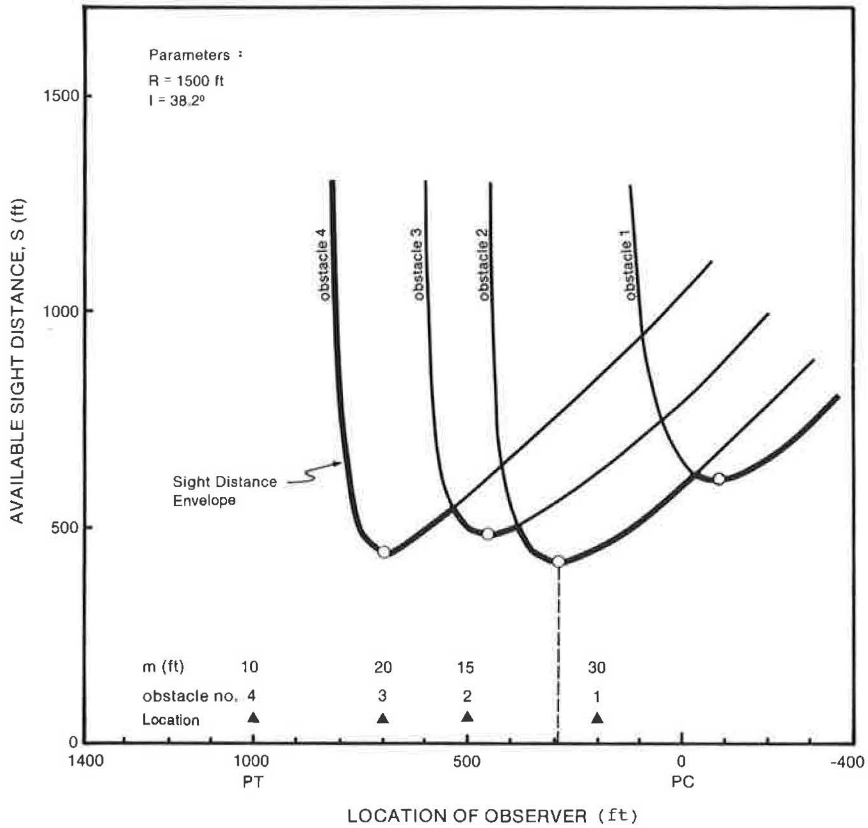

FIGURE 6 Sight distance envelope on horizontal curve with multiple obstacles.

Comparison with NCHRP Method

As previously indicated, the geometric relationships of the NCHRP report give the available sight distance for different locations of observer, object, and obstacle (*9,13*). Using these relationships, the available sight distance was computed for consecutive locations of the observer, and the sight distance profile was plotted as shown in Figure 7 (R = 1,000 ft, m = 80 ft, and $I_1/I = 0.1$). The sight distance profile based on the relationships presented in this paper is also shown. The S_m values of the NCHRP and presented methods are 1,020 ft and 950 ft, respectively. Thus, the NCHRP method overestimates S_m by about 7 percent. The differences in S_m were found to be much larger for $I_1/I = 0$ and smaller radii. However, the differences decrease as I_1/I approaches 0.5. The two methods give almost identical results of S_m for $I_1/I = 0.5$ in Case 1 and for any value of I_1/I in Case 3.

Although the difference between the minimum sight distances of the two methods is not large, the respective sight distance profiles are considerably different. If the required sight distance at the location is 1,100 ft, for example, the lengths of the restricted sight distance provided by the NCHRP and presented methods will be about 250 ft and 400 ft, respectively. Such a difference may affect the operational and cost-effectiveness analysis of restricted locations (*10,14*).

The difference between the two methods is caused by an assumption in the NCHRP relationships. The relationships implicitly assume that the lines connecting the observer and object to the curve center form equal angles with a perpendicular line drawn from the center to the line of sight. This assumption is not generally valid for computing the available sight distance. In addition, for computing S_m, this assumption is exact only for Case 1 (when $I_1/I = 0.5$) and Case 3.

EVALUATION AND DESIGN VALUES

To facilitate evaluation of the sight distance adequacy on horizontal curves, the developed relationships were used to establish values of the minimum sight distance for different characteristics of the curve and obstacle. The values are given in Tables 1 and 2, which are applicable to stopping, decision, and passing sight distances. Tables 1 and 2 can be used to

1. Determine the minimum sight distance on an existing curve and obstacles,
2. Determine the required lateral clearances to maintain a required minimum sight distance, and
3. Determine the critical lateral clearance (for design) that maintains a required minimum sight distance.

The critical lateral clearance, M, is the largest value of m for given S_m, R, and I. For $S_m \leq L$, the critical value is presented by AASHTO (*4*). For $S_m > L$, a formula and a nomograph for determining M were presented by Waissi and Cleveland (*13*). In Tables 1 and 2, the critical lateral clearance for $S_m \leq L$ or $S_m > L$ is the lateral clearance for $I_1/I = 0.5$. Figure 8 illustrates the results for R = 1,500 ft and I = 40°. As noted, there is a great difference between the lateral clearance requirements when the obstacle lies at PC (or PT) and

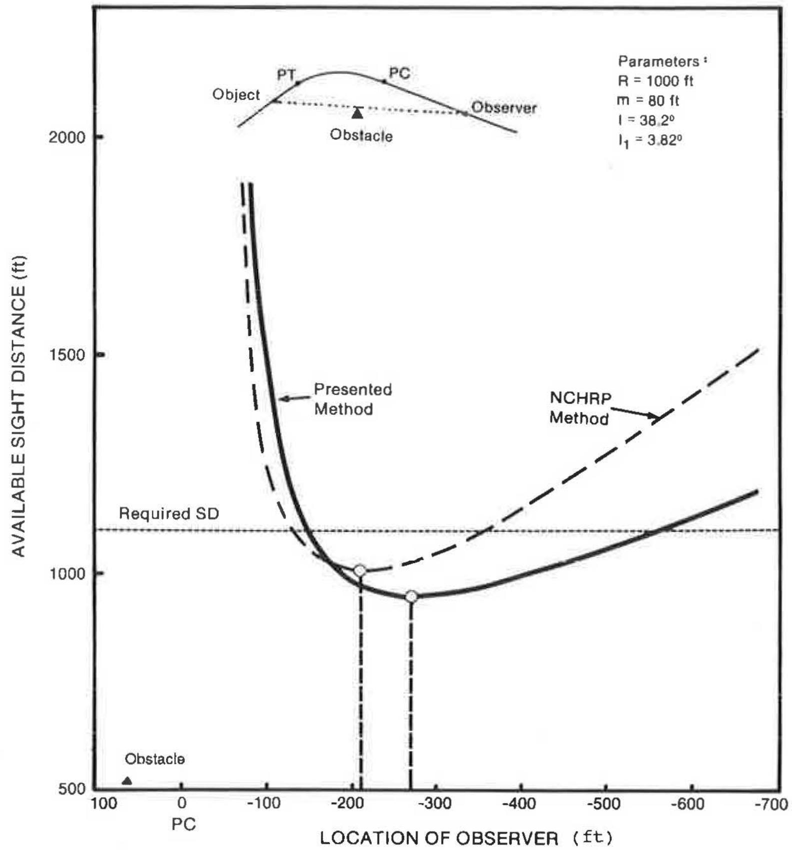

FIGURE 7 Comparison between sight distance profiles of presented and NCHRP methods.

TABLE 1 MINIMUM SIGHT DISTANCE ON HORIZONTAL CURVE WITH SINGLE OBSTACLE (R = 200 TO 800 ft)

Cent. Angle (deg)	Obst. Loc. (I_1/I)	Curve Radius (ft)																			
		200					400					600					800				
		m=20[a]	40	60	80	100	20	40	60	80	100	20	40	60	80	100	20	40	60	80	100
5	0.0	940	1850	2770	3690	4600	950	1870	2790	3700	4620	970	1890	2800	3720	4640	990	1910	2820	3740	4650
5	0.1	930	1850	2770	3680	4600	950	1870	2780	3700	4610	960	1880	2800	3710	4630	980	1890	2810	3730	4640
5	0.2	930	1850	2770	3680	4600	940	1860	2780	3690	4610	960	1870	2790	3710	4620	970	1880	2800	3720	4630
5	0.3	930	1850	2760	3680	4600	940	1860	2770	3690	4610	950	1870	2790	3700	4620	960	1880	2800	3710	4630
5	0.4	930	1850	2760	3680	4600	940	1860	2770	3690	4610	950	1870	2780	3700	4620	960	1870	2790	3710	4630
5	0.5	930	1850	2760	3680	4600	940	1860	2770	3690	4610	950	1860	2780	3700	4620	960	1870	2790	3710	4620
10	0.0	500	950	1410	1870	2320	530	990	1440	1900	2360	560	1020	1480	1940	2390	590	1050	1510	1970	2430
10	0.1	490	950	1410	1860	2320	520	980	1430	1890	2350	540	1000	1460	1920	2380	570	1030	1490	1950	2410
10	0.2	490	940	1400	1860	2320	510	970	1430	1880	2340	530	990	1450	1910	2370	550	1010	1470	1930	2390
10	0.3	480	940	1400	1860	2320	500	960	1420	1880	2340	520	980	1440	1900	2360	540	1000	1460	1920	2380
10	0.4	480	940	1400	1860	2320	500	960	1420	1880	2330	520	980	1430	1890	2350	540	990	1450	1910	2370
10	0.5	480	940	1400	1860	2320	500	960	1420	1870	2330	520	970	1430	1890	2350	530	990	1450	1910	2370
20	0.0	300	530	750	980	1210	360	590	820	1050	1280	410	650	880	1110	1340	470	710	950	1180	1410
20	0.1	290	520	750	970	1200	340	570	800	1030	1260	390	620	850	1080	1310	430	670	910	1140	1370
20	0.2	280	510	740	970	1200	320	550	780	1010	1240	360	600	830	1060	1290	410	640	870	1100	1340
20	0.3	270	500	730	960	1190	310	540	770	1000	1230	350	580	810	1040	1270	390	620	850	1080	1310
20	0.4	270	500	730	960	1190	310	540	770	1000	1230	340	570	800	1030	1260	380	610	840	1070	1300
20	0.5	270	500	730	960	1190	300	530	760	1000	1230	340	570	800	1030	1260	370	600	830	1060	1300
30	0.0	250	400	550	700	850	330	490	650	800	950	410	580	740	890	1050	470	660	820	980	1140
30	0.1	230	390	540	690	850	300	470	620	770	930	370	540	700	850	1010	420	610	770	930	1080
30	0.2	220	380	530	690	840	290	440	600	750	910	340	510	660	820	970	390	570	730	880	1040
30	0.3	220	370	530	680	830	270	430	580	740	890	320	490	640	800	950	370	540	700	850	1010
30	0.4	210	370	520	680	830	270	420	570	730	880	320	470	630	780	940	360	530	680	840	990
30	0.5	210	370	520	670	830	260	420	570	730	880	310	470	620	780	930	360	520	680	830	980
40	0.0	240	350	470	580	690	330	470	590	700	820	410	570	700	820	940	470	660	800	930	1050
40	0.1	220	340	450	570	680	300	430	550	670	790	360	520	650	770	890	400	590	740	860	980
40	0.2	210	320	440	550	670	280	410	530	640	760	330	480	610	730	850	370	550	690	810	930
40	0.3	200	310	430	550	660	260	390	510	620	740	320	460	580	700	820	360	520	660	770	890
40	0.4	190	310	430	540	660	260	380	500	610	730	310	450	570	680	800	360	510	640	750	870
40	0.5	190	310	420	540	660	260	380	490	610	730	310	450	560	680	800	360	510	630	750	870

[a] Lateral clearance (ft)

Note: minimum sight distances are expressed in feet.

TABLE 2 MINIMUM SIGHT DISTANCE ON HORIZONTAL CURVE WITH SINGLE OBSTACLE (*R* = 1,000 TO 3,000 ft)

Cent. Angle (deg)	Obst. Loc. (I_1/I)	Curve Radius (ft) 1000					1500					2000					3000				
		m=20[a]	40	60	80	100	20	40	60	80	100	20	40	60	80	100	20	40	60	80	100
5	0.0	1010	1920	2840	3760	4670	1050	1970	2880	3800	4710	1090	2010	2920	3840	4760	1170	2090	3010	3930	4840
5	0.1	990	1910	2820	3740	4660	1030	1940	2860	3780	4690	1060	1980	2890	3810	4730	1130	2050	2960	3880	4800
5	0.2	980	1900	2810	3730	4650	1010	1930	2840	3760	4680	1040	1950	2870	3790	4710	1090	2010	2930	3850	4760
5	0.3	970	1890	2810	3720	4640	1000	1910	2830	3750	4660	1020	1940	2860	3770	4690	1070	1990	2910	3820	4740
5	0.4	970	1880	2800	3720	4630	990	1910	2820	3740	4660	1010	1930	2850	3760	4680	1060	1970	2890	3810	4730
5	0.5	960	1880	2800	3720	4630	990	1900	2820	3740	4650	1010	1930	2840	3760	4680	1050	1970	2890	3800	4720
10	0.0	620	1090	1550	2000	2460	700	1170	1630	2090	2550	770	1240	1710	2170	2630	910	1390	1860	2330	2790
10	0.1	600	1060	1520	1980	2430	660	1130	1590	2050	2500	720	1190	1650	2110	2570	840	1320	1780	2250	2710
10	0.2	580	1040	1500	1950	2410	630	1090	1550	2010	2470	680	1150	1610	2070	2530	790	1260	1720	2180	2640
10	0.3	560	1020	1480	1940	2400	610	1070	1530	1990	2450	660	1120	1580	2040	2500	750	1220	1680	2140	2600
10	0.4	550	1010	1470	1930	2390	600	1060	1520	1980	2430	640	1100	1560	2020	2480	730	1190	1650	2110	2570
10	0.5	550	1010	1470	1930	2390	590	1050	1510	1970	2430	640	1100	1560	2010	2470	720	1180	1640	2100	2560
20	0.0	520	770	1010	1240	1470	640	910	1150	1390	1620	740	1040	1290	1540	1770	900	1280	1560	1810	2060
20	0.1	480	720	960	1190	1420	570	840	1080	1320	1550	650	950	1200	1440	1670	780	1140	1430	1670	1920
20	0.2	450	680	920	1150	1380	530	790	1020	1260	1490	600	890	1130	1360	1600	720	1060	1330	1570	1810
20	0.3	420	660	890	1120	1350	510	750	990	1220	1450	580	850	1080	1310	1550	700	1010	1270	1500	1740
20	0.4	410	640	870	1100	1340	500	730	960	1190	1420	570	820	1050	1280	1510	700	990	1230	1460	1690
20	0.5	410	640	870	1100	1330	490	730	960	1190	1420	570	810	1040	1270	1500	700	980	1220	1450	1680
30	0.0	520	740	910	1070	1230	640	900	1100	1270	1440	740	1040	1270	1470	1640	900	1280	1560	1800	2010
30	0.1	460	670	840	1000	1160	550	810	1000	1170	1340	620	920	1140	1340	1510	740	1100	1370	1610	1820
30	0.2	420	620	790	950	1100	510	750	940	1100	1260	580	850	1060	1250	1410	700	1010	1270	1490	1690
30	0.3	410	600	750	910	1070	490	710	890	1050	1210	570	810	1010	1190	1350	700	990	1220	1420	1610
30	0.4	400	580	730	890	1040	490	700	870	1020	1180	570	810	990	1150	1310	700	980	1210	1390	1570
30	0.5	400	570	730	880	1040	490	700	860	1010	1170	570	810	990	1140	1300	700	980	1210	1390	1560
40	0.0	520	740	900	1040	1160	640	900	1100	1270	1420	740	1040	1270	1470	1640	900	1280	1560	1800	2010
40	0.1	450	650	810	950	1070	530	780	980	1140	1290	600	890	1110	1300	1470	720	1060	1330	1560	1770
40	0.2	410	600	760	890	1010	500	720	900	1060	1200	570	820	1020	1200	1360	700	990	1230	1440	1630
40	0.3	400	580	720	850	970	490	700	870	1010	1150	570	810	990	1150	1300	700	980	1210	1390	1570
40	0.4	400	570	710	820	940	490	700	860	990	1120	570	810	990	1140	1280	700	980	1210	1390	1560
40	0.5	400	570	700	820	930	490	700	860	990	1110	570	810	990	1140	1270	700	980	1210	1390	1560

[a] Lateral clearance (ft)

Note: minimum sight distances are expressed in feet.

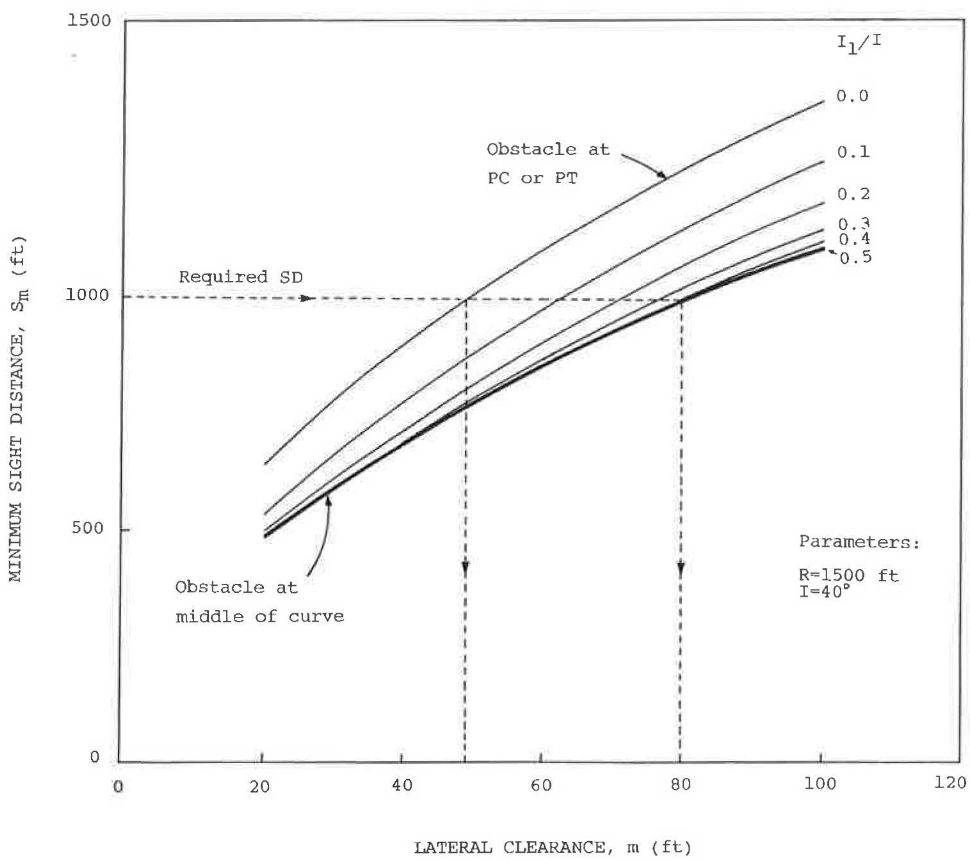

FIGURE 8 Comparison of lateral clearance requirements for different obstacle locations.

the middle of the curve; for example, for a sight distance of 1,000 ft, the lateral clearances needed are 49 ft and 80 ft, respectively.

Example

The following example illustrates the application of Tables 1 and 2. Given a horizontal curve with a single obstacle, R = 1,300 ft, m = 40 ft, I = 35°, and I_1 = 9.5°. The ratio I_1/I = 0.27. Then,

1. Determine S_m: Using Table 2, interpolate the values of S_m for R = 1,000 ft and 1,500 ft and I_1/I = 0.2 and 0.3 at I = 30° to obtain S_m = 682 ft. Repeat the interpolation for I = 40° to obtain S_m = 664 ft. Therefore, for I = 35°, S_m = 673 ft (the exact value of S_m computed by the presented relationships is 661 ft).
2. Determine m if the required sight distance is 800 ft: Using Table 2, the corresponding lateral clearance can be interpolated in a similar manner as m = 70 ft. The exact value of S_m (computed by the presented relationships) corresponding to this lateral clearance is 803 ft, which is very close to the required value.
3. Determine the critical lateral clearance: For S_m = 800 ft, M is determined from Table 2 as 75 ft (for I_1/I = 0.5). Using the Waissi and Cleveland formula (*13*), M is computed as 76 ft.

DISCUSSION OF RESULTS

Application of the AASHTO sight distance model for $S_m \leq L$ to situations in which $S_m > L$ results in overestimation of the maximum required lateral clearance M. Situations in which the sight distance is greater than the curve length may arise because of the following factors:

1. AASHTO policy (*4*) and NCHRP research (*9*) have presented increased values of SSD. In addition, Neuman (*5*) recommends greater SSD values than those of AASHTO for most of the highway classifications.
2. At locations with special geometry or conditions, the DSD should be provided. The AASHTO design values of DSD (*4*) and those recommended by Neuman (*5*) and McGee (*8*) are twice to three times the SSD design values.
3. Where AASHTO PSDs are provided, these distances will in most cases be greater than the curve length.

Even when $S_m \leq L$, the vision obstacle on the horizontal curve may lie near the ends of the curve so that the needed lateral clearance is less than the maximum value M. For these cases ($S_m \leq L$, $S_m > L$), the developed relationships provide exact values of the minimum sight distance or the lateral clearance that satisfy sight distance needs. Application of the presented method should result in cost savings from roadside clearing and perhaps land acquisition.

SUMMARY

Exact relationships for establishing the sight distance profiles for highway horizontal curves with a single obstacle or multiple obstacles on the inside of the curve are presented. Closed-form solutions of the minimum sight distance are also presented for any location of the obstacle, and design values for practical use are established. These values can be used to determine the adequacy of sight distance at a particular location. It is no longer necessary to plot the entire sight distance profile to determine whether the location has a restricted sight distance. Only for restricted locations is the sight distance profile plotted to determine the length of the road with restricted sight distance and to evaluate alternative improvements. The results of this research should be useful for the design, operation, and safety of critical highway locations.

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APPENDIX A
Derivation of the Formulas for S_m

CASE 1

Differentiating Equation 10 with respect to θ_1, then

$$\frac{dS}{d\theta_1} = m_1[\sin(\theta_1 + \alpha)\cos\theta_1 - \sin\theta_1\cos(\theta_1 + \alpha)] \div \sin^2(\theta_1 + \alpha) + m_2[\sin(\theta_1 + J - \beta)\cos(\theta_1 + J) - \sin(\theta_1 + J)\cos(\theta_1 + J - \beta)]/\sin^2(\theta_1 + J - \beta) \quad (29)$$

Consider the following identity (*15*):

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \quad (30)$$

Based on this identity, the first and second expressions in brackets equal $\sin\alpha$ and $\sin(-\beta)$, respectively. Thus,

$$\frac{dS}{d\theta_1} = \frac{m_1 \sin\alpha}{\sin^2(\theta_1 + \alpha)} + \frac{m_2 \sin(-\beta)}{\sin^2(\theta_1 + J - \beta)} \quad (31)$$

Equating Equation 31 to zero and noting that $\sin(-\beta) = -\sin\beta$,

$$\frac{\sin(\theta_1^* + \alpha)}{\sin(\theta_1^* + J - \beta)} = \left(\frac{m_1 \sin\alpha}{m_2 \sin\beta}\right)^{1/2} \quad (32)$$

in which θ_1^* is the angle corresponding to the minimum sight distance. Equation 32 is the same as Equation 11.

CASE 2

Substituting for γ from Equation 16 into Equation 15 and then substituting for ϕ from Equation 17 gives

$$I_2 = 270° - \sin^{-1}(f) - \alpha - \theta_1 \quad (33)$$

where f is a function of θ_1 given by

$$f = \left(\frac{R - m}{R}\right) \sin(I_1 + \alpha + \theta_1 - 90°) \quad (34)$$

Substituting for I_2 from Equation 33 into Equation 18 and differentiating Equation 18 with respect to θ_1,

$$\frac{dS}{d\theta_1} = \frac{-f'}{(1 - f^2)^{1/2}} + \frac{m_1 \sin(\theta_1 + \alpha) \cos\theta_1}{\sin^2(\theta_1 + \alpha)} - \frac{m_1 \sin\theta_1 \cos(\theta_1 + \alpha)}{\sin^2(\theta_1 + \alpha)} - 1 \quad (35)$$

where f' equals $df/d\theta_1$, which is given by

$$f' = \left(\frac{R - m}{R}\right) \sin(\theta_1 + \alpha + I_1) \quad (36)$$

The expression in brackets in Equation 35 equals $\sin\alpha$ based on the identity of Equation 30. Substituting for f and f' from Equations 34 and 36 into Equation 35 and equating $dS/d\theta_1$ to zero gives

$$\frac{m_1 \sin\alpha}{R \sin^2(\theta_1^* + \alpha)} - \frac{(R - m) \sin(\theta_1^* + \alpha + I_1)}{[R^2 - (R - m)^2 \cos^2(\theta_1^* + \alpha + I_1)]^{1/2}} = 1 \quad (37)$$

which is Equation 19.

CASE 3

Equation 23 is written as

$$S = 2R[I_3 - \tan^{-1}(f)] \quad (38)$$

where f is a function of I_3 given by

$$f = [R(1 - \cos I_3) - m]/(R \sin I_3) \quad (39)$$

Differentiating Equation 38 with respect to I_3,

$$\frac{dS}{dI_3} = 2R\left[1 - \frac{f'}{(1 + f^2)}\right] \quad (40)$$

where $f' = df/dI_3$, which is given by

$$f' = \frac{(R \sin I_3)^2 - [R(1 - \cos I_3) - m]R \cos I_3}{R^2 \sin^2 I_3} \quad (41)$$

Substituting for f and f' from Equations 39 and 41 into Equation 40 and equating dS/dI_3 to zero gives

$$[R(1 - \cos I_3^*) - m]^2 + [R(1 - \cos I_3^*) - m]R \cos I_3^* = 0 \quad (42)$$

After rearranging, Equation 42 becomes

$$[R(1 - \cos I_3^*) - m](R - m) = 0 \quad (43)$$

which is Equation 24.

APPENDIX B
Numerical Solution of Equations 11 and 19

The critical angle θ_1^* of Equations 11 and 19 can be obtained by successive approximations using the method of linear interpolation (*16*). Equation 11 or 19 is written as

$$f(u) = 0 \quad (44)$$

where u is used instead of θ_1^*. To determine the root of Equation 44, select two values u_1 and u_2 for which $f(u_1)$ and $f(u_2)$ have opposite signs. The following steps are then performed:

1. Set

$$u_3 = u_2 - f(u_2)\frac{u_2 - u_1}{f(u_2) - f(u_1)}$$

2. If $f(u_3)$ has an opposite sign to $f(u_1)$, set $u_2 = u_3$. Otherwise, set $u_1 = u_3$
3. Repeat Steps 1 and 2 until

$$|u_2 - u_1| \leq \varepsilon_1$$

$$|f(u_3)| \leq \varepsilon_2$$

where ε_1 and ε_2 are specified tolerance values.

This method guarantees convergence. A modified linear interpolation method, which converges faster, may also be used (*16*).

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