Lateral Clearance to Vision Obstacles on Horizontal Curves

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Evaluation of the sight distance adequacy on highway horizontal curves with single or multiple obstacles requires determination of the minimum sight distance on the curve. The current method of establishing such a minimum is to plot the sight distance profile for the given curve and obstacle. Approximate relationships have been developed for establishing the sight distance profile. However, there is no explicit, exact solution available for determining the minimum sight distance on the curve. Exact formulas have been derived to relate the available sight distance to the circular curve parameters, lateral clearance of the obstacle, its location along the curve, and the locations of observer and object. These relationships are then used to derive closed-form solutions of the minimum sight distance, $S_m$. To facilitate practical use, values of $S_m$ are established for typical ranges of the curve parameters, lateral clearance, and obstacle location. Values of the maximum lateral clearance, which is required in design, are also provided.

The methodology and results should be valuable in the operational and cost-effectiveness analysis of highway locations with restricted sight distances.

The sight distance on highway horizontal curves may be restricted by such physical features as longitudinal barriers, cut slopes, foliage, and other structures. For safe operations, the available sight distance at any point on the traveled way must be greater than the sight distance needed for stopping, passing, or decision at complex locations. The available sight distance is a function of the horizontal curve parameters, locations of the observer and object, and the location of the vision-limiting obstacle inside the curve.

The stopping sight distance (SSD), presented by AASHTO (1–4), is one of the basic considerations in the design of highways. Design values for SSD applicable to all highways are presented by AASHTO. A new approach to SSD that considers the functional classifications of highways was recently presented by Neuman (5). Design values for passing sight distance (PSD) involving passenger cars on two-lane highways are presented in AASHTO’s 1984 Policy on Geometric Design of Highways and Streets (Green Book) (4). Design values for PSD for all combinations of passing involving a passenger car and a truck have been developed by Harwood and Glennon (6). These design values are based on a model developed by Glennon (7) that logically accounts for the kinematic relationships among the passing, passed, and opposing vehicles and explicitly contains vehicle-length variables. Design values for decision sight distance (DSD) are presented by AASHTO (4) and Neuman (5), and their usefulness and application have been evaluated by McGee (8).

A number of models exist that relate the available sight distance and the lateral clearance on horizontal curves. AASHTO presents a model that relates the sight distance, $S_m$, and maximum lateral clearance, $M$, for $S_m \leq L$, where $L$ is the length of the curve (4). The case $S_m > L$ is not presented by AASHTO. When either the vehicle or vision obstacle is situated near the ends of the curve, the less clearance is needed. The studies by Olson et al. (9), Neuman and Glennon (10), and Glennon (11) show that when the vehicle is on the tangent within a distance $S$ from the point of curvature, PC, the maximum lateral clearance needed varies from 0 to $M$ (when the vehicle is at PC). The required lateral clearance for all points within $S/2$ beyond PC is also less than $M$. For these situations, AASHTO recommends the use of a graphical procedure or the curves empirically developed by Raymond (12).

To eliminate the need for the graphical procedure, Waissi and Cleveland (13), on the basis of the results of the NCHRP report by Olson et al. (9), derived approximate relationships that relate the available sight distance to the horizontal curve parameters, locations of observer and road object, location of obstacle, and lateral clearance to a single obstacle to vision located inside the curve. Relationships for determining the maximum lateral clearance for $S_m > L$ are also presented.

For a given obstacle on the curve, the available sight distance varies as the observer moves along the tangent and curve. Clearly, there is a minimum value of sight distance on the traveled path that determines the adequacy of sight distance on the curve. There is no explicit, exact solution available for determining this minimum sight distance. The purpose of this paper is threefold:

1. To derive exact relationships for determining the available sight distance for arbitrary locations of the observer and obstacle;
2. On the basis of the preceding, to develop relationships for determining the minimum sight distance, and
3. To establish evaluation and design values for practical use.

THEORETICAL DEVELOPMENT

Relationships for determining the minimum sight distance, $S_m$, are developed for a single obstacle located on the inside of a simple horizontal curve between PC and PT (point of tangency). Both the observer and object are assumed to be located on the centerline of the inside lane. Three cases are considered:

- Case 1: Observer before PC and object beyond PT,
Case 1: Observer Before PC and Object Beyond PT

The geometry of this case is shown in Figure 1. As the observer moves toward PC, the available sight distance decreases, reaches a minimum value, and then increases again. The lateral clearance requirements should be based on this minimum value. In Figure 1, $x_1$ is the distance from the observer to PC, and $x_2$ is the distance from the object to PT.

Available Sight Distance

The available sight distance is given by

$$S = L + x_1 + x_2$$

(1)

where $S$ equals available sight distance, and $L$ equals curve length.

With the law of sines for triangles $abPT$ and $acPC$, $x_1$ and $x_2$ can be expressed in terms of the angles $\theta_1$ and $\theta_2$, shown in Figure 1, as

$$x_1 = m_1 \sin \theta_1 / \sin(\theta_1 + \alpha)$$

(2)

$$x_2 = m_2 \sin \theta_2 / \sin(\theta_2 + \beta)$$

(3)

where

$m_1 =$ distance from the obstacle to PC,

$m_2 =$ distance from the obstacle to PT,

$\alpha =$ angle at PC between the tangent and the line to the obstacle, and

$\beta =$ angle at PT between the tangent and the line to the obstacle.

Note that in Equation 2, $\sin(\theta_1 + \alpha) = \sin(180 - \theta_1 - \alpha)$, and similarly for Equation 3. These four elements, which are constant for a given curve and obstacle, are computed using triangles $oaPC$ and $oaPT$ as follows:

$$m_1 = \frac{R^2 + (R - m)^2 - 2R(R - m) \cos I_1}{2}$$

(4)

$$m_2 = \frac{R^2 + (R - m)^2 - 2R(R - m) \cos(I - I_1)}{2}$$

(5)

$$\alpha = 90^\circ + \cos^{-1}(\frac{-(R - m)^2 + R^2 + m_I^2}{2Rm})$$

(6)

$$\beta = 90^\circ + \cos^{-1}(\frac{-(R - m)^2 + R^2 + m_J^2}{2Rm})$$

(7)

where

$R =$ curve radius,

$m =$ lateral clearance between the centerline of the inside lane and obstacle,

$I_1 =$ central angle from PC to the obstacle, and

$I =$ central angle from PC to PT.

The angles $\theta_1$ and $\theta_2$ are related by

$$\theta_2 = 180^\circ - \theta_1 - J$$

(8)

Since $\angle aPCo = \alpha - 90^\circ$ and $\angle aPTo = \beta - 90^\circ$, the angle $J$ is obtained as follows:

$$J = \alpha + \beta + I - 180^\circ$$

(9)

Substituting $\theta_2$ of Equation 8 into Equation 3, the available sight distance of Equation 1 can be written as

$$S = L + [m_1 \sin \theta_1 / \sin(\theta_1 + \alpha)] + [m_2 \sin(\theta_2 + J) / \sin(\theta_1 + J - \beta)]$$

(10)

in which the curve length $L$ equals $R\pi I/180$.

Condition for $S_m$

Differentiating Equation 10 with respect to $\theta_1$ and equating $dS/d\theta_1$ to zero gives

$$\sin(\theta_1 + \alpha) / \sin(\theta_1 + J - \beta) = (m_1 \sin \alpha / m_2 \sin \beta)^{1/2}$$

(11)

in which $\theta_1^* \alpha$m is the critical angle corresponding to the minimum sight distance, $S_m$. The derivation of Equation 11 is included in Appendix A. A successive approximation method for solving Equation 11 to determine $\theta_1^*$ is given in Appendix B.

Note that Equation 11 implies that $\theta_1^*$ must be greater than $(\beta - J)$. For equal or smaller values, the line of sight from the observer to the obstacle does not intersect with the tangent beyond PT. After determining $\theta_1^*$, $S_m$ is computed by substituting $\theta_1^*$ into Equation 10.

If the obstacle lies at the midpoint of the curve ($I/I = 0.5$), then $m_1 = m_2$ and $\alpha = \beta$. With these values, Equation 11 yields $\theta_1^* + \alpha = 180 - (\theta_1^* + J - \beta)$ (note that $\alpha + J - \beta$ is based on Equation 9). Thus,

$$\theta_1^* = (180^\circ - J) / 2 \text{ for } I/I = 0.5$$

(12)

which implies that $x_1 = x_2$, as expected.

Case 2: Observer Before PC and Object on Curve

In this case, the observer is on the tangent at a distance $x_1$ from PC, and the object is on the curve at a distance $L_1$ from PC (Figure 2).
Available Sight Distance

The available sight distance is given by

\[ S = L_1 + x_1 \]  \hspace{1cm} (13)

where \( L_1 \), the distance from PC to the object, is given by

\[ L_1 = R I_2 \]  \hspace{1cm} (14)

and \( x_1 \) is given by Equation 2 (\( m_i \) and \( \alpha \) are given by Equations 4 and 6). \( I_2 \) is the central angle between PC and the object (in radians). The angle \( oda = \theta_1 + \alpha - 90^\circ \). Therefore, using triangle \( obd \), \( I_2 \) is obtained:

\[ I_2 = 270^\circ - \gamma - \alpha - \theta_1 \]  \hspace{1cm} (15)

Using triangles \( oab \) and \( oad \), respectively, \( \gamma \) and \( \phi \) are obtained:

\[ \gamma = \sin^{-1}\left(\frac{(R - m) \sin \theta}{R}\right) \]  \hspace{1cm} (16)

\[ \phi = I_1 + \alpha + \theta_1 - 90^\circ \]  \hspace{1cm} (17)

Now Equation 13 can be written as follows:

\[ S = R I_2 + [m_i \sin \theta_1 / \sin(\theta_1 + \alpha)] \]  \hspace{1cm} (18)

**Condition for \( S_m \)**

Differentiating Equation 18 with respect to \( \theta_1 \) and equating \( dS/d\theta \), to zero give (Appendix A)

\[ \frac{m_1 \sin \alpha}{R \sin^2(\theta_1 + \alpha)} - \frac{(R - m) \sin(\theta_1 + \alpha + I_2)}{[R^2 - (R - m)^2 \cos^2(\theta_1 + \alpha + I_2)]^{1/2}} = 1 \]  \hspace{1cm} (19)

Solving Equation 19 by successive approximations gives the critical angle \( \theta_1^* \) (Appendix B). \( S_m \) is then computed by substituting \( \theta_1^* \) into Equation 18.

Because of the symmetry of the horizontal curve, Case 2 may occur only when \( I_i/I \) is less than 0.5. For \( I_i/I \) greater than 0.5, the minimum sight distance occurs when the observer is on the curve and the object is beyond PT. The solution of this situation is also given by Equations 13–19 after switching the positions of PC and PT, and of the observer and object.

**Case 3: Observer and Object on Curve**

In Case 3, both observer and object are on the curve. Figure 3 shows the geometry of this case. Let \( I_3 \) denote the central angle from the observer to the obstacle and \( I_4 \) the central angle from the obstacle to the perpendicular line \( od \). Using triangle \( ace \), \( \tan I_4 = \tan \theta \). But \( \tan \theta = (R - m) / \alpha e \). From triangle \( oec \), \( ec = R \sin I_3 \) and \( oe = R \cos I_3 \). Thus,

\[ I_4 = \tan^{-1}\left([R - m - R \cos I_3]/(R \sin I_3)\right) \]  \hspace{1cm} (20)

where \( I_4 \), which equals \( I_1 - \angle coPC \), is given by

\[ I_4 = I_1 - (180 \times / R) \]  \hspace{1cm} (21)

and \( x_1 \) is the distance along the curve between the observer and PC.

**Available Sight Distance**

The available sight distance is given by (Figure 3)

\[ S = 2R(I_3 - I_4) \]  \hspace{1cm} (22)

where \( I_3 \) and \( I_4 \) are in radians. Substituting for \( I_4 \) from Equation 20 into Equation 22 gives

\[ S = 2R(I_3 - \tan^{-1}\left([R - m - R \cos I_3]/(R \sin I_3)\right)) \]  \hspace{1cm} (23)

**Condition for \( S_m \)**

Differentiating Equation 23 with respect to \( I_3 \) and equating \( dS/dI_3 \), to zero gives (Appendix A)
\[ [R(1 - \cos l') - m](R - m) = 0 \]  \hspace{1cm} (24)

from which

\[ m = R(1 - \cos l') \]  \hspace{1cm} (25)

Substituting for \( m \) into Equation 20 gives \( I_4 = 0 \), which implies that \( S_m \) occurs when the observer and object are at equal distances from the obstacle. Thus, from Equation 22, \( S_m = \pi R/90 \) or \( I_3 = 90S_m/\pi R \) (where \( I_3 \) is in degrees). Substituting for \( I_3 \) into Equation 25 gives

\[ m = R[1 - \cos(90S_m/\pi R)] \]  \hspace{1cm} (26)

which is the formula presented by AASHTO (4). It is clear that for Case 3, \( S_m \) is independent of the location of the obstacle for any given value of \( m \).

**Conditions for Case Determination**

The geometry of the conditions for different cases is shown in Figure 4. Line \( oc \) is a radial line passing through the obstacle. The line from \( PC \) to \( d \) is perpendicular to this radial line. If \( m \) is less than \( cb \), this is Case 2. If \( m \) is greater than \( cb \) but less than \( ca \), this is Case 2. If \( m \) is equal to or greater than \( ca \), this may be Case 1 or 2.

Given \( R \), \( I \), \( m \), and \( I_1 \), the following steps are used to determine the respective case and the minimum sight distance:

1. \( S_m \) corresponds to Case 3 if the following condition is satisfied:

\[ m \leq R[1 - \cos l'] \]  \hspace{1cm} (27)

The right-hand side equals \( cb \) in Figure 4. If this condition is not satisfied, go to the next step.

2. \( S_m \) corresponds to Case 2 if the following condition is satisfied:

\[ m < R[1 - \cos(90S_m/\pi R)] \]  \hspace{1cm} (28)

The right-hand side of Equation 28 equals \( ca \) in Figure 4. This is the radial distance from the curve center to the line of sight when the observer is at \( PC \) and object is at \( PT \). If this condition is not satisfied, \( S_m \) may correspond to Case 1 or 2. Go to the next step.

3. To determine whether \( S_m \) corresponds to Case 1 or 2, first calculate \( \theta_i \), \( x_i \), and \( x_2 \) for Case 1. If \( x_1 \) and \( x_2 \) are positive or zero, then \( S_m \) corresponds to Case 1. Otherwise, \( S_m \) corresponds to Case 2.

The following numerical example illustrates these steps. Suppose that \( R = 1,500 \) ft, \( I = 38.2^\circ \), \( m = 30 \) ft, and \( I_1 = 7.64^\circ \). In Step 1, the right-hand side of Equation 27 equals 13.32 ft and Equation 27 is not satisfied. Therefore, this is not Case 3. In Step 2, the right-hand side of Equation 28 equals 53.74 ft. Therefore, Equation 28 is satisfied and \( S_m \) corresponds to Case 2. For Case 2, calculate \( m_1 = 200.12 \) ft (Equation 4) and \( \alpha = 167.58^\circ \) (Equation 6). Solving Equation 19 by successive approximations (Appendix B) gives \( \theta_i = 3.25^\circ \). Then, \( \phi = 88.47^\circ \) (Equation 17), \( \gamma = 78.42^\circ \) (Equation 16), \( I_2 = 20.75^\circ \) (Equation 15), and \( S_m = 614 \) ft (Equation 18).

**PRACTICAL ASPECTS**

The minimum sight distance on a horizontal curve must be determined to know whether the required sight distance (stopping, decision, or passing) is satisfied. The sight distance profile is a necessary input to the cost-effectiveness analysis of locations with restricted sight distances. In this section the
sight distance profile, the application of the presented methodology to multiple obstacles, and a comparison with the NCHRP method are discussed.

Sight Distance Profile

The sight distance profile for a given obstacle for different values of the lateral clearance is shown in Figure 5. The obstacle is located at \( l/l = 0.3 \). The horizontal axis shows the location of the observer at various points of the tangent and curve. The PC is designated as the reference point, with the locations before it being negative and the locations beyond it being positive. The vertical axis shows the available sight distance for any given location of the observer. For example, for \( m = 15 \) ft, the minimum sight distance (435 ft) occurs when the observer is about 90 ft beyond PC. The sight distance profile is established using the developed relationships by computing the available sight distance for successive values of \( x_1 \). Unlike vertical curves, the minimum sight distance on a horizontal curve with a single obstacle occurs at a specific point on the traveled way rather than through a section of the traveled way.

For locations with restricted sight distances, the sight distance profile provides the length of the road within which the sight distance is restricted. This length is required for the operational and cost-effectiveness analysis developed by Neuman et al. (14). The probability that a critical event will occur at the location is directly proportional to the length of restricted sight distance. For example, if the required sight distance in Figure 5 is 600 ft and \( m = 15 \) ft, the SD profile shows that the length with a restricted sight distance is about 400 ft.

Application to Multiple Obstacles

For a horizontal curve with multiple obstacles, the sight distance profile of one obstacle interferes with the profiles of other obstacles. The actual sight distance profile is an envelope of the individual profiles, as shown in Figure 6. The horizontal curve has four obstacles with the indicated lateral clearances and locations on the curve. It is clear that Obstacle 2 is critical because it gives the least value of \( S_m \) (430 ft). The minimum sight distances can be determined using the developed relationships by considering each obstacle as a single obstacle (note that some obstacles may not have their minimum values on the sight distance envelope).

The interface among the profiles of various obstacles is an important element that should be considered in improving the sight distance on the curve. As noted in Figure 6, \( S_m \) on the curve can be improved by increasing the lateral clearance at Obstacle 2, but only to \( S_m = 450 \), which corresponds to Obstacle 4. Any further improvement in sight distance would require increasing the lateral clearances at both Obstacles 2 and 4, and so on. Thus, an obstacle that is currently not critical may become critical as other obstacles are displaced.

![FIGURE 5 Sight distance profile on horizontal curve for different lateral clearances.](image-url)
Comparison with NCHRP Method

As previously indicated, the geometric relationships of the NCHRP report give the available sight distance for different locations of observer, object, and obstacle (9,13). Using these relationships, the available sight distance was computed for consecutive locations of the observer, and the sight distance profile was plotted as shown in Figure 7 \( (R = 1,000 \text{ ft}, m = 80 \text{ ft}, \text{ and } I/I = 0.1) \). The sight distance profile based on the relationships presented in this paper is also shown. The \( S_m \) values of the NCHRP and presented methods are 1,020 ft and 950 ft, respectively. Thus, the NCHRP method overestimates \( S_m \) by about 7 percent. The differences in \( S_m \) were found to be much larger for \( I/I = 0 \) and smaller radii. However, the differences decrease as \( I/I \) approaches 0.5. The two methods give almost identical results of \( S_m \) for \( I/I = 0.5 \) in Case 1 and for any value of \( I/I \) in Case 3.

Although the difference between the minimum sight distances of the two methods is not large, the respective sight distance profiles are considerably different. If the required sight distance at the location is 1,100 ft, for example, the lengths of the restricted sight distance provided by the NCHRP and presented methods will be about 250 ft and 400 ft, respectively. Such a difference may affect the operational and cost-effectiveness analysis of restricted locations (10,14).

The difference between the two methods is caused by an assumption in the NCHRP relationships. The relationships implicitly assume that the lines connecting the observer and object to the curve center form equal angles with a perpendicular line drawn from the center to the line of sight. This assumption is not generally valid for computing the available sight distance. In addition, for computing \( S_m \), this assumption is exact only for Case 1 (when \( I/I = 0.5 \)) and Case 3.

EVALUATION AND DESIGN VALUES

To facilitate evaluation of the sight distance adequacy on horizontal curves, the developed relationships were used to establish values of the minimum sight distance for different characteristics of the curve and obstacle. The values are given in Tables 1 and 2, which are applicable to stopping, decision, and passing sight distances. Tables 1 and 2 can be used to

1. Determine the minimum sight distance on an existing curve and obstacles,
2. Determine the required lateral clearances to maintain a required minimum sight distance, and
3. Determine the critical lateral clearance (for design) that maintains a required minimum sight distance.

The critical lateral clearance, \( M \), is the largest value of \( m \) for given \( S_m \), \( R \), and \( I \). For \( S_m \leq L \), the critical value is presented by AASHTO (4). For \( S_m > L \), a formula and a nomograph for determining \( M \) were presented by Waissi and Cleveland (13). In Tables 1 and 2, the critical lateral clearance for \( S_m \leq L \) or \( S_m > L \) is the lateral clearance for \( I/I = 0.5 \). Figure 8 illustrates the results for \( R = 1,500 \text{ ft and } I = 40° \). As noted, there is a great difference between the lateral clearance requirements when the obstacle lies at PC (or PT) and
FIGURE 7 Comparison between sight distance profiles of presented and NCHRP methods.

TABLE 1 MINIMUM SIGHT DISTANCE ON HORIZONTAL CURVE WITH SINGLE OBSTACLE (R = 200 TO 800 ft)

<table>
<thead>
<tr>
<th>Cent. Angle (deg)</th>
<th>Obstr. (ft)</th>
<th>Present Method</th>
<th>Required SD</th>
<th>NC HAP Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0</td>
<td>500</td>
<td>950</td>
<td>1000</td>
</tr>
<tr>
<td>40</td>
<td>0.1</td>
<td>290</td>
<td>520</td>
<td>1000</td>
</tr>
<tr>
<td>60</td>
<td>0.2</td>
<td>220</td>
<td>370</td>
<td>1000</td>
</tr>
<tr>
<td>80</td>
<td>0.3</td>
<td>210</td>
<td>370</td>
<td>1000</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>200</td>
<td>370</td>
<td>1000</td>
</tr>
</tbody>
</table>

Note: minimum sight distances are expressed in feet.
### TABLE 2 MINIMUM SIGHT DISTANCE ON HORIZONTAL CURVE WITH SINGLE OBSTACLE (R = 1,000 TO 3,000 ft)

<table>
<thead>
<tr>
<th>Cent. Angle Loc. (deg)</th>
<th>500</th>
<th>1,000</th>
<th>1,500</th>
<th>2,000</th>
<th>3,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-20² 40 60 80 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 0.0 1010 1920 2840 3760 4670 1050 1970 2880 3790 4710 1090 2010 2920 3840 4760</td>
<td>1170 2090 3010 3930 4840</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 0.1 980 1910 2820 3740 4660 1030 1940 2860 3780 4700 1070 2000 2920 3860 4790</td>
<td>1130 2050 2960 3880 4800</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>1090 2010 2930 3850 4760</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5 0.3 970 1930 2810 3720 4640 1010 1910 2830 3750 4660 1020 1940 2860 3770 4690</td>
<td>1070 1990 2910 3820 4740</td>
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<td></td>
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<td>1050 1970 2890 3800 4720</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

² Lateral clearance (ft)

Note: minimum sight distances are expressed in feet.

![FIGURE 8](image-url) Comparison of lateral clearance requirements for different obstacle locations.
the middle of the curve; for example, for a sight distance of 1,000 ft, the lateral clearances needed are 49 ft and 80 ft, respectively.

Example

The following example illustrates the application of Tables 1 and 2. Given a horizontal curve with a single obstacle, \( R = 1,300 \) ft, \( m = 40 \) ft, \( I = 35^\circ \), and \( I_2 = 9.5^\circ \). The ratio \( I_1/I = 0.27 \). Then,

1. Determine \( S_m \): Using Table 2, interpolate the values of \( S_m \) for \( R = 1,000 \) ft and 1,500 ft and \( I_1/I = 0.2 \) and 0.3 at \( I = 30^\circ \) to obtain \( S_m \) = 682 ft. Repeat the interpolation for \( I = 40^\circ \) to obtain \( S_m \) = 664 ft. Therefore, for \( I = 35^\circ \), \( S_m \) = 673 ft (the exact value of \( S_m \) computed by the presented relationships is 661 ft).
2. Determine \( m \) if the required sight distance is 800 ft: Using Table 2, the corresponding lateral clearance can be interpolated in a similar manner as \( m = 70 \) ft. The exact value of \( S_m \) (computed by the presented relationships) corresponding to this lateral clearance is 803 ft, which is very close to the required value.
3. Determine the critical lateral clearance: For \( S_m = 800 \) ft, \( M \) is determined from Table 2 as 75 ft (for \( I_1/I = 0.5 \)). Using the Waissi and Cleveland formula (13), \( M \) is computed as 76 ft.

DISCUSSION OF RESULTS

Application of the AASHTO sight distance model for \( S_m \leq L \) to situations in which \( S_m > L \) results in overestimation of the maximum required lateral clearance \( M \). Situations in which the sight distance is greater than the curve length may arise because of the following factors:

1. AASHTO policy (4) and NCHRP research (9) have presented increased values of SSD. In addition, Neuman (5) recommends greater SSD values than those of AASHTO for most of the highway classifications.
2. At locations with special geometry or conditions, the DSD should be provided. The AASHTO design values of DSD (4) and those recommended by Neuman (5) and McGee (8) are twice to three times the SSD design.
3. Where AASHTO PSDs are provided, these distances will in most cases be greater than the curve length.

Even when \( S_m \leq L \), the vision obstacle on the horizontal curve may lie near the ends of the curve so that the needed lateral clearance is less than the maximum value \( M \). For these cases (\( S_m \leq L \), \( S_m > L \)), the developed relationships provide exact values of the minimum sight distance or the lateral clearance that satisfy sight distance needs. Application of the presented method should result in cost savings from roadside clearing and perhaps land acquisition.

SUMMARY

Exact relationships for establishing the sight distance profiles for highway horizontal curves with a single obstacle or multiple obstacles on the inside of the curve are presented. Closed-form solutions of the minimum sight distance are also presented for any location of the obstacle, and design values for practical use are established. These values can be used to determine the adequacy of sight distance at a particular location. It is no longer necessary to plot the entire sight distance profile to determine whether the location has a restricted sight distance. Only for restricted locations is the sight distance profile plotted to determine the length of the road with restricted sight distance and to evaluate alternative improvements. The results of this research should be useful for the design, operation, and safety of critical highway locations.

ACKNOWLEDGMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

APPENDIX A

Derivation of the Formulas for \( S_m \)

CASE 1

Differentiating Equation 10 with respect to \( \theta_1 \), then

\[
\frac{dS}{d\theta_1} = m_1[\sin(\theta_1 + \alpha)\cos\theta - \sin\theta_1\cos(\theta_1 + \alpha)]
\]

\[
+ \sin^2(\theta_1 + \alpha) + m_2[\sin(\theta_1 + J - \beta)\cos(\theta_1 + J) - \sin(\theta_1 + J)\cos(\theta_1 + J - \beta)]/\sin^2(\theta_1 + J - \beta)
\]

Consider the following identity (15):

\[
\sin(x - y) = \sin x \cos y - \cos x \sin y
\]

Based on this identity, the first and second expressions in brackets equal \( \sin \alpha \) and \( \sin(-\beta) \), respectively. Thus,

\[
\frac{dS}{d\theta_1} = \frac{m_1 \sin \alpha}{\sin^2(\theta_1 + \alpha)} + \frac{m_2 \sin(-\beta)}{\sin^2(\theta_1 + J - \beta)}
\]

Equating Equation 31 to zero and noting that \( \sin(-\beta) = -\sin \beta \),

\[
\sin(\theta_1 + \alpha) = \frac{m_1 \sin \alpha}{m_2 \sin \beta}
\]

in which \( \theta_1^* \) is the angle corresponding to the minimum sight distance. Equation 32 is the same as Equation 11.

CASE 2

Substituting for \( \gamma \) from Equation 16 into Equation 15 and then substituting for \( \phi \) from Equation 17 gives

\[
I_2 = 270^\circ - \sin^{-1}(f) - \alpha - \theta_1
\]
where \( f \) is a function of \( \theta_i \) given by

\[
f = \left( \frac{R - m}{R} \right) \sin(I_s + \alpha + \theta_i - 90^\circ)
\]  

(34)

Substituting for \( I_s \) from Equation 33 into Equation 18 and differentiating Equation 18 with respect to \( \theta_i \),

\[
\frac{dS}{d\theta_i} = \frac{-f'}{1 - f^2} + \frac{m\sin(\theta_i + \alpha) \cos \theta_i}{\sin^2(\theta_i + \alpha)}
\]

\[
- \frac{m\sin \theta_i \cos (\theta_i + \alpha)}{\sin^2(\theta_i + \alpha)} - 1
\]  

(35)

where \( f' \) equals \( df/d\theta_i \), which is given by

\[
f' = \left( \frac{R - m}{R} \right) \sin(\theta_i + \alpha + I_s)
\]  

(36)

The expression in brackets in Equation 35 equals \( \sin \alpha \) based on the identity of Equation 30. Substituting for \( f \) and \( f' \) from Equations 34 and 36 into Equation 35 and equating \( dS/d\theta_i \) to zero gives

\[
\frac{m_i \sin \alpha}{R \sin^2(\theta_i + \alpha)} - \frac{(R - m) \sin(\theta_i + \alpha + I_s)}{[R^2 - (R - m)^2 \cos^2(\theta_i + \alpha + I_s)]^{1/2}} = 1
\]  

(37)

which is Equation 19.

**CASE 3**

Equation 23 is written as

\[
S = 2RI_s - \tan^{-1}(f)
\]  

(38)

where \( f \) is a function of \( I_s \) given by

\[
f = \frac{[R(1 - \cos I_s) - m][R \sin I_s]}{R^2 \sin^2 I_s}
\]  

(39)

Differentiating Equation 38 with respect to \( I_s \),

\[
\frac{dS}{dI_s} = 2R \left[ 1 - \frac{f'}{1 + f^2} \right]
\]  

(40)

where \( f' = df/dI_s \), which is given by

\[
f' = \frac{[R \sin I_s]^2 - [R(1 - \cos I_s) - m][R \cos I_s]}{R^2 \sin^2 I_s}
\]  

(41)

Substituting for \( f \) and \( f' \) from Equations 39 and 41 into Equation 40 and equating \( dS/dI_s \) to zero gives

\[
[R(1 - \cos I_s) - m]^2 + [R(1 - \cos I_s) - m][R \cos I_s] = 0
\]  

(42)

After rearranging, Equation 42 becomes

\[
[R(1 - \cos I_s) - m][R - m] = 0
\]  

(43)

which is Equation 24.

**APPENDIX B**

**Numerical Solution of Equations 11 and 19**

The critical angle \( \theta_i \) of Equations 11 and 19 can be obtained by successive approximations using the method of linear interpolation (16). Equation 11 or 19 is written as

\[
f(u) = 0
\]  

(44)

where \( u \) is used instead of \( \theta_i \). To determine the root of Equation 44, select two values \( u_1 \) and \( u_2 \) for which \( f(u_1) \) and \( f(u_2) \) have opposite signs. The following steps are then performed:

1. Set

\[
u_3 = u_2 - f(u_2) \frac{u_2 - u_1}{f(u_2) - f(u_1)}
\]

(45)

2. If \( f(u_3) \) has an opposite sign to \( f(u_1) \), set \( u_2 = u_3 \). Otherwise, set \( u_1 = u_3 \)

3. Repeat Steps 1 and 2 until

\[
|u_2 - u_1| \leq \varepsilon_1 \]

\[
|f(u_3)| \leq \varepsilon_2
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are specified tolerance values.

This method guarantees convergence. A modified linear interpolation method, which converges faster, may also be used (16).

**REFERENCES**


*Publication of this paper sponsored by Committee on Geometric Design.*