# Sight Distance Model for Unsymmetrical Crest Curves

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In the AASHTO geometric design policy, the need for using unsymmetrical vertical curves because of clearance restrictions and other design controls is pointed out. Formulas for laying out these curves are presented in the highway engineering literature. However, no relationships are available concerning sight distance characteristics on these curves. A sight distance model for unsymmetrical crest curves has been developed to relate the available sight distance to the curve parameters, driver and object heights, and their locations along the curve. These relationships are used in a procedure for determining the available minimum sight distance. The model is used to explore the distinct features of sight distance profiles on unsymmetrical crest curves. To facilitate practical use, the model is used to establish design length requirements of unsymmetrical crest curves based on the stopping, decision, and passing sight distance needs presented by recent innovative approaches and by AASHTO. The model should prove useful in the design and safety evaluation of critical highway locations.

Three types of sight distances are considered on highways and streets: (a) stopping sight distance (SSD), applicable to all highways; (b) passing sight distance (PSD), applicable only to two-lane highways; and (c) decision sight distance (DSD), needed at complex locations [AASHTO (*1–4*), Neuman and Glennon (*5*), Olson et al. (*6*)]. Sight distance is one of the most fundamental criteria affecting the design of horizontal and vertical curves and their construction cost and safety. The effect of sight distance on highway safety has been addressed by Glennon (*7*) and Urbanik et al. (*8*). To meet this criterion, the available sight distance at any point on the curve must be greater than the required sight distance. For vertical crest curves, the available sight distance depends on the curve design parameters, the driver's eye height, the height of the road object, and the positions of the driver and the object.

The AASHTO sight distance models for crest (and sag) vertical curves (*1–4*) are based on a parabolic curve with an equivalent vertical axis centered on the vertical point of intersection (PVI). For simplicity, this symmetrical curve, which has equal horizontal projections of the tangents, is usually used in roadway profile design. In AASHTO's *Policy on Geometric Design of Highways and Streets* (Green Book) (*4*) it is pointed out that on certain occasions, because of critical clearance or other controls, the use of unsymmetrical curves may be required. Because the need for these curves is infrequent, no information on them has been included in the Green Book; for limited instances, this information is available in highway engineering texts.

A number of existing highway and surveying engineering texts (*9–11*) derive or present the formulas required for laying out an unsymmetrical curve (which consists of two unequal parabolic arcs with a common tangent). These formulas relate the rates of change in grade of the two arcs to the total curve length, the algebraic difference in grade, and the lengths of the arcs. Apparently, however, no information has been presented in the literature concerning the relationships between sight distance and the parameters of an unsymmetrical curve. These relationships are needed to design the curve length that satisfies a required sight distance or to evaluate the adequacy of sight distance on existing unsymmetrical curves.

A sight distance model for unsymmetrical crest curves has been developed and used to establish design length requirements for these curves based on SSD, DSD, and PSD. Before the model is presented, it is useful to describe the characteristics of an unsymmetrical curve.

The unsymmetrical vertical curve connects two tangents of the grade line and consists of two parabolic arcs with a common tangent point, PCC, located at PVI as shown in Figure 1. The first and second tangents have grades $g_1$ and $g_2$ (in decimals) and intersect at PVI. The grade is positive if it is upward to the right and negative if it is downward to the right. The beginning and end points of the curve are BVC (beginning of vertical curve) and EVC (end of vertical curve). The length of the vertical curve is $L$ and the lengths of its first and second parabolic arcs are $L_1$ and $L_2$. The algebraic difference in grade of the vertical curve, $A$, equals $(g_1 - g_2)$.

The formulas for the rates of change in grade of the first and second parabolic arcs, $r_1$ and $r_2$, are given by Hickerson (*9*):

$$r_1 = AL_2/LL_1 \tag{1}$$

$$r_2 = AL_1/LL_2 \tag{2}$$

The ratio $(A/L)$ is the rate of change in grade for the vertical curve if it were symmetrical ($L_1 = L_2 = L/2$). Therefore, if $L_1 > L_2$ for an unsymmetrical curve, $r_1$ would be smaller than the rate of change in grade of the respective symmetrical curve and $r_2$ would be greater. This means that the parabolic arc with a smaller length is sharper and the arc with a larger length is flatter. For this reason, the minimum sight distance on an unsymmetrical curve would be smaller than that of a symmetrical curve of the same length.

## GEOMETRIC RELATIONSHIPS

Suppose that the second parabolic arc of the unsymmetrical curve has a shorter length. To determine the minimum sight

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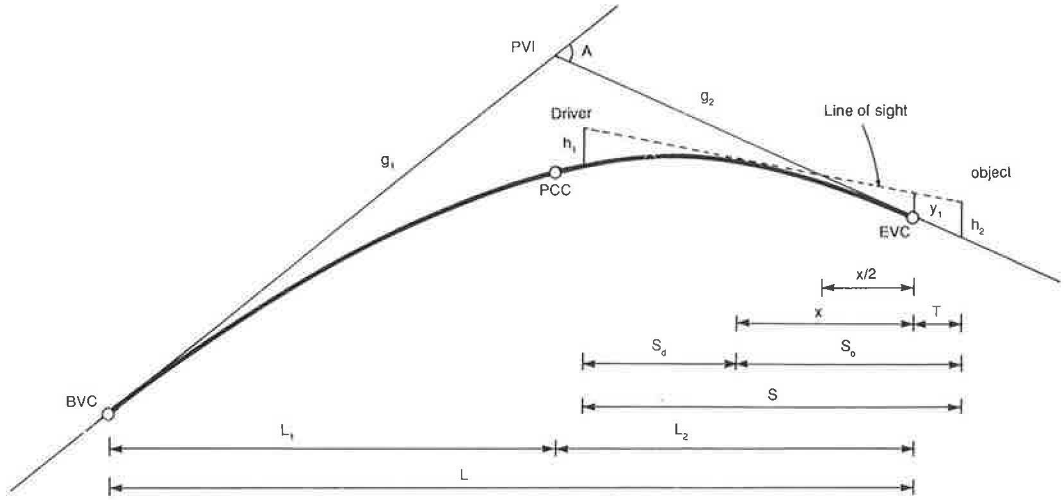


**FIGURE 1 Case 1: Object beyond EVC and driver on second arc.**

distance, $S_m$, on the curve, the following five cases are considered:

- Case 1: Object beyond EVC and driver on second arc,
- Case 2: Object beyond EVC and driver on first arc,
- Case 3: Object beyond EVC and driver before BVC,
- Case 4: Object before EVC and driver on first arc, and
- Case 5: Object before EVC and driver before BVC.

The relationships between the available sight distance $S$ and the vertical curve length are derived for each of these cases for a specified location of the object. These relations are used later to determine $S_m$. The derivation is divided into the following three parts:

1. Derivation of the distance between the object and the tangent point of the line of sight, $S_o$;
2. Derivation of the distance between the driver and the tangent point of the line of sight, $S_d$; and
3. Obtaining the sight distance, $S = S_o + S_d$.

### Case 1: Object Beyond EVC and Driver on Second Arc

In Case 1, the object lies beyond (or at) EVC and the driver is on the second arc. The distance between the object and EVC is denoted by $T$. Figure 1 shows the geometry of this case.

*Component $S_o$*

On the basis of the property of a parabola, the vertical distance from EVC to the line of sight, $y_1$, is given by

$$y_1 = r_2x^2/2 \tag{3}$$

where $x$ is the distance from EVC to the tangent point of the line of sight. Based on the similarity of the two triangles with bases $h_2$ and $y_1$, a quadratic equation in $x$ is formed and the following relationship can be obtained:

$$x = -T + [T^2 + (2h_2/r_2)]^{1/2} \tag{4}$$

The distance $S_o$, which equals $T + x$, becomes

$$S_o = [T^2 + (2h_2/r_2)\ ]\ 1/2 \tag{5}$$

*Component $S_d$*

On the basis of the property of a parabola, $S_d$ is given by

$$S_d = (2h_1/r_2)^{1/2} \tag{6}$$

*Sight Distance S*

The sight distance $S$ is the sum of the components of Equations 5 and 6, which gives

$$S = [T^2 + (2h_2/r_2)]^{1/2} + (2h_1/r_2)^{1/2} \tag{7}$$

If the object is at EVC ($T = 0$), Equation 7 indicates that $S$ will be constant and will remain so even if the object is before EVC, as long as both the driver and object are on the second arc.

### Case 2: Object Beyond EVC and Driver on First Arc

The geometry of Case 2 is shown in Figure 2. Assume for now that the line of sight is tangent to the second arc. (The situation when the line of sight is tangent to the first arc is addressed later.)

*Component $S_o$*

The derivation of $S_o$ is similar to Case 1. Thus,

$$S_o = [T^2 + (2h_2/r_2)]^{1/2} \tag{8}$$

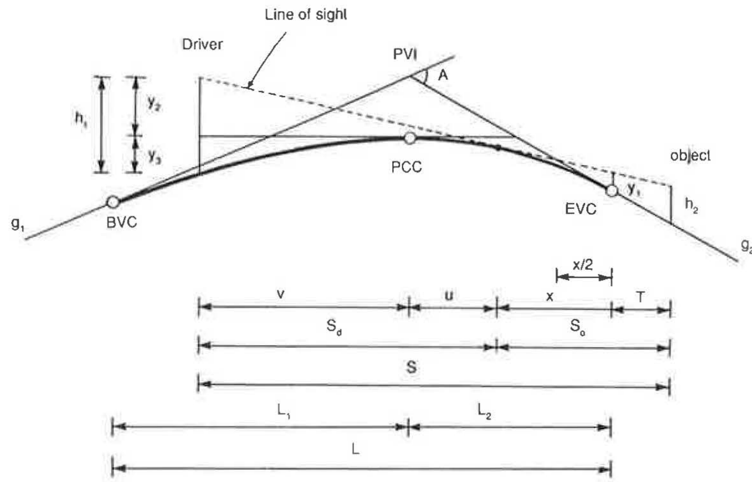


**FIGURE 2 Case 1: Object beyond EVC and driver on first arc.**

*Components $S_d$*

The distance $S_d$ consists of two components, $u$ and $v$. The distance $u$ equals $(L_2 - x)$, which after substituting for $x$ from Equation 4 becomes

$$u = L_2 + T - [T^2 + (2h_2/r_2)]^{1/2} \tag{9}$$

The component $v$ can be derived by equating $h_1$ to its two parts, $y_2$ and $y_3$, shown in Figure 2. These parts are given by

$$y_2 = r_2u[(u/2) + v] \tag{10}$$

$$y_3 = r_1v^2/2 \tag{11}$$

The right-hand side of Equation 10 is the product of the difference in grade between the line of sight and the tangent at PCC, $r_2u$, and the respective horizontal distance, $(u/2) + v$. Thus,

$$h_1 = (r_2u^2/2) + r_2uv + (r_1v^2/2) \tag{12}$$

Solving Equation 12 for $v$ and considering the positive root,

$$v = [-r_2u + (r_2^2u^2 - r_1r_2u^2 + 2r_1h_1)^{1/2}]/r_1 \tag{13}$$

Thus, the sight distance compound $S_d$ is given by

$$S_d = u + [-r_2u + (r_2^2u^2 - r_1r_2u^2 + 2r_1h_1)^{1/2}]/r_1 \tag{14}$$

*Sight Distance S*

The available sight distance when the object is beyond EVC and the driver is on the first arc is the sum of the components of Equations 8 and 14. Thus,

$$S = L_2 + T + [- r_2u + (r_2^2u^2 - r_1r_2u^2 + 2r_1h_1)^{1/2}]/r_1 \tag{15}$$

where $u$ is a function of $T$ given by Equation 9.

**Case 3: Object Beyond EVC and Driver Before BVC**

The geometry of Case 3 is shown in Figure 3. Assume again that the line of sight is tangent to the second arc. The distance from the object to EVC is $T$.

*Component $S_o$*

The component $S_o$ is derived, as it was for Case 2, as

$$S_o = [T^2 + (2h_2/r_2)]^{1/2} \tag{16}$$

*Component $S_d$*

As shown in Figure 3, the component $S_d$ consists of three parts, $u$, $v$, and $w$. The distance $u$ is given by Equation 9, and the derivation of $v$ and $w$ follows. The distance from PVI to the line of sight, $y_2$, equals the distance from PVI to PCC minus the distance from PCC to the line of sight. Thus,

$$y_2 = r_2(L_2^2 - u^2)/2 \tag{17}$$

The horizontal distance $v$ equals $y_2$ divided by the difference in grade between the line of sight and the first tangent, $A - r_2$ $(L_2 - u)$. Thus,

$$v = r_2(L_2^2 - u^2)/2\ [A - r_2(L_2 - u)] \tag{18}$$

Similarly, the distance $w$ equals $h_1$ divided by the difference in grade between the line of sight and the first tangent. Thus,

$$w = h_1/[A - r_2(L_2 - u)] \tag{19}$$

The sight distance component, $S_d$, equals the sum of $u$, $v$, and $w$, giving

$$S_d = u + \{[r_2(L_2^2 - u^2)/2] + h_1\}/[A - r_2(L_2 - u)\ ] \tag{20}$$

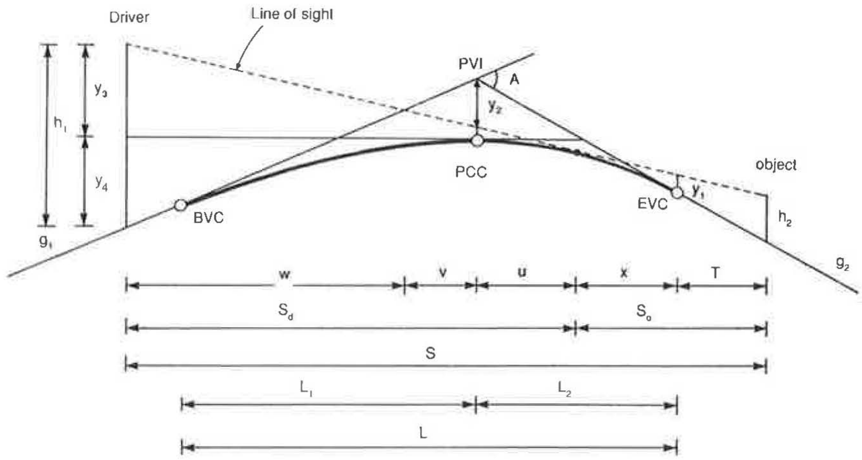


**FIGURE 3 Case 1: Object beyond EVC and driver before bvc.**

*Sight Distance S*

The available sight distance when the object is beyond EVC and the driver is before BVC is the sum of the components of Equations 16 and 20. Thus,

$$S = T + L_2 + \{[r_2(L_2^2 - u^2)/2] + h_1\}/[A - r_2(L_2 - u)] \quad (21)$$

where $u$ is a function of $T$ given by Equation 9.

**Case 4: Object Before EVC and Driver on First Arc**

The geometry of Case 4 is similar to that of Case 2, except that the object is on the second arc at a distance $T'$ from EVC. The component $S_o$ is given by Equation 8 for $T$ equals zero, and $S_d$ is given by Equation 14. Thus, the sight distance can be obtained as

$$S = L_2 - T' + [- r_2u + (r_2^2u^2 - r_1r_2u^2 + 2r_1h_1)^{1/2}]/r_1 \quad (22)$$

in which $u$ is given by

$$u = L_2 - T' - (2h_2/r_2)^{1/2} \quad (23)$$

**Case 5: Object Before EVC and Driver Before BVC**

The geometry of Case 5 is similar to that of Case 3, except that the object is on the second arc at a distance $T'$. Again, the component $S_o$ is given by Equation 8 for $T = 0$ and $S_d$ is given by Equation 20. Thus, the sight distance becomes

$$S = L_2 - T' + \{[r_2(L_2^2 - u^2)/2] + h_1\}/[A - r_2(L_2 - u)] \quad (24)$$

in which $u$ is given by Equation 23.

**SIGHT DISTANCE CHARACTERISTICS**

The minimum sight distance can occur only for Case 1, 2, or 3. For Case 1, the minimum value occurs when the object is at EVC ($T = 0$). For Cases 2 and 3 the object generally would be somewhere beyond EVC. For Cases 4 and 5, the available sight distance decreases as the driver and object move toward EVC, because the second arc is sharper than the first arc. $S_m$ then occurs when the object is beyond EVC, which corresponds to Case 2 or 3.

Cases 4 and 5, however, are considered if the line of sight for Cases 2 and 3 is tangent to the first arc, which occurs when $u$ of Equation 9 is negative. This situation is handled by defining $T$ as the distance between the driver and BVC and applying the relationships of the five cases after replacing $h_1$ by $h_2$, $L_1$ by $L_2$, and $r_1$ by $r_2$ (and vice versa). A comparison of the sight distance characteristics for symmetrical and unsymmetrical curves and a procedure for determining $S_m$ follow.

**Comparison with Symmetrical Curves**

For symmetrical crest curves, the sight distance relationships have been developed for $S \leq L$ and $S \geq L$ (6). The relationships of Cases 1, 2, and 3 are reduced to the known relationships for symmetrical crest curves for $r_1 = r_2 = A/L$ and $L_1 = L_2 = L/2$. Substituting these values into Equations 7 and 15 of Cases 1 and 2 (for $T = 0$) yields the relationship for $S \leq L$. Similarly, substituting these values into Equation 21 of Case 3, expressing $u$ and $T$ in terms of $x$, the known relationship for $S \geqslant L$ is obtained.

The sight distance profile for an unsymmetrical curve differs from that of a symmetrical curve (with the same length) in several respects, as shown in Figure 4. Note that $R$ denotes the ratio of the shorter arc to the total curve length, $L_2/L$. For the unsymmetrical curve, the available sight distance varies along the curve even when both the driver and object are on the curve. The sight distance profile for the unsymmetrical curve also varies with the direction of travel, unlike that for the symmetrical curve. This significant aspect of sight dis-

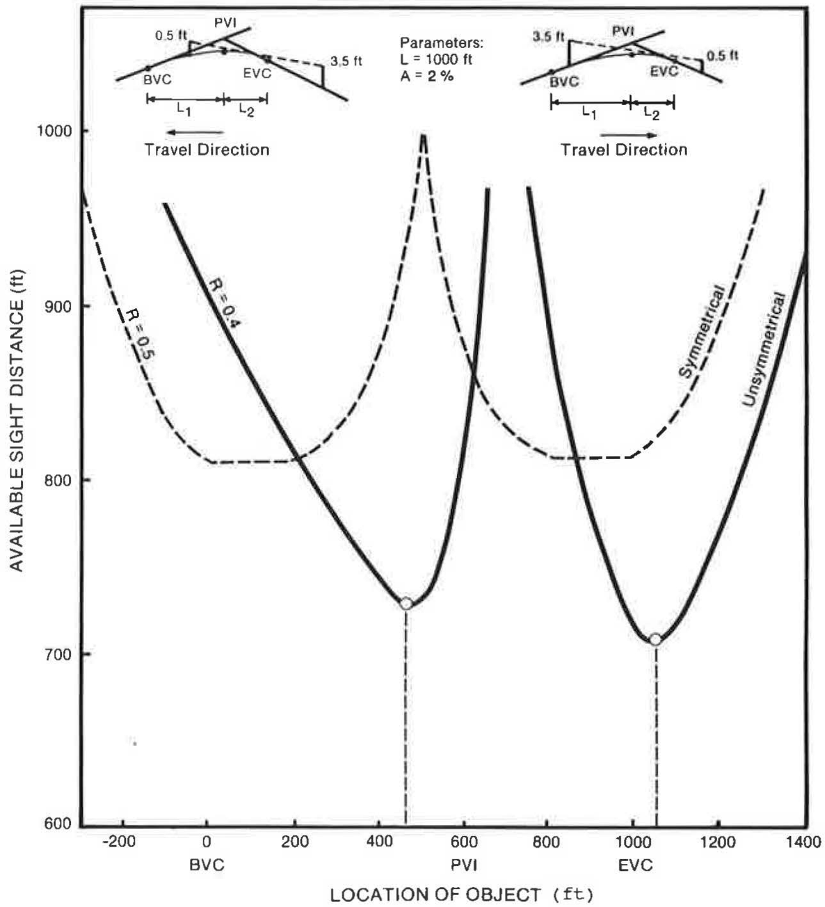


**FIGURE 4 Sight distance profile of an unsymmetrical crest curve.**

tances on unsymmetrical crest curves has implications for the operational and cost-effectiveness analysis of critical locations [Neuman and Glennon (*5*), Neuman et al. (*12*) ]. As noted in Figure 4, the minimum sight distance is less when the driver travels from the flatter to the sharper arc of the unsymmetrical curve ($S$ = 710 ft). This value is 13 percent less than the minimum sight distance of the symmetrical curve (815 ft).

### Procedure for Calculating $S_m$

The minimum sight distance, $S_m$, can be determined by differentiating $S$ (for Cases 2 and 3) with respect to $T$ and equating the derivative to zero. The resulting expression, however, is too complicated to be useful. Therefore, a simple iterative procedure was used to determine the available sight distance for consecutive values of $T$ until $S_m$ is obtained (see Figure 5).

The procedure starts with an initial (sufficiently large) value of $T$ along with an increment $\Delta T$. For each $T$, the available sight distance $S$ is computed and compared with the previously computed value, $S'$. The procedure continues until $S > S'$; at this point the minimum sight distance has just been reached ($S_m = S'$). If $u < 0$ (the line of sight is tangent to the first arc), the curve and sight distance variables are switched and all five cases are considered.

## DESIGN CREST CURVE LENGTH FOR SSD

Design length requirements of unsymmetrical crest curves are developed based on the SSD design values, object height, and driver's eye height presented by Neuman (*13*) and AASHTO (*4*).

### Neuman's Approach

Neuman's approach abandons the concept that a single design model of SSD is appropriate for all highway types under all conditions (*13*). It suggests a fresh approach that considers the functional highway classification in determining SSD design policy and values. The following five types of highways are considered:

1. Low-volume roads,
2. Two-lane primary rural highways,
3. Multilane urban arterials,
4. Urban freeways, and
5. Rural freeways.

SSD requirements by highway type were developed by Neuman on the basis of highway-related perception-reaction time and friction characteristics. The design values of the object

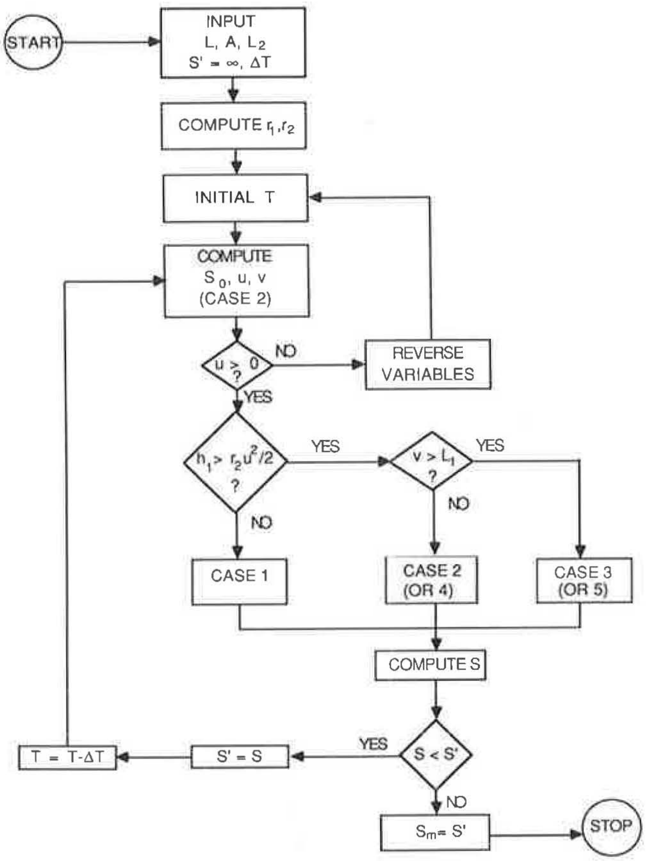


**FIGURE 5 Calculating minimum sight distance on an unsymmetrical crest-curve–logical flow.**

and driver eye heights also vary according to the highway type. These different values reflect the frequency of occurrence and severity of the consequences of events on various highways. The design values are based on the following critical events: (a) a single-vehicle encounter with a large object (1-ft high) for low-volume roads; (b) a single-vehicle encounter with a small object (6-in. high) for rural highways; and (c) vehicle-vehicle conflict (2-ft object height) for other highway types.

## Length Requirements

### *Based on Neuman's Approach*

Crest curve length requirements are calculated using the developed relationships. The crest curve length, $L$, is varied and the minimum sight distance is determined for each assumed value of $L$. The required design length is the one for which the minimum sight distance equals the required SSD.

Tables 1–5 show the length requirements of crest curves for $R = 0.3$, 0.4, and 0.5, where $R = L_2/L$ ($L_2$ is the length of the shorter parabolic arc and therefore $R \leq 0.5$). These requirements are based on Neuman's SSD values, which are shown in the column heads. The values for $R = 0.5$ are the same as those presented by Neuman for symmetrical curves (*13*). As noted, the length requirements of unsymmetrical curves can be more than twice those of symmetrical curves. For small SSD and $A$, the required curve length is generally small. A minimum value equal to three times the design speed in miles per hour is used. It is also noted that some length requirements are not practical.

### *Based on AASHTO Approach*

Table 6 shows the length requirements based on the required SSD values of AASHTO, a driver's eye height of 3.5 ft, and an object height of 6 in. These requirements are applicable to all highways. The values for $R = 0.5$ correspond to symmetrical crest curves and are the same as those of AASHTO (*4*).

A comparison of the length requirements for symmetrical and unsymmetrical crest curves is shown in Figure 6. The length of the symmetrical curve is expressed as a percentage of the design length of the unsymmetrical curve. The solid curves correspond to the low-volume roads (Table 1) for $V = 50$ mph. For $R = 0.3$, the length of the symmetrical curve represents 69 percent of the required design length for $A = 3$ percent and only 43 percent for $A = 8$ to 10 percent. The results for $A = 8$ to 10 percent are the same because for these values $S < L_2$ and the ratio of $L_s$ and $L$ depends only on $R$. These results clearly show that the sight distance model for symmetrical curves would greatly underestimate the length

TABLE 1 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON LOW-VOLUME ROADS BASED ON SSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 30 mph (SSD= 141 ft) | | | 40 mph (SSD= 236 ft) | | | 50 mph (SSD= 363 ft) | | | 60 mph (SSD= 507 ft) | | |
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 90 | 90 | 90 | 120 | 120 | 120 | 150 | 150 | 150 | 280 | 220 | 190 |
| 4 | 90 | 90 | 90 | 120 | 120 | 120 | 560 | 370 | 320 | 1430 | 880 | 630 |
| 6 | 90 | 90 | 90 | 350 | 240 | 200 | 1120 | 700 | 480 | 2190 | 1410 | 940 |
| 8 | 120 | 90 | 90 | 600 | 370 | 280 | 1500 | 960 | 640 | 2920 | 1880 | 1250 |
| 10 | 210 | 140 | 120 | 790 | 500 | 340 | 1870 | 1200 | 800 | 3640 | 2340 | 1560 |

a Driver eye height = 3.5 ft
Object height = 1.0 ft
SSD values are based on Neuman (13)

b Ratio of shorter arc to total curve length

TABLE 2 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON TWO-LANE PRIMARY RURAL ROADS BASED ON SSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed: 40 mph (SSD= 343 ft) | | | 50 mph (SSD= 498 ft) | | | 60 mph (SSD= 680 ft) | | | 70 mph (SSD= 891 ft) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 120 | 120 | 120 | 150 | 150 | 150 | 390 | 320 | 290 | 1140 | 790 | 710 |
| 4 | 210 | 170 | 150 | 850 | 540 | 460 | 1970 | 1220 | 860 | 3440 | 2210 | 1480 |
| 6 | 630 | 400 | 330 | 1610 | 1010 | 690 | 3000 | 1930 | 1290 | 5150 | 3320 | 2210 |
| 8 | 1010 | 620 | 440 | 2150 | 1380 | 920 | 4000 | 2580 | 1720 | 6870 | 4420 | 2950 |
| 10 | 1280 | 820 | 550 | 2690 | 1730 | 1150 | 5000 | 3220 | 2150 | 8590 | 5520 | 3680 |

[a] Driver eye height = 3.5 ft
Object height = 2.0 ft
SSD values are based on Neuman (13)
[b] Ratio of shorter arc to total curve length

TABLE 3 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES FOR MULTILANE URBAN ARTERIALS BASED ON SSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed: 30 mph (SSD= 189 ft) | | | 40 mph (SSD= 304 ft) | | | 50 mph (SSD= 452 ft) | | |
|---|---|---|---|---|---|---|---|---|---|
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 90 | 90 | 90 | 120 | 120 | 120 | 150 | 150 | 150 |
| 4 | 90 | 90 | 90 | 120 | 120 | 120 | 610 | 420 | 370 |
| 6 | 90 | 90 | 90 | 420 | 290 | 250 | 1310 | 810 | 570 |
| 8 | 160 | 130 | 110 | 760 | 460 | 350 | 1770 | 1140 | 760 |
| 10 | 290 | 190 | 170 | 1000 | 630 | 430 | 2210 | 1420 | 950 |

[a] Driver eye height = 3.5 ft
Object height = 2.0 ft
SSD values are based on Neuman (13)
[b] Ratio of shorter arc to total curve length

TABLE 5 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON RURAL FREEWAYS BASED ON SSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed: 50 mph (SSD= 545 ft) | | | 60 mph (SSD= 765 ft) | | | 70 mph (SSD=1074 ft) | | |
|---|---|---|---|---|---|---|---|---|---|
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 720 | 510 | 430 | 1970 | 1210 | 890 | 4050 | 2610 | 1740 |
| 4 | 2090 | 1350 | 900 | 4110 | 2650 | 1770 | 8100 | 5210 | 3480 |
| 6 | 3130 | 2020 | 1350 | 6170 | 3970 | 2650 | 12150 | 7820 | 5210 |
| 8 | 4180 | 2690 | 1790 | 8220 | 5290 | 3530 | 16200 | 10420 | 6950 |
| 10 | 5220 | 3360 | 2240 | 10280 | 6610 | 4410 | 20250 | 13020 | 8680 |

[a] Driver eye height = 3.5 ft
Object height = 0.5 ft
SSD values are based on Neuman (13)
[b] Ratio of shorter arc to total curve length

TABLE 4 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON URBAN FREEWAYS BASED ON SSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed: 50 mph (SSD= 518 ft) | | | 60 mph (SSD= 726 ft) | | | 70 mph (SSD= 989 ft) | | |
|---|---|---|---|---|---|---|---|---|---|
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 150 | 150 | 150 | 520 | 420 | 380 | 1650 | 1060 | 900 |
| 4 | 960 | 600 | 500 | 2280 | 1420 | 980 | 4230 | 2720 | 1820 |
| 6 | 1750 | 1110 | 750 | 3420 | 2200 | 1470 | 6350 | 4080 | 2720 |
| 8 | 2330 | 1500 | 1000 | 4560 | 2940 | 1960 | 8460 | 5440 | 3630 |
| 10 | 2910 | 1870 | 1250 | 5700 | 3670 | 2450 | 10580 | 6800 | 4540 |

[a] Driver eye height = 3.5 ft
Object height = 2.0 ft
SSD values are based on Neuman (13)
[b] Ratio of shorter arc to total curve length

if it were used for unsymmetrical curves. The dashed curves, which correspond to Table 6 (based on AASHTO's SSD), exhibit similar characteristics.

## DESIGN CREST CURVE LENGTH FOR DSD

For locations with special geometry or conditions, where DSD should be provided, object and eye heights of 0 and 3.5 ft, respectively, are used to develop the design length requirements from crest curves. The results are presented in Table 7 for DSD ranging from 200 to 800 ft. For larger values of DSD, the length requirements are generally impractical, except for very flat curves. It should be noted that Table 7 is applicable to all highway types. One need only specify the required DSD value [AASHTO (*4*), Neuman (*13*), McGee (*14*) ] and interpolate the curve length from the table.

TABLE 6 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON ALL HIGHWAYS BASED ON SSD OF AASHTO[a]

| Algeb. Diff. grade (%) | Design Speed | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 mph (SSD= 125 ft) | | | 30 mph (SSD= 200 ft) | | | 40 mph (SSD= 275 ft) | | | 50 mph (SSD= 400 ft) | | | 60 mph (SSD= 525 ft) | | | 70 mph (SSD= 625 ft) | | |
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 210 | 160 | 150 | 630 | 460 | 390 | 1120 | 720 | 590 |
| 4 | 60 | 60 | 60 | 110 | 90 | 90 | 370 | 260 | 220 | 1100 | 680 | 490 | 1940 | 1250 | 830 | 2750 | 1770 | 1180 |
| 6 | 60 | 60 | 60 | 330 | 220 | 180 | 790 | 490 | 350 | 1690 | 1090 | 730 | 2910 | 1870 | 1250 | 4120 | 2650 | 1770 |
| 8 | 140 | 100 | 90 | 550 | 340 | 250 | 1070 | 690 | 460 | 2250 | 1450 | 970 | 3880 | 2490 | 1660 | 5490 | 3530 | 2360 |
| 10 | 230 | 150 | 120 | 710 | 450 | 310 | 1330 | 860 | 570 | 2810 | 1810 | 1210 | 4840 | 3120 | 2080 | 6860 | 4410 | 2940 |

[a] Driver eye height = 3.5 ft
Object height =0.5 ft
[b] Ratio of shorter arc to total curve length

Note: curve lengths are expressed in feet.

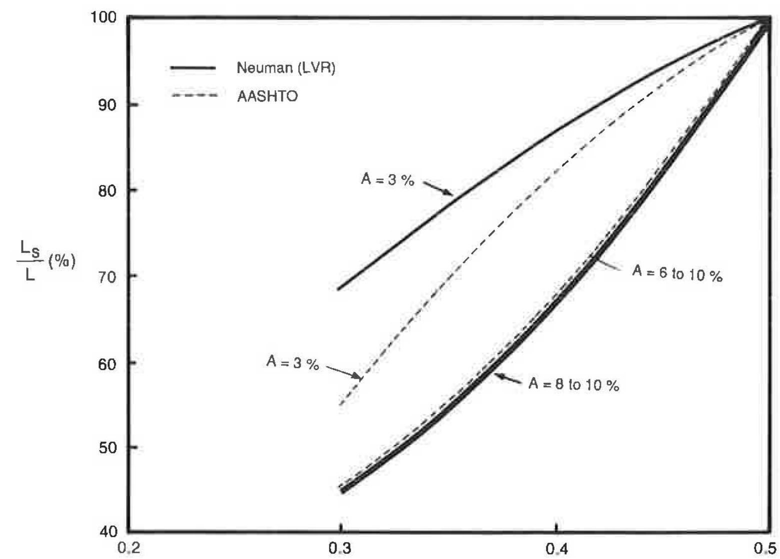


**FIGURE 6 Comparison of length requirements of symmetrical and unsymmetrical crest curves ($V$ = 50 mph).**

TABLE 7 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES ON ALL HIGHWAYS BASED ON DSD[a]

| Algeb. Diff. grade (%) | Decision Sight Distance (ft) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 200 | | | 400 | | | 600 | | | 800 | | |
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 2 | 90 | 70 | 50 | 1020 | 640 | 460 | 2400 | 1550 | 1030 | 4270 | 2750 | 1830 |
| 4 | 510 | 320 | 230 | 2140 | 1380 | 920 | 4800 | 3090 | 2060 | 8540 | 5490 | 3660 |
| 6 | 800 | 520 | 350 | 3200 | 2060 | 1380 | 7200 | 4630 | 3090 | 12800 | 8230 | 5490 |
| 8 | 1070 | 690 | 460 | 4270 | 2750 | 1830 | 9600 | 6180 | 4120 | 17070 | 10980 | 7320 |
| 10 | 1340 | 860 | 580 | 5340 | 3430 | 2290 | 12000 | 7720 | 5150 | 21340 | 13720 | 9150 |

[a] Driver eye height = 3.5 ft
Object height = 0 ft
[b] Ratio of shorter arc to total curve length

Note: curve lengths are expressed in feet.

TABLE 8 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES BASED ON PSD (IN FEET)[a]

| Algeb. Diff. grade (%) | Design Speed | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 mph | | | 30 mph | | | 40 mph | | | 50 mph | | | 60 mph | | | 70 mph | | |
| | R=.3[d] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| **Passenger Car Passing Passenger Car[b]** | | | | | | | | | | | | | | | | | | |
| | (PSD= 325 ft) | | | (PSD= 525 ft) | | | (PSD= 700 ft) | | | (PSD= 875 ft) | | | (PSD= 1025 ft) | | | (PSD= 1200 ft) | | |
| 2 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 260 | 220 | 200 | 650 | 540 | 500 | 1250 | 920 | 850 |
| 4 | 60 | 60 | 60 | 360 | 300 | 280 | 1120 | 710 | 630 | 2180 | 1320 | 1000 | 3160 | 1960 | 1360 | 4350 | 2790 | 1870 |
| 6 | 180 | 150 | 140 | 1090 | 660 | 540 | 2220 | 1380 | 960 | 3470 | 2230 | 1490 | 4760 | 3060 | 2040 | 6520 | 4200 | 2800 |
| 8 | 430 | 290 | 270 | 1660 | 1040 | 720 | 2960 | 1910 | 1270 | 4630 | 2980 | 1990 | 6350 | 4080 | 2720 | 8700 | 5590 | 3730 |
| 10 | 720 | 440 | 350 | 2080 | 1340 | 900 | 3700 | 2380 | 1590 | 5780 | 3720 | 2480 | 7930 | 5100 | 3400 | 10870 | 6990 | 4660 |
| **Passenger Car Passing Truck[b]** | | | | | | | | | | | | | | | | | | |
| | (PSD= 350 ft) | | | (PSD= 575 ft) | | | (PSD= 800 ft) | | | (PSD= 1025 ft) | | | (PSD= 1250 ft) | | | (PSD= 1450 ft) | | |
| 2 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 650 | 540 | 500 | 1470 | 1030 | 950 | 2520 | 1570 | 1350 |
| 4 | 60 | 60 | 60 | 530 | 410 | 380 | 1710 | 1040 | 830 | 3160 | 1960 | 1360 | 4720 | 3030 | 2030 | 6350 | 4080 | 2720 |
| 6 | 240 | 200 | 190 | 1400 | 850 | 650 | 1900 | 1860 | 1250 | 4760 | 3060 | 2040 | 7080 | 4550 | 3040 | 9520 | 6120 | 4080 |
| 8 | 560 | 360 | 320 | 2000 | 1280 | 860 | 3870 | 2490 | 1660 | 6350 | 4080 | 2720 | 9440 | 6070 | 4050 | 12690 | 8160 | 5440 |
| 10 | 880 | 530 | 400 | 2500 | 1610 | 1070 | 4830 | 3110 | 2070 | 7930 | 5100 | 3400 | 11790 | 7580 | 5060 | 15870 | 10200 | 6800 |
| **Truck Passing Passenger Car[c]** | | | | | | | | | | | | | | | | | | |
| | (PSD= 350 ft) | | | (PSD= 600 ft) | | | (PSD= 875 ft) | | | (PSD= 1125 ft) | | | (PSD= 1375 ft) | | | (PSD= 1625 ft) | | |
| 2 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 230 | 190 | 170 | 920 | 740 | 670 | 1770 | 1300 | 1170 |
| 4 | 60 | 60 | 60 | 220 | 180 | 160 | 1170 | 800 | 710 | 2610 | 1590 | 1220 | 4220 | 2620 | 1820 | 5930 | 3800 | 2540 |
| 6 | 60 | 60 | 60 | 870 | 580 | 510 | 2540 | 1570 | 1110 | 4260 | 2740 | 1830 | 6370 | 4090 | 2730 | 8890 | 5720 | 3810 |
| 8 | 250 | 200 | 180 | 1540 | 940 | 700 | 3440 | 2210 | 1480 | 5680 | 3650 | 2440 | 8490 | 5460 | 3640 | 11850 | 7620 | 5080 |
| 10 | 470 | 320 | 290 | 2020 | 1280 | 870 | 4300 | 2760 | 1840 | 7100 | 4570 | 3050 | 10610 | 6820 | 4550 | 14810 | 9520 | 6350 |
| **Truck Passing Truck[c]** | | | | | | | | | | | | | | | | | | |
| | (PSD= 350 ft) | | | (PSD= 675 ft) | | | (PSD= 975 ft) | | | (PSD= 1275 ft) | | | (PSD= 1575 ft) | | | (PSD= 1875 ft) | | |
| 2 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 640 | 520 | 470 | 1570 | 1190 | 1070 | 2990 | 1930 | 1670 |
| 4 | 60 | 60 | 60 | 430 | 350 | 310 | 1710 | 1080 | 910 | 3570 | 2190 | 1570 | 5570 | 3560 | 2390 | 7890 | 5070 | 3380 |
| 6 | 60 | 60 | 60 | 1280 | 800 | 660 | 3200 | 2020 | 1380 | 5470 | 3520 | 2350 | 8350 | 5370 | 3580 | 11830 | 7610 | 5070 |
| 8 | 250 | 200 | 180 | 2030 | 1260 | 880 | 4270 | 2750 | 1830 | 7300 | 4690 | 3130 | 11130 | 7160 | 4770 | 15770 | 10140 | 6760 |
| 10 | 470 | 320 | 290 | 2560 | 1650 | 1100 | 5340 | 3430 | 2290 | 9120 | 5860 | 3910 | 13910 | 8950 | 5970 | 19720 | 12680 | 8450 |

[a] PSD values are based on Harwood and Glennon (15)
[b] Driver eye height = 3.5 ft
Object height = 4.25 ft
[c] Truck driver eye height = 6.25 ft
Object height = 4.25 ft
[d] Ratio of shorter arc to total curve length

## DESIGN CREST CURVE LENGTH FOR PSD

Design length requirements of unsymmetrical crest curves for PSD are established based on the PSD design requirements presented by Harwood and Glennon (*15*) and AASHTO (*4*).

### Harwood-Glennon Approach

Glennon (*16*) developed a model for estimating PSD that accounts for the kinematic relationships among the passing, passed, and opposing vehicles. The model not only involves a more logical formulation than the AASHTO and other similar models, it also explicitly contains vehicle length terms. The Glennon model was used by Harwood and Glennon (*15*) to develop sight distance requirements for passing in the following cases:

1. Passenger car passing passenger car,
2. Passenger car passing truck,
3. Truck passing passenger car, and
4. Truck passing truck.

The PSD requirements for these four cases are shown in parentheses in Table 8.

### Length Requirements

*Based on Harwood-Glennon Approach*

The minimum length requirements of unsymmetrical crest curves, established using the developed relationships, are shown in Table 8. For any design or prevailing speed, the length requirements are given for $R = 0.3$, 0.4, and 0.5. The values for $R = 0.5$ are the length requirements for symmetrical crest curves.

Table 8 is based on a passenger car driver eye height of 42 in., truck driver eye height of 75 in., and object height of 51 in. These are the same values used by Harwood and Glennon (*15*). The use of 75 in. to represent truck driver eye height is conservative because the literature shows that truck driver eye height ranges from 71.5 to 112.5 in. (*17–19*). The object height of 51 in. suggested by AASHTO (*4*) corresponds to an opposing passenger car and therefore is also conservative.

*Based on AASHTO Approach*

Table 9 shows the length requirements based on the required PSD of AASHTO, a driver's eye height of 3.5 ft, and an object height of 4.25 ft (which corresponds to passenger cars).

TABLE 9 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES BASED ON PSD OF AASHTO (PASSENGER CARS)[a]

| Algeb. Diff. grade (%) | 20 mph (PSD= 800 ft) | | | 30 mph (PSD=1100 ft) | | | 40 mph (PSD=1500 ft) | | | 50 mph (PSD=1800 ft) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R=.3[b] | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 | R=.3 | R=.4 | R=.5 |
| 1 | 60 | 60 | 60 | 90 | 90 | 90 | 120 | 120 | 120 | 650 | 540 | 500 |
| 2 | 70 | 60 | 60 | 870 | 700 | 650 | 2810 | 1730 | 1450 | 4680 | 2840 | 2100 |
| 3 | 830 | 610 | 570 | 2480 | 1500 | 1180 | 5100 | 3230 | 2190 | 7340 | 4720 | 3150 |
| 4 | 1710 | 1040 | 830 | 3660 | 2310 | 1570 | 6800 | 4370 | 2920 | 9780 | 6290 | 4200 |
| 5 | 2400 | 1480 | 1040 | 4570 | 2940 | 1960 | 8490 | 5460 | 3640 | 12230 | 7860 | 5240 |
| 6 | 2900 | 1860 | 1250 | 5480 | 3530 | 2350 | 10190 | 6550 | 4370 | 14670 | 9430 | 6290 |

[a] Driver eye height = 3.5 ft
Object height = 4.25 ft
[b] Ratio of shorter arc to total curve length

Note: curve lengths are expressed in feet.

The values for $R = 0.5$ are the same as those obtained by the AASHTO equations (*4*). Table 9 includes only moderate values of algebraic difference in grades and design speeds up to 50 mph. Design for PSD may be feasible only for special combinations of high design speeds and very small grades, or low design speeds with moderate grades.

## SUMMARY AND CONCLUSIONS

Unsymmetrical crest curves may be required because of vertical clearance and other design controls. No relationships are available concerning the available and minimum sight distances on these curves. Such relationships are derived here and are used to establish design length requirements of unsymmetrical crest curves based on the SSD, DSD, and PSD needs presented by recent innovative approaches (*13,15*) and by AASHTO (*4*). A computer program implementing these relationships was prepared and can be used to generate the sight distance profiles on both travel directions and the minimum sight distance. Such profiles are useful for evaluating the length of the road with restricted sight distances and the locations on the crest curve where the minimum sight distance occurs.

The developed model can be used to design or evaluate unsymmetrical crest curves to satisfy sight distance needs. The length requirements presented for SSD and DSD are based on passenger cars. In recent years, however, attention has been given to sight distance needs for large trucks (*20,21*). Crest curve lengths needed to provide SSD for trucks can be examined using the model.

The results show that, for a given sight distance, the length requirements of unsymmetrical curves are as great as twice or three times those of symmetrical curves. This finding strongly supports the use of the developed model in new design and in evaluating the adequacy of sight distance on existing unsymmetrical curves. Although the use of these curves in practice is infrequent, their design must satisfy sight distance needs to maintain or achieve safe operations.

## ACKNOWLEDGMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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# DISCUSSION

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Easa has added to the knowledge of sight distance on vertical curves with this paper. The ability to develop sight distance profiles, as shown in Figure 4, will be valuable in assessing sight distance conditions on existing highways.

This discussion is concerned with the design requirements for unsymmetrical crest vertical curves, and in particular the length of curve necessary to provide a specified length of sight distance. In keeping with the nomenclature of the paper (Figure 1), an unsymmetrical vertical curve is made up of two symmetrical vertical curves of length $L_1$ and $L_2$ (where $L_1 > L_2$) with the common point PCC under the PVI. A line tangent to the curve at PCC is parallel to a line connecting BVC to EVC and has a grade $g_3$ given by

$$g_3 = \frac{g_1L_1 + g_2L_2}{L_1 + L_2} \tag{25}$$

The algebraic difference in grade for the unsymmetrical vertical curve is $A$ equal to $g_2 - g_1$. Note that this is the negative of $A$ as given in the paper. The algebraic differences in grades of the two symmetrical curves are given by

$$A_1 = g_3 - g_1 \tag{26}$$

and

$$A_2 = g_2 - g_3 \tag{27}$$

In this discussion, $g$ and $A$ are given in percent.

The symmetrical vertical curve of length $L_2$ is the critical one for sight distance because it is the shorter of the two. Therefore the length of this curve must satisfy the design requirement that

$$L_2 > K{\cdot}A_2 \tag{28}$$

where $K$ is the rate of vertical curvature as given, for example, in Tables III-40 and III-41 of AASHTO (*1*) for stopping and passing sight distance. Substituting $g_3$ from Equation 25 into $A_2$ in Equation 27 and recognizing that $L_1$ plus $L_2$ is equal to $L$, the total length of the unsymmetrical curve, gives

$$A_2 = L_1{\cdot}A/L = (L - L_2){\cdot}A/L \tag{29}$$

Substituting $L = L_2/R$, as defined by the author, into Equation 29 and then substituting this $A_2$ into Equation 28 gives

$$L_2 > K{\cdot}A{\cdot}(1 - R) \tag{30}$$

Substitution into $L = L_2/R$ gives

$$L > K{\cdot}A{\cdot}(1 - R)/R \tag{31}$$

Equation 30 gives the required length of the shorter symmetrical vertical curve, and Equation 31 gives the required total length of the unsymmetrical curve in terms of parameters familiar to designers and the additional parameter $R$:

$A$ = algebraic difference in grade,
$K$ = required rate of vertical curvature as given in AASHTO tables (*1*), and
$R$ = ratio of length of shorter symmetrical curve to total length of the unsymmetrical curve.

It should be noted that when using the tabulated values of $K$ as given by AASHTO (*1*) with small values of $A$, the calculated length of the vertical curve is greater than actually required for sight distance. This occurs when the sight distance is greater than the required length of the shorter symmetrical vertical curve. For this reason the values of $L$ computed by Equation 31 will be greater than the values given in the paper in Tables 6 and 9 for small values of $A$. Also note that the author did not used the tabulated $K$-values in AASHTO (*1*) Tables III-40 and III-41 associated with the design speeds in the author's Tables 6 and 9. The corresponding $K$-values for the paper's sight distances can be determined from AASHTO

TABLE 10 DESIGN LENGTH REQUIREMENTS FOR UNSYMMETRICAL CREST CURVES AT 50-MPH DESIGN SPEED

| | SSD = 400 ft | | | | PSD = 1,800 ft | | | |
|---|---|---|---|---|---|---|---|---|
| | Paper, Table 6 | | Discussion, Equation 31[a] | | Paper, Table 9 | | Discussion, Equation 31[b] | |
| $A$(%) | $R = 0.3$ | $R = 0.4$ | $R = 0.3$ | $R = 0.4$ | $R = 0.3$ | $R = 0.4$ | $R = 0.3$ | $R = 0.4$ |
| 2 | 210 | 160 | 562 | 361 | 4,680 | 2,840 | 4,888 | 3,143 |
| 3 | | | 843 | 542 | 7,340 | 4,720 | 7,333 | 4,714 |
| 4 | 1,100 | 680 | 1,124 | 722 | 9,780 | 6,290 | 9,777 | 6,285 |
| 5 | | | 1,405 | 903 | 12,230 | 7,860 | 12,221 | 7,856 |
| 6 | 1,690 | 1,090 | 1,686 | 1,084 | 14,670 | 9,430 | 14,665 | 9,428 |
| 8 | 2,250 | 1,450 | 2,247 | 1,445 | | | | |
| 10 | 2,810 | 1,810 | 2,809 | 1,805 | | | | |

[a]With $K = S^2/1{,}329 = 120.4$
[b]With $K = S^2/3{,}093 = 1{,}047.5$.

Equation 3 for stopping sight distance ($K = S^2/1329$) (*1*, p. 283) and Equation 5 for passing sight distance ($K = S^2/3093$) (*1*, p. 288).

Table 10 compares the design length for unsymmetrical vertical curves as determined by the method of the paper and the method of this discussion for 50-mph design speed. The lengths are essentially the same except for small values of $A$.

### REFERENCE

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## AUTHOR'S CLOSURE

The author thanks Professor Guell for his interest in the paper and for his thoughtful comments regarding establishment of the design length requirements of unsymmetrical crest curves based on the shorter arc.

The formula derived in his discussion for establishing length requirements (Equation 31) assumes that both the driver and object are on the shorter arc, which corresponds to Case 1 of the paper. The discussion indicates that the lengths calculated using this formula will be greater than actually required when $A$ is small. The purpose of this closure is twofold: (a) to derive a general expression for Equation 31 and the condition for applying it, and (b) to show that this equation may overestimate the length requirements even when $A$ is large.

For Case 1, the minimum sight distance, $S_m$, occurs when the object is at EVC. Setting $T = 0$, substituting for $r_2$ from Equation 2 into Equation 7, and nothing that $L_1 = (1 - R)L$, one obtains

$$L_2 = A\,(1 - R)\left\{\frac{S_m^2}{[(2h_1)^{1/2} + (2h_2)^{1/2}]^2}\right\} \tag{32}$$

where the term in brackets equals the rate of vertical curvature $K$ (Equation 32 is similar to Equation 30). Note that $A$ is defined in the paper as $g_1 - g_2$, which always yields a positive value for crest curves. Since $L_2 = LR$, Equation 32 gives

$$L = A\,\frac{(1 - R)}{R}\left\{\frac{S_m^2}{[(2h_1)^{1/2} + (2h_2)^{1/2}]^2}\right\} \quad \text{(Case 1)} \tag{33}$$

which is a general expression for the length requirements for Case 1 (Equation 33 is similar to Equation 3). For Equation 33 to be valid, however, $S_m$ must be less than or equal to $L_2$. That is,

$$S_m \le A\,(1 - R)\left\{\frac{S_m^2}{[(2h_1)^{1/2} + (2h_2)^{1/2}]^2}\right\} \tag{34}$$

from which

$$A \ge \frac{[(2h_1)^{1/2} + (2h_2)^{1/2}]^2}{(1 - R)S_m} \tag{35}$$

Equation 35 is the condition of $A$ for which Equation 33 gives exact length requirements. For values of $A$ less than those given by Equation 35, Equation 33 overestimates the length requirements.

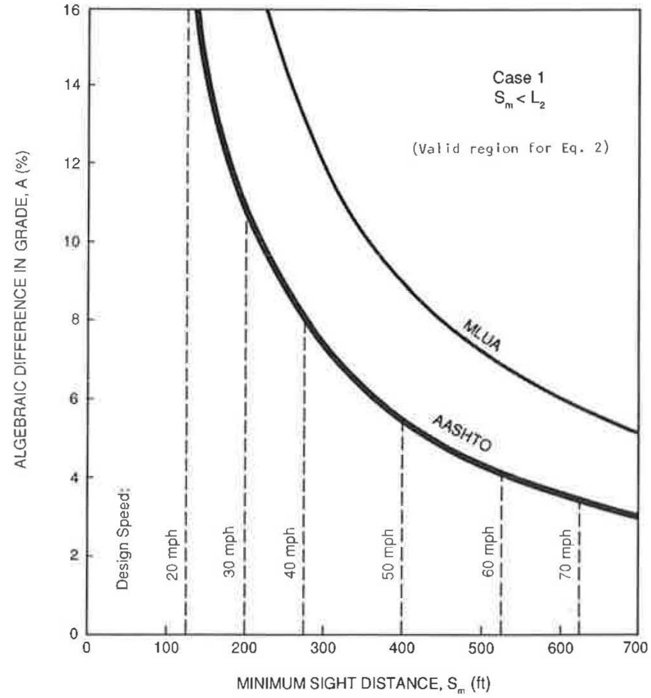


**FIGURE 7 Range of $A$ for which Equation 33 overestimates length requirements based on SSD ($R = 0.4$).**

A graphical representation of Equation 35 using the AASHTO design parameters of SSD ($h_1 = 3.5$ ft, $h_2 = 0.5$ ft) and $R = 0.4$ is shown in Figure 7. For a given $S_m$, Equation 33 overestimates the length requirements for the values of $A$ below the shaded region. For $S_m = 400$ ft (50-mph design speed), the length requirements are overestimated when $A < 5.5$ percent. The overestimation may be more than 100 percent, as noted from Table 10. For lower design speeds, the overestimation occurs for larger values of $A$. For example, the length requirements are overestimated when $A < 11.1$ percent for $S_m = 200$ ft (30 mph) and when $A < 17.7$ percent for $S_m = 125$ ft (20 mph). For other design parameters of SSD, the region of $A$ for which overestimation occurs may be larger than that of AASHTO. This is illustrated in Figure 7 by the upper curve, which corresponds to the design parameters of multilane urban arterials (MLUA) of Table 3 ($h_1 = 3.5$ ft, $h_2 = 2.0$ ft). Applying Equation 35 using the AASHTO design parameters of PSD ($h_1 = 3.5$ ft, $h_2 = 4.25$ ft) shows that overestimation occurs when $A < 2$ percent for $S_m = 1{,}800$ ft (50 mph), as also noted in Table 10. For $S_m = 800$ ft (20 mph), overestimation occurs when $A < 4.5$ percent.

In summary, the length requirements of unsymmetrical crest curves may be computed using Equation 33 (which corresponds to Case 1) only if the condition of Equation 35 holds. If this condition does not hold, this means the analysis corresponds to other sight distance cases and the length requirements should be established using the procedure presented in the paper.

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