Application of Markov Decision Process to Level-of-Service-Based Maintenance Systems

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Systems based on level of service (LOS) currently implemented to manage highway maintenance use extensive subjective data. Collection of these data is tedious and expensive and the inherent uncertainties in the data render the results imprecise. A modified method using the Markov decision process (MDP), which overcomes most of the drawbacks of the LOS-based systems, is described. The adoption of the MDP is consistent with progressive evolution in the field of highway maintenance management. It introduces measures of performance benefits that are less subjective than those used in the NCHRP LOS model, which rely on attributes and utility functions. The modified model uses three types of key input data: transition probabilities, costs, and relative-importance weights. The transition probabilities are computed analytically using sample deterioration models and quality standards. An approach is described for computing the cost of each alternative from historical data. An analytic approach is also described for computing relative-importance weights using simple ranking of and comparison scores for the highway elements. This method was tested with 58 highway elements in 12 strata and 3 levels of service each. The resulting problem, which had 2,088 variables and 697 constraints, required less than 15 min on an IBM PC using an off-the-shelf linear programming package. The results of the test were consistent with the input data and demonstrated that the objectives set for the method were being met. Although the method was tested with mostly roadside elements, it can generally be used with any LOS-based system.

Under NCHRP sponsorship, a method was developed to assist in selecting optimum levels of service (LOS) for those highway elements that are subject to the constraints of available resources (*1*). LOS values are discrete condition state thresholds or maintenance intervals for highway elements. This method is supposed to be based on theoretically sound principles of decision analysis and to be implementable in a well-defined, step-by-step procedure. The method uses both objective data and subjective expert opinion. The subjective data are required for the following areas:

- Specification of alternate LOS values;
- Estimation of effects of alternate LOS values on various user considerations such as safety, preservation of investment, etc.;
- Assessment of individual value functions of different attributes used to measure the user considerations; and
- Assessments of relative-importance weights among the individual attributes.

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Further, considerable amount of subjective input is needed from experienced maintenance engineers in estimating the resource requirements for each LOS of individual elements. Availability of significant amounts of objective maintenance cost data is unlikely as records are not kept in such detail. Although deterioration models for the highway elements are not explicitly determined in this method, they certainly play a role in the experts' minds in the establishment of explicit LOS values and the corresponding resource requirements.

As discussed earlier, the implementation and operation of this model largely depend on extensive use of subjective expert opinion and management input. The potential for large variability is expected to be high in both these inputs: but the model is deterministic and does not implicitly or explicitly handle these variabilities.

Further, the integration of this model with the existing pavement management system (PMS) and bridges and structures management systems (BSMS) for the purpose of budget allocation requires top management (TM) to make adjustments to performance standards for the PMS and BSMS and budget for the LOS-based nonpavement management system (NPMS). The models can then be analyzed again; the expected costs can be summed across pavements, bridges, and structures and added to the NPMS budget; and the total can be judged for acceptability on the long-term basis. This approach to integration has the advantage that TM has direct control over the performance and budget-setting process. The disadvantage is that a large number of parameters are involved in setting all standards, and a considerable amount of adjustments to parameters may be needed to achieve an affordable solution.

Therefore, this method needs significant changes to overcome some of its inherent drawbacks:

1. Deterministic models fail to account for the variability inherent in the process.
2. Extensive dependency of the models on subjective expert opinion is a potential source of significant modeling errors.
3. Incompatibility of the models with those of other state-of-the-art management systems for integration with limited amount of adjustments by the TM in setting the performance standards.

A modified method overcomes most of these drawbacks. This method will be described with its input and output requirements. This description will then be followed by procedures to generate the input data required.

METHODOLOGY

The development of this modified model is in keeping with the progressive evolution that has occurred in the field of highway maintenance management as LOS models have given way to a Markov decision process (MDP) model, in particular, for pavements and bridges. Although retaining some of the basic features of the LOS models, this model assumes the deterioration of the highway elements to be a Markov process.

This model introduces measures of performance benefits that are less subjective than those used in the NCHRP LOS model and that rely on attributes and utility functions. The assumption of a Markov process replaces deterministic deterioration of elements by probabilistic deterioration. Considering the overall need for an integrated highway maintenance management system, this model is compatible with other MDP-based pavement and bridges and structures management systems, making such integration feasible.

The measures of performance require definition of QS values, which are discrete ranges of condition states corresponding to the alternate LOS values. These QS values should take into account the regional, climatic, geographical, and road classification differences in the network of highways. This accounting is accomplished by stratifying the highways using some or all of these factors and defining the alternate LOS (and QS) value for each stratum. Figure 1 shows the concepts of LOS and QS for a single stratum or a group of similar strata; n_1, n_2 and n_3 are the corresponding maintenance intervals.

Model Description

In order to describe the new model, some definitions and notation are needed. Let

$f_{sj}(l|t)$ = probability that an element j in Stratum s would reach LOS l, in t time periods after the last action. These time periods may be in days, weeks, or months;

T_{sjl} = number of time periods between actions for the element j in Stratum s at LOS l;

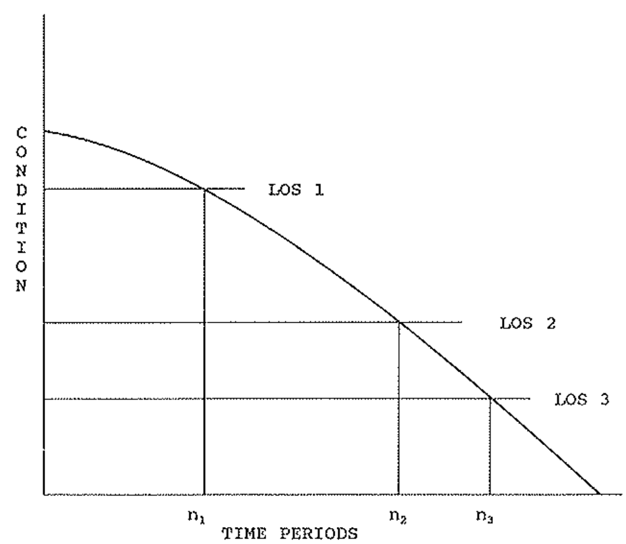

FIGURE 1 Deterioration curve for a highway element.

$w_{sj}(d)$ = relative-importance weight for being within a desirable QS for Element j in Stratum s;

$w_{sj}(u)$ = relative-importance weight for not being within an undesirable QS for Element j in Stratum s;

D_{sjl} = probability that Element j in Stratum s will be within a desirable QS when the cycle time corresponding to LOS l is selected;

U_{sjl} = probability that an Element j in Stratum s will be within an undesirable QS when the cycle time corresponding to LOS l is selected;

$\overline{N}_{sj}$ = normalized number of units of Element j in Stratum s (i.e., the fraction);

X_{sjl} = logical variable representing the choice of cycle time (1 if selected, 0 otherwise); and

C_{sjl} = annual cost of applying an action at LOS l to all the units of Element j in Stratum s.

In this notation, it is assumed that an element requiring different actions for different conditions will be considered a separate element for each condition. For example, a retaining wall with poor appearance is different from a retaining wall with damaged structure.

Given this notation, the following relations may be defined:

$$D_{sjl} = (1/T_{sjl}) \sum_{i \in d} \sum_{t=1}^{T_{sjl}} f_{sj}(i|t) \qquad (1)$$

$$U_{sjl} = (1/T_{sjl}) \sum_{i \in u} \sum_{t=1}^{T_{sjl}} f_{sj}(i|t) \qquad (2)$$

With these equations, the probabilities of finding an element in a desirable or undesirable QS, given a particular cycle time T_{sjl}, can be computed. The model to select the cycle times that maximize the performance is an assignment model with a budget constraint as follows:

$$\text{Max} \sum_s \sum_j \overline{N}_{sj} \times \left[w_{sj}(d) \sum_l D_{sjl} X_{sjl} + w_{sj}(u) \sum_l (1 - U_{sjl}) X_{sjl} \right] \qquad (3)$$

subject to

$$\sum_l X_{sjl} = 1 \qquad \text{for all } s \text{ and } j \qquad (4)$$

$$\sum_s \sum_j \sum_l C_{sjl} X_{sjl} \leq B_p \qquad (5)$$

$$X_{sjl} = 0 \text{ or } 1 \qquad \text{for all } s, j, \text{ and } l \qquad (6)$$

B_p is the budget for periodic maintenance of the nonpavement elements.

The objective function of Expression 3 is the weighted sum of transition probabilities. Equation 4 ensures that one and only one cycle time is selected for each element in each stratum, Inequality 5 is the budget constraint, and Equation 6 defines the decision variables to be (0 or 1) integer variables.

As can be seen from the objective function, the functional performance of the nonpavement elements are represented by stochastic variables D_{sjl} and U_{sjl}. In the NCHRP LOS-based model, the deterioration functions are implicitly deterministic and the benefit functions are utility functions based on subjective expert opinion. The stochastic variables can incorporate the variability of performance among different units of the same element.

The MDP-LOS-based model will still need subjective data for relative-importance weights $w_{sj}(d)$ and $w_{sj}(u)$, as discussed later. However, these parameters are signficantly fewer than those needed to define the user consideration attributes and the various utility functions. Further, these relative-importance weights are thought to be easily obtainable because the experts have a better feel for these weights than for the consideration attributes by LOS values.

This model is formulated as a (0 or 1) linear integer problem with a special structure. The number of generalized upper bound (GUB) type constraints, as in Equation 4, greatly exceeds the number of linear variables plus the general constraints, Inequality 5. Consequently, when the integer constraints are replaced by nonnegativity constraints, the resulting linear program has an optimal solution with mostly integer values (*2*). This approximation can provide a satisfactory solution of the original problem.

Model Implementation

The solution of this model requires several input data, assuming that the network of highway elements are identified and stratified, and that alternative LOS and QS values are defined:

1. Number ($\overline{N}_{sj}$) of units of Element j in Stratum s as a fraction of the units in the network,
2. Probability D_{sjl} that an element in Stratum s will be within a desirable QS when maintained at LOS l,
3. Probability U_{sjl} that an Element j in Stratum s will be within an undesirable QS when maintained at LOS l,
4. The maintenance cost C_{sjl} for Element j in Stratum s at LOS l,
5. Relative-importance weights $w_{sj}(d)$ of an Element j in Stratum s to be within a desirable QS, and
6. Relative-importance weights $w_{sj}(u)$ of an Element j in Stratum s to be outside an undesirable QS.

The output of the system is an optimum set of LOS values to be implemented.

Item 1 is obtained from the inventory. Estimation of D_{sjl} and U_{sjl} is based on Equations 1 and 2, where the unknown function is $f_{sj}(l|t)$, the probability that an Element j in Stratum s would reach LOS l in t time periods after the last action. Methods for determining function f_{sj}, for estimating probabilities D_{sjl} and U_{sjl}, and for estimating costs C_{sjl}, and the subjective expert input for assessing the relative-importance weights $w_{sj}(d)$ and $w_{sj}(u)$ are described in later sections.

DETERMINATION OF TRANSITION PROBABILITIES

The approach to determining the transition probabilities requires knowledge of how the highway element deteriorates with time and the definition of the QS values. Defining the deterioration functions and the quality standards for the highway elements has some similarity to Step 1 of the NCHRP method (*1*). In this step, elements are selected, maintenance conditions are identified, and alternate LOS values are specified. These concepts are shown in Figure 1, in which the vertical axis indicates the condition and the horizontal lines represent the various LOS values for a given element. The only difference is that the deterioration of this element is indicated by the curve, which explicitly defines the condition as a function of time.

Specification of LOS values implies quality standards as ranges of condition between these LOS values. In the NCHRP method, the LOS values were defined in relation to budget levels. Alternatively, quality standards can be defined using engineering judgment with management input. For example, the full range of conditions can be divided into excellent, acceptable, barely acceptable, and unacceptable standards. Then the boundaries of these standards will be the alternate LOS values, and the intersections of these boundaries with the deterioration curve will define alternate maintenance intervals, such as n_1, n_2, and n_3 on the time axis shown in Figure 1.

Ideally, deterioration functions can be defined from historical maintenance data, but in reality such data are not available for all elements. Then, engineering judgment would be relied on. Assume that a group of experienced maintenance engineers could provide a series of deterioration times for a given set of conditions. These conditions may be thresholds of quality standards or some other condition values that would make it easier for the engineers to estimate the corresponding times.

Deterioration functions are obtained either by fitting these data to simple mathematical functions or by simply joining adjacent points with straight lines. Then, the maintenance times corresponding to the LOS values can be determined by inverting the functions for the conditions implied by these levels.

To compute the transition probability matrix elements (**tp**) from the deterioration functions and the quality standards, the deterioration of an element is assumed to be a Markov process in which the transition probability from the current LOS value to any of the LOS values in time period t depends only on the current condition and is independent of how it got there. If the time period is chosen small enough, then it can be assumed that the element can transition only into the next level down, if at all. That is, if the probability of this element's staying in the same level is p, the probability of its transitioning into the next level is $(1 - p)$. Assuming four quality standards, the transition probability matrix for Element j in Stratum s for one period can be written as

$$\mathbf{tp} = \begin{vmatrix} p_1 & 1 - p_1 & 0 & 0 \\ 0 & p_2 & 1 - p_2 & 0 \\ 0 & 0 & p_3 & 1 - p_3 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (7)$$

where Subscripts 1, 2, and 3 refer to the first three quality standards. Elements in the last quality standard will stay in that level with probability 1. After n periods, this matrix will be

$$\mathbf{tp}^n = \begin{vmatrix} p_1 & 1-p_1 & 0 & 0 \\ 0 & p_2 & 1-p_2 & 0 \\ 0 & 0 & p_3 & 1-p_3 \\ 0 & 0 & 0 & 1 \end{vmatrix}^n \tag{8}$$

At time $t = 0$, the element is assumed to be in the best QS. After $t = n$ periods, the probabilities $P_{\cdot n}$ of this element's being in QS 1, 2, 3, or 4 are given by

$$(P_{1n}, P_{2n}, P_{3n}, P_{4n}) = (1, 0, 0, 0)(\mathbf{tp}^n)_{sj} \tag{9}$$

If the maintenance interval corresponding to LOS l is T_{sjl}, then

$$D_{sjl} = \frac{1}{T_{sjl}} \sum_{i \in d} \sum_{n=1}^{T_{sjl}} P_{in} \tag{10}$$

and

$$U_{sjl} = \frac{1}{T_{sjl}} \sum_{i \in u} \sum_{n=1}^{T_{sjl}} P_{in} \tag{11}$$

The only information needed to define the probability functions is the transition probability matrix elements $\mathbf{tp}_{sj}$, in which the specific unknowns are the individual probabilities p_1, p_2, and p_3.

Determining these individual probabilities depends on the availability of field data. As could be expected, no historical maintenance data are available. Consequently, establishing the matrix element $\mathbf{tp}_{sj}$ for Element j in Stratum s is proposed to be accomplished in two steps: (a) determining initial matrices, and (b) updating these matrices from the field data. The details of these steps are discussed in the next section.

Transition Probability Calculation

The calculation of transition probabilities is based on some form of deterioration models developed either from expert opinion or from historical maintenance data. How these probabilities are computed from the deterioration models is discussed in this section.

The structure of the **tp** matrix for n periods (Equation 8) is as follows:

$$\mathbf{tp}^n = \begin{vmatrix} p_1^n & q_n & z_n & * \\ - & p_2^n & * & * \\ - & - & p_3^n & * \\ - & - & - & 1 \end{vmatrix} \tag{12}$$

where p_1^n, p_2^n, and p_3^n are the nth powers of p_1, p_2, and p_3, respectively; q_n and z_n are unknown terms; terms denoted with an asterisk are also unknown, but their exact values are not important to this discussion. By induction,

$$q_n = (1 - p_1)(p_1^{n-1} + p_1^{n-2}p_2 + p_1^{n-3}p_2^2 + \cdots + p_1p_2^{n-2} + p_2^{n-1}) \tag{13}$$

and

$$z_n = (1 - p_2)(q_{n-1} + q_{n-2}p_3 + \cdots q_2p_3^{n-3} + q_1p_3^{n-2}) \tag{14}$$

With these expressions, the deterioration model for an Element j in Stratum s shown in Figure 1 can be examined.

The deterioration curve is an expression of condition as a function of time defined from data solicited from expert maintenance engineers or from historical data. Therefore, it is conjectured that these engineers considered an average or a median unit of an element in providing this opinion. The latter means that the deterioration curves for 50 percent of the units are below the median curve and the other 50 percent are above it. With this assumption, the following equations can be written:

$$p_1^{n1} = 0.5 \tag{15}$$

$$p_1^{n2} + q_{n2} = 0.5 \tag{16}$$

$$p_1^{n3} + q_{n3} + z_{n3} = 0.5 \tag{17}$$

Equation 15 means that after n_1 periods the probability of a unit of element being in QS 1 is 0.5; after n_2 periods, the probability of this unit being in QS 1 or 2 is also 0.5; similarly, after n_3 periods, the probability of this unit being in QS 1, 2, or 3 is 0.5.

Equation 15 provides a value of p_1; a combination of Equations 13 and 16 can be solved for p_2; and Equations 13, 14, and 17 can be solved for p_3. Consequently, $(\mathbf{tp}^n)_{sj}$ can be evaluated for any n. Then a combination of Equations 9–11 can be used to compute the necessary probabilities to be used in the objective function in Equation 13.

Update Considerations for the tp Matrices

Initially, the **tp** matrices are based on a deterioration model developed using subjective expert opinion. When historical data become available, these data will be used to generate more realistic deterioration models from which new **tp** matrices will be computed.

RESOURCE REQUIREMENTS

Maintenance cost is a necessary input to the budget constraint in Equation 5 for each element in each stratum by LOS value. This cost depends on unit cost and other quantities depending on the stratum and element. This is true for the LOS value for which the unit cost and quantities are applicable. In order to generalize it for other LOS values, an intensity factor needs to be considered. This factor reflects the change in the level of effort required to accomplish the maintenance each time. In order to estimate the annual cost, this cost must be multiplied by the frequency of maintenance per year. This process can be expressed as follows:

$$C_{sjl} = U_{sj}Q_{sj}I_{sjl}F_{sjl} \tag{18}$$

where

U_{sj} = unit cost of action on Element j in Stratum s,
Q_{sj} = quantities involved,
I_{sjl} = intensity factor for LOS l (= 1 for base case), and
F_{sjl} = frequency of maintenance for the year.

U_{sj}, Q_{sj}, and F_{sjl} are expected to be estimated from historical data. The intensity factor could come from engineering judgment. Alternatively, if the level effort is assumed proportional to the range of conditions over which the action restores the element, I_{sjl} may be defined as

$$I_{sjl} = \frac{\text{LOS}(*) - \text{LOS}(l)}{\text{LOS}(1) - \text{LOS}(3)} \tag{19}$$

where LOS(*) is the base LOS, for which $I_{sjl} = 1.0$.

RELATIVE-IMPORTANCE WEIGHTS

The objective function of Expression 3 uses relative-importance weights $w_{sj}(d)$ and $w_{sj}(u)$, where

$w_{sj}(d)$ = relative-importance weight for having an Element j in Stratum s within desirable quality standards, and
$w_{sj}(u)$ = relative-importance weight for having an Element j in Stratum s not within undesirable quality standards.

In defining the quality standards, the differences caused by road stratification criteria such as traffic volume and terrain have been accounted for by specifying varying QS values for different strata. The objective function in the optimization model accounts for the quantities of elements by the fraction $\overline{N}_{sj}$, which is the normalized number of units of Element j in Stratum s. Consequently, these factors are not used in estimating the relative-importance weights $w_{sj}(d)$ and $w_{sj}(u)$ (because they are already included in the QS values), the subscript s can be dropped. In the rest of the discussion, these weights are designated as $w_j(d)$ and $w_j(u)$.

The approach chosen in estimating the relative-importance weights is a general expert opinion elicitation procedure called the "analytic hierarchy procedure" (*3*). This approach consists of two sequential steps. In the first step, the nonpavement elements are divided into a hierarchy of groups of 4 to 8 elements each and a set of ranking and comparison criteria are identified. Eight is considered a reasonable maximum number of items that an engineer can be expected to rank and score among as a group. Then the experts are expected to rank each item of these groups at all levels. In the second step, a relative ratio comparison score is given to each item of these groups. These scores are then reduced to obtain the relative-importance weights.

Concepts of Ranking and Comparison Scores

Suppose there are n items $I_1, I_2, \ldots, I_n$ that need to be evaluated with respect to some Criterion C. An expert is asked to make judgments about these items with respect to Criterion C. The first step is for the expert to rank-order the items from best to worst, with respect to Criterion C. Next, suppose that the items have been reindexed in this order so that the index order and the rank order are the same. The third step is for the expert to compare each pair (I_i, I_j) where $j > i$ to determine how much better I_i is than I_j with respect to Criterion C. Then the expert is asked to respond with ratio-scale comparisons rather than interval-scale comparisons. For example, in comparing I_i with I_j, a response of 3 would mean that I_i is three times as good as I_j with respect to Criterion C (rather than three additional units better). After some empirical testing and much experience, Saaty (*3*) suggests that these ratio comparisons be based on a standard scale such as the following:

Ratio	*Comparison Significance*
1	I_i and I_j are about the same with respect to C.
3	I_i is slightly better than I_j with respect to C.
5	I_i is much better than I_j with respect to C.
7	I_i is considerably better than I_j with respect to C.
9	I_i is so much better than I_j with respect to C that there is almost no comparison between the two.

Values of ratios of 2, 4, 6, and 8 may be used to strike a compromise between adjacent categories.

This process may be organized in the upper part of a matrix such as the example shown in Figure 2. Such matrices are referred to as "pairwise-comparison" matrices.

In Figure 2, a value at the intersection of, say, Row I_2 and Column I_4 would represent how much better I_2 is over I_4 with respect to Criterion C. Because the items were first rank-ordered with respect to Criterion C, the numbers provided by the expert should all be greater than or equal to 1 and tend to increase from left to right.

For a given set of highway elements, a number of matrices like Figure 2 must be elicited from experts. These matrices may differ from each other in two ways. First, each deals with different types of items to be compared; second, the comparisons may be made with respect to different criteria. These items and criteria form a hierarchy the structure of which can be used to synthesize the elicited values into the relative importance weights needed to establish the optimal NPMS policy.

First Level of the Hierarchy

At the first level, Criterion C is thought of as the importance of the considerations to the overall goal or objective of the NPMS. This overall objective may not be articulated, but the expert engineers should have an adequate sense for making these comparisons. Among others, the items that must be

	I_1	I_2	I_3	I_4	I_5
I_1	1	3	5	5	9
I_2	–	1	2	3	7
I_3	–	–	1	5	8
I_4	–	–	–	1	3
I_5	–	–	–	–	1

FIGURE 2 Example of pairwise-comparison matrix that an expert participant might provide for five items.

compared may include the following four considerations:

- Safety,
- Aesthetics,
- User convenience, and
- Preservation of investment in highway elements.

In order to accomplish the first step, the task asked of the expert would be to rank-order these items with respect to how important they are for the overall nonpavement system. A matrix is then set up with these items listed as row labels in the order of most important to least important, and with columns labeled in the same order. The expert then compares the items that label the rows and columns that intersect in the upper half of the table. The comparisons are scored on the scale of 1 to 9, with the score indicating how much more important for the NPMS the row item consideration is than the column item consideration. This effort results in a single pairwise-comparison matrix.

Second Level of the Hierarchy

At the second level, the problem expands because the elicitation process needs to be applied for each of four considerations. The items to be compared are the groups of nonpavement elements that have already been identified. If the groups contain about 14 or 15 elements, they need to be split into two groups of 7 or 8 elements each. The groups are then rank-ordered with respect to one of the considerations, and then the pairwise-comparison matrix is established.

This level results in one pairwise-comparison matrix for each consideration.

Third Level of the Hierarchy

The bulk of the effort is in establishing a pairwise-comparison matrix for the elements in each group with respect to each of the considerations.

Comparisons are not necessary between elements in different groups. The number of pairwise-comparison matrices in this level is the number of considerations, N_c, times the number of groups, N_g.

Fourth Level of the Hierarchy

For this level, it is determined for each element and consideration whether being in a desirable QS is more or less important than not being in an undesirable QS and by how much.

As an example, one may pose the question for a given element, say, cateyes; and for consideration, suppose safety, as follows: for purposes of *safety*, is it more important for a segment of *cateyes* to be in a desirable QS than for it to not be in an undesirable QS, and by how much? The how much? answer should be expressed in the 1 to 9 scale defined earlier. The italicized words *safety* and *cateyes* are then replaced by another consideration and element, respectively, and the question is posed again. The process continues until all combinations of considerations and elements have been examined.

Relative-Importance Weight Calculations

Each pairwise-comparison matrix is made into a square matrix. The diagonal is filled with 1s and the upper triangular part of the matrix is filled in with the evaluated elements. The area below the diagonal is filled in with the reciprocals of the numbers above the diagonal, each in the reflected position across the diagonal. For example, suppose a_{rc} is an element in Row r and Column c and suppose that it is below the diagonal, then its value should be set to $1/a_{rc}$, which is the reciprocal of an element above the diagonal. For a numerical example, the matrix of Figure 2 is converted into the following appropriate matrix:

$$\begin{vmatrix} 1 & 3 & 5 & 5 & 9 \\ 1/3 & 1 & 2 & 3 & 7 \\ 1/5 & 1/2 & 1 & 5 & 8 \\ 1/5 & 1/3 & 1/5 & 1 & 3 \\ 1/9 & 1/7 & 1/8 & 1/3 & 1 \end{vmatrix}$$

This matrix contains some redundant information, so that the consistency of the participant expert can be checked. If the participant expert was entirely consistent, then the largest eigenvalue of this matrix would be 5 (because the matrix is 5 × 5). Therefore, the check is to see that the largest eigenvalue is close to 5.

Assuming that the consistency is satisfactory, the eigenvector corresponding to the largest eigenvalue is normalized so that the sum of its components equals 1. The components of this normalized eigenvector are interpreted by Saaty (*3*) to be priority weights for the elements involved.

In the case of the given matrix, the eigenvalue was 5.36, indicating a reasonable degree of consistency. The normalized eigenvector is (0.50, 0.22, 0.18, 0.07, 0.03), the components of which are the priority weights for the items I_1, I_2, I_3, I_4, and I_5.

Even Level 4 of the hierarchy produces a series of 2 × 2 matrices that have an eigenvalue of 2 and an eigenvector with two components representing priority weights one for the desirable condition and one for the not undesirable condition. Thus, each level of the hierarchy results in a set of priority weights that sum to 1 for each set of items that is directly compared at that level. The weights at each level are distributed and combined at the next level down in the hierarchy. This accomplishment results from an averaging process described in the following section.

Weight Distribution for Analytical Hierarchy Procedure

At each level of the hierarchy, weights are developed from each matrix by calculating the eigenvector corresponding to the largest eigenvalue for the matrix. The weights are then combined as follows:

At Level 1, weights are found for the criteria. Let α_c be those weights for $c = 1, 2, \ldots, N_c$.

At Level 2, weights are found for groups of elements for each criterion. Let β_{cg} be those weights for $c = 1, 2, \ldots,$

N_c; and $g = 1, 2, \ldots, N_g$. In this case, for each c, the sum over g of the weights is 1.

At Level 3, weights are found for elements within groups for each criterion. Let γ_{cge} be those weights for $c = 1, 2, \ldots, N_c$: $g = 1, 2, 3, \ldots, N_g$; and $e = 1, 2, \ldots, n_g$, where n_g is the number of elements in Group g. These weights sum to 1 when summed over the elements of the specific group and criterion.

At Level 4, weights are found for each of two states of elements within groups for each criterion. The two states are (a) a desirable QS and (b) a not undesirable QS. Let δ_{ggeq} be those weights for which the indices c, g, and e range as before, while $q = 1, 2$. As before, for fixed c, g, and e, the sum of these weights over q is equal to 1.

In the mathematical models proposed earlier, the elements were not grouped but simply indexed sequentially. Thus, let j be the element index so that j assumes a value for each feasible pair (g, e). The state also had index $q = d$ or u instead of 1 and 2. The weights to be used in the optimization model were therefore w_d and w_u. They were calculated as follows:

$$w_d = \sum \alpha_c \beta_{cg} \gamma_{cge} \delta_{cge1} \quad (20)$$

$$w_u = \sum \alpha_c \beta_{cg} \gamma_{cge} \delta_{cge2} \quad (21)$$

TEST RESULTS AND CONCLUSIONS

The method described was tested with a system of 58 highway elements in 12 strata with 3 LOS values each. The resulting linear integer programming problem had 2,088 variables and 697 constraints. An off-the-shelf linear programming package (LP83) solved this problem on an IBM PC (286) in less than 15 min. Only one element in one stratum had a noninteger solution.

Both the deterioration models as well as the relative-importance weights were developed with subjective engineering judgments. Several maintenance engineers were used to collect the data. Significant variations were observed among the deterioration time estimates provided by these engineers. Improvement in the solicitation process and the use of formal analysis techniques such as Delphi to get consensus-based opinions would reduce such variabilities. Also, if a maintenance management system already exists, the development of deterioration models can use input from historical maintenance data.

The results of this test indicated that the maintenance policy generated was consistent with the input data. Detailed discussion of these results would be the subject of another paper. However, the results indicated that the objectives set for this method at the outset of this paper were being met. This method uses stochastic deterioration models and relies on relatively fewer and easier-to-get subjective data. Further, the optimization model formulated here is consistent with the other systems based on the MDP so that these models can be integrated for budget allocation exercises.

Even though this method is tested with mostly nonpavement elements, it can generally be used with any LOS-based system.

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