Evaluation of Control Strategies Through a Doubly Dynamic Assignment Model

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A within-day and day-to-day dynamic assignment model for a general network has been proposed recently. The model follows a nonequilibrium approach, in which flow fluctuations are modeled as a stochastic process. It includes a model of dynamic network loading for computing within-day variable arc flows from path flows. In this paper, the sensitivity and the operational characteristics of the model are tested by analyzing some effects of control measures on a small realistic network. The results of these applications show that the proposed model is a valid tool to estimate the effectiveness of some traffic engineering measures and informative systems. It also appears that some control measures cannot be assessed correctly without the explicit simulation of the demand elasticity over departure times and of the day-to-day adjustment process determined by users' memory and forecasting.

Recently, the stochastic process approach to the analysis of transportation system dynamics (*1,2*) has been extended to account for both within-day and day-to-day temporal fluctuations of demand and flows (*3*). Following this approach the evolution of the system over time is analyzed rather than seeking an equilibrium or self-reproducing solution (if any), as in within-day constant (*4,5*) and within-day dynamic equilibrium models. The latter can be developed on the basis of deterministic (*6,7*) or stochastic (*8,9*) users' behavior models.

The stochastic doubly dynamic model, described in this paper, allows the simulation of system adjustments following network modifications, the role of habit in users' choices, and the effects of some informative systems and control strategies. In addition, the model can be extended to cover the case of real-time informative strategies, allowing users to change their path en route.

In this paper, the general structure of the model is briefly outlined. Potential applications to a small real-size network are then presented to show the effects of traffic engineering measures, different demand structures and levels, and different types of informative systems.

DYNAMIC DEMAND/SUPPLY INTERACTION MODEL

Day-to-day dynamics refers to system variations occurring between successive reference periods, which can be part of the day, (e.g., the morning peak period) or the whole day. In the following, the reference period will be called "day" and denoted by index "t".

Within-day dynamics refers to variations taking place within the day and is analyzed by dividing the day into n_h subperiods. In the following, the generic subperiod will be called "interval" and denoted by index "h"; with no loss of generality, the interval length will be assumed constant and equal to T units of time.

Definitions and Notations

The transportation system is represented by a network; the generic origin-destination (O-D) pair is denoted by i, the generic path of the network by k, and the set of indexes relative to paths connecting the O-D pair i by K_i.

The total number of users deciding whether, when, and how to travel between each O-D pair i is denoted by d_i; it is assumed to be known and constant in each day t. Elasticity of the demand level; that is, changes in the number of users traveling each day, can be simulated by defining a fictitious interval ($h = n_h + 1$) corresponding to the choice of not moving at all. The disutility associated with this alternative is defined by the utility of not moving at all.

Let F^t_{hk} be the flow of users following path k and leaving during interval h of day t. These values, arranged in a vector $\mathbf{F}^t$, are assumed to define the state of the system at day t. Demand and path flows are related, since

$$d_i = \sum_h \sum_{k \varepsilon K_i} F^t_{hk} \tag{1}$$

If $\bar{p}^t(h,k)$ denotes the fraction of users leaving during interval h of day t and following path $k \varepsilon K_i$ between the O-D pair i, the path flow can be expressed as:

$$F^t_{hk} = d_i \cdot \bar{p}^t(h,k) \qquad k \varepsilon K_i \tag{2}$$

An obvious simplification occurs if the demand temporal profile or the departure time fractions $\bar{p}_i(h)$ are fixed. In this case, the choice fractions can be expressed as $\bar{p}^t(h,k) = \bar{p}_i(h) \cdot \bar{p}^t(k/h)$, $k \varepsilon K_i$. Elasticity of the demand level; that is, changes in the number of users traveling each day, can be simulated also in this case, by introducing a fictitious path ($k = 0$) between each O-D pair to simulate, though only descriptively, the choice of not moving at all.

Let f^t_{ah} be the flow on arc a during interval h of day t, and $\mathbf{f}^t$ the corresponding arc flow vector. In a within-day dynamic

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framework, the definition of arc flow is not unique because time and space averages do not coincide any longer. The operative definition of arc flow then depends on the modeling framework adopted for network loading, as reported in the section on the model of users' behavior.

The correspondence between arc flows, however defined, and path flows (network loading mapping) can be formally expressed as

$$f^t = f^t(F^t) \quad (3)$$

Let C^t_{hk} be the average generalized travel cost on path k leaving during interval h of day t, and $\mathbf{C}^t$ be the corresponding travel cost vector. Path travel costs are generally functions of the arc travel costs, which in turn are functions of arc flow vector; therefore, the following formal relationship holds:

$$\mathbf{C}^t = \mathbf{C}^t(\mathbf{f}^t) \quad (4)$$

Strictly speaking, travel costs are random variables with average values that can be expressed as a function of arc flows by means of arc cost functions. The probability of the occurrence of a given cost vector conditional to the path flow vector can thus be expressed by Equation 3 as

$$Pr[\mathbf{C}^t/\mathbf{f}^t] = Pr[\mathbf{C}^t/\mathbf{f}^t(\mathbf{F}^t)] \quad (5)$$

On the other hand, most assignment models ignore the dispersion of travel times around their mean values; in this case, the conditional probabilities in Equation 5 can be substituted by the usual expression

$$\mathbf{C}^t = \mathbf{C}^t(\mathbf{F}^t) \quad (6)$$

obtained by combining Equations 3 and 4.

Stochastic Process Model

It appears realistic to assume that the number of users F^t_{hk} is an integer. Then, the number of feasible states, that is, path flow vectors with nonnegative components and consistent with the demand as defined by Equation 1, is finite.

For the dispersion of users' behavior and the intrisic randomness of some parameters (number of users, network conditions, travel costs, times, etc.), it is assumed that the system takes different states in different days. Furthermore, these states cannot be exactly forecasted by the analyst.

In other words, the actual values of fractions $\bar{p}^t(h,k)$ for a given day cannot be predicted in advance. Therefore, the evolution of the system among feasible states in successive days can be described as a stochastic process, with properties depending on the hypotheses made on users' behavior and network configuration. Interval/path fractions are realizations of random variables, whose average values are the choice probabilities $p^t(h,k)$, which can be obtained by a properly defined model (see the next section for an example).

The probability $Pr[\mathbf{F}^t]$ that the system is in a given state $\mathbf{F}^t$ at day t can be computed, at least theoretically, from choice probabilities:

$$Pr[\mathbf{F}^t] = Pr[\mathbf{F}^t/p^t(h,k)\ \forall h,k] \quad (7)$$

It can be reasonably assumed that users choose paths and departure times using information about times and costs that have occurred in previous days $\mathbf{C}^{t-1}$, C^{t-2}, . . . , either because this is the only available information or because it complements information supplied by an informative system.

If some travelers choose using a real-time informative system, choice probabilities depend also on times and costs incurred in the current day. More precisely, in the case of a real-time trip planning system, departure time and path probabilities depend on costs incurred at most up to the departure interval, in the case of a real-time route guidance system, path choice probabilities depend on costs up to the arrival time (computed from the departure time and the travel time). Similar conditions occur if an adaptive route choice behavior is assumed for users. Then the following will result:

$$p^t(h,k) = p^t(h,k)[\mathbf{C}^t, \mathbf{C}^{t-1}, \mathbf{C}^{t-2}, \ldots]$$

Moreover, if it is assumed that users have a limited memory, that is they are significantly influenced in their choices at most by a limited number (m) of past days, then the following results are obtained:

$$p^t(hk) = p^t(h,k)[\mathbf{C}^{t-i}, i = 0, \ldots, m] \quad (8)$$

Since path travel times and costs in congested networks depend deterministically or stochastically on arc flows, it turns out that the probability that the system is in a given state $\mathbf{F}^t$ at day t depends on the states occupied by the system in m previous days. Combining Equations 5, 7, and 8

$$\begin{aligned} &Pr[\mathbf{F}^t/\mathbf{F}^t, \mathbf{F}^{t-1}, \ldots, \mathbf{F}^{t-m}] \\ &= Pr[\mathbf{F}^t/p^t(h,k)[\mathbf{C}^{t-i}, i = 0, \ldots, m]] \cdot Pr[\mathbf{C}^t/\mathbf{F}^t] \\ &\cdot Pr[\mathbf{C}^{t-1}/\mathbf{F}^{t-1}] \cdot \ldots \cdot Pr[\mathbf{C}^{t-m}/\mathbf{F}^{t-m}] \end{aligned} \quad (9)$$

If path costs are assumed not to be random variables, then

$$\begin{aligned} &Pr[\mathbf{F}^t/\mathbf{F}^{t-1}, \ldots, \mathbf{F}^{t-m}] \\ &= Pr\{\mathbf{F}^t/p^t(hk)[\mathbf{C}^{t-1}(\mathbf{F}^{t-1}), \quad 0 = 1, \ldots, m]\} \end{aligned}$$

It can be proven, by using results of m-dependent Markov chains (*2*) that the process admits a unique stationary probability distribution and it is ergotic if, in addition to the limited memory assumption (Equation 8), the following (sufficient) conditions hold:

1. Choice probabilities, given the same sequence of costs relative to the previous day, and possibly to the same day, are time homogeneous, that is, invariant with respect to a time translation:

$$p^t(hk)[\mathbf{C}^{t-i}, i = 0, \ldots, m] = p^{t'}(hk)[\mathbf{C}^{t'-i}, i = 0, \ldots, m]$$
$$\text{if } \mathbf{C}^{t-i} = \mathbf{C}^{t'-i}, i = 0, \ldots, m$$

2. For each pair of different states there exists at least one sequence of feasible states, with strictly positive transaction probabilities, from one to the other.

The stated conditions depend only on the assumption that behavioral rules are constant over time, and do not depend on any assumptions about the particular type of users' choice behavior and the information available to them, apart from that of finite memory. In other words, different types of choice models can be used for departure time and path choice, such as random utility or noncompensatory models in so far they do not assume that users' experience influence their behavior thereafter.

It is also worth noting that random events modifying network performances, such as accidents and bad weather conditions, can be included in the proposed framework if their occurrence probability law is stable over time.

Because of the existence and unicity of the stationary distribution, one distribution of path flows can be associated to each demand-supply system, independent of the starting configuration. Process ergodicity allows the computation of flow means and moments through the simulation of only one realization of the process. It is worth noting that the mentioned properties are satisfied regardless of the type of arc cost functions.

Obviously it is still possible to study transitions between two stationary states of the system. In this case, the ergodicity property no longer applies, and flow moments must be computed over repeated simulations of transients.

The stationary probability distribution of flows could have different modes, denoting a situation comparable to that of multiple equilibria. However, in the proposed stochastic process approach, the whole probability distribution for each arc flow can in principle be computed, although in the multiple equilibria case no method known to the authors guarantees information on all equilibrium configurations.

Solution Approach

In the following, a solution approach for computing expected values and moments of time-varying arc flows, both in steady state and transient conditions, will be described briefly.

Full specification of the assignment model requires a modeling of departure time and path choice, of users' learning and forecasting mechanisms, and possibly of the informative strategy and users' reactions. Most models proposed in the literature to simulate the aspects given previously could be adapted to fit into the proposed framework. For instance, departure time choice could be simulated by a random utility model (*10*) or by the "bounded rationality" model proposed by Mahmassani and Chang (*11,12*). A possible specification will be described in the next section.

The number of users choosing each departure interval/path alternative in a given day can be obtained from choice probabilities by using a Monte Carlo simulation or by substituting probabilities to fractions. In the latter case, the resulting sequence is a pseudo realization of a stochastic process. An obvious simplification occurs when fractions $p_i^t(h)$ are exogenously given (prefixed demand profile).

Once that choice fractions and, consequently, path flows are known for the current day, arc flows can be computed by a dynamic network loading method, as described in a following section. Arc flows allow the computation of travel times and thus the update of travelers' information and forecasts to be used for modeling choice probabilities of the next day.

Flows for each arc of the network and for each interval in the reference period can be used for two purposes. The first is to estimate stationary means and moments for arc flows; the second is to estimate means and moments during transients caused by any modification in the network, in the demand, or in both the network and the demand.

The reaching of stationarity can be checked by performing a Student's-*t* test on the differences between average arc flows in each interval over two successive sequences of days.

MODEL OF USERS' BEHAVIOR

Users' Choice Behavior

Users' choice behavior has been modeled by the random utility model proposed by Small (*10*) and reformulated by Ben Akiva et al. (*13*). This model has been slightly modified to explicitly introduce a "habit effect" in users' choices. In particular it is assumed that each day only a fraction Ω of users reconsider the previous days' choice, and that they give an extra utility to the alternative chosen the previous day.

It is assumed that each user deciding how (path k) and when (departure interval h) to travel at day t associates to each alternative (h,k) a perceived utility expressed by the sum of a systematic utility and a random residual error.

The systematic utility represents the average predicted utility, whereas the random residual takes into account different perception errors made by users (e.g., relative to travel times and costs) and the dispersion of some characteristics within the population of decision makers (desired arrival times, reciprocal substitution coefficients, missing attributes, etc.).

The systematic utility $\overline{V}_{hk}^t$ can be expressed using a modified version of the model proposed by Small (*10*) as the sum of disutilities relative to the generalized transportation cost and to early or late arrival penalty:

$$\begin{aligned}\overline{V}_{hk}^t = & -(1 - \mu_{hk}) \cdot (\beta_{1i}\overline{C}_{hk}^t \\ & + \mathrm{MAX}\{\beta_{2i}[(D_i - \delta_{1i}) - \overline{B}_{hk}^t], 0\} \\ & + \mathrm{MAX}\{\beta_{3i}[\overline{B}_{hk}^t - (D_i + \delta_{2i})], 0\}) \qquad (10)\end{aligned}$$

μ_{hk} = $\mu \varepsilon [0,1]$ if (h,k) is the alternative chosen the previous day,
= 0, otherwise this parameter reduces proportionally the disutility for the choice of the same path and departure time as in the previous day $t - 1$, and attempts to capture the conservative behavior of users;

$\overline{C}_{hk}^t$ = average predicted generalized transportation cost along path k starting during interval h, on day t;

$\overline{B}_{hk}^t$ = average predicted arrival time, starting during interval h and moving along path k, on day t, computed as $(h - 1) \cdot T + \overline{T}_{hk}^t$;

T_{hk}^t = average predicted travel time along path k starting during interval h, on day t;

D_i = desired arrival time, variable with the O-D pair (and category) i; and

β_{1i}, β_{2i}, β_{3i} = reciprocal substitution coefficients, variable with the O-D pair (and category) i.

This expression assumes that users of O-D pair i have a tolerance interval $[-\delta_{1i}, \delta_{2i}]$ around the desired arrival time, and early or late arrivals cause a disutility proportional to the advance or the delay.

On day t average values of predicted travel time and generalized transportation cost of path k leaving during interval h can be computed through duly defined filters, which model the learning and forecasting mechanisms used by the average traveler, including the interaction with an informative system, if any. Different filters can be used for different kinds of users, for example, commuters versus noncommuters, informed versus noninformed, and so on.

Systematic utility has been defined assuming the generalized transportation cost to be equal to the travel time. The average perceived travel time has been computed as a weighted average of the previous day actual travel time T_{hk}^{t-1} and of the previous day average perceived travel time $\overline{T}_{hk}^{t-1}$:

$$\overline{T}_{hk}^{t} = \tau \cdot \sum_{i=1}^{t-1} (1-\tau)^{i-1} \cdot T_{hk}^{t-i} + (1-\tau)^{t} \cdot T_{hk}^{0}$$
$$= \tau \cdot T_{hk}^{t-1} + (1-\tau) \cdot \overline{T_{hk}^{t-1}} \qquad (11)$$

where $\overline{T}_{hk}^{0} = T_{hk}^{0}$ is a starting value. Values of τ close to one denote a stronger influence of the previous day travel time. Similar filters have been proposed by Mahmassani and Chang (*12*) and Iida et al. (*14*).

It has been assumed that each day a prefixed fraction of users Ω takes into consideration the possibility of modifying their previous day choice (but they do not necessarily have to). The choice probabilities of the users that reconsider their choice is simulated through a path/departure time nested logit model:

$$p^t(h,k) = p^t(h) \cdot p^t(k/h) \qquad (12)$$

$$p^t(k/h) = \exp[\Theta_1 \overline{V}_{hk}^t] / \sum_j \exp[\Theta_1 \overline{V}_{hj}^t] \qquad (13)$$

$$p^t(h) = \exp[\Theta_2 \overline{Y}_h^t] / \sum_j \exp[\Theta_2 \overline{Y}_j^t] \qquad (14)$$

where

Θ_1 = Weibull-Gumble parameter of the random residual relative to the pair (h,k),
Θ_2 = $(1/\Theta_1^2 + 1/\Theta^2)^{(-1/2)}$,
Θ = Weibull-Gumble parameter of the random residual relative to the interval h only, and
Y_h^t = $(1/\Theta_1) \cdot \ln \Sigma_j \exp[\Theta_1 V_{hj}^t]$ is the logsum variable relative to interval h.

If $\Theta_1 = \Theta_2$ the above model reduces to the factorialization of a multinomial logit over the pair (h,k).

As it is known from the logit theory, coefficients Θ_i are inversely proportional to the standard deviation σ_i of the perception error in path and departure time choice, respectively. For each O-D pair values of coefficients Θ_i have been computed by assuming a prefixed value of the variation coefficient Cv_i for each of them:

$$\Theta_1 = \Pi/(\sqrt{6} \cdot \sigma_i) \cong 1.282/\sigma_i = 1.282/(Cv_i \cdot \overline{V}) \qquad (15)$$

where $\overline{V}$ is the value of utility obtained by averaging across all the alternatives, the average perceived utility as given by Equation 10.

Thus different quality in information can be simulated by differentiating all the users of each O-D pair in two or more types (e.g., informed and not informed) with different variation coefficients, and consequently, different values of Θ_i.

Users' Behavior and Informative Systems

Generally users moving between an O-D pair are assumed to choose at their origin the departure interval and a path k_0 to reach their destination. After leaving the origin, rerouting during the trip may occur because of adaptive behavior at duly located diversion nodes (possibly at each node of the network). Therefore, the actually used path k may be different from the initially chosen k_0. The choice probability at day t, $p^t(h,k)$, can be expressed as

$$p^t(h,k) = p_i^t(h) \cdot p^t(k/h) \qquad k \in K_i$$

where

$p_i^t(h)$ = probability of choosing departure interval h for a user traveling between O-D pair i; and
$p^t(k/h)$ = probability to follow path $k \in K_i$ from the origin to the destination of O-D pair i, once departure interval h has been chosen (it can be expressed as the joint probability that the sequence of paths from the origin to the first diversion node, between each pair of successive diversion nodes, and from the last diversion node to the destination forms path k).

An informative system may supply information or indications to users before they start (pretrip) or while they are traveling (en route). Moreover, the informative system can use exclusively information about the network conditions in the previous days (static) or combine them with information about the network conditions that occurred during day t (dynamic or real time) before the departure time (pretrip) or the arrival time (en route). Different classes of users with different types of available information can be taken into account.

The behavior of users not advised by an informative system can be modeled through usually adopted users' choice behavior models, as the one described in the previous subsection. The same kind of models, with a different parameter specification, can be adopted to deal with users advised by a static pretrip informative system (an example is given in a subsequent section).

A slight modification is needed in the case of a real-time pretrip informative system: Let

$$p_i^t[j = h] = p_i^t(h)$$

be the choice probability that begins the trip between O-D pair i during interval h of day t;

$$p_i^t[j \geq h] \begin{cases} = 1 & \text{if } h = 1 \\ = 1 - \sum_{m=1}^{h-1} p_i^t[j = m] & \text{if } h > 1 \end{cases}$$

be the choice probability that a user begins the trip during interval h or later; and

$$p_i^t[j = h/j \geq h] = p_i^t(h/\{h, h + 1, \ldots\})$$

be the choice probability that a user begins the trip during interval h conditional to leaving not earlier than interval h. This can be computed using a choice behavior model (as the one described in the previous section) assuming as choice set $\{h, h + 1, \ldots\}$.

Since

$$p_i^t[j = h/j \geq h] \cdot p_i^t[j \geq h] = p_i^t[j = h \cap j \geq h] = p_i^t[j = h]$$

the choice probabilities $p^t(h)$ can be recursively computed as

$$p_i^t(h) \begin{cases} = p_i^t(1/\{1, 2, \ldots\}) & \text{if } h = 1 \\ = p_i^t(h/\{h, h + 1, \ldots\}) \cdot \left(1 - \sum_{m=1}^{h-1} p_i^t(m)\right) & \text{if } h > 1 \end{cases}$$

Once choice probability $p_i^t(h)$ has been computed, the path choice probabilities $p^t(k/h)$ can be computed through a users' behavior model. The resulting stochastic process maintains all the previously mentioned stationarity and ergodicity properties.

If a static or dynamic en route informative (or route guidance) system is operating for some users, the choice probabilities for the departure interval h and the initially chosen path k_0 can be computed as described previously. Then during the dynamic network loading stage at duly located diversion nodes (beacons of the route guidance system, or each node of the network), users are allowed to reroute onto a new path, and so on, until the destination is reached, according to a behavior model and the information or indications supplied by the route guidance system (as better explained in the next section).

A DYNAMIC NETWORK LOADING METHOD

In this section, a model for dynamic network loading; that is, computation of time-varying arc flows from a given path flow pattern in a given day t, is described and compared with other state-of-art models. A solution algorithm is also presented (*3*). In the following, superscript t will be omitted for the sake of simplicity.

Notation

Notations used in this section are as follows. Vectors and arrays are not explicitly cited.

k = path,
a = arc,
a' = arc (if any) following arc a on path k,
h = interval,
j = interval,
(j,k) = group leaving during interval j and traveling on path k,
f_{ah} = arc flow,
F_{jk} = path flow,
l_a = running arc length,
q_{ah} = queue at the end of interval,
Q_{ah} = capacity,
s_{ah}^{jk} = abscissa on running arc,
$u_{ah} = \overline{u}_{ah}$ = undersaturation delay,
$v_{ah} = V_{ah}(f_h)$ = running speed,
w_{ah}^{jk} = waiting time for queuing arc,
$y_{\substack{jk\\ah}}$ = arrival time at arc a,
$y_{\substack{jk\\a'h}}$ = arrival time at arc a',
$z_{\substack{jk\\ah}} = z_{ah}(y_{\substack{jk\\ah}})$ = oversaturation delay, and
$\alpha_{\substack{jk\\ah}}$ = crossing fraction.

Statement of the Problem

Previously it was stated that the relationship between arc and path flows in a within-day dynamic context is not trivial. It was also observed that arc flows are not even uniquely defined in this case. It is still possible to express the relationship between path and arc flows, however defined, in a way that is formally similar to the within-day uniform case.

Denoted by $\alpha_{ah}^{jk} \varepsilon [0,1]$, the fraction of *path flow* F_{jk} contributing to *arc flow* f_{ah}, named *crossing fraction*, the flow on arc a can be expressed by

$$f_{ah} = \sum_{jk} \alpha_{ah}^{jk} F_{jk} \tag{16}$$

If arc a does not belong to path k, or the starting interval of flow F_{jk} follows interval $h (j > h)$, or flow F_{jk} does not occupy arc a during interval h, the fraction α_{ah}^{jk} is equal to zero.

Crossing fractions can be arranged in matrices such as $\boldsymbol{A}_{hj} = \{\alpha_{ah}^{jk}\}_a^k$, arc-path crossing matrix between arc-flows during interval h and path flows leaving in interval j. Denoted by $\boldsymbol{f}_h$, the arc flow vector during interval h and by $\boldsymbol{F}_j$, the path flow vector leaving during interval j, Equation 16 can be stated in matrix form as follows:

$$\boldsymbol{f}_h = \sum_{j=1}^{h} \boldsymbol{A}_{hj} \cdot \boldsymbol{F}_j \tag{17}$$

For the whole day Equation 16 can be expressed as

$$\boldsymbol{f} = \boldsymbol{A} \cdot \boldsymbol{F} \tag{18}$$

where

$\boldsymbol{f}$ = arc flow vector for the whole day,
$\boldsymbol{F}$ = path flow vector for the whole day, and
$\boldsymbol{A}$ = arc-path crossing matrix for the whole day formed by matrices $\boldsymbol{A}_{hj}$ with $\boldsymbol{A}_{hj} = \boldsymbol{O}$ (if $h < j$, $\boldsymbol{A}$ is a low triangular block matrix)

It is worth noting that the arc-path crossing matrix $\boldsymbol{A}$ is a generalization of the usual arc-path incidence matrix, because the following should result:

$$\sum_{m=1}^{nh} \alpha_{am}^{jk} = 1 \tag{19}$$

if all the users entering the network during interval j leave it in some interval $h \geq j$.

Generally, crossing fractions α_{ah}^{jk} depend on the definition adopted for arc flows, the network topology, and the time needed to reach arc a traveling on path k. Therefore, they depend on the travel times on arcs preceding arc a along path k, which in congested networks are function of the arc flows. Hence, it generally results that

$$A = A(f) \tag{20}$$

Therefore combining Equations 18 and 20 the following fixed-point problem is obtained:

$$f^* = A(f^*) \cdot F \tag{21}$$

A dynamic network loading method is essentially an algorithmic definition of crossing fractions, that is of relationship $\alpha_{ah}^{jk} = \alpha_{ah}^{jk}(f)$ needed to solve Equation 21.

It is worth noting that in noncongested networks in which travel times and delays are constant and independent of arc flows, the arc-path crossing matrix does not depend on arc flows and Equation 21 reduces to $f^* = AF$, as in the case of within-day constant demand.

Several methods have been proposed to solve the dynamic network loading problem, which in any case could be solved by a discrete simulation technique (microsimulation), although at the expense of a large computational effort. Generally, these methods give different results depending on the different hypotheses adopted.

Some methods do not explicitly formulate and solve the fixed-point problem (Equation 21). They are based on a generalization of network loading procedures used in static deterministic user equilibrium (*6,7*). A different method to indirectly solve the problem (Equation 21) has been proposed recently by Vythoulkas (*9*). This model is based on the discretization of a differential equation for each arc, expressing the relationship between the time derivative of the number of users on the arc and the difference between inflow and outflow.

All the preceding methods rely on assumptions that do not rule out some counterintuitive results such as overtaking between vehicles traveling on the same path and vehicles leaving in different times. In addition, these methods can hardly be extended to include en route diversions from the initially chosen path, because of the computational burden of keeping the identity of diverted path flows.

In the following, a new method for dynamic network loading is described, which directly solves the fixed-point problem (Equation 21), thus overcoming some of the drawbacks of the other proposed methods.

General Hypotheses and Definitions

The set of all users leaving in the same interval j and following the same path k is called *group* or packet (j,k). All users of a group are assumed to experience the same trip as the group leader, whose departure occurs at a prefixed instant (the middle or the beginning) of the interval. Hence, if an arc is occupied by the leader of a group during an interval, it is occupied by all the users belonging to that group (*grouping hypothesis*).

This assumption appears acceptable for usual O-D demand flows and interval lengths. In any case its realism can be improved by reducing the interval duration, or by subdividing a group into smaller units. Currently, an enhanced model for dynamic network loading relaxing this hypothesis is being developed by the authors.

In the following it is also assumed, for simplicity, that a user group follows the path chosen before starting the trip until its destination is reached. However, the described method can be easily extended to include while-trip rerouting. In this case, at duly located diversion nodes (eventually each node can be a diversion node), group (j,k) traveling on path k may reroute on a new path k' from the diversion node to the destination, thus becoming group $(j,k - k')$. This can be the result of a simple adaptive behavior or of the interaction with a route guidance system, providing indications or information. In both cases, a choice model is needed to represented users' reactions resulting in a change from path k to path k'.

Two types of arcs requiring different modeling approaches will be considered in the following:

- *Running arcs* (e.g., a stretch of a street): for which the time needed to leave the arc is continuously spent along its length; and
- *Queuing or waiting arcs* or bottlenecks (e.g., a junction approach): for which delay occurs only at the end of the arc, assumed of null length, because of queuing due to capacity constraints.

Obviously the simulated network can include both types of arc.

Let v_{ah} be the *average running speed* on running arc a, with length l_a, during interval h. Running speed is assumed to be the same for all users traveling on the arc during interval h (*equal running speed hypothesis*), regardless of when they have entered the arc.

Let u_{ah} be the *undersaturation delay* for queuing arc a, during interval h. Undersaturation delay is assumed to be the same for all users entering the arc during interval h (*equal undersaturation delay hypothesis*) regardless of when they have entered the arc.

Let z_{ah}^{jk} be the *oversaturation delay* for group (j,k) joining the queue at bottleneck arc a during interval h. It is assumed to be equal for all the users of group (j,k) (*group specific oversaturation delay hypothesis*) and depending on the arrival time of group (j,k) at arc a during interval h. For undersaturated conditions, the oversaturation delay is equal to zero.

Group Movements on the Network

Let $y_{ah}^{jk} \in [0,T]$ be the *arrival time* of group (j,k) at arc a during interval h. It is meaningfully defined only if arc a belongs to path k and interval j precedes interval h, that is $j \leq h$ (otherwise it is assumed equal to zero by convention). In the following it is always assumed that arc a belongs to path k, and interval j precedes interval h, that is $j \leq h$. Let a' be the arc (if any) following arc a on path k. Naturally the exit time from arc a of group (j,k) is equal to the arrival time at arc a'.

If group (j,k) has not yet reached arc a during interval h, $y^{jk}_{ah} = 0$; vice versa if group (j,k) has already left arc a before interval h, $y^{jk}_{ah} = T$. Moreover, it is assumed that the arrival time is equal to zero for the first arc occupied during interval h.

The difference, $T - y^{jk}_{ah}$, between the interval length T and the arrival time y^{jk}_{ah} at arc a is the time still available to group (j,k) to move on arc a during interval h.

A group (j,k) arriving at a *running arc a* during interval h needs a time l_a/v_{ah} to entirely cover the arc. It can be proven that the exit time from arc a [or the arrival time at the next arc a' (if any) on path k)] and the abscissa reached on arc a by group (j,k) by the end of interval h are given by (also shown in Figure 1)

$$y^{jk}_{a'h} = \mathrm{MIN}[T, y^{jk}_{ah} + (l_a - s^{jk}_{ah-1})/v_{ah}] \tag{22}$$

$$s^{jk}_{ah} = \mathrm{MIN}[l_a, s^{jk}_{ah-1} + (T - y^{jk}_{ah}) \cdot v_{ah}] \tag{23}$$

assuming that

$y^{jk}_{ah} = 0$ if arc a is the first occupied by group (j,k) during interval h;
$y^{jk}_{ah} = 0$ if group (j,k) has left from arc a before interval h;
$y^{jk}_{ah} = T$ if group (j,k) has not yet reached arc a by the end of interval h;
$s^{jk}_{a0} = 0$
$s^{jk}_{ah} = 0$ if group (j,k) has not yet reached arc a by the end of interval h; and
$s^{jk}_{ah} = l_a$ if group (j,k) has left from arc a by the end of interval h.

If group (j,k) enters *queuing arc* a during interval h, the time needed to exit the arc is equal to the total delay: $u_{ah} + z^{jk}_{ah}$, whereas the time still available to move is given by the difference $T - y^{jk}_{ah}$.

If $u_{ah} + z^{jk}_{ah} \leq T - y^{jk}_{ah}$, group (j,k) leaves arc a during interval h and enters the next arc a' (if any) on path k at time

$$y^{jk}_{a'h} = y^{jk}_{ah} + u_{ah} + z^{jk}_{ah} \tag{24}$$

Vice versa, if $T - y^{jk}_{ah} < u_{ah} + z^{jk}_{ah}$, it remains on the arc. Let

$$w^{jk}_{ah} = u_{ah} + z^{jk}_{ah} - (T - y^{jk}_{ah}) > 0 \tag{25}$$

be the time needed by group (j,k) to leave arc a at the end of interval h (*residual waiting time*).

If group (j,k) is still on queuing arc a at the beginning of interval $h + 1$, it should stay queuing for a time w^{jk}_{ah}. If it results that $T \leq w^{jk}_{ah}$, the exit time from arc a for group (j,k) or the arrival time on the next arc a' (if any) on path k is given by

$$y^{jk}_{a'h+1} = w^{jk}_{ah} \tag{26}$$

otherwise it should stay on arc a for a time

$$w^{jk}_{ah+1} = w^{jk}_{ah} - T \tag{27}$$

If group (j,k) reaches its destination during interval h, it leaves the network and it turns out that $s^{jk}_{ah} = l_a$ or $w^{jk}_{ah} = 0$ for all arcs on path k, and $m \geq h$. Groups that do not reach their final destination by the end of the reference period can be left on the network without any loss of generality.

Computation of Crossing Fractions

To compute crossing fractions from running speeds and delays, an operational definition of arc flows has to be formulated.

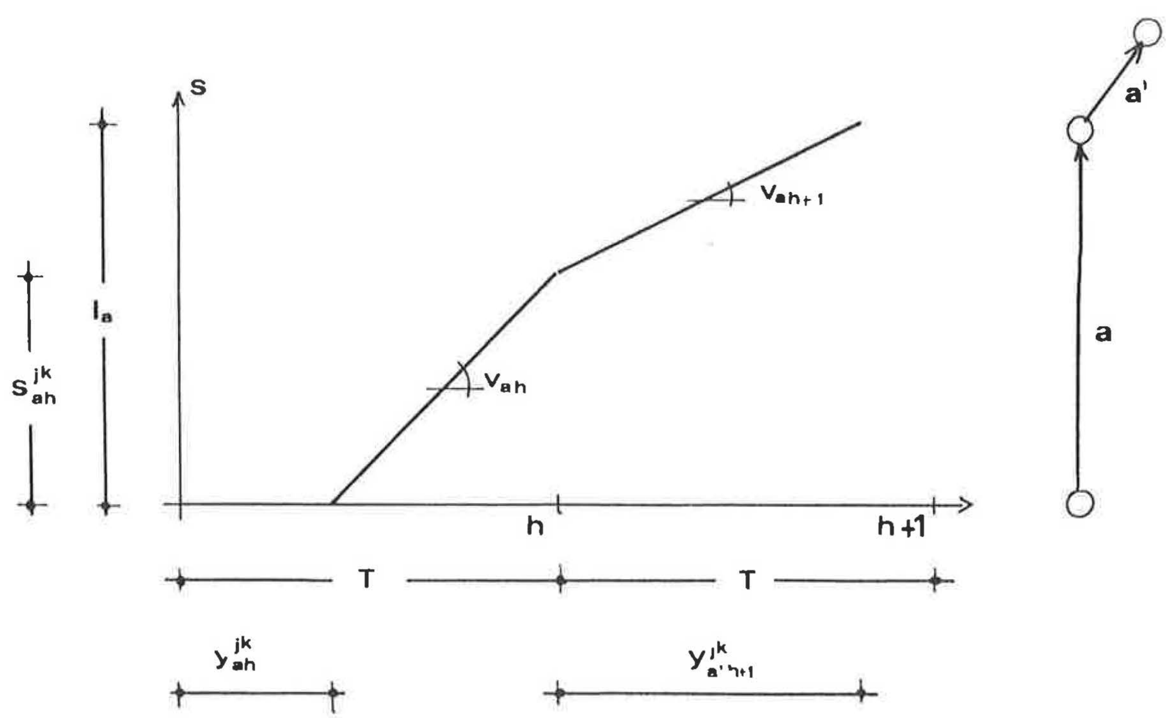

FIGURE 1 Group movement for a running arc.

For the grouping hypothesis, the average number of users per unit of time on *running arc a* during interval h is given by

$$\left(\sum_{jk} F_{jk} \cdot T \cdot \int_0^T \delta_{ah}^{jk}(t)\, dt\right) \Big/ T$$

Where

$\delta_{ah}^{jk}(t) = 1$ if group (j,k) is on arc a at time t; and
$\delta_{ah}^{jk}(t) = 0$ otherwise.

Moreover, for the equal running speed hypothesis, the time spent by group (j,k) on arc a during interval h is equal to the traveled length divided by the average running speed, that is

$$\int_0^T \delta_{ah}^{jk}(t)\, dt = (s_{ah}^{jk} - s_{ah-1}^{jk})/v_{ah}$$

Therefore, the average density on arc a during interval h, which is equal to the average number of users per unit of time divided by the arc length, is given by the expression $\sum_{jk} F_{jk} \cdot (s_{ah}^{jk} - s_{ah-1}^{jk})/(v_{ah} \cdot l_a)$

The flow f_{ah} on *running arc a* during interval h can be defined as the product of the average density and the average speed:

$$f_{ah} = \sum_{jk} F_{jk} \cdot (s_{ah}^{jk} - s_{ah-1}^{jk})/l_a$$

Therefore from Equation 16, it turns out that

$$\alpha_{ah}^{jk} = (s_{ah}^{jk} - s_{a(h-1)}^{jk})/l_a$$

Also from Equation 23 it turns out that

$$\alpha_{ah}^{jk}(s_{ah-1}^{jk}, v_{ah}, y_{ah}^{jk}) = \mathrm{MIN}\{(T - y_{ah}^{jk}) \cdot v_{ah}/l_a, (l - s_{ah-1}^{jk}/l_a)\} \quad (28)$$

Moreover it results that

$$s_{ah}^{jk} = s_{ah-1}^{jk} + \alpha_{ah}^{jk} \cdot l_a \quad (29)$$

Similarly for a *queuing arc*, the flow can be defined as the time average in flow. For the grouping hypothesis, it turns out that if group (j,k) reaches arc a during interval h, the corresponding crossing fraction is equal to one, it is equal to zero otherwise:

$$\alpha_{ah}^{jk}(y_{ah}^{jk}) \begin{cases} = 1 \text{ if } T - y_{ah}^{jk} > 0 \\ = 0 \text{ if } T - y_{ah}^{jk} = 0 \end{cases} \quad (30)$$

For both types of arcs, if group (j,k) stops on arc a during interval h, it turns out that $\alpha_{ih}^{jk} = 0$ for any arc i following arc a on path k. If group (j,k) reaches its destination, it exits from the network and it turns out that $\alpha_{am}^{jk} = 0$, $m > h$, $a \in k$.

It can be proven that this relationship is satisfied by the preceding proposed definition of crossing fractions, for groups leaving the network by the end of the simulation period.

Solution Approach

In this section, the crossing fraction definition described in the previous section is used to build up a fixed-point formulation of the dynamic network loading problem, according to the considerations in section, "Statement of the Problem."

Summarizing the results of the preceding section, the crossing fraction of group (j,k) on a running arc depends on the running speed and the arrival time at that arc. On the other hand, for a queuing arc, the crossing fraction of group (j,k) depends on the arrival time at that arc.

In turn, according to next preceding section, the arrival time of group (j,k) at arc a can be sequentially computed for each arc of a given path from running speeds and delays, by Equations 22 through 27.

Hence crossing fraction α_{ah}^{jk} is a function of running speeds and delays on arcs preceding arc a on path k. In the case of a running arc it is also a function of the running speed on that arc, whereas if it is relative to a queuing arc it does not depend on the delay on the same arc. Then, it generally results that

$$\alpha_{ah}^{jk} = \alpha_{ah}^{jk}(s_{ah-1}^{jk}, w_{ah-1}^{jk}, \mathbf{v}_h, \mathbf{u}_h, \mathbf{z}_h) \quad (31)$$

where $\mathbf{v}_h$, $\mathbf{u}_h$, and $\mathbf{z}_h$ are, respectively, the vectors of running speeds, undersaturation, and oversaturation delays occurred on the network during interval h.

Average running speed is assumed to be function of the vector of the arc flows during interval h, $\mathbf{f}_h$, through usually adopted cost-flow functions:

$$v_{ah} = v_{ah}(\mathbf{f}_h) \quad (32)$$

The undersaturation delay is assumed flow-independent:

$$u_{ah} = \bar{u}_{ah} \quad (33)$$

According to a fluid approximation deterministic model, it turns out that

$$\begin{aligned} z_{ah}^{jk} &= z_{ah}(q_{ah-1}, f_{ah}, y_{ah}^{jk}) \\ &= \mathrm{MAX}[(q_{ah-1}/Q_{ah} + (f_{ah}/Q_{ah} - 1) \cdot y_{ah}^{jk}), 0] \end{aligned} \quad (34)$$

where

q_{ah} = *queue* on arc a at the end of interval h (assuming $q_{a0} = 0$);
Q_{ah} = *capacity* of arc a during interval h (it can vary in different intervals, for example in the case of a variable traffic light system).

Therefore, the expression for crossing fractions can be formally rewritten as

$$\alpha_{ah}^{jk} = \alpha_{ah}^{jk}(s_{ah-1}^{jk}, w_{ah-1}^{jk}, q_{ah-1}, \mathbf{f}_h) \quad (35)$$

If crossing fractions are sequentially computed through this equation for each interval h, values s_{ah-1}^{jk}, w_{ah-1}^{jk}, and q_{ah-1} relative to previous intervals are known. In this case, therefore, the only unknown arguments are arc flows relative to interval h, and it results that

$$\alpha_{ah}^{jk} = \alpha_{ah}^{jk}(\mathbf{f}_h) \quad (36)$$

Combining Equations 36 and 16, the fixed-point problem (Equation 21) is obtained, which can be decomposed in a sequence of fixed-point problems since A is a low-triangular block matrix:

$$f_h^* = \sum_{j=1}^{h} A_{hj}(f_{h'}^*) \cdot F_j \tag{37}$$

Each of the problems (Equation 37) can be solved by usual fixed-point methods, as described in the following algorithm:

$i: = 0; f^{(0)}: = f_0$

REPEAT

$i: = i + 1;$

$e^{(i)}: = \sum_{j=1}^{h} A_{hj}(f^{(i-1)})\, F_j$

$f^{(i)}: = (\alpha + \beta/i) \cdot e^{(i)} + (1 - (\alpha + \beta/i)) \cdot f^{(i-1)}$

UNTIL $f^{(i)} - f^{(i-1)} \leq \varepsilon$

$f_h: = f^{(i)}$

where

f_0 = assigned arc flow pattern,
ε = assigned tolerance vector, and
α, β = parameters in the range [0,1].

It is worth noting that the results of the algorithm are not affected by the sequence in which groups are examined and loaded on the network.

Comparisons of this algorithm with alternative specifications and an analysis of effective values for parameters α and β are reported elsewhere (*15*). In the following, $\alpha = 0.10$ and $\beta = 0.00$ will be adopted, and the initial flow pattern is assumed equal to the average flow pattern over the previous days for the same interval.

NUMERICAL EXAMPLES

In this section some results relative to an application of the proposed procedure to a realistic network are described briefly. The test network refers to the town of Battipaglia, Italy, with about 30,000 inhabitants. Supply data are relative to the real network, global O-D demand has been generated through a simple gravity model, and choice behavior has been modeled by adapting literature models, as described previously.

This example aims to test the proposed procedure on a real case. For this reason no specific conclusions about the case studied or comparisons with observational data are reported.

Test Network

The network used to test the model is shown in Figure 2. It has 62 nodes, 168 arcs, 269 O-D pairs, and is connected by 891 paths. A total travel demand of 4,970 users was considered. The parameters adopted for the choice behavior model described previously are as follows:

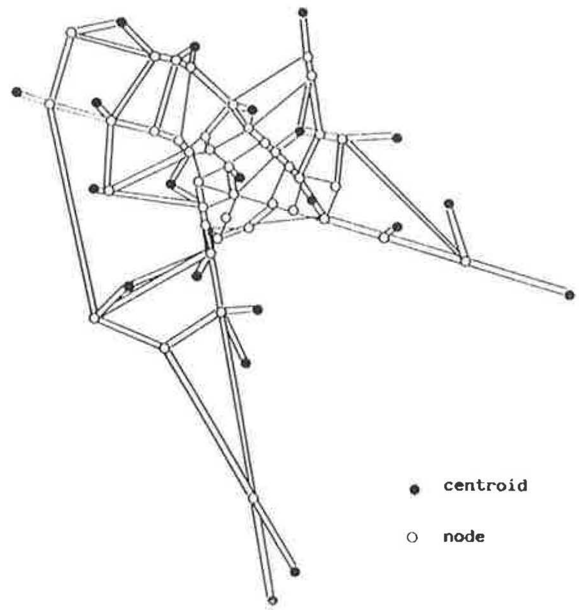

FIGURE 2 Test network.

Fraction of users reconsidering	*Parameter*
Their previous day choice	$\Omega = 0.5$
Reciprocal substitution coefficients	$\beta_1 = 1$, $\beta_2 = 1$, and $\beta_3 = 4$
Logit variation coefficients	$Cv_1 = Cv_2 = 0.20 \div 0.40$
Extra utility weight	$\mu = 0.10$
Filter parameter	$\tau = 0.90$

Davidson's cost functions were adopted for running arcs giving:

$$v_{ah}(f_{ah}) = v_a/[1 + 0.2 \cdot f_{ah}/(Q_{ah} - f_{ah})]$$

where v_a is the free-flow speed and Q_{ah} is the arc capacity. The value 0.2 is a calibration parameter that should be estimated by using actual data. For $f_{ah}/Q_{ah} \geq b_a$ (with b_a positive and less than one), the tangent approximation has been considered to avoid computational problems with asympotic functions. The greater the value b_a, the more sensitive to congestion the function is. In the following a value 0.80 is used, unless otherwise stated. For simplicity, no bottleneck was introduced in the network.

The simulation period lasts 60 min (the morning peak hour from 7:30 a.m. to 8:30 a.m.). It has been divided in 12 intervals with a length $T = 5$ min. Users are allowed to leave in the first 6 intervals (from 7:30 a.m. to 8:00 a.m.); the last 6 intervals have been included for system clearing—to allow all users to reach their final destination. A common decided arrival time has been assumed equal to 7:50, with $\delta_1 = 2.5$ and $\delta_2 = 2.5$.

The stationarity test was adopted on the basis of the comparison of the arc flow time means over two successive 10-day periods. Longer periods are not efficient, because they delay the time at which stationarity is recognized. On the other hand, using shorter periods the results of the test can be affected by periodic solutions.

Simulation of Traditional Control Strategies

The effects of some modifications in demand and supply have been simulated to evaluate the capabilities of the proposed model. Simulation scenarios were generated as follows:

- N1—an increase of total demand from 4,970 to 6,608; and
- N2—an increase of tolerance $\delta_1 = \delta_2 = 7.5$.

Case N1 is aimed at showing the effects of an increase of congestion, and case N2 represents a demand management measure (flexible work times).

The excess generalized cost (computed according to Equation 10) and travel time per user—difference between the actual and the zero-flow values—are compared with the results obtained without any modifications (STD) in Table 1, together with the average percentual changes in departure time and path demand patterns over successive days. Two different values of the variation coefficient of perception error (Equation 15), $Cv = 0.20 \div 0.40$, were used.

As expected, an increase of travel demand causes an increase of the generalized cost and travel time per user (case N1).

An increase of the tolerance band causes a significant reduction of generalized cost per user, and a smaller decrease of travel time. The first effect is a result of the reduction of late or early arrival penalty for users keeping their departure time, whereas the second one can be attributed to elasticity over departure times and a reduction of congestion (case N2), as shown in Figures 3 and 4, which report the departure (continuous line) and arrival (dashed line) profiles for cases N1 and N2.

Generally it seems that quite different values of the coefficient of variation of perception errors cause not great modifications of costs.

In all cases, the modifications in the demand structure with respect to the previous day values are very small, confirming the substantial stability of the day-to-day adjustment process adopted in spite of the quite high influence of the previous

TABLE 1 COMPARISON OF DEMAND MANAGEMENT STRATEGIES

	Excess General Cost (min)		Excess Travel time (min)		Changing users (fraction)	
Cv	0.20	0.40	0.20	0.40	0.20	0.40
STD	5.39	6.77	3.48	2.37	.002	.000
N1	6.94	8.36	3.40	3.65	.010	.000
N2	4.46	5.14	3.00	3.25	.000	.010

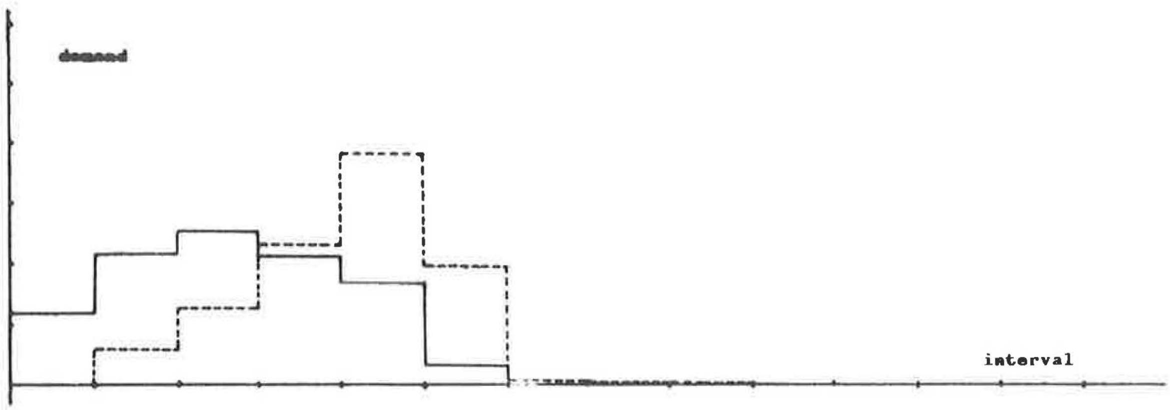

FIGURE 3 Departure and arrival profiles for case N1.

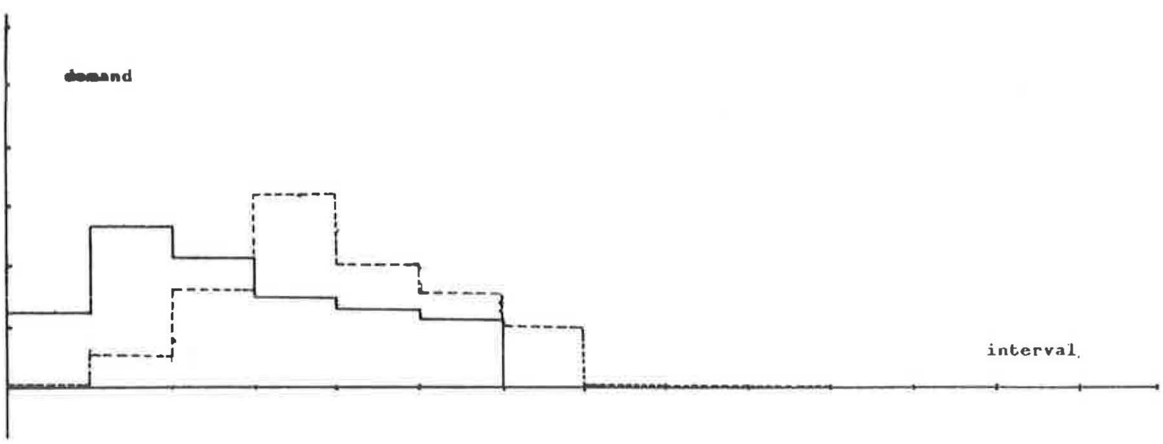

FIGURE 4 Departure and arrival profiles for case N2.

day information in the memory filter (τ = 0.9 in Equation 20).

Simulation of Informative Control Strategies

Using the travel demand value equal to 6,608, the effects of the introduction of a pretrip informative system based on historical data has been modeled. The drivers' reactions to the information provided was simulated by eliminating their inertia to change (the fraction Ω of users reconsidering the previous day choice was set equal to one, and no "habit" externality was considered, μ = 0) and by assuming a much lower dispersion in their choices with respect to the "predicted costs" given by the system, Cv = 0.05. Moreover it has been assumed that time and cost forecasts supplied by the informative system are less dependent on the recent past (filter parameter τ = 0.50 in Equation 20).

Some scenarios were examined considering different "market penetration" of the informative system:

- T1—1 percent of users are informed,
- T2—10 percent of users are informed,
- T3—50 percent of users are informed,
- T4—90 percent of users are informed, and
- T5—100 percent of users are informed.

The results are reported in Table 2, together with the results of case N1 as reference (*i* denotes informed users).

As expected, informed users experience lower generalized costs than noninformed ones (with a reduction of about 20 percent), and a small increase of travel time, since they have a better perception of the trade-off between travel time, even in congested intervals, and early or late arrival penalty. As an example, Figure 5 shows the departure and arrival profiles for case T2 for noninformed users (above) and informed users (below).

TABLE 2 COMPARISON OF INFORMATIVE STRATEGIES

	Excess General cost (min)		Excess Travel time (min)		Changing users Fraction	
Cv	0.20	0.40	0.20	0.40	0.20	0.40
N1	6.94	8.36	3.40	3.65	.010	.000
T1	6.94	8.35	3.39	3.65	.010	.000
i[a]	5.45	5.51	3.72	3.97	.020	.000
T2	6.79	8.39	3.30	3.53	.005	.010
i	5.35	5.57	3.67	3.88	.018	.024
T3	6.37	7.67	2.93	2.93	.002	.000
i	4.88	4.91	3.27	3.33	.006	.000
T4	6.40	7.53	2.47	2.38	.000	.000
i	4.87	4.85	2.90	2.90	.006	.003
T5 *i*	6.51	6.51	2.65	2.65	.010	.010

NOTE: [a]*i* denotes informed users.

In addition, for more than 50 percent of informed users, generalized costs decrease for both types of users, as the fraction of informed users increases, leading to conditions which are better than the case N1. However, the difference between noninformed and informed users is not greatly affected by the fraction of informed users.

All these effects occur both for low (0.20) and high (0.40) values of the variation coefficient of the perception error of noninformed users; the value of this parameter does not affect the level of costs of informed users, a reduction of about 20 percent and 30 percent, respectively, occurs in comparison with noninformed users.

To show the sensitivity of the network to travel time function specification, the same scenarios have been simulated

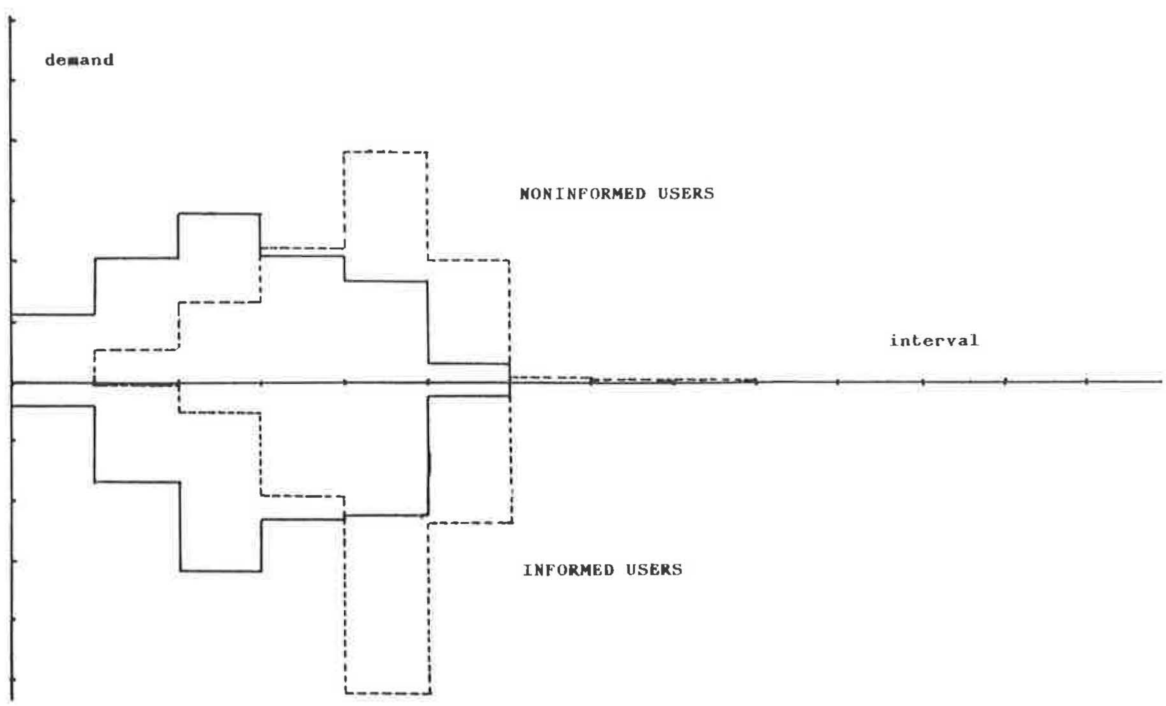

FIGURE 5 Departure and arrival profiles for case T2 (b_a = 0.80).

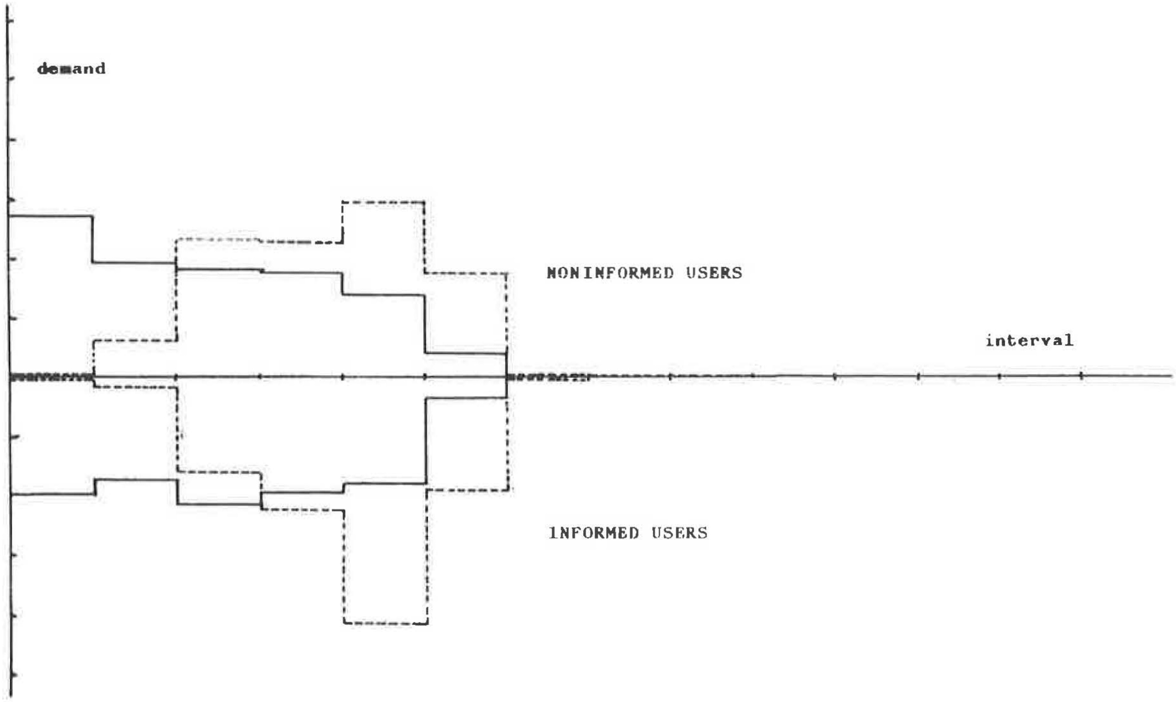

FIGURE 6 Departure and arrival profiles for case T2 (b_a = 0.90).

assuming a value $b_a = 0.90$ instead of 0.80, considering only the lowest value for the variation coefficient for noninformed users. These results are shown in Table 3. The results of the previously examined cases are generally confirmed. However, as a comparison with the results in Table 2, an increase occurs more in generalized cost than in travel time, since the values of travel times for arc flows close to capacity are higher and the users should spread their departure times to avoid congestion, thus less users can be on time. As an example, Figure 6 shows the departure and arrival profiles for case T2 for noninformed users (above) and informed users (below).

The preceding results, although inconclusive, indicate that informative control strategies may lead to better system conditions, in addition, as the fraction of informed users increases the performance of the system as a whole may become better both for informed and noninformed users.

TABLE 3 COMPARISON OF INFORMATIVE STRATEGIES

	Excess General cost (minutes)	Excess Travel time (minutes)	Changing users Fraction
Cv	0.20	0.20	0.20
N1	8.70	3.84	.010
T1	8.70	3.83	.010
i[a]	7.27	4.14	.015
T2	8.66	3.78	.000
i	7.24	4.10	.000
T3	9.07	3.48	.036
i	7.62	3.81	.074
T4	7.68	2.82	.021
i	6.04	3.41	.060
T5 *i*	6.08	3.38	.048

NOTE: [a]*i* denotes informed users

CONCLUSIONS

In this paper some applications of a model recently proposed for the doubly dynamic traffic assignment to a transportation network are described. In particular, the model has been specified and used to simulate the effects of different control measures, ranging from flexible working times to a trip planning informative system on a small but realistic network.

Although the results are by no means conclusive as a result of the exogenous assumptions made especially about users' behavior, they appear to give some insights about both the potential of the model and the effectiveness of alternative control measures.

The results show that the proposed model is a valid tool to simulate the relevant effects of control strategies in different scenarios. It also appears that some control measures cannot be correctly assessed without the explicit simulation of the demand elasticity over departure times and of the day-to-day adjustment process based on users' memory and forecasting.

The effectiveness of an informative system appears to be greatly affected by the type of users' behavior and by the informative strategy followed. Also, the type of control strategy and memory depth play an important role on the network performance.

The results suggest that a careful evaluation is needed to assess the effects on the network performance and the benefits of informative control strategies, and that the impacts on both informed and noninformed users should be taken into account.

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REFERENCES

1. E. Cascetta. Static and Dynamic Models of Stochastic Assignment to Transportation Networks. In *Flow Control of Congested Networks* (G. Szaego, L. Bianco, and A. Odoni, eds.). Springer Verlag, Series F, Vol. 38, 1987.
2. E. Cascetta. A Stochastic Process Approach to the Analysis of Temporal Dynamics in Transportation Networks. *Transportation Research*, Vol. 23B, No. 1, 1989, pp. 1–17.
3. E. Cascetta and G. Cantarella. A Day-To-Day and Within-Day Dynamic Stochastic Assignment Model. *Transportation Research*, forthcoming.
4. J. L. Horowitz. The Stability of Stochastic Equilibrium in a Two-Link Transportation Network. *Transportation Research*, 18B, 1984, pp. 13–28.
5. Y. Sheffi. *Urban Transportation Networks*. Prentice-Hall, Englewood Cliffs, N.J., 1985.
6. R. Hammerslag. A Three-Dimensional Assignment in the Time-Space. UTSG Annual Conference 1988, London, England, 1988.
7. B. N. Janson. *Dynamic Traffic Assignment for Urban Road Networks*. Working Paper. Oak Ridge National Laboratory. Oak Ridge, Tenn, 1989.
8. M. Ben Akiva and A. De Palma. *Dynamic Models of Transportation Networks*. PTRC Transport and Planning Summer Annual Meeting, 1987.
9. P. K. Vythoulkas. (1990). A Dynamic Stochastic Assignment Model for the Analysis of General Networks. *Transportation Research*, Part B, forthcoming.
10. K. Small. The Scheduling of Consumer Activities: Work Trips. *The American Economic Review*, Vol. 72, No. 3, 1982, pp. 467–479.
11. H. S. Mahmassani and G. L. Chang. Experiments With Departure Time Choice Dynamics of Urban Commuters. *Transportation Research*, Vol. 20B, No. 1, 1986, pp. 297–320.
12. H. S. Mahmassani and G. L. Chang. Travel Time Prediction and Departure Time Adjustment Behavior Dynamics in a Congested Traffic System. *Transportation Research*, Vol. 22B, 1988, pp. 217–232.
13. M. Ben Akiva, M. Cyna, and A. De Palma. Dynamic Model of Peak Period Congestion. *Transportation Research*, Vol. 18B, Nos. 4–5, 1984, pp. 339–355.
14. Y. Iida, T. Akiyama, and T. Uchida. *Experimental Analysis of Dynamic Route Choice Behaviour*. Preprints of International Conference on Dynamic Travel Behavior Analysis. Kyoto, Japan, July 1989.
15. G. E. Cantarella, E. Cascetta, and M. Di Gangi. An algorithm for a dynamic traffic assignment model. *Proceedings of the Annual Conference of the Operational Research Society of Italy*, Sorrento, Italy, Oct. 1990.

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