Multiple User Class Assignment Model for Route Guidance

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The application of the concept of multiple user class equilibrium assignment to the modeling of route guidance systems is the concern of this paper. In particular, its role in modeling guided and unguided drivers is discussed, as well as its ability to lead to guidance strategies that are effective even with a large proportion of drivers equipped. A review of previous route guidance model work is given. A multiple user class model of route guidance is then proposed and the properties of such a model are discussed. Finally, a presentation of simulation results obtained from such a model, for two real-life networks and for a number of route guidance scenarios, is given.

One of the basic implicit assumptions in standard assignment methods is that drivers and vehicle attributes are identical; they do not differ from one another in either their travel cost definition or their vehicle size or vehicle performance.

APPLICATION OF MULTIPLE USER CLASS ASSIGNMENT TO ROUTE GUIDANCE

Dafermos (*1*) was probably the first to realize the limitations of this assumption, and to propose as a remedy a multiple user class (MUC) model, which takes differences between drivers and between vehicles into account. These classes may differ in (*a*) vehicle type or size, (*b*) travel cost definition, and (*c*) network restrictions (*2*). Within each class, however, driver and vehicle attributes are identical. Typical classes could be lorries (particularly in conjunction with lorry bans), commuters (minimizing some measure of generalized cost), business travelers (minimizing travel time), and tourists (following road signs).

In a MUC assignment model all classes are to be assigned to the network in interaction with each other, so that in equilibrium for each class "no-one can improve his or her (perceived) travel cost by unilaterally changing route," and in that respect MUC assignment is clearly an extension to the standard, single class assignment model.

The relevance of the MUC concept for route guidance modeling is evident. In a situation with some kind of in-vehicle route guidance at least two user classes can be defined: those who are equipped, and those who are not. In fact three groups could even be distinguished; namely, those who follow complete guidance, those who follow partial guidance (because they either lose their way or their confidence in the advice), and those who do not follow guidance at all. Each of these user classes would have a different cost definition and possibly even different network restrictions (e.g., if the guidance network does not include all existing roads for environmental or computational reasons).

A number of cost definitions for each user class can be distinguished on the basis of one's representation of route choice in reality and the assumed routing criterion in the guidance system. The assumptions made in our model are illustrated in the following table and will be discussed next.

	Case		
	A	*B*	*C*
Guided	UE	SO	SUE
Unguided	SUE	SUE	SUE

The authors believe that drivers currently make perceptual errors in their routing decisions. If it is assumed that these will be removed by the guidance system, a combination of stochastic user equilibrium (SUE) and user equilibrium (UE) assignment, as in case A, will occur. Along the same lines, a combination of guided drivers following a system optimum could be envisaged, so as to improve network conditions explicitly, and unguided ones following a SUE route pattern, as in case B.

Then, if it is expected that the system not provide flawless information (because of communication delays or forecasting errors), the two classes could both follow a SUE (case C), but the variance in randomized link travel times would be lower for the equipped vehicles. In fact, the model presented here allows for all three guidance criteria to be implemented simultaneously, to given proportions of the equipped drivers, although for the purposes of the numerical results described later, the three cases will only be examined individually. For brevity, throughout this paper cases A, B, and C will often be referred to as the UE routing strategy, the SO routing strategy, and the SUE routing strategy, respectively.

The tool of MUC assignment now allows the investigation of the various future scenarios by comparing resulting network costs of different guidance strategies, and by changing the proportion of drivers per user class, the spread in link time errors, and the level of congestion, using real-life networks. In this paper, the development of such a route guidance simulation model, based on MUC assignment, will be described and selected simulation results will be presented.

PREVIOUS ROUTE GUIDANCE MODEL WORK

Earliest reported route guidance related model work was carried out by Kobayashi (*3*) who used a simulation model to

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assess benefits of the Comprehensive Automobile traffic Control System (CACS) route guidance system by comparing shortest routes for guided drivers with routes for nonguided drivers on the basis of road length, number of lanes, percentage of trunk lanes, and number of right and left turns. The network consisted of 99 intersections and 286 directional links; Kobayashi estimated a maximum total possible reduction in travel time in this network of 6 percent, at a level of take-up of 50 to 75 percent, compared with observed travel times in Tokyo. Tsuji et al. (*4*) set up a mathematical model, on the basis of the stochastic nature of travel time, even under guidance. The proportion of guided vehicles was assumed so small, that no influence on nonguided vehicles was expected. Comparing the expected travel time of guided routes with those of alternative routes, they estimated an expected reduction in travel time for guided drivers of approximately 11 percent, which compared well with the observed reduction in the CACS system of some 12 percent. Al-Deek et al. (*5*) compared shortest route travel times with the observed route pattern using TRANSYT-7F in the Los Angeles SMART corridor, and found that for recurring congestion the travel time savings of route guidance would be negligible (less than 3 min per trip of on average 25-min length). A similar approach was adopted by Rakha et al. (*6*) who compared routes based on free flow costs (for unguided drivers) and those based on minimum costs (for guided drivers). These assumptions are clearly not valid in congested situations, which probably accounts for the possible total network travel time savings they recorded of up to 21 percent. An interesting finding of their simulations was, however, that a large proportion (85 percent) of total possible savings was achieved with the first 20 percent of equipped vehicles.

Breheret et al. (*7*) used the heuristic dynamic assignment model CONTRAM (*8*). They assumed unguided drivers to follow an approximate stochastic user equilibrium on the basis of prevailing conditions, whereas guided drivers followed optimum routes on the basis of current conditions. If multiple routes were calculated in an attempt to find a user equilibrium reassignment, guided drivers obtained travel time benefits of up to 15 percent; most of these benefits were obtained before a level of take up of guidance of 10 percent of the driver population. In this model, the reassignment of guided drivers benefited nonequipped drivers also with travel time reductions of up to approximately 4 percent. Finally, under these assumptions most of the total network travel time savings (of up to some 5 percent of total travel time) was achieved at a level of take-up of approximately 20 percent [see for e.g., Rakha et al. (*6*)]. If, however, reassignment for guided drivers was calculated by a single, shortest route, total network, travel times invariably increased, which indicated possible problems with systems that advise single routes. Even when unguided drivers were allowed to reassign because of the changed conditions, resulting network travel time savings of such a system were negligible or negative. Smith and Russam (*9*) also reported on a CONTRAM-based model study, in their case of the possible benefits of AUTOGUIDE in London. Whereas unguided vehicles were assumed to base their routes on the average demand pattern and subsequent link costs, guided vehicles were routed along actual optimum routes (for a randomly perturbed trip matrix). They found an estimated average journey time saving of 6 to 7 percent for guided vehicles, which actually decreased with an increase in take-up. Unequipped vehicles benefited also by the guidance system, with travel time reductions of up to 3 percent, resulting in overall network travel time savings of 2.5 to 6.0 percent.

Van Vuren et al. (*10*) employed a MUC assignment model to investigate the situation in which unguided drivers follow a user equilibrium while guided drivers are assigned according to a system optimum. These researchers were mainly concerned with conditions on the cost function that guaranteed existence and uniqueness of a solution. Their main finding was that for a combination of system optimal and user equilibrium drivers and for the family of cost functions considered, only a polynomial cost function gave rise to a convex minimization problem (hence guaranteeing existence and uniqueness properties). Further discussion of these findings appears subsequently. Numerical results given by Van Vuren et al. were limited. Koutsopoulos and Lotan (*11*) assumed that route guidance would reduce the perception errors in link travel time estimates by participating drivers, so that their model consists of a stochastic user equilibrium assignment of two user classes with different variances in the (normal) distribution of random perturbations in perceived link costs; these cost functions did not satisfy conditions for existence and uniqueness, as derived by Daganzo (*12*). Scenarios they investigated on a 204-node network were level of information (influencing the perception errors by guided drivers), percentage of take-up, and the level of recurring congestion. Clearly, an increase in the quality of information resulted in a reduction in perception errors by guided drivers, and therefore in a reduction in their travel times. The advantage of guided drivers over unguided drivers in average travel time was roughly 4 percent, independent of the level of take-up, and an increase in congestion actually reduced the benefits of route guidance. Although these results are obviously rather limited, the most important finding by Koutsopoulos and Lotan is that in their model unguided drivers did not benefit at all from the improved route choice by the guided vehicles. This finding conflicts with the generally held belief [see for e.g., Jeffery (*13*) and Smith and Russam (*9*)] that route guidance benefits nonusers too.

For a corridor consisting of three parallel highways plus connecting links, Mahmassani and Jayakrishnan (*14*) built a model based on route-switching assumptions for drivers that receive dynamic network information. The main conclusions for this simple network were as follows. For optimum resulting travel times both for the equipped drivers and for the system as a whole, route switching should only take place if the alternative route for the trip remainder is at least 20 percent shorter than the existing route, indicating possible instability problems for systems that advise optimum routes (like ALI-SCOUT and AUTOGUIDE), as compared with systems that provide in-vehicle information (such as PATHFINDER). Second, benefits for individual drivers decreased with an increase in participation, whereas benefits for the system as a whole (generally) increased with such an increase; above 50 percent participation the increase in benefits was negligible.

The results of these various model studies are clearly rather ambiguous. Hypotheses about the route choice and interaction of guided and unguided drivers strongly influence the model outcomes. Often the models used in these studies are heuristic or they are only valid under rather strong assump-

tions. This is not to belittle the importance of these model studies: it merely shows the current problems in understanding and anticipating the behavior of future route guidance systems.

DEVELOPMENT OF THE MODEL

The MUC model of a route guidance system will now be introduced, building on the concepts described in the first section of this paper. The demand for travel (as represented by the mean origin-destination matrix) is assumed to be fixed, as are the network supply conditions. It is also assumed that the whole network is available to the guidance system. Average cost-flow relationships are supplied for each link. In all cases, "cost" is measured in terms of travel time and so the words cost and time will be used interchangeably.

The model consists of four user classes, the demand level for each being a fixed (known) proportion of the origin-destination matrix. Three of the user classes correspond to vehicles equipped with a guidance device, and for two of these three equipped classes it is assumed further that the guidance system is provided with perfect information and that the guided drivers adhere totally to the route recommendations. The first class consists of unguided drivers, each of which aims to minimize his or her own personal cost of travel, but in general fails to do so because of imperfect knowledge of the traffic conditions. This class is modeled by a SUE, the "perceived cost" for each link following some specified distribution (discussed later). The second class is a subset of the equipped vehicles wherein each driver is guided so as to minimize his own personal travel cost. The perfect information assumed to be available to the guidance system is used to eliminate the perception errors, that is, they follow a Wardrop user equilibrium. The third class consists of a second subset of the equipped drivers, which are guided so as to minimize the total system cost [system optimal (SO)], again using the perfect information available. The fourth class consists of the remaining equipped drivers. The aim of the guidance system for this class is again to recommend routes according to a UE pattern; however, to represent the effect of errors in the journey time prediction methods or of drivers not adhering completely to the recommendations, they are modeled by a SUE, but with a distribution for the stochastic variations, which is different from that of the unguided drivers.

The four user classes interact with one another, in the sense that the flows of one user class affect the costs, and hence the route choice, of the other user classes. In this way, the assumption is that under such steady-state conditions, the unguided drivers will tend to change their routes in response to the new route choice of the guided drivers.

Van Vuren (*10*) concluded that for a guidance system with user equilibrium unguided drivers and system optimal guided drivers, the only link cost functions c_a of the family which was established by Van Vliet et al. (*2*) to ensure existence and uniqueness of a multiple user class equilibrium were of the polynomial form:

$$c_a = d_a + b_a F_a^k \tag{1}$$

where

F_a = the total flow on link a,
d_a = a constant representing fixed effects such as free flow travel time,
b_a = a constant, and
k (>0) = a link independent constant.

In the more general four user class model considered here, the same result of Van Vliet et al. (*2*) cannot be used because the properties were established only for the deterministic cost case, whereas here a mixture of stochastic and deterministic costs are used. Results established by Daganzo (*12*) may be used, however, for a similar (though more general) family of cost functions to those of Van Vliet et al., but for the case in which some of the classes may have stochastic costs. Then, in a similar way to Van Vuren et al., by applying the work of Daganzo, it follows that the equilibrium for our more general four user class model is guaranteed to exist and be unique (with respect to link flows and user class/link costs) for cost functions of the form in Equation 1. In this case, c_a is the actual link travel time for all drivers. In the assignment, however, each class will be associated with a different cost: the unguided drivers making random perception errors with Equation 1 as the mean; the guided SUE drivers experiencing different random errors because of imperfect recommendations, and so on; the guided SO drivers using marginal costs corresponding to the actual costs (Equation 1); and the guided UE drivers using the actual costs (Equation 1). That Daganzo's results may be applied to guarantee the above conditions on the equilibrium may be verified as follows.

It is well known that a system optimal assignment in the one user class case with link costs c_a may be obtained by a user equilibrium assignment with marginal link costs c'_a given by

$$c'_a = c_a + F_a \frac{dc_a}{dF_a}$$

To obtain, then, the required routing pattern with actual link costs (Equation 1), the user class costs c_{ai} for link a and user class i must be

$$c_{a1} = c_{a2} = c_{a4} = d_a + [b_a F_a^k]$$

$$c_{a3} = d_a + (k + 1)\,[b_a F_a^k]$$

where user class 1 consists of the unguided drivers, and the remaining classes are the guided drivers, following (respectively) UE, SO, and SUE routing; perceived costs are therefore stochastic for user classes 1 and 4, and deterministic for user classes 2 and 3. It may be seen that the user class cost functions above are indeed of the form required to apply Daganzo's work. Furthermore, Daganzo's conditions require that the variance of the perceived journey time distribution is flow independent. This condition has been noted variously by authors investigating the single user class stochastic user equilibrium case: Sheffi and Powell (*15*), Daganzo (*16*) and Sheffi (*17*). In the last reference, Sheffi suggests—for a probit-based route choice model—the use of a standard deviation of link a perceived cost of θc_{oa}, where c_{oa} is the free-flow

travel time and $\theta(>0)$ is a constant. In the guidance model proposed here, normally distributed perception errors for the unguided drivers are used, but with a standard deviation of θc_a^{UE}, where c_a^{UE} is the travel cost for link a corresponding to a (deterministic) user equilibrium flow pattern for all drivers. This is preferred because it is more closely related to the idea that larger perception errors are made with larger travel times and greater congestion, rather than using the free-flow travel time which may be more related to the physical characteristics of the link (for example, an uncongested freeway would have a relatively large free-flow travel time and would thus counter-intuitively tend to result in large perception errors). The guided SUE drivers are modeled in the same way, except that their link travel time standard deviation is Ψc_a^{UE}, where $0 < \Psi < \theta$ (i.e., guidance tends to reduce the size of the errors made by equipped drivers). The errors are distributed independently among user classes and among links. It is noted that this model is somewhat unrealistic in one respect, because the journey time prediction methods will tend to be more precise with larger levels of take-up—data on actual travel times relayed to the guidance system from the beacons will relate only to equipped vehicles, and so an increase in level of take-up will essentially lead to an increase in sample size. It would be expected, then, that the variance of the random errors would be a decreasing function of the level of take-up. Because no suitable relationship of this kind was available, however, it was necessary to retain the assumption of a constant error variance relative to the proportion of vehicles equipped.

Finally, Daganzo proposes a solution algorithm for the multiple user class equilibrium problem with stochastic costs, which is essentially an extension of the method of successive averages, as introduced by Sheffi and Powell (*15*). Convergence of this algorithm to the equilibrium solution is guaranteed for the cost functions considered here. Daganzo's scheme was implemented as follows:

1. Set $f_{ai}^{(0)} = 0$, a, i, where $f_{ai}^{(r)}$ refers to the estimate of the equilibrium user class flows at iteration r. Set $r = 0$.
2. Calculate $\underline{F}^{(r)}$ from

$$\underline{F}^{(r)} = \sum_j f_{aj}^{(r)}$$

and hence the costs $c_{ai}^{(r)}$ corresponding to $\underline{F}^{(r)}$ ($\forall\ a, i$).

3. For each user class i
   (a) Sample a set of link error terms θ_{ai} ($\forall\ a$) from the specified probability distribution, by a pseudo-randomization process, and set

$$C_{ai}^{(r)} = c_{ai}^{(r)} + \theta_{ai} \forall\ a$$

   (b) Perform an all-or-nothing assignment for this user class using the randomized costs $C_{ai}^{(r)}$—yielding a set of user class link flows $g_{ai}^{(r)}$ ($\forall\ a$)
   (c) Set

$$f_{ai}^{(r+1)} = (1 - 1/r) f_{ai}^{(r)} + 1/r g_{ai}^{(r)}\ (\forall\ a)$$

   (d) Set

$$F_a^{(r)} = F_a^{(r)} - f_{ai}^{(r)} + f_{ai}^{(r+1)}\ (\forall\ a)$$

and recompute the costs corresponding to $\underline{F}^{(r)}$—store again in $c_{ai}^{(r)}$ ($\forall\ a$)

(4) Set $r = r + 1$ and return to Step 2 until the predetermined number of iterations are complete.

The main advantage of the guidance strategy described in the preceding may be seen to be that it anticipates the effect of rerouted traffic on travel times in the network, and in this way, directs traffic to one or more routes. It would be hoped that such an approach would be effective even with a high proportion of equipped vehicles (and this will be investigated for the test networks studied here). The single route strategies currently being considered for field trials [Von Tomkewitsch (*18*) and Belcher and Catling (*19*)] are likely, on the other hand, only to be of use when the proportion of equipped vehicles is small, so that rerouted traffic has only a relatively small effect on delays [see the earlier comments on the findings of Breheret et al. (*7*)]. Increasing the frequency at which such single route strategies are updated as the level of equipped vehicles increases is clearly one possibility, but there are limitations on this frequency imposed, for example, by the time it takes to communicate information from the beacons to the guidance system. In the future, then, as the popularity of route guidance grows, multiple route strategies are likely to be an essential component of the guidance system.

A second notable feature of the model is that it supposes unguided drivers will respond to the new routes taken by the equipped vehicles, and will aim to choose new routes which minimize their personal (perceived) travel cost. Given that an equilibrium-based assignment is accepted here as a reasonable approximation to the long-term average route choice under steady state conditions, then the assumption seems natural that unguided drivers will—in the long run—still seek minimum cost routes when guided drivers are in the network. It is recognized, however, that this is a strong behavioral assumption, neglecting any loyalty drivers might have had to their chosen routes before guided drivers were introduced [compare with Mahmassani and Jayakrishnan (*14*)], and would in any case be much more difficult to justify in situations where a frequently updated dynamic route guidance system was in operation.

TEST RESULTS

The guidance strategy was implemented using an adaptation of the simulation model SATURN (*20*) and the solution algorithm of Daganzo, as described previously. The two real-life networks considered were those of Weetwood (an area of Leeds) consisting of 70 zones, 104 intersections, and 440 links; and of Barcelona consisting of 110 zones, 820 intersections, and 2,547 links. The cost functions used were of the form of Equation 1, where k was given the value 5 for both networks. (The two networks had been calibrated originally using different powers in the cost functions on different links—the value 5 was chosen as it was approximately the average of all these powers, in both cases).

For each network, the guidance model was implemented under the following:

1. Three different demand levels: 100 percent, 130 percent, and 160 percent of the observed origin-destination flows, cor-

responding to an average network speed (before guidance) of approximately 15, 25, and 35 km/h, respectively;

2. Nine different levels of equipped vehicles: 0 percent, 5 percent, 10 percent, 20 percent, 30 percent, 50 percent, 70 percent, 90 percent, and 100 percent; and

3. Three different routing criteria: with equipped drivers either all guided as a UE, all guided as a SO, or all guided as a SUE (with two different levels of error in this latter case).

Finally, to decide upon a suitable value for a parameter θ, which determines the link travel time variances for the unguided drivers, an idea from Breheret et al. (*7*) is used. For a number of values of θ, the average inefficiency $I(\theta)$ is calculated by

$$I(\theta) = 100\,(p(\theta) - 1)\ \%$$

where

$$p(\theta) = \frac{\text{Total system travel time under SUE}(\theta)}{\text{Total system travel time under UE}}$$

and where SUE(θ) means an SUE assignment for the whole origin-destination matrix, with parameter θ. That is, assuming that drivers are aiming to follow a UE, $I(\theta)$ is a measure of the average excess travel time incurred by their perception errors.

For the purposes of this paper, for a given network, a value for θ is then chosen which gives rise to an inefficiency of the order of 5 to 6 percent for each of the demand levels considered. The reasoning behind this is that various studies have shown that the percentage wastage caused by drivers not fulfilling their objective of choosing the minimum time or minimum distance route is of this order—for example, Lunn (*21*) estimated the average excess on all journeys in Great Britain, excluding commuter trips, to be at least 5 percent of total costs; Wootton et al. (*22*) arrived at figures of 4 to 6.5 percent inefficiency; and Jeffrey (*13*), from analyzing times and distances corresponding to a sample of journeys made in the United Kingdom, concluded that the average inefficiency of drivers was around 6 percent. The use of inefficiency to build a suitable route choice model for unguided drivers is appealing, in that it allows the calibration of the model against observed data (though in a very coarse manner).

It should be emphasized here that although the use of inefficiency in this way appears to be a sensible one, it has the disadvantage that in specifying a value for θ, the system benefit of UE guidance at 100 percent take-up is directly determined, and the benefits of other scenarios are clearly greatly affected by the value chosen. It means therefore that the model should not be used to infer absolute measures of the effects of route guidance; its purpose is to compare the effects of different levels of take-up, demand levels, routing strategies, and so on.

The values of I(θ) for a number of values of θ are given in Figures 1 and 2. For Weetwood, there is a clear pattern of an increased I(θ) with increased θ, or greater demand. The value $\theta = 0.3$ is chosen for the purposes of further investigation, giving an average inefficiency of 6 percent over the three demand levels. For the Barcelona network, the pattern is somewhat different, with much less difference between demand levels and with the possibility of I(θ) decreasing with greater demand (demand level 1 showing greatest inefficiencies). It is still the case, however, that I(θ) is an increasing function of θ. The value of $\theta = 0.4$ is chosen for future study.

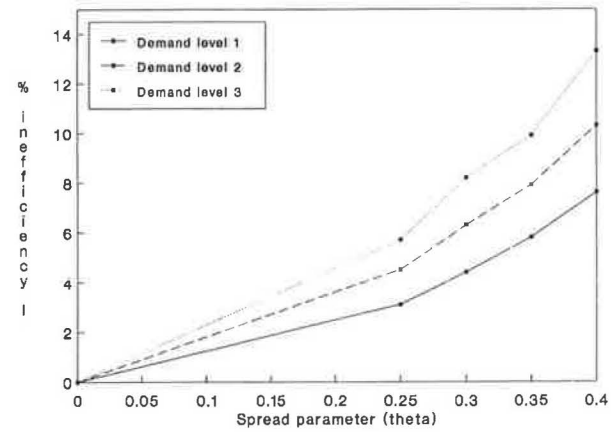

FIGURE 1 Inefficiency for Weetwood.

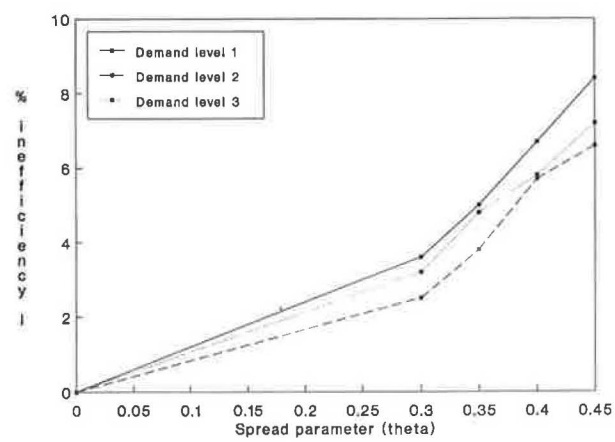

FIGURE 2 Inefficiency for Barcelona.

There are two studies with which some comparison may be drawn on this point of modeling unguided drivers. Breheret et al. (*7*), in using a uniform error structure for perceived costs (with flow dependent range), found the relationship between inefficiency and spread parameter to be highly network dependent and demand dependent—because of this, and because they give no indication as to the size of the networks or the absolute levels of congestion, it is difficult to draw any further parallels with this work. Koutsopoulos and Lotan (*11*), on the other hand, used a very similar "before guidance" model to that considered here, the most notable disparity being their use of a flow dependent perception error variance of $\phi c_a(F_a)$. For their study on a network of a similar size to Weetwood, they used a value of $\phi = 0.5$, which gave rise to an inefficiency of around 4 percent (relative to the UE, $\phi = 0$, case) for the three demand levels considered.

It is evident that quite large values of the spread parameters are required to give realistic inefficiencies. In one respect this is unappealing, since—as it makes sense to truncate the perceived travel time distributions at zero—the randomization may be biased (although for the values considered here the bias will tend to be very small). A factor of greater concern

is that in using the approach proposed in this paper to model unguided drivers, some of the deficiencies of the model in reproducing a realistic route choice criterion are contained in θ. There is part of the inefficiency, therefore, which cannot be recovered by route guidance. In routing all guided drivers as a SUE, a third component (in addition to errors in the information used by the guidance system and the nonadherence of equipped drivers to the recommended routes), which contributes to the error terms, may be regarded as lack-of-fit of the model to the observed value of 6 percent inefficiency—the observed value was calculated by comparing minimum time routes with what drivers actually did, and so also accounts for drivers who were not intending to follow minimum time routes.

The model was applied to the networks described, with unguided drivers modeled by a SUE with $\theta = 0.3$ for Weetwood and $\theta = 0.4$ for Barcelona. From initial studies of the Weetwood network, it appeared that to obtain a reasonable degree of convergence, a large number of iterations would be required—the stopping criterion chosen was the completion of 200 iterations. Although this may have been the case too for Barcelona, the size of the network meant that such a large number of iterations would be computationally prohibitive, and so only 30 iterations were carried out in this case. The results are given in Figures 3 to 14. In the figures on

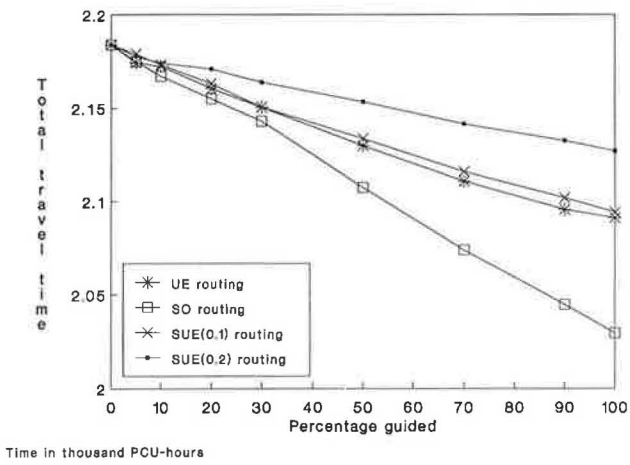

FIGURE 3 System benefit for Weetwood demand level 1.

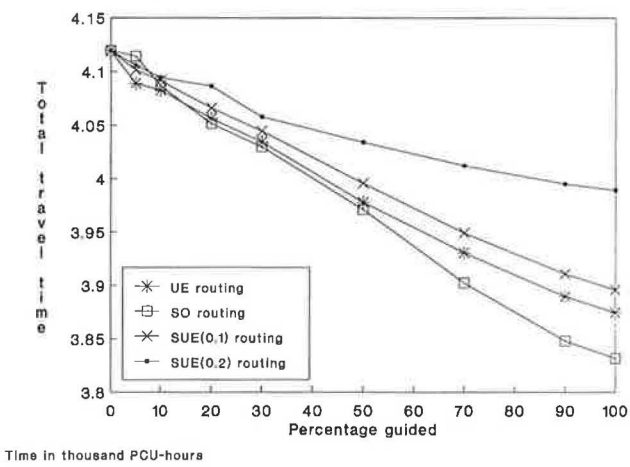

FIGURE 4 System benefit for Weetwood demand level 2.

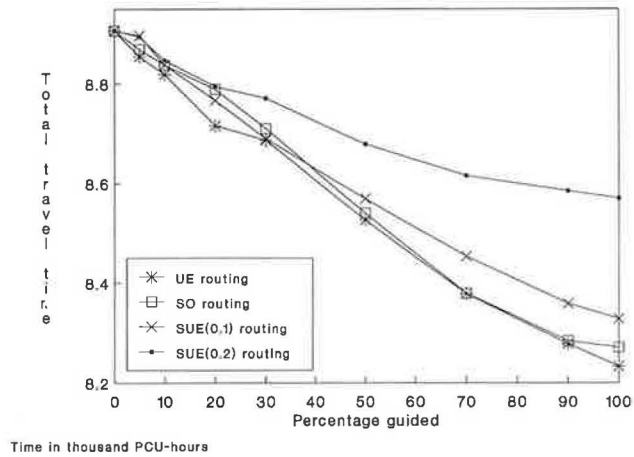

FIGURE 5 System benefit for Weetwood demand level 3.

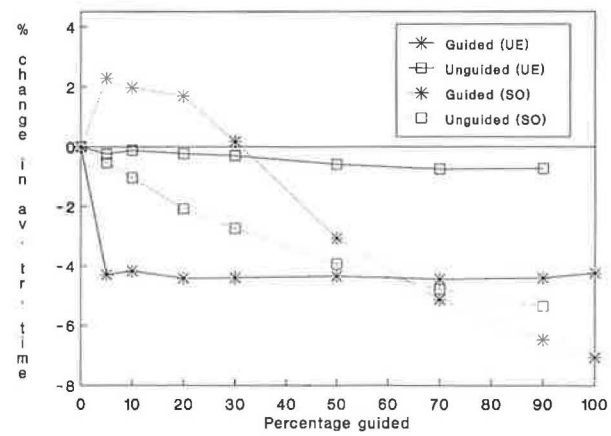

FIGURE 6 Individual benefits for Weetwood demand level 1.

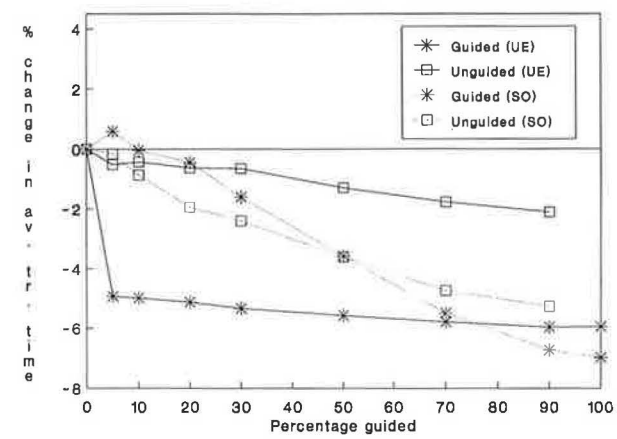

FIGURE 7 Individual benefits for Weetwood demand level 2.

system benefit (expressed in terms of total travel time, Figures 3 to 5 and 9 to 11), each of the four strategies is represented—for example, the trend labeled SUE(0.2) refers to the strategy in which all equipped vehicles are guided according to a SUE with $\Psi = 0.2$. In the figures on individual benefits (Figures 6 to 8 and 12 to 14), results relating to the UE (broken line)

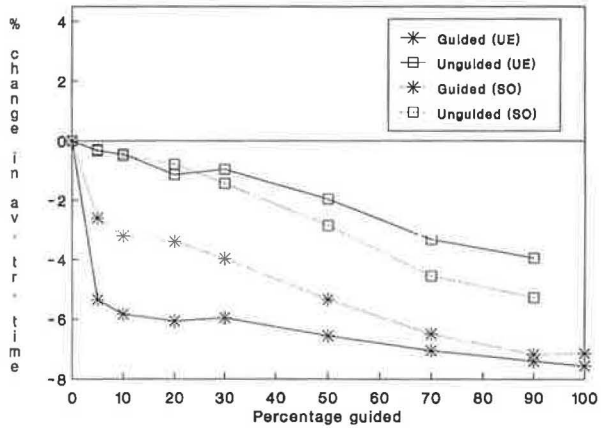

FIGURE 8 Individual benefits for Weetwood demand level 3.

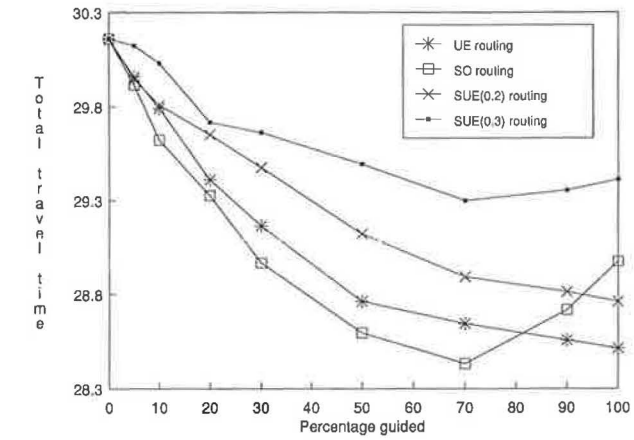

FIGURE 11 System benefit for Barcelona demand level 3.

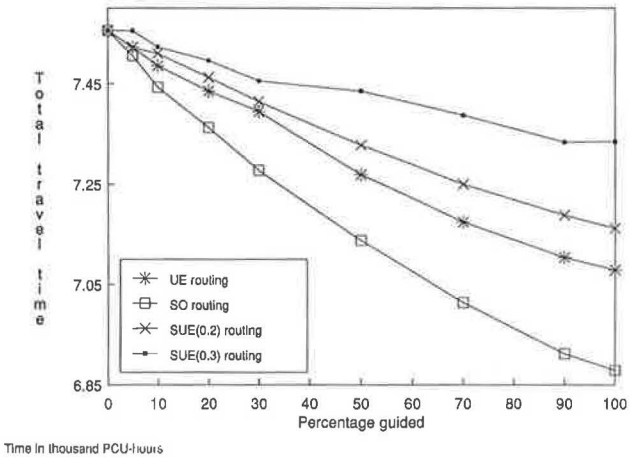

FIGURE 9 System benefit for Barcelona demand level 1.

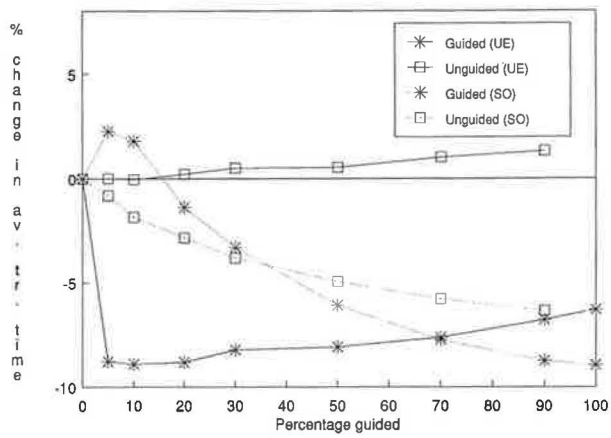

FIGURE 12 Individual benefits for Barcelona demand level 1.

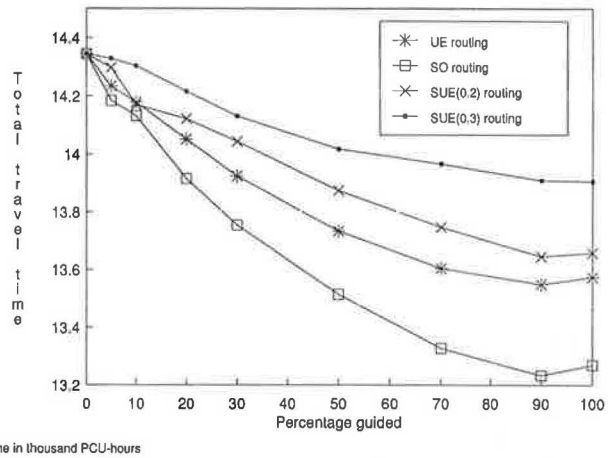

FIGURE 10 System benefit for Barcelona demand level 2.

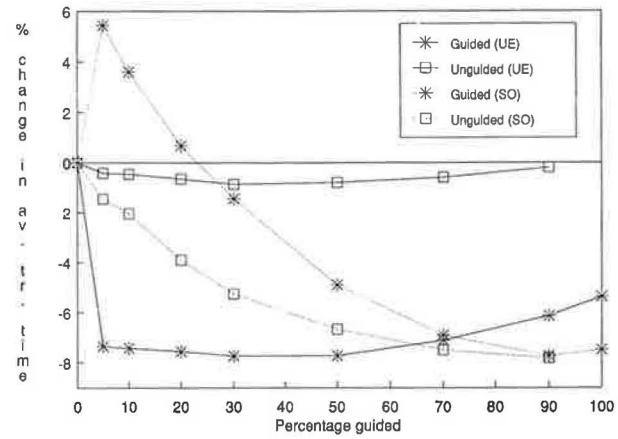

FIGURE 13 Individual benefits for Barcelona demand level 2.

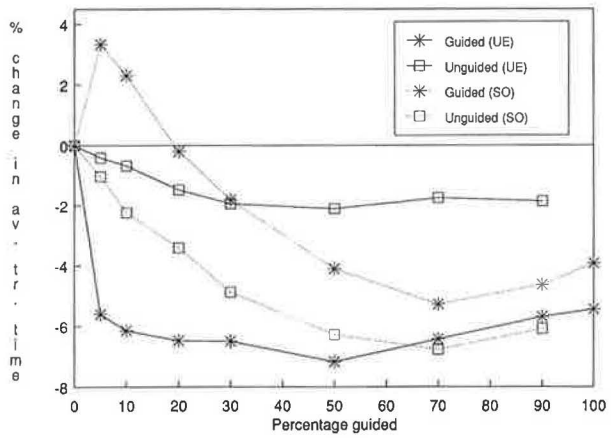

FIGURE 14 Individual benefits for Barcelona demand level 3.

and SO (continuous line) routing strategies only are given, with each separated into guided (star symbol) and unguided (square symbol) vehicles. So that, for example, in Figure 6 the label "Guided (UE)" refers to guided drivers under the strategy in which all equipped drivers are guided as a UE—that is, case A in Table 1—and "Unguided (UE)" refers to unguided drivers under the same strategy.

The first point to note is that in a small number of situations, there is evidence of strange behaviour—first, in Figure 5, for Weetwood at demand level 3 with 100 percent of vehicles equipped, the total travel time arising from SO routing is greater than that arising from UE routing, which clearly should not be the case. For Barcelona at higher demand levels (Figures 10 and 11) a similar problem is evident—for example, in Figure 11, an assignment with 70 percent guided according to a SO routing gives a smaller total travel time than a 100 percent SO routing. This point will be addressed further in the next section.

For the system benefit (as measured by total travel time), it can be seen that for all of the routing strategies considered and for both networks, guidance offers an improvement over the base (no guidance) situation, for all levels of equipped vehicles and all demand levels considered. In all cases, the travel time saving becomes greater as the level of take-up increases (with one or two exceptions—see preceding comments on convergence), following an almost linear trend in the Weetwood case. Below a 50 percent take-up, the percentage saving in total travel time tends to be higher with higher demand levels, although in all cases the differences between demand levels are not great. Concentrating specifically on the UE and SO routing strategies, Figures 3 to 5 show that for Weetwood, the percentage saving in total travel time due to guidance increases with demand for most levels of take-up—for example, between 10 and 90 percent take-up, UE routing gives savings of 0.5–4.0 percent, 0.9–5.6 percent, and 1.0–7.1 percent, respectively, for demand levels 1, 2, and 3; SO routing, on the other hand, gives respective savings of 0.7–6.4 percent, 0.8–6.6 percent, and 0.8–7.0 percent. The differences between demand levels are, however, not great, and the same is true for the Barcelona network. In this latter case, though, there is a slight decrease in the benefits attainable at higher levels of take-up as demand increases (Figures 9 to 11)—the savings for 10–90 percent take-up are 0.9–6.0 percent, 1.2–5.5 percent, and 1.2–5.3 percent under UE routing for demand levels 1, 2 and 3 respectively, and under SO routing the respective savings are 1.5–8.5 percent, 1.5–7.7 percent, and 1.8–5.8 percent. On the whole, for both networks, the pattern is as one may expect, with an increase in total travel time savings as decreases for SUE routing (down to $\Psi = 0$ for UE routing), all of these giving rise to larger total travel times than SO routing.

For the benefit to individuals (the percentage decrease in average travel time with the guidance system in operation), it may be seen that for a UE routing, equipped drivers are always better off with guidance. A striking feature is that this benefit is approximately constant for all levels of take-up—notably, the benefit is achieved at a very low percentage of equipped vehicles (in fact at 1 percent take-up, not shown in the graphs), and does not decrease notably at higher participation levels. From Figures 6 to 8, it may be seen that under UE routing, guided drivers save around 4 percent, 5 percent, and 6 percent respectively for Weetwood demand levels 1, 2 and 3, whereas (Figures 12 to 14) the savings are around 8 percent, 7 percent, and 6 percent for Barcelona demand levels 1, 2 and 3. Under such a routing scenario, the change in travel time for unequipped drivers is always small relative to the benefit to guided drivers (always less than 3 percent, and usually less than 1 percent, with an actual disbenefit for the Barcelona network at demand level 1), although there appears to be slightly greater benefit to them as congestion increases from demand level 1 to demand level 3. In most of the situations, the benefit to individual unguided drivers also tends to increase with level of take-up.

With SO routing, there is a disbenefit to individual guided drivers on average, for lower levels of take-up (low being levels of equipped vehicles less than of the order of 10 to 30 percent); for higher levels of take-up, on the other hand, guided drivers experience a saving in travel time which increases with level of take-up. Above 50 percent take-up the savings for equipped drivers under SO routing are 3 to 7 percent for Weetwood and 5 to 9 percent for Barcelona, depending on level of take-up and demand level. The journey time saving for unguided drivers tends to be somewhat larger here than with UE routing, particularly at lower levels of take-up, with guided and unguided drivers benefiting similar amounts at higher levels of take-up.

Comparing the UE and SO routing strategies for both networks, it may be seen that UE guidance will always benefit the equipped drivers most, with only limited benefits to the unequipped drivers, but giving rise to considerable system benefits. Not surprisingly, SO routing primarily benefits the unguided drivers—at the expense of the guided drivers—at lower levels of equipped vehicles. However, equipped drivers start benefiting too when their numbers increase. At the highest levels of equipped vehicles (more than 50 to 70 percent), under such a routing strategy the guided drivers may even benefit more than the unguided drivers, despite being guided to minimum marginal cost routes. The system benefits of SO routing are higher than with UE routing, but may not warrant the disbenefits to equipped drivers at lower participation levels.

In this paper, there has only been space to discuss a relatively small number of features of the guidance model; in

various other papers (*23–25*), a description of many other aspects is given, such as the effect on distance traveled; the performance in situations of unforeseen variability; the influence of the reactions of unguided drivers; properties of the recommended routes; and guided drivers' acceptance of SO advice.

CONVERGENCE

The simulations reported in this paper have used the completion of a specified number of iterations of the solution algorithm as a stopping criterion. In this section, the aim is to briefly investigate the influence of this stopping criterion, particularly with respect to the problem results identified earlier. As examples, Barcelona demand level 3 (100 percent take-up) and Weetwood demand level 2 (5 percent take-up) will be considered under UE and SO routing.

Although convergence indicators do not appear to have been proposed for use with Daganzo's algorithm, some work has been done in this area with the standard method of successive averages in the one user class SUE case. It is noted that a choice of indicator on the basis of similarity of link flows between iterations is not altogether straightforward, because the search direction is a random variable and the step size ($1/r$) is fixed (unlike, for example, the Frank-Wolfe algorithm for the deterministic UE problem)—a good discussion of this point is given by Sheffi (*17*). Sheffi and Powell (*15*) have, however, proposed an indicator I_r for this problem:

$$I_r = \sum_a s_a^{(r)} / \sum_a F_a^{(r)}$$

where for each link a, $F_a^{(r)}$ and $s_a^{(r)}$ are, respectively, the sample mean and sample standard deviation of the flow estimates for that link on the last m iterations, for some user-specified m. The indicator was used as a stopping criterion ($I_r < 0.0005$, with $m = 5$) for Barcelona demand level 3 with 100 percent take-up, for each of UE and SO routing; the results at convergence were:

	No. of iterations	*Total travel time*
UE	25	28,836.0
SO	33	28,692.9

The SO strategy now gives rise to a smaller total travel time than the UE one, needing a larger number of iterations to reach the same degree of convergence. Extending the definition of I_n to the multiple user class case was then achieved by applying it to the *total* link flows; this makes more sense than studying the user class flows separately, since our results only guarantee uniqueness of the total link flows. For the Weetwood network with 5 percent guided ($m = 5$, $I_n < 0.00005$), the results obtained were:

	No. of iterations	*Total travel time*
UE	217	4,088.5
SO	203	4,113.4

and the progress of the algorithm is illustrated in Figure 15 (with respect to the indicator I_n and Figure 16 (with respect to total travel time). Although total travel times again appear to suggest a slower convergence for the SO strategy than the

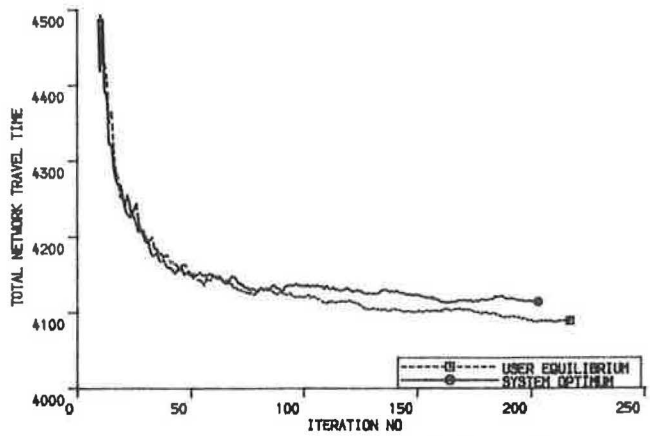

FIGURE 15 Convergence with respect to indicator, I_r (Weetwood, 5 percent guided, demand level 2).

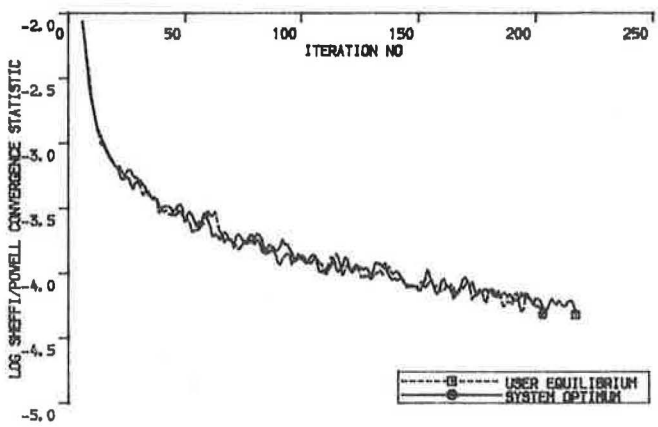

FIGURE 16 Convergence with respect to total travel time (Weetwood, 5 percent guided, demand level 2).

UE strategy, this is not apparent in the indicator I_n, and in fact the algorithm under SO routing is terminated *before* that under UE routing. The same problem encountered with the fixed number of iterations case is evident here, with SO routing giving rise to a higher total travel time than UE guidance. Because of this inconsistency in the performance of the indicator (other scenarios tested reinforced this inconsistency), it was decided to retain the stopping criterion of a fixed number of iterations, although this is clearly an area in which more work is needed.

CONCLUSION

A model of a route guidance system has been proposed in terms of a multiple user class equilibrium assignment, with vehicles divided into equipped and unequipped classes, the former being subdivided further dependent on the routing criterion used and the quality of the information supplied. Guidance is used to route vehicles either to a user equilibrium or to a system optimum flow pattern, assuming that without guidance drivers aim to follow a user equilibrium but fail to do so because of perception errors in their evaluation of travel times. Furthermore, unequipped drivers are assumed to re-

spond to the new route choice of guided drivers, and seek a new user equilibrium routing.

For cost functions of a particular polynomial form, it is shown that such routing strategies, in combination with the route choice of unequipped drivers, are guaranteed to lead to a unique and stable equilibrium flow pattern with respect to total link flows.

The main advantage of such an equilibrium-based strategy is that it spreads the traffic between multiple routes on each origin-destination movement, and so would be expected to lead to effective guidance even when a high proportion of drivers are equipped with a guidance device. The test runs on two real-life networks were used to investigate such a property, as well as the performance of the strategies in several different scenarios. It was seen that both user equilibrium and system optimal guidance generally reduced the total system travel time, the benefit being an increasing function of the level of take-up. The level of congestion appeared to have less effect on the benefits than the percentage guided, although there was some indication of slightly greater percentage savings in more congested situations below 50 percent take-up.

Under UE routing, individual guided drivers experienced a significant reduction in average travel time, this being approximately constant (on the order of 5 percent, varying with demand level) for all levels of take-up. Unguided drivers also tended to benefit from such guidance, but always to a much lesser degree than guided drivers (usually less than 1 percent reduction in average travel time); their savings tended to increase as the percentage of equipped drivers or the level of congestion increased.

SO routing was found to lead to a slightly greater reduction in total travel time than UE routing, particularly in the least congested scenarios. The effects on individual groups of drivers are, however, quite different in the two cases. SO routing was seen to primarily benefit unequipped drivers, significantly improving their position in comparison with UE routing. At lower levels of take-up (10 percent or less) this saving tends to be at the expense of equipped drivers, who may experience an increase (of as much as 5 percent in the extreme) in average travel time due to guidance. For higher levels of take-up, equipped drivers will benefit too from SO routing, with the saving in average travel time growing in similarity (lending to the order of 5 percent) as the level of take-up increases.

In terms of future research, if the model proposed here is to be used further there is a need to determine a suitable convergence indicator for the solution algorithm—possibly on the basis of development of an objective function for the problem, or on a "gap" function monitoring route-based properties—to overcome the problem of apparently different convergence rates of UE and SO routing.

There is also a need for a better modeling both of unguided drivers—in particular, their response to the implementation of route guidance—and of guided drivers, with quite different reactions likely to occur when system optimal as opposed to user equilibrium routing is recommended. Furthermore, relationships are needed between the level of take-up of guidance and the level of errors in the journey time prediction algorithms, the latter tending to decrease as the sample from which the prediction is made (i.e., the guided drivers) increases in size.

Taking a wider view, the limitations of the model used here have to be recognized; the strategies have been assessed in a steady-state framework, under long-term average network conditions and demand. Under a rolling program of research funded by the Science and Engineering Research Council of Great Britain, *Fundamental Aspects of Full-Scale Dynamic Route Guidance*, one strand of the work being carried out at Leeds is a fundamental assessment of network models, with an emphasis on the representation of dynamics, variability, and driver behavior. The specification, and subsequent design, of such a model is seen as an important stage in the future evaluation of route guidance systems.

ACKNOWLEDGMENTS

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