Dynamic Network Traffic Assignment and Route Guidance Via Feedback Regulation

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A deterministic, macroscopic modeling framework for dynamic traffic phenomena on networks consisting of freeways and urban streets is presented for nonelastic but time-varying traffic demands. A feedback methodology is applied to the network model to establish dynamic traffic assignment conditions. Specifically, a multivariable feedback regulator with integral parts and a simple bang-bang controller are developed and tested for a particular network traffic model. Because of three fundamental features (low computational effort, low sensitivity with respect to unknown demands and compliance rates, and integrated design procedure), the feedback concept appears attractive for a broad class of traffic control problems including route guidance systems.

Dynamic modeling and control of a multidestination traffic network is generally considered to be a highly complex problem. There is no generally applicable macroscopic mathematical model describing dynamic phenomena of traffic flow on street or freeway networks to the best of the authors' knowledge. Nevertheless, traffic network models are urgently needed both as simulation tools and as a basis for developing efficient route guidance strategies. Perhaps the most advanced concept so far for macroscopic dynamic modeling of multidestination networks is the one proposed by D'Ans and Gazis (*1*). In their work, however, D'Ans and Gazis assume that the route choice of drivers with a given origin and destination is fixed and known. In this paper, dynamic modeling and control of traffic networks including traffic assignment are considered. A basic assumption is that traffic demand at the origins of the network is considered to be deterministic and independent of the traffic conditions in the network. Consideration of elastic demands is left to future investigations.

The model presented in this paper was developed on the basis of a dynamic traffic network model framework that was initially presented elsewhere (*2*). The model consists of three interacting parts:

1. A traffic flow part describing traffic flow evolution along network links;
2. A traffic composition part describing propagation of traffic composition for substreams with different destinations; and
3. A dynamic assignment part, which routes traffic substreams so as to guarantee dynamic user optimum conditions in real time.

The dynamic assignment part may be used both for modeling and for control purposes (e.g., in the context of a route guidance system). A feedback concept is proposed for development of the dynamic assignment part (Item 3).

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DYNAMIC MODELING OF TRAFFIC NETWORKS

A precise mathematical framework for deterministic, macroscopic modeling of traffic networks has been published elsewhere (*2*). Therefore, in this paper only the basic approach and the resulting model structure will be outlined.

Definitions

Consider a traffic network represented by a directed graph. (See Figure 1.) N and M denote the sets of network nodes and links, respectively. Let I_n and O_n be the sets of links entering and leaving the nth node, respectively. It is assumed that traffic demands d_{ij} (veh/h) arriving at origin nodes $i \in D$ and being routed through the traffic network to some destination nodes $j \in S$, where D and S denote the sets of origin and destination nodes, respectively. A node may belong either to D or to S or to both or to none of them. The traffic demand being routed through the network is denoted by q_m and Q_m, $m \in M$, the traffic volume (veh/h) entering and leaving the link m, respectively. In each link there may be traffic subflows destinated to different destinations j. We denote by γ_{mj} (Γ_{mj}), $j \in S_m$, the composition rate, that portion of q_m (Q_m) destinated to Node j, where S_m is the set of destination nodes, which are reachable via Link m. Note that

$$\sum_{j \in S_m} \gamma_{mj} = 1, \sum_{j \in S_m} \Gamma_{mj} = 1 \tag{1}$$

always holds. Hence, the number of independent composition rates for a link is equal to the cardinality of S_m minus one.

Modeling of Network Nodes

A model of a network node n should be capable of calculating q_m, γ_{mj} for $m \in O_n$ on the basis of d_{nj}, $j \in S^n$, and of Q_m, Γ_{mj}, $m \in I_n$, where S^n is the union of all S_m, $m \in O_n$. To make this possible, an additional variable that reflects the route choice behavior of drivers needs to be introduced. Hence, the splitting rates β_{nj}^m, $m \in O_n$, denote the portion of traffic flow which arrives at node n (regardless its origin), is destined to j, and is exiting node n by link m. In other words, splitting rates are turning rates by destination. Note that

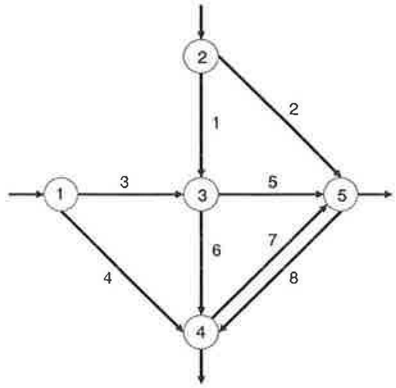

FIGURE 1 Example network.

$$\sum_{m \in \Lambda_{nj}} \beta_{nj}^{m} = 1, \; 0 \leq \beta_{nj}^{m} \leq 1 \quad (2)$$

always hold, where Λ_{nj} is the set of output links of node n for which $j \in S_m$ holds. Hence, if λ_{nj} is the cardinality of the set Λ_{nj}, the number of independent splitting rates is $\lambda_{nj} - 1$ for each couple (n, j).

With this definition we obtain at each node $n \in N$

$$q_m = \sum_{j \in S^n} \beta_{nj}^{m} q_{nj} \qquad m \in O_n \quad (3)$$

$$\gamma_{mj} = \beta_{nj}^{m} q_{nj}/q_m \qquad m \in O_n, \; j \in S^n \quad (4)$$

where q_{nj} is the traffic volume (veh/h) arriving at node n (regardless of its origin) and destinated to j, that is

$$q_{nj} = \sum_{m \in I_n} Q_m \Gamma_{mj} + d_{nj} \qquad j \in S^n \quad (5)$$

Note that Equations 3 through 5 distribute the traffic flow entering a network node among the leaving links according to the destination of the involved subflows and according to the splitting rates β_{nj}^{m} (see Figure 2).

Modeling of Network Links

The evolution of the traffic state inside a link and at the link's output depends entirely upon the traffic conditions at the link's boundaries. If the variables q_m, γ_{mj}, $m \in M$, $j \in S_m$, are organized in an input vector $\underline{U}$ and the variables Q_m, Γ_m, $m \in M$, $j \in S_m$, in an output vector $\underline{Y}$, the very general link model structure can be written as

$$\underline{Y}(k) = \underline{G}[\underline{x}(k), \underline{U}(k)] \quad (6)$$

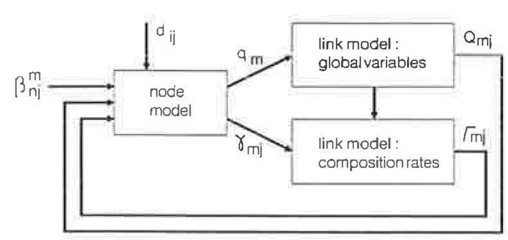

FIGURE 2 The overall network model.

$$\underline{x}(k + 1) = \underline{F}[\underline{x}(k), \underline{U}(k)] \quad (7)$$

where $k = 0, 1, 2, \ldots$ is the discrete time index, that is, $\underline{x}(k) = \underline{x}(k \cdot T)$, T being the sample time interval. The dimension and the composition of the state vector $\underline{x}$ depends upon the particular link model used.

For the development of dynamic link models, the global traffic variables that do not depend upon a particular destination are concentrated on first. A relatively simple dynamic model utilized by Merchant and Nemhauser (*3*) and by Wie (*4*) requires introduction of traffic density ρ_m (veh/km) in link m and makes use of the conservation equation

$$\rho_m(k + 1) = \rho_m(k) + (T/\Delta_m)[q_m(k) - Q_m(k)] \quad (8)$$

where Δ_m is the length of link m. Furthermore it is assumed that Q_m is given in terms of ρ_m by a nonlinear algebraic relation. As an example, consider the exponential relationship.

$$Q_m(k) = q_{\max,m} \left[1 - \exp(-\rho_m(k)/R_m)\right] \quad (9)$$

where $q_{\max,m}$ and R_m are constant parameters. For stability reasons, the sample time interval should be chosen such that $T < \min(\Delta_m R_m/q_{\max,m}, \; m \in M)$. Equations 8 and 9 provide the form required by Equations 6 and 7.

The choice of a dynamic link flow model depends upon the physical background of the corresponding network link. Equations 8 and 9 are very similar to the platoon dispersion model used in TRANSYT for links that represent urban streets [see Robertson (*5*)]. If some of the network links represent freeway axes, more sophisticated dynamic models are required [see Papageorgiou (*6*)]. Sophisticated models of freeways links consider subdivision of links into a number of segments and apply hydrodynamic equations to each of the segments. A general freeway network modeling computer program based on the presented network framework and on sophisticated dynamic modeling of traffic flow along the links is now available (*7*).

Note that the network modeling framework presented here allows for using different models for corresponding groups of links, which is an essential feature in modeling corridor or other mixed-traffic networks.

A traffic network may include control inputs such as urban traffic lights or freeway ramp metering. For the sake of simplicity, these control inputs have not been considered in the general Equations 6 and 7 because the description of route choice phenomena in the network will be concentrated on. A suitable extension of the general modeling structure to include traffic control measures is given elsewhere (*2*).

The dynamic modeling of composition rates $\gamma_{mj}(k)$ along a link will now be considered. There is no sound theoretical basis for development of a macroscopic model that propagates the composition rates $\gamma_{mj}(k)$ along a link. A possible approximation reads

$$\Gamma_{mj}(k + 1) = \alpha_m \gamma_{mj}(k) + (1 - \alpha_m)\Gamma_{mj}(k) \quad (10)$$

where α_m may be either constant or dependent upon the travel time along the link, that is

$$\alpha_m(k) = T/\tau_m(k) \quad (11)$$

The travel time is given by $\tau_m(k) = \Delta_m/v_m(k)$, and the mean speed reads $v_m(k) = Q_m(k)/\rho_m(k)$. Further composition rate models are presented and discussed in Papageorgiou (*2*).

Integrated Dynamic Network Model

The overall network model consists of the following interacting modules (see Figure 2):

- The node modeling Equations 3 through 5, and
- The chosen link models for traffic flow and for composition rates.

The overall model can be expressed by the general nonlinear, discrete-time vector equation:

$$\underline{x}(k+1) = \underline{f}[\underline{x}(k), \underline{\beta}(k), D(k)],$$
$$k = 0, \ldots, K-1 \tag{12}$$

where $\underline{x}$ is the state vector and $D = (d_{ij})$ is the origin-destination demand matrix. The vector $\underline{\beta}(k) \in R^p$ includes all independent splitting rates and obeys

$$\underline{0} \le \underline{\beta}(k) \le \underline{1} \tag{13}$$

Figure 3 illustrates Equation 12 from a system theoretic viewpoint; $\underline{\beta}(k)$'s are input variables and $D(k)$'s are disturbances. Note that Equation 12 can be resolved for $\underline{x}(k)$, $k = 0, \ldots, K$, if the trajectories $\underline{\beta}(k)$, $D(k)$, $k = 0, \ldots, K-1$, and the initial condition $\underline{x}(0)$ are given.

For example, using the dynamic Equations 8 and 10 for a link model and replacing the static Equations 3, 4, and 5 (node model) by Equations 9 and 11, a state vector consisting of traffic densities and composition rates is obtained. In the case of the example network presented in Figure 1, the corresponding state vector reads

$$\underline{x} = [\rho_1 \ldots \rho_8 \gamma_1 \ldots \gamma_6]^T \tag{14}$$

where $\gamma_1, \ldots, \gamma_6$ are the independent composition rates for links 1, ..., 6. The p independent splitting rates for this example are listed in Table 1 (here $p = 6$).

Physical Significance of Splitting Rates

The independent splitting rates $\underline{\beta}$ reflect the drivers' behavior with respect to alternative route choice. Clearly, the drivers' behavior may be influenced by real-time information or route recommendation provided to them either by use of suitably located variable message signs or by individual communication with suitably equipped vehicles. The authors' interest in the independent splitting rates is twofold:

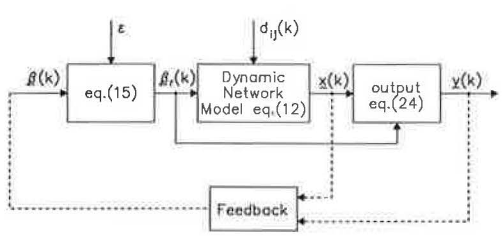

FIGURE 3 Network process considering compliance rate ε and feedback for dynamic assignment.

TABLE 1 SPLITTING RATES FOR EXAMPLE NETWORK

From Network Node	To Destination Node	Splitting Rates	Independent Splitting Rates	Number
2	4	$\beta^1_{24}, \beta^2_{24}$	β^1_{24}	1
2	5	$\beta^1_{25}, \beta^2_{25}$	β^2_{25}	2
1	5	$\beta^3_{15}, \beta^4_{15}$	β^3_{15}	3
1	4	$\beta^3_{14}, \beta^4_{14}$	β^4_{14}	4
3	5	$\beta^5_{35}, \beta^6_{35}$	β^5_{35}	5
3	4	$\beta^5_{34}, \beta^6_{34}$	β^6_{34}	6

1. Modeling: How should $\underline{\beta}$ be calculated in absence of any communication to the drivers so as to reflect their natural behavior?
2. Control: If $\underline{\beta}$ is manipulable through suitable communications to the drivers, what is the best choice of $\underline{\beta}$?

The next sections present a feedback mechanism that leads to the specification of $\underline{\beta}$ so as to satisfy some generalized dynamic user optimal conditions.

Assume that $\underline{\beta}$ is manipulable by use of variable message signs which recommend route choice to the drivers. In this case, the modeling results of this section suggest that one variable message sign should be installed for each independent β_i. More specifically, at each node n of the network, the number of required variable message signs equals the number of destination nodes j, which are reachable from node n and for which a splitting at node n is possible.

The case where only a portion of the vehicles are equipped and/or only a portion of the drivers follow the recommendations provided will now be discussed. A parameter E, $0 \le \varepsilon \le 1$, reflecting the compliance rate and/or the rate of equipped vehicles such that for $\varepsilon = 0$ none follows the recommendations, and for $\varepsilon = 1$ everybody follows the recommendations. If β is the splitting rate ordered by the control system and β_r is the resulting real splitting rate, it may be written

$$\beta_r = 1 - (1 - \beta)\varepsilon \tag{15}$$

Equation 15 may be integrated into the general state space model (Equation 12). Figure 3 illustrates that—from a system theoretic viewpoint—ε can be interpreted as a disturbance acting on the process under control.

DYNAMIC USER OPTIMUM

Dynamic User Optimum Definition

For simplicity, the single-origin–single-destination network of Figure 4 will first be considered. A generalization of the obtained results will be considered later. The demand arriving at node 1 (Figure 4) is distributed between the links according

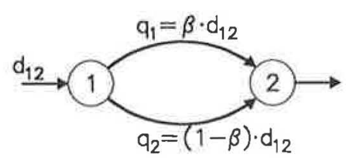

FIGURE 4 A single-origin-single-destination network.

to the independent splitting rate β such that $q_1 = \beta d_{12}$, $q_2 = (1 - \beta)d_{12}$.

As it is well known, steady-state user optimal assignment conditions for the single-origin–single-destination network of Figure 4 are present if and only if β obeys the following conditions

$$\begin{aligned} &\beta = 1 && \text{if } C_1 < C_2 \\ &0 < \beta < 1 && \text{if } C_1 = C_2 \\ &\beta = 0 && \text{if } C_1 > C_2 \end{aligned} \tag{16}$$

where C_m is a measure of the individual cost along link m. For example, it can be assumed that C_m is the travel time along link m, in which case

$$C_m = \tau_m = \Delta_m / v_m \tag{17}$$

where v_m is the mean speed on link m.

Equation 16 may be readily expanded into a dynamic user assignment condition applying for $k = 0, \ldots, K$:

$$\begin{aligned} &\beta(k) = 1 && \text{if } C_1(k) < C_2(k) \\ &0 < \beta(k) < 1 && \text{if } C_1(k) = C_2(k) \\ &\beta(k) = 0 && \text{if } C_1(k) > C_2(k) \end{aligned} \tag{18}$$

An equivalent form of these conditions may be expressed in terms of the quantity

$$y = \beta \Psi(C_1 - C_2) + (1 - \beta)\Psi(C_2 - C_1) \tag{19}$$

where the function Ψ is defined $\Psi(\cdot) = \max(0, \cdot)$. With this definition, the conditions (Equation 18) are equivalent to

$$y(k) = 0 \qquad k = 0, \ldots, K. \tag{20}$$

A precise mathematical description of dynamic user optimal conditions now requires an adequate definition of the individual cost $C_m(k)$ along link m at time k. For simplicity but without loss of generality, it will be assumed that the individual cost corresponds to a notion of travel time.

A particular dynamic generalization of the static user optimum is provided by the "reactive user optimum," which is defined by

$$C_m(k) = \tau_m(k) = \Delta_m / v_m(k) \tag{21}$$

By its definition, $\tau_m(k)$ depends upon the current traffic conditions on link m. In other words, $\tau_m(k)$ is an ideal travel time spent by an ideal vehicle that travels along the link m under traffic conditions that correspond to the current traffic conditions.

The reactive user optimum relies on the assumptions that

1. Traffic conditions in the network are not predictable because of, for example, incidents, variable demands, and stochastics.
2. Complete real-time information is available to the decision makers.

In fact, under these assumptions, a driver arriving at a bifurcation location, will choose the route which, according to reliable real-time information, currently appears to be shorter. A connection with an alternative, predictive dynamic assignment definition is discussed by Papageorgiou (*2*).

Generalization

The preceding statements will now be generalized for a multiple origin–multiple destination network, concentrating on the connection of a network node $n \in N$ with a destination node $j \in S^n$. There are three complications when compared with the simple network of Figure 4:

1. The number of output links of node n may be greater than two.
2. Each alternative route may consist of more than one link.
3. Some output links of node n may belong to more than one alternative route (because of farther downstream bifurcations).

As far as the first complication is concerned, it will be assumed that the number of output links of each node does not exceed two. This is without loss of generality because any node with more than two output links may be decomposed as indicated in Figure 5, by introducing artificial links without dynamics and with zero costs. This simplification gives $\lambda_{nj} \leq 2$ for all pairs $(n \in N, j \in S^n)$. Eventually there is at most one independent splitting rate β_{nj} for each pair $(n \in N, j \in S^n)$.

To handle the other two complications, the shortest travel time between nodes n and j through link $m \in \Lambda_{nj}$ is introduced.

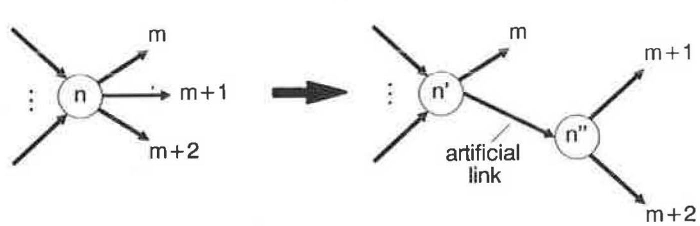

FIGURE 5 Decomposition of a complex node.

More precisely

$$\tau_{nj}^{m} = \min_{z \in Z} \sum_{v \in L_{nj}^{z}} \tau_v \qquad m \in \Lambda_{nj} \tag{22}$$

where $Z = \{z | m \in L_{nj}^{z}\}$.

Note that Equation 22 may be used as a generalization of Equation 21 for the general network case. Note further that the definition (Equation 22) implies the execution of a shortest path algorithm for the calculation of τ_{nj}^{m} from known link costs τ_v.

As an example, consider the corresponding formulas for τ_{14}^{3}, τ_{14}^{4} in the example network of Figure 1:

$$\tau_{14}^{3} = \min \{\tau_3 + \tau_6, \tau_3 + \tau_5 + \tau_8\}$$

$$\tau_{14}^{4} = \tau_4$$

With these definitions, generalization of Equations 18, 19, and 20 is straightforward and the statements made for the simple network of Figure 4 apply to general networks as well. More precisely, perform the following replacements in Equations 18, 19, and 20:

β by β_{nj}, C_1 by τ_{nj}^{m}, C_2 by τ_{nj}^{1}, and y by y_{nj}

The resulting conditions are required to hold: $\forall\ k \in [0,K]$; $\forall\ n \in N$; $\forall\ j \in S^n$; $m,\mu \in \Lambda_{nj}$.

Note that there is exactly one variable y_{nj} assigned to each independent splitting rate β_{nj} of the general network. Thus a vector $\underline{y} \in R^p$ comprising all y_{nj}, $n \in N, j \in S^n$ may be defined. With the preceding generalizations it may be stated that

Reactive dynamic user optimal conditions in a general traffic network are present if and only if

$$\underline{y}(k) = \underline{0}, k = 0, \ldots, K - 1. \tag{23}$$

Because the travel times $\tau_m(k)$ depend upon the system state $\underline{x}(k)$, general notation for the overall network (see Figure 3) may be written as

$$\underline{y}(k), = \underline{g}[\underline{x}(k), \underline{\beta}(k)] \tag{24}$$

There is no guarantee that there is a unique $\underline{\beta}(k)$ trajectory satisfying Equation 23 under a given demand in a given traffic network. Hence more than one solution may generally be present for the dynamic traffic assignment problem defined in this paper.

Dynamic User Optimum Via Feedback Regulation

The question to be treated in this section reads: is it possible to establish a dynamic user optimum by use of feedback regulation, that is, by a relationship $\underline{\beta}(k) = R[\underline{x}(k)]$? The significance of a real-time feedback law for dynamic assignment and for route guidance is obvious and will be further discussed later.

First note that Equation 18 is satisfied (and hence a reactive user optimum is reached) if the following simple feedback law is applied to a general traffic network.

$$\beta_{nj}(k) = \begin{cases} 1 \text{ if } \tau_{nj}^{m}(k) > \tau_{nj}^{\mu}(k) \\ 0 \text{ if } \tau_{nj}^{m}(k) < \tau_{nj}^{\mu}(k) \end{cases} \tag{25}$$

This feedback law is a bang-bang one, that is, the input variable $\beta_{nj}(k)$ takes values only on its bounds. Such a bang-bang controller may be adequate in the case of collective route guidance in which no values other than 0 and 1 can be implemented.

In some cases, a bang-bang solution may not be satisfactory. In fact, for individual route guidance with a high rate of equipped vehicles, bang-bang control may lead to strong perturbations of traffic flow. A smooth regulation may be achieved if $\underline{y}(k)$ is understood as the output of a process with input $\underline{\beta}(k)$ (see Figure 3). In this case, $\underline{y}(k)$ might be kept near or equal to zero by introducing a feedback law

$$\underline{\beta}(k) = \underline{\beta}(k-1) + K_p[\underline{x}(k) - \underline{x}(k-1)] + K_I\underline{y}(k) \tag{26}$$

where K_p is the proportional gain matrix and K_I is the integral gain matrix, which is assumed to have full rank p. Equation 26 describes a multivariable feedback regulator with integral parts.

To investigate the properties of the closed-loop system, it is first assumed (as a theoretical experiment) that the compliance rate ε demands to be constant, that is, $\varepsilon(k) = \bar{\varepsilon}$ and $D(k) = \overline{D}$, $k = 0, \ldots, K - 1$. If the closed-loop system is stable, a steady-state solution of Equation 26 then reads

$$K_I\underline{\bar{y}} = \underline{0} \tag{27}$$

where bars denote steady-state values. Since K_I is chosen to have full rank, $\underline{\bar{y}} = \underline{0}$ results from Equation 27 which is a well-known result in automatic control theory. Thus for constant demands and constant compliance rates, the multivariable feedback regulator (Equation 26) leads automatically to dynamic user optimum conditions without knowledge of the compliance rates and of the demands.

If disturbances $D(k)$, $\varepsilon(k)$ are not constant, as is usually the case, the feedback regulation will keep $\underline{y}(k)$ near zero for a reasonable choice of the gain matrices. It is important to underline again that the feedback laws (Equations 25 and 26) do not include any information on the present or future values of the demand and of the compliance rate.

The gain matrices K_p, K_I of the feedback law (Equation 26) should be chosen such that the overall closed-loop system be stable in a reasonable operating region around a theoretical steady-state. For example, specification of the gain matrices may be achieved by linearization of the system equations around a theoretical steady-state and by application of linear-quadratic (LQ) optimization methodology [see Papageorgiou (2) or other suitable methods; e.g., Kwakernaak and Sivan (8)]. Suitable gain matrices can be developed by the LQ method by means of a systematic trial-and-error procedure. Although not trivial, this development can be performed efficiently with some experience and basic knowledge of the LQ approach even for large-scale networks. The resulting LQ regulator is known to have excellent robustness properties for a wide range of process conditions.

The preceding results will now be illustrated on the basis of the example network of Figure 1 and the modeling Equations 8 through 11. Appropriate gain matrices K_p, K_I were selected by application of the LQ method, see Senninger (9) for details and see Papageorgiou (2) or Senninger (9) for the matrix values. The feedback law (Equation 26) was applied to the nonlinear network traffic modeling equations for different demand scenarios. First consider the rectangular demand scenario depicted in Figure 6. Figure 7 shows the resulting travel time differences $\Delta\tau_i(k)$ (in percent) for the six pairs of alternative routes corresponding to the six independent splitting rates of Table 1. Figure 8 depicts the corresponding trajectories of $\underline{\beta}(k)$. The following remarks may be stated:

1. The required condition $\underline{y}(k) = \underline{0}$ is satisfied for most $k \in [0,K]$ through the action of the feedback regulator. In fact, for most $k \in [0,K]$, either $\beta_i(k) = 1$ (no splitting) and $\Delta\tau_i(0) < 0$ (routes are not competitive), or $0 < \beta_i(k) < 1$ (splitting of the corresponding substream occurs) and $\Delta\tau_i(k) = 0$ (equal travel times on alternative routs) hold.
2. A steady-state is achieved for each of the three sets of constant demand values included in the demand scenario of Figure 6. Note that each steady-state is equivalent to a corresponding static user optimal equilibrium.

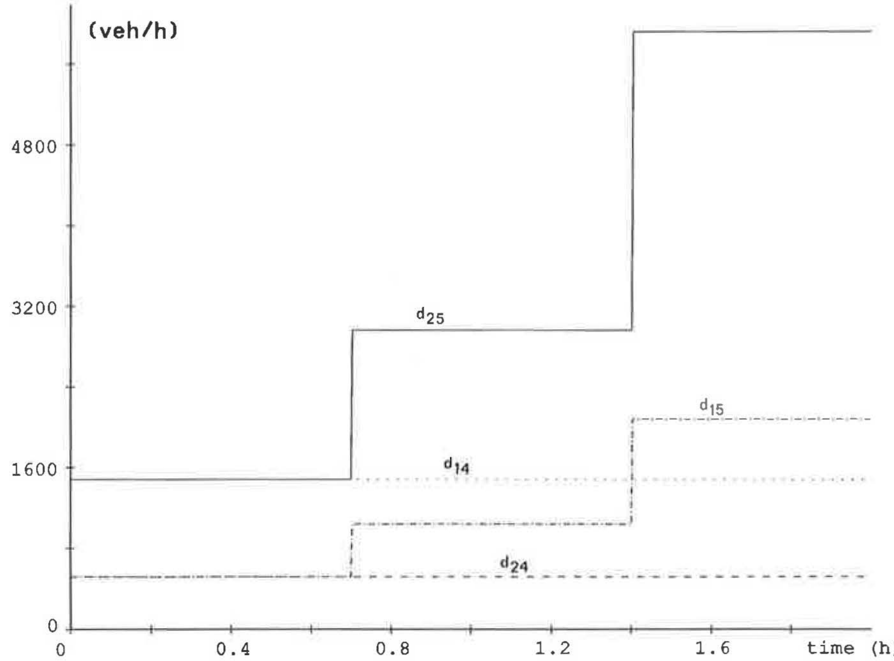

FIGURE 6 A rectangular demand scenario.

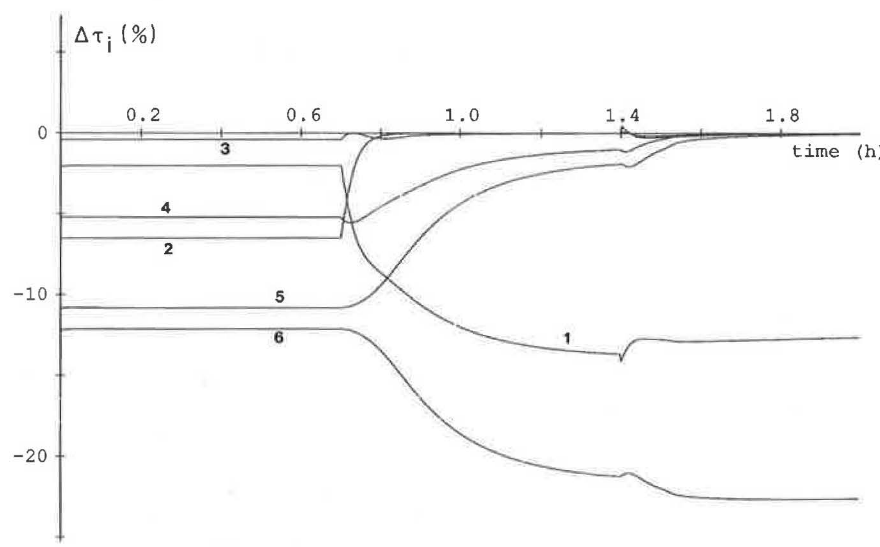

FIGURE 7 Travel time differences $\Delta\tau_i$ for six pairs of alternative routes for multivariable feedback law (Equation 26).

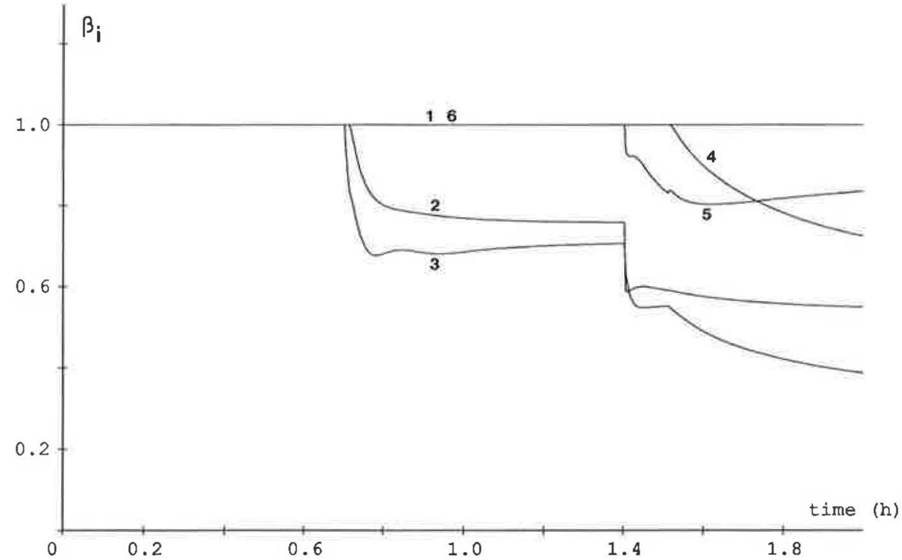

FIGURE 8 Splitting rates provided by multivariable feedback law (Equation 26).

Figure 9 depicts the travel time differences $\Delta\tau_i(k)$ resulting by application of the bang-bang controller (Equation 25) to the same network with the same rectangular demand of Figure 6. Note that the bang-bang controller

1. Leads to a slightly oscillatory behavior for some $\Delta\tau_i(k)$.
2. Equalizes travel times on alternative routes for a smaller number of alternative route pairs as compared to the multivariable regulator.

Figure 10 depicts a triangular demand scenario and Figures 11 and 12 depict the resulting travel time differences $\Delta\tau_i(k)$ and the splitting rates $\beta_i(k)$, $i = 1, \ldots, 6$, for the multivariable regulator. Again the feedback regulator (Equation 26) succeeds in keeping $\underline{y}(k)$ close (but not exactly equal) to zero although the demands $d_{ij}(k)$ are unknown to the feedback law. Figure 13 depicts the corresponding results of the bang-bang controller.

The advantages of the feedback concept when applied to traffic networks in the aim of establishing reactive dynamic user optimal conditions will now be summarized:

1. The feedback concept requires only few calculations at each time instant k. Moreover, it is a real-time procedure such that no iterations or other time consuming algorithms are required.
2. The feedback law does not utilize current or future values of the process disturbances of the origin-destination demands $D(k)$ and of the compliance rate ε (see Figure 3). Nevertheless, its sensitivity with respect to variations of these disturbances seems low as demonstrated by the preceding results.
3. Note that for a real-life control application, only the feedback portion of Figure 3 is implemented using measurements from the real traffic process, that is, no model calculations are required in real time.

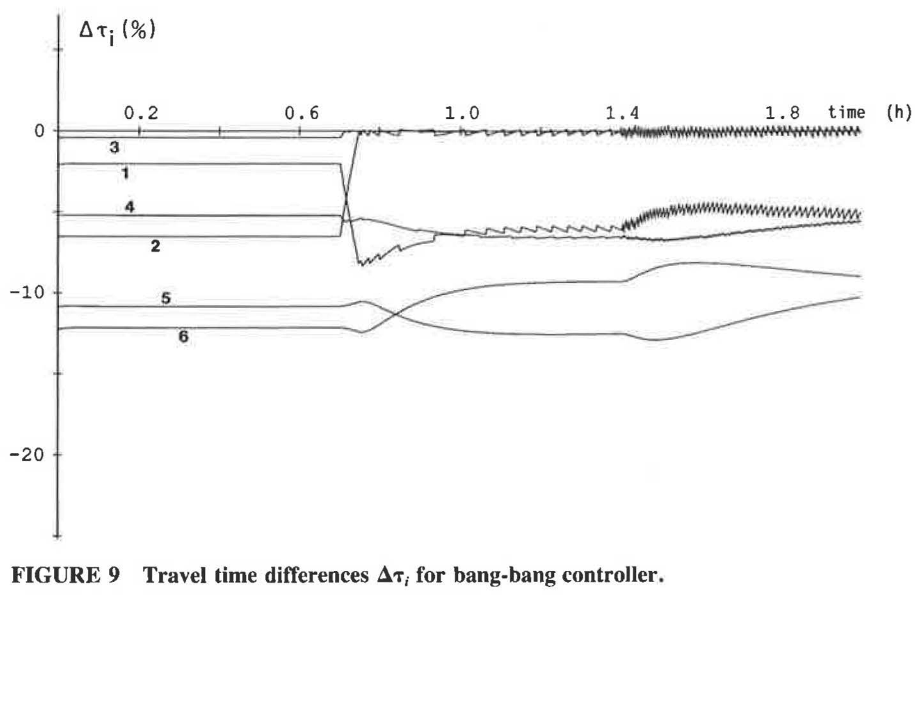

FIGURE 9 Travel time differences $\Delta\tau_i$ for bang-bang controller.

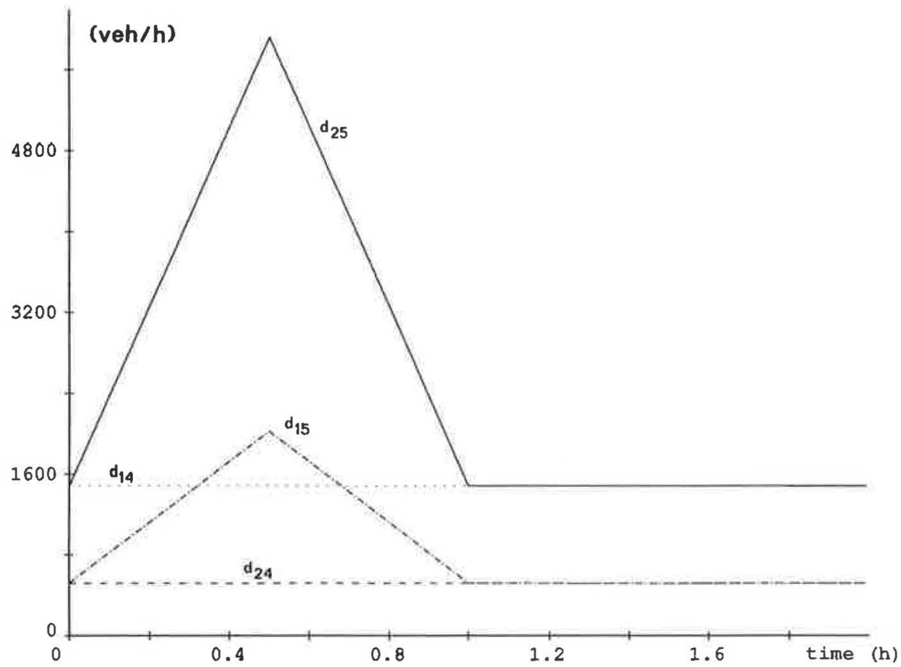

FIGURE 10 **A triangular demand scenario.**

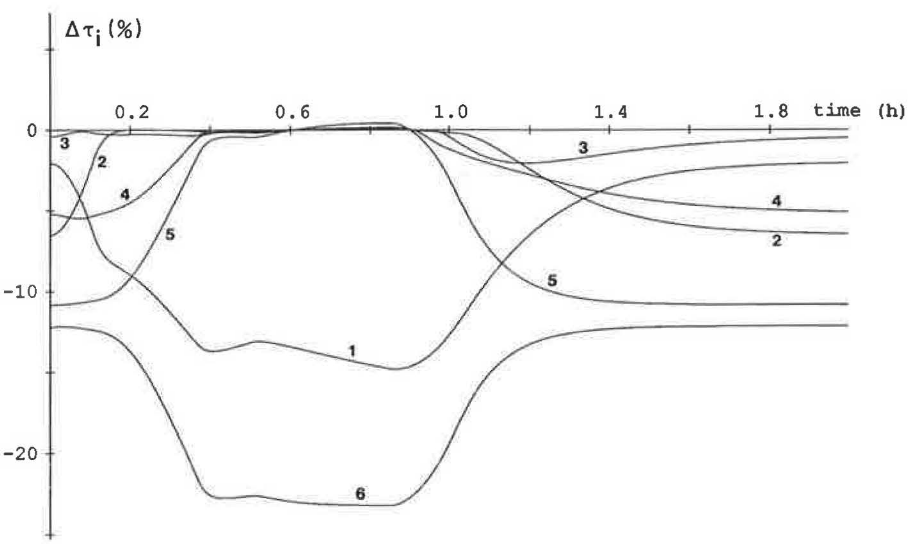

FIGURE 11 **Travel time differences $\Delta\tau_i$ for triangular demand and multivariable feedback.**

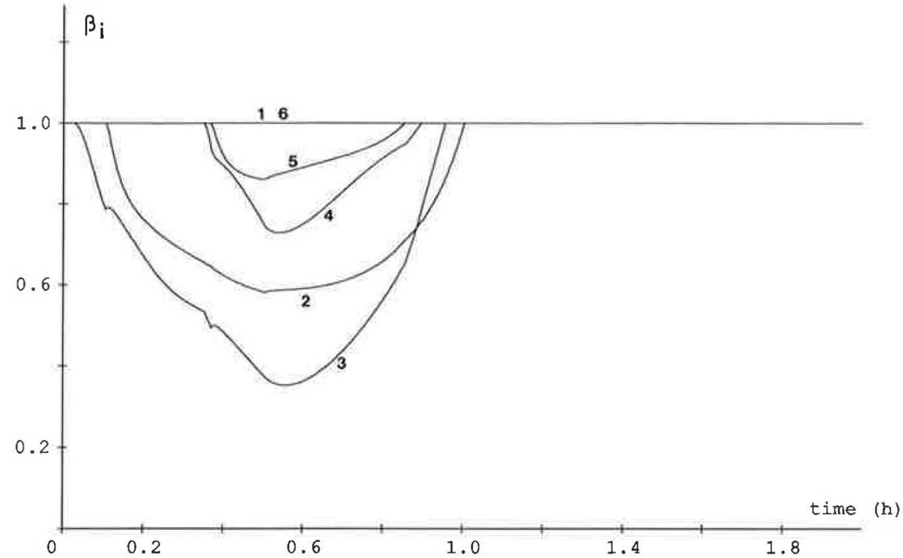

FIGURE 12 Splitting rates for triangular demand scenario and multivariable feedback.

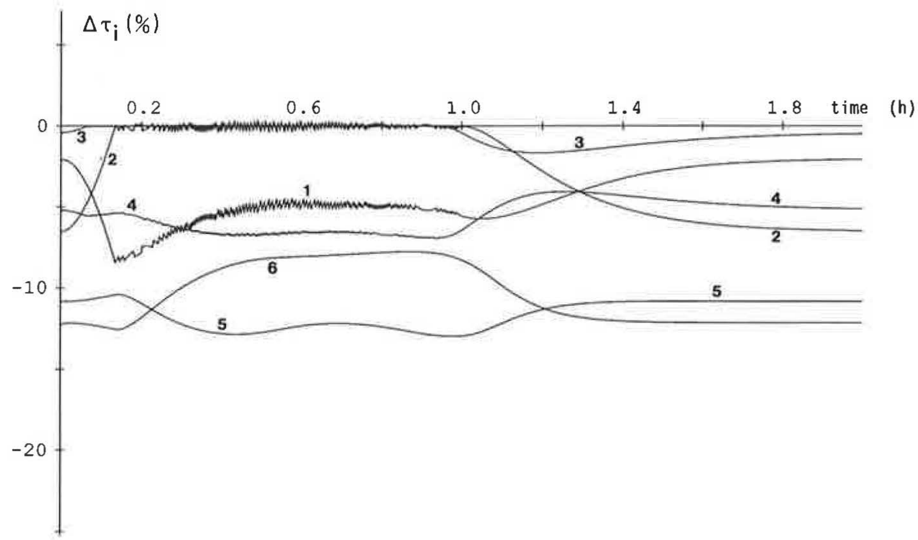

FIGURE 13 Travel time differences $\Delta\tau_i$ for triangular demand and bang-bang controller.

It should be emphasized that the proposed feedback concept is of a reactive character—it reacts indirectly to the disturbances (namely via their impact on the traffic state)—and this is the reason why it does not need disturbance predictions. For example, if the compliance rate ε is too low (high) this will have an impact on the traffic state and will lead automatically the feedback law to according modification of the input $\boldsymbol{\beta}$ so as to approach the goal $\underline{y}(k) \approx \underline{\mathbf{0}}$.

Under realistic conditions, with $D(k)$ and ε varying strongly with time, the feedback concept cannot lead exactly to $\underline{y}(k) = \underline{\mathbf{0}}$ but hopefully to $\underline{y}(k) \approx \underline{\mathbf{0}}$ as demonstrated in the example tests. Furthermore, for very strong variations of the traffic state from the linearization conditions (severe congestion!), the linear regulator may need a long time to lead the output $\underline{y}(k)$ near zero, although it will react in the right sense. But what could be an alternative approach? One should be able to predict the origin-destination demands and compliance rates (which is rather unrealistic) and to apply a mathematical traffic model (accuracy?) in order to calculate iteratively (effort!) the route recommendations so as to achieve $\underline{y}(k) = \underline{\mathbf{0}}$ in the computer (in real life?). In contrast to such an approach, the feedback philosophy is to react to real-life measurements rather than to rely on predictions and mathematical models.

The tests of this paper are certainly not significant for practical applications with more realistic models or under real-life conditions. Nevertheless they do provide a very encouraging first step towards application of the innovative concept of feedback to a fairly complicated traffic problem, which opens the way to consideration of more realistic conditions. Anyhow, the particular link model used for the reported tests is not simpler than the ones used in previous research work on traffic assignment as cited in the references. Investigations of the feedback concept for route recommendation by variable message signs on freeway networks is currently under way, see Wolf (*10*) using realistic high-order link models of freeway traffic like METANET (*7*).

CONCLUSIONS

A general framework for deterministic dynamic modeling and control of traffic networks has been presented under non-elastic but time-varying demand conditions. The traffic network may include both freeways and urban roads. The presented methodology may be readily extended to consider control measures like ramp metering and signal settings, see Papageorgiou (*2*).

A feedback concept has been applied to the traffic network to achieve dynamic user optimum conditions. Because of three fundamental features—low computational effort, low sensitivity with respect to unknown origin-destination demands and unknown compliance rates, and integrated design procedure—the feedback concept appears particularly attractive for a broad class of traffic control problems, which include [see Papageorgiou (*2*), for more details]:

- Dynamic network traffic modeling including traffic assignment;
- Static user optimal traffic assignment;
- Integrated strategy development for route guidance and traffic control systems;
- Development of optimal traffic control strategies subject to dynamic assignment conditions; and
- Development of feedback strategies for a variety of traffic control problems including route guidance.

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