Theoretical Implications of the AASHTO 1986 Nondestructive Testing Method 2 for Pavement Evaluation

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The conventional classification of pavement design procedures into "empirical" and "mechanistic" is reexamined. It is submitted that the term "mechanistic" originally denoted a data interpretation methodology based on the laws of engineering mechanics, proposed as an alternative to the statistical interpretation techniques used extensively in the interpretation of AASHO Road Test data. Current mechanistic design procedures retain statistical/empirical correlations, but this is a matter of practical expediency, pending improvements in analytical capabilities. On the other hand, incorporating mechanistic results into a statistical/empirical framework rarely leads to reliable conclusions. A case in point is provided by AASHTO 1986 Nondestructive Testing Method 2 (NDTM2), which combines the mechanistic layered elastic theory with the purely statistical/empirical structural number concept. The derivation of the NDTM2 equations is traced and simplified, and a number of theoretical shortcomings are highlighted. It is recommended that use of NDTM2 be discouraged and that efforts for the gradual elimination of statistical/empirical constructs [e.g., structural number, equivalent single-axle load (ESAL), and Miner's fatigue concepts] be intensified. Attempts to define statistical/empirical parameters [e.g., layer coefficients, present serviceability index (PSI), and load equivalency factors] using mechanistic theoretical tools should be abandoned.

Following the AASHO Road Test (1958–1960), a conventional nomenclature evolved according to which pavement design procedures are generally classified as either "empirical" or "mechanistic." Despite widespread use of these two terms, a surprising lack of agreement exists today among practicing pavement engineers as to the terms' precise meaning. Even a brief review of the pertinent literature reveals that relatively little effort has been expended in clarifying the distinguishing features of each approach. Investigators adopt instead the term that lends most credibility to a proposed new design procedure, with "mechanistic" being considered somehow superior. Since both "empirical" and "mechanistic" design procedures were in existence long before the adoption of the current terminology, it must be recognized that the meaning ascribed today to these terms has been greatly influenced by the concepts stemming from the AASHO Road Test experience.

Consider first the dictionary definitions of the two terms. "Empirical" means "relying upon or derived from observation or experiment," or "guided by experience and not theory" (*1*). Since observation and experience are indispensible ingredients of any credible and reliable scientific endeavor, it is not immediately evident why "empirical" pavement design procedures are often nowadays considered largely out of date and are described as based on rules of thumb dictating the development of "a more rational approach" (*2*).

On the other hand, a "mechanistic" procedure is one "of or pertaining to mechanics as a branch of physics," or "of or tending to explain phenomena only by reference to physical or biological causes" (*1*). The term "rational" is no longer used very often in current pavement literature, perhaps in view of its judgmental undertones. In its technical meaning, however, "rational" appears to have been an early synonym for "mechanistic," "in the sense of having reason and understanding, of properly relating causes and effects" (*3*), and implying the use "of accurate predictions of stresses and strains in various parts of the layered system through the use of structural analysis techniques" (*4*). "Theoretical" is another term that has been used quite often as a synonym to "mechanistic," although its diminishing popularity probably reflects "a certain disenchantment in the attitude of investigators toward the application of pure elastic theory" (*5*). The pioneers of the theoretical approach, however, never advocated the use of "pure" theory stripped of observation, experience, or common sense. Westergaard (*6*) dismissed as "irrelevant" the comments of a certain Turner, who compared the use of the theory of elasticity with the use of the discredited phlogiston theory in the middle ages, but would probably have endorsed the following statement of Burmister (*7*):

> Every important advance in science and engineering has stemmed from theoretical working hypotheses, which have brought phenomena into the realm of greater certainty and have served as a guide to experimentation and investigation. It should be realized, however, that no theory or statement of physical laws in any science or engineering is complete in its present form. It cannot fully and adequately include, explain, and take into account present apparent "exceptions to the rule," outside its realm of validity of limiting boundary conditions. Yet such theories and statements of physical laws have provided the essential stimulus and guide to major scientific advances, and have established the nature and basic form of the physical laws governing phenomena.

Why then did the term "theoretical"—used extensively in other civil engineering design disciplines—need to be replaced by the term "mechanistic," one used almost exclusively by pavement engineers?

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SCOPE OF PAPER

It is submitted that the term "mechanistic" was coined in the late 1960s to denote an alternative methodology to the statistical approach used in interpreting the data collected during the AASHO Road Test. Thus, it was not originally intended to be in opposition to the empirical process of collecting data to validate an engineering theory. Consider, for example, the comments of Vesic and Saxena (*8*), who were among the first to use the term "mechanistic" in the sense usually ascribed to it today:

> The aim of this study is to provide a rational, mechanistic interpretation of measurements and observations made on rigid pavements during the AASHO Road Test. . . . Unfortunately in most [previous] analyses of AASHO Road Test data, [the present serviceability index] has been related to wheel loads on a purely empirical basis, though with elaborate statistical analyses . . . [The finding of this study] confirms the soundness of a rational, mechanistic approach to design of rigid pavements. It demonstrates beyond doubt that failure in pavement performance is not a phenomenon of chance, as some statistical approaches tend to suggest, but a phenomenon that has a definite mechanical cause.

Such comments reveal that the term "mechanistic" originally referred to the approach used in data interpretation rather than to the nature or source of the information to be interpreted. Vesic and Saxena used the same "empirical" data collected during the AASHO Road Test that previous statistical analyses had also employed. Furthermore, they applied computational tools available at the time (namely, the discrete element method) to generate additional "numerical" or "theoretical" data. The main departure of their "mechanistic" interpretation method lay precisely in the fact that instead of statistical correlations, they sought to develop cause-and-effect relationships based on reasoned understanding of the physical laws governing the mechanisms of the distresses observed in the field. Such relationships may be arrived at by adhering to the principles of structural or continuum mechanics, which provide the working hypotheses to be verified by observation and experience. In contrast, current experimental factorial designs and empirical interpretation techniques focus almost exclusively on testing statistical hypotheses, which in many instances are not substantiated—or are sometimes contradicted—by engineering analysis and design concepts.

With respect to the source of data, "empirical" may be juxtaposed to "analytical" or "numerical," but not in the sense of mutually exclusive opposites, because comparisons between measurements in the field or in the laboratory and computational results have been a powerful method of unraveling the phenomena observed and validating engineering working hypotheses. In regard to approaches to data interpretation, "mechanistic" denotes a procedure that relies on the use of engineering theory and mechanics, and should be contrasted to "statistical/empirical." This composite term defines more succintly the cardinal difference between the two approaches.

The main issue upon which this paper focusses is whether and under what conditions "mechanistic" and "statistical/empirical" data interpretation techniques can be synergistic and when they are mutually exclusive and immiscible. The discussion centers around Nondestructive Testing (NDT) Method 2 (NDTM2) of the 1986 AASHTO Design Guide (*9*), not so much because of the practical significance of this particular aspect of the Guide, but because NDTM2 provides a good case in point for the development of broad conclusions that could have enormous repercussions on prevailing experimental design philosophies and data interpretation approaches. In addition, the paper addresses some of the concerns raised by practicing professionals with respect to the application of NDTM2. The development of the pertinent formidable equations is traced and documented, and a corrected and considerably simpler form of the equations is presented.

To facilitate the flow of the arguments presented, the following convention is adopted. The 1986 *AASHTO Guide for Design of Pavement Structures* (*9*) is referred to herein simply as "the Guide." References to page, equation, figure, and section numbers, as well as appendixes to the Guide are preceded by the letter *G*. For example, "p. GIII-31" refers to page III-31 in the Guide; "Equation (GPP.4)" refers to Equation (PP.4), found in Appendix PP of the Guide; and so forth. The Guide itself provides no numbers for the equations in Chapter 5 of Part III. To clarify references to these equations, they will be numbered consecutively, starting with Equation G1 on p. GIII-77. Symbols used in the Guide are often complicated by superfluous subscripts. A simplified, uniform system of symbols has been adopted in this paper. For easy reference, Table 1 presents a comparison of the symbols used in this paper and those in Part III (Chapter 5) of the Guide, as well as in Appendixes GN, GNN, and GPP.

NDT METHODS IN AASHTO OVERLAY DESIGN

The AASHTO overlay design methodology is presented in Chapter 5 of Part III of the Guide. Procedures are described for "all types of overlay placed on any type of pavement

TABLE 1 COMPARISON OF SYMBOLS USED

	PAPER	PT.III	APP.GN	APP.GNN	APP.GPP
In situ layer modulus	E_i	E_i	–	E_{im}	E_i
In situ layer thickness	h_i	h_i	–	h_{im}	h_i
In situ layer Poisson ratio	μ_i	u_i	–	μ_{im}	u_i
Transformed top layer modulus	E_1	–	–	E_{sgb}	E_e
Transformed top layer thickness	h	h_t	h_t	T_{egb}	h_t
Transformed top layer Poisson ratio	μ_1	–	–	μ_{sgb}	u_e
Equivalent thickness	h_e, g_e	H_e	h_e	h_{1e}	h_e
Subgrade modulus	E_s	E_{sg}	E_{Rs}	E_{sg}	E_{Rs}
Subgrade Poisson ratio	$\mu_s(=0.5)$	u_{sg}	u_{Rs}	μ_{sg}	u_{Rs}

structure," and nondestructive testing is endorsed for use in material characterization for determining the in situ structural capacity of the existing pavement. Two different NDT methods are described for this purpose. In contrast to earlier methodologies that relied exclusively on maximum deflection measurements, both these methods employ measurements at additional locations.

NDT Method 1 is termed "Pavement Layer Moduli Prediction Technique" (p. GIII-32) and corresponds to the conventional NDT backcalculation schemes that have recently become very popular. Several sensor measurements are made, and an iterative computer code is used to match the measured deflection basin with a theoretically predicted one, thereby leading to estimates of the in situ layer moduli.

On the other hand, NDT Method 2 is offered as an alternative when a "manual" (noncomputerized) method is desired (p. GIII-86), or "in the event that the concept/philosophy/analysis using deflection basins to interpret the E_i values is not considered feasible" (p. GNN-16). The method employs only two deflection measurements, one under the center of the load and one at a sufficiently large radial distance ("outer geophone"). This "Direct Structural Capacity Prediction Technique" bypasses the backcalculation of each layer modulus, E_i, relying instead on an approximate approach first proposed by Ullidtz (*10*) for determining the soil modulus, E_s, on the basis of one distant deflection measurement.

DEVELOPMENT OF NDTM2 EQUATIONS

Reduction of Multilayered Pavement to Two-Layered System

The basic equations governing the application of NDTM2 are summarized on p. GN-8. Their derivation is presented in Appendix GPP, in which additional references are made to Appendix GNN, where a somewhat different derivation is presented. The review of NDTM2 presented below considers both these appendixes simultaneously, highlighting their essential similarity and areas of disagreement.

The theoretical basis for NDTM2 consists of a generalization of the classical method of equivalent thicknesses (MET), most definitively described by Odemark in 1949 (*11*). It appears, however, that because this important study was published in Swedish, only a few investigators have had access to more than scant (and not always accurate) references to Odemark's work, in a small number of publications in English. As a result, the derivation presented in the Guide is not based directly on Odemark's work. There is a single reference to Thenn de Barros (p. GPP-3), but it appears that the method employed is more akin to that suggested by Barber.

The derivation of the pertinent equations begins with the assertion that "a layered pavement system can be viewed as an equivalent thickness of any arbitrary selected material type" (p. GNN-11). Thus, the in situ multilayered pavement may be reduced to a two-layered system, the top layer of which is characterized by "transformed" modulus, thickness, and Poisson ratio values whereas the subgrade characteristics remain unchanged. The following expression (slightly modified for clarity) is given in Appendix GNN for the transformed top-layer thickness:

$$T_{\mathrm{egb}} = \sum_{i=1}^{n} h_{im}\left[\frac{E_{im}\,(1-\mu_{\mathrm{sgb}}^2)}{E_{\mathrm{sgb}}\,(1-\mu_{im}^2)}\right]^{1/3} \tag{1}$$

(See Equation GNN.23.) That this is not entirely in accordance with the MET concept as proposed by Odemark (*11*) becomes evident when one compares this equation with Equation G5, given on p. GIII-86:

$$H_e = 0.9\sum_{i=1}^{n} h_i\left\{\frac{[E_i(1-u_{\mathrm{sg}}^2)]}{[E_{\mathrm{sg}}(1-u_i^2)]}\right\}^{1/3} \tag{2}$$

(See Equation G5.)

In Equation G5, modular ratios are determined with respect to the actual in situ modulus of elasticity of the subgrade, E_{sg}, rather than that of an "arbitrary material," E_{sgb}, and the multilayered pavement system is transformed into a homogeneous foundation. Equation G5 is readily recognized as a generalization of the formula proposed by Odemark (*11*), intended to transform the overlying layer in a two-layer system into an equivalent thickness of the same material as the subgrade, rendering the pavement a single-layer system. A correction factor, f, of 0.9 was introduced by Odemark (*11*) to ensure better agreement between the MET and the more accurate layered elastic theory (*12*).

It should be noted, however, that Odemark's suggestion was specifically intended only for the reduction of an in situ two-layered system, presumably because of the approximation involved in a transformation of this type. Furthermore, $f = 0.9$ pertains only to a two-layered system in which $\mu_1 = \mu_s = 0.5$. Odemark indicated that $f = 0.83$ for $\mu_1 = 1/6$ and $\mu_s = 0.5$. Use of $f = 0.9$ for a general multilayer system is not justified, especially if Poisson ratios are retained. The mere omission of f in Equation GNN.23 may alone account for a discrepancy of up to 20 percent. This would certainly justify neglecting the contribution of the Poisson ratio terms in MET applications. For these reasons, the following form of Equation 2 is recommended:

$$h_e = 0.9\sum_{i=1}^{n} h_i\left(\frac{E_i}{E_s}\right)^{1/3} \tag{3}$$

Use of Equation 3 when $n > 3$ should be discouraged, except to provide a rough indication of the "equivalent thickness."

In contrast, Appendix GPP suggests a transformation resulting in an equivalent modulus rather than an equivalent thickness. Thus, the $(n - 1)$ pavement layers are reduced to a single one, whose thickness is the same total thickness, h_t, as in the real system, whereas its modulus is the "equivalent transformed material modulus," E_e (p. GPP-3). The following formula is proposed for the calculation of E_e (appropriately corrected):

$$E_e = \left\{\sum_{i=1}^{n-1}\frac{h_i}{h_t}\left[\frac{E_i(1-u_e^2)}{(1-u_i^2)}\right]^{1/3}\right\}^3 \tag{4}$$

(See Equation GPP.1.) This expression is a generalization of the concept proposed by Thenn de Barros (*14*), who described his original formula as "approximate," and only intended to reduce a three-layered system to a two-layered system (i.e.,

$n \leq 3$). The Thenn de Barros formula can be written as follows:

$$E_e = \left(\sum_{i=1}^{2} \frac{h_i}{h_t} E_i^{1/3} \right)^3 \qquad (5)$$

Omission of the μ-terms in Equation 5 is consistent with the approximate nature of such a transformation and the uncertainties involved in estimating Poisson ratios for pavement materials. Thus, Equation 5 is recommended as a simpler form of Equation 4 (GPP.1).

The formulation finally adopted in NDTM2 (as revealed by the equations presented in Appendix GN, p. GN-8) follows the "equivalent modulus" transformation of Appendix GPP. This choice is justified, because Appendix GPP follows the original Thenn de Barros proposal quite closely, whereas a considerable extrapolation of the Odemark formulation is represented by the Appendix GNN approach. Yet, except for the difference in the manner in which the actual n-layered system is reduced to a two-layered one, the derivations presented in both Appendixes GNN and GPP are identical. For the sake of clarity and generality, therefore, the evaluation of the derivations presented below will consider a two-layered system consisting of a top layer (h, E_1, μ_1) resting on a semiinfinite foundation (E_s, μ_s).

Reduction of Two-Layered System to Homogeneous Half-Space

The derivation begins by considering plate load tests performed on the real (multilayered) as well as on the equivalent (two-layered) pavement systems. For the latter, Burmister (*13*) gives the maximum (central) deflection as

$$\Delta_0 = 2pa \frac{(1 - \mu_s^2)}{E_s} F_w \qquad (6)$$

(see Equations GPP.19 and GNN.26), in which a is the radius of the plate and p is the applied (uniform) pressure. For $\mu_s = 0.5$, this expression is simplified to

$$\Delta_0 = 1.5 \frac{pa}{E_s} F_w \qquad (7)$$

The parameter F_w is the so-called "Burmister two-layer deflection term." It is a nondimensional correction factor that, applied to the corresponding one-layer expression by Boussinesq (*15*), permits the determination of Δ_0 in a two-layered system. Burmister (*13*) presented values of F_w in a chart, assuming $\mu_1 = \mu_s = 0.5$, and showed that F_w is a function only of the ratios h/a and E_1/E_s. A similar chart for $\mu_1 = 0.2$ and $\mu_s = 0.4$ was presented by Burmister (*16*), whereas Thenn de Barros (*14*) derived the corresponding chart for $\mu_1 = \mu_s = 0.35$. The latter also notes the following concerning the Poisson ratio values:

> The numerical value of the factor [F_w] is different [depending on the assumed values for μ_1 and μ_s] for the same values of the parameters [(h/a) and (E_1/E_s)], but the deflections computed by [Equation 6 (GNN.26 and GPP.19) in each case] are very close. The reduction of Poisson's ratio from 0.5 to 0.35 increases the deflection of layered systems by less than 10 percent, for the practical range of the parameters. The average increase is about 7 percent. The actual value of Poisson's ratio of pavement structures is not known, but it is likely to be between 0.35 and 0.5. This difference can be ignored in practical applications.

These comments justify adopting Equation 7 instead of Equation 6 (GNN.26 and GPP.19). An additional consideration favoring such a simplification relates to the nature of the conventional plate load test. This is conducted using a rigid plate for which slightly different expressions are more appropriate (*17*). Adopting the "flexible load" expressions given by Equations 6 (GNN.26 and GPP.19) and 7 introduces a discrepancy of about 20 percent. Once again, the comments of Thenn de Barros (*14*) are enlightening:

> [Deflections] computed for the case of a flexible bearing area . . . should be multiplied by a "bearing factor" [in] the case of a rigid plate. The exact value of this factor cannot be determined at this time. It can be safely stated that for the layered systems of interest in pavement design the bearing factor must be between $\pi/4$ and 1, probably closer to 1. From analogy with the uniform medium, it is evident that the surface layers have a greater influence on the difference between deflections of rigid and flexible bearing areas. If the surface layers are relatively stiff, this difference should be small. Taking into account the overall inaccuracies of modeling the pavement by an elastic layered system, it is suggested that the same deflection factors [i.e., flexible load] should be tentatively used in deflection analysis of pavement systems loaded with rigid plates."

Returning now to the Burmister deflection term, F_w, the Guide quotes the following equation for its explicit calculation:

$$F_w = \frac{E_s}{E_1} \frac{(1 - \mu_1^2)}{(1 - \mu_s^2)} + F_b\left(1 - \frac{E_s}{E_1}\right) \qquad (8)$$

(See Equations GPP.20 and GNN.27.)

The Guide suggests the following equation for the term F_b:

$$F_b = \{[1 + (h_e/a)^2]^{1/2} - (h_e/a)\} \times \left\{1 + \frac{(h_e/a)}{2(1 - \mu_s)[1 + (h_e/a)^2]^{1/2}}\right\} \qquad (9)$$

(see Equations GPP.21 and GNN.27) with

$$h_e = 0.9h\left(\frac{E_1}{E_s} \frac{(1 - \mu_s^2)}{(1 - \mu_1^2)}\right)^{1/3} \qquad (10)$$

(see Equations GPP.22 and GNN.29).

Equation 10 (GPP.22 and GNN.29) can be readily recognized as an "extension" (by the inclusion of the μ-terms) of Equation 3 for $n = 2$. Through the use of this equation, the original in situ multilayered pavement, which had been reduced according to Thenn de Barros into a two-layered system, is now transformed according to Odemark into a homogeneous half-space. This double transformation alone is justification for dropping the μ-terms. Thus, Equation 10 (GPP.22 and GNN.29) may be simplified to

$$h_e = 0.9h\left(\frac{E_1}{E_s}\right)^{1/3} \tag{11}$$

which is the exact form proposed by Odemark (*11*).

Now, the term F_b defined by Equation 9 (GPP.21) is similarly recognized as the so-called "Boussinesq deflection factor" describing the attenuation of the center-line deflection with depth in a homogeneous half-space (*18*). Equation 9 (GPP.21) gives the value of this factor at $r = 0$, $z = h_e$, and by setting $\mu_s = 0.5$, the equation reduces to

$$F_b = \frac{1}{[1 + (h_e/a)^2]^{1/2}} \tag{12}$$

It is evident that the complexity of Equation 9 (GPP.21) is all but eliminated by assuming $\mu_s = 0.5$.

The derivation of Equation 8 (GPP.20 and GNN.27) is more perplexing. Poulos and Davis (*18*) present a similar formula, which they attribute to Palmer and Barber (*12*). Their equation assumes $\mu_1 = \mu_s = 0.5$ and results in the following expression for F_w:

$$F_w = (E_s/E_1) + \frac{a}{[a^2 + h^2(E_1/E_s)^{2/3}]^{1/2}}[1 - (E_s/E_1)] \tag{13}$$

Implicit in Equation 13 is a transformation of the two-layered system into a homogeneous half-space by deriving an equivalent thickness as follows [after Poulos and Davis (*18*), corrected for an evident typographical error]:

$$h_e = h\left[(E_1/E_s)\frac{(1 - \mu_s^2)}{(1 - \mu_1^2)}\right]^{1/3} \tag{14}$$

Equation 14 assumes that $f = 1.0$, as was common before the publication of Odemark's work in 1949. If $\mu_1 = \mu_s = 0.5$, Equation 14 can be introduced into Equation 13 to yield

$$F_w = (E_s/E_1) + F_b(h_e/a)[1 - (E_s/E_1)] \tag{15}$$

in which F_b is a function of (h_e/a) as defined by Equation 12. Thus, it is concluded that Equation 8 (GPP.20) constitutes once again an extension through the introduction of the μ-terms of Equation 13, whereas Equation 15 is its simpler, preferable form.

A thorough review of the pertinent literature reveals that Equation 14 was first presented by Barber (*12*, Closure), whereas the derivation of Equation 13 was outlined very briefly by Barber in a subsequent discussion (*19*). Examination of this derivation reveals a discrepancy between Equation 15 and the corresponding formula proposed by Odemark (*11*). Both Barber and Odemark begin by asserting that the maximum deflection in a two-layered system, Δ_0, consists of two parts:

1. The deflection due to the subgrade, Δ_s; and
2. The compression of the upper layer, Δ_1, that is, the difference in the deflection observed at its surface and the corresponding one experienced at its underside.

Both these investigators are in essential agreement concerning the calculation of Δ_s, as the deflection at depth $z = h_e$ in the homogeneous half-space with modulus, E_s. It is in the calculation of Δ_1 that the aforementioned discrepancy arises. Odemark (*11*) provides more details as to how Δ_1 is determined. This is accomplished by transforming the two-layered system into a homogeneous half-space of modulus E_1, which reduces the thickness of the top layer from h to

$$g_e = n_1 h \tag{16}$$

where n_1 is equal to about 0.9 for $\mu_1 = \mu_s = 0.5$. According to Odemark,

$$\Delta_1 = 1.5\frac{pa}{E_1}[1 - F_b(g_e/a)] \tag{17}$$

Note that F_b in Equation 17 is a function of the equivalent thickness, g_e, rather than of h_e, which was used in the calculation of Δ_s. Barber, on the other hand, uses h_e for Δ_1 as well. It is evident, therefore, that Equation 15 expresses the sum of Δ_s and Δ_1, both determined as functions of h_e, as suggested by Barber. The consequence of this discrepancy, however, on the calculation of Δ_0 is quite benign. Δ_s is significantly larger than Δ_1 in most cases, which perhaps explains in part why the discrepancy has gone unnoticed until now. Comparisons with Burmister (*13*) indicate that the error introduced by Equation 15 increases as E_1/E_s increases, that is, as h_e diverges from h. This is the case of a stronger top layer on a weaker subgrade. The discrepancy also increases as h/a decreases, that is, when a larger loaded area is applied on a thinner top layer. Note, however, that in both these cases, the contribution of Δ_1 to the total deflection, Δ_0, is relatively minor. For example, for $E_1/E_s = 10{,}000$ and $h/a = 0.1$, the error in Δ_1 itself is about 17 percent, but the resulting error in Δ_0 is less than 0.01 percent. It is not surprising, therefore, that Barber reported that his method is "practically the same numerically" as that of Burmister (*13*).

Use of h_e in the calculation of both Δ_s and Δ_1 fortuitously permits writing the relatively simpler expression for F_w, given by Equations 8 or 15. The more rigorous form (assuming $\mu_s = 0.5$) should have been

$$F_w = (E_s/E_1)[1 - F_b(g_e/a)] + F_b(h_e/a) \tag{18}$$

which is identical to the corresponding formula presented by Odemark (*11*).

In light of the above discussion, it is recommended that the equations presented in Table 2 be adopted instead of the corresponding ones in the Guide.

Derivation of Structural Number Equations

The structural number (SN) equations presented in Appendixes GPP and GNN (and summarized on p. GN-8) can be simplified considerably by the adoption of the proposed equations and by neglecting the Poisson ratio terms. The derivation presented below follows Appendixes GPP and GNN closely in all other respects and yields equations in which the inch is the unit of length and the pound is the unit of force.

TABLE 2 RECOMMENDED EQUATIONS

Argument	Equation Recommended	Instead of Equation
Δ_0	7	GPP.19 or GNN.26
F_w	18	GPP.20 or GNN.27
F_b	12	GPP.21 or GNN.28
h_e	11	GPP.22 or GNN.29
g_e	16	–

By definition,

$$\mathrm{SN} = \sum_{i=1}^{n-1} h_i a_i \tag{19}$$

(see Equations GPP.2 and GNN.8). For two "structurally equivalent" layers, that is, layers making the same contribution to SN,

$$h_g = \frac{a_i}{a_g} h_i \qquad i, g = 1, \ldots n - 1; g \neq i \tag{20}$$

(see Equations GPP.3 and GNN.9) or

$$a_i = \frac{h_g}{h_i} a_g \tag{21}$$

(see Equations GPP.4 and GNN.10).

According to the Guide (pp. GPP-3 and GNN-8), the structural number of any layer, SN_i, is directly proportional to the layer flexural stiffness. Thus, ignoring Poisson effects,

$$\mathrm{SN}_i = h_i a_i \propto E_i h_i^3 \tag{22}$$

whence

$$\frac{h_g}{h_i} = \frac{a_i}{a_g} = \left(\frac{E_i}{E_g}\right)^{1/3} \tag{23}$$

(Cf. Equations GPP.7 and GNN.12.)

Substituting into Equation 19 (Equations GPP.2 or GNN.8),

$$\mathrm{SN} = \sum_{i=1}^{n-1} h_i a_i = \sum_{i=1}^{n-1} h_i a_g \left(\frac{E_i}{E_g}\right)^{1/3} \tag{24}$$

in which E_g and a_g are constants describing an arbitrary "standard" material. Thus, each in situ layer can be transformed into an equivalent thickness of the standard material using Equation 20 (GPP.3). Ignoring the loss in accuracy involved in each such transformation, Equation 24 (GPP.9) can be written as

$$\mathrm{SN} = \frac{a_g}{E_g^{1/3}} \sum_{i=1}^{n-1} h_i E_i^{1/3} \tag{25}$$

(Cf. Equations GPP.16 and GNN.21.)

The values of E_g = 30,000 psi and a_g = 0.14 are adopted in the Guide, which correspond to the AASHO Road Test results for the crushed stone granular base:

$$\mathrm{SN} = \frac{0.14}{30{,}000^{1/3}} \sum_{i=1}^{n-1} h_i E_i^{1/3} \tag{26}$$

(cf. Equation GPP.16; see Equation GNN.25) or

$$\mathrm{SN} = 0.0045 \sum_{i=1}^{n-1} h_i E_i^{1/3} \tag{27}$$

(cf. Equation GPP.17; see Equation GNN.25).

For the two-layer system considered above (h, E_l, E_s), the last equation results in

$$E_1 = \left(\frac{\mathrm{SN}}{0.0045h}\right)^3 \tag{28}$$

(cf. Equation GPP.18) or

$$h = \frac{\mathrm{SN}}{0.0045 E_1^{1/3}} \tag{29}$$

Combining Equation 29 with Equation 11 yields, upon division of both sides by the radius of the applied (plate) load, a,

$$(h_e/a) = 200 \frac{\mathrm{SN}}{a} \frac{1}{E_s^{1/3}} \tag{30}$$

(cf. Equations GPP.23 and GNN.32), whence

$$(h_e/a)^2 = 40{,}000 \frac{\mathrm{SN}^2}{a^2} \frac{1}{E_s^{2/3}} \tag{31}$$

(cf. Equations GPP.24 and GNN.33). Substituting this expression into Equation 12 results in

$$F_b = \frac{1}{\left(1 + 40{,}000 \dfrac{\mathrm{SN}^2}{a^2} \dfrac{1}{E_s^{2/3}}\right)^{1/2}} \tag{32}$$

(cf. Equation GPP.27). Equation 18 therefore gives (assuming $g_e \approx h$)

$$F_w = (E_s/E_1)\left\{1 - \frac{1}{[1 + (h/a)^2]^{1/2}}\right\} + \frac{1}{\left(1 + 40{,}000 \dfrac{\mathrm{SN}^2}{a^2} \dfrac{1}{E_s^{2/3}}\right)} \tag{33}$$

Now Equation 7 may be introduced into Equation 33 to yield

$$\Delta_0 = 1.5\frac{P}{\pi a}\left(\frac{1}{E_1}\left\{1 - \frac{1}{[1 + (h/a)^2]^{1/2}}\right\} + \frac{1}{E_s\left(1 + 40{,}000\dfrac{\mathrm{SN}^2}{a^2}\dfrac{1}{E_s^{2/3}}\right)^{1/2}}\right) \tag{34}$$

Substituting for E_1 according to the modified Equation 28 (GPP.18),

$$\Delta_0 = 1.5\frac{P}{\pi a}\left\{\frac{(0.0045h)^3}{\mathrm{SN}^3}\left[1 - \frac{1}{[1 + (h/a)^2]^{1/2}}\right] + \frac{1}{E_s\left(1 + 40{,}000\dfrac{\mathrm{SN}^2}{a^2}\dfrac{1}{E_s^{2/3}}\right)^{1/2}}\right\} \tag{35}$$

(cf. Equation GPP.26).

STRUCTURAL NUMBER CONCEPT: ASSUMPTIONS AND DEFINITIONS

NDT-Method 2 provides an illustration of the problems that may arise when statistical/empirical and mechanistic data interpretation procedures are combined. NDT methods and backcalculation schemes are sophisticated mechanistic tools, developed recently and employed successfully in pavement studies. Notwithstanding any approximations they introduce, the equations derived on the basis of layered elastic theory (including MET) are also mechanistic in nature. The fundamental weakness of the mathematical derivations presented in Appendixes GNN and GPP lies in the fact that such mechanistic, rigorous, and analytical methods are combined—by necessity—with the entirely statistical/empirical structural number concept, developed on the basis of AASHO Road Test data. It is ironic that this very fact is heralded in the Guide as one of the strengths of the design procedure presented (p. GNN-11). The fundamental presuppositions underlying the structural number concept are discussed below, and their incompatibility with a mechanistic understanding of the pavement system is highlighted.

The assumptions and definitions constituting the concept of the structural number may be stated explicitly as follows. The pavement consists of series of interchangeable individual layers, whose contribution to the structural behavior of the pavement system is uniquely defined by each layer's intrinsic, individual, and unchanging characteristics, namely, thickness and layer coefficient. Deriving from its statistical/empirical nature is the fact that the structural number concept ignores the effect of the interactions between the various layers of the pavement system. Instead, it considers that a given layer behaves (or contributes to the structural capacity of the system) in exactly the same manner, independent of the pavement layer sequence it finds itself in. The adequacy of the structural number concept has been the subject of considerable debate (*20–24*). On the other hand, the simplification it affords in pavement design is unquestionable.

The major weakness of the structural number concept is that emphasis is placed exclusively on pavement materials, rather than on the behavior of the pavement as a system of interacting components. This limitation is also inherent in the conventional classification of all pavements as "flexible" or "rigid," primarily on account of the material of the surface layer. Furthermore, the structural number concept ignores the influence on pavement system behavior of two very important factors, namely, subgrade support and geometry of the applied load. In real in situ pavement systems exhibiting nonlinear or stress-dependent behavior, the concept also ignores the effect of load level. Thus, this statistical/empirical concept may be expected to serve its intended purpose as a design tool adequately only as long as these factors are similar to those prevailing at the AASHO Road Test, which provided the original data from which the structural number concept was developed.

Consider now the understanding of the pavement as a structural system, inherent in layered elastic theory. This is illustrated by the solution to the two-layered system problem, first presented by Burmister (*13*), who showed on the basis of mathematical formulations that the behavior of such a pavement system is uniquely defined by two dimensionless ratios, namely, h/a and E_1/E_s. Note that the effect of load level is essentially eliminated by the use of linear elasticity, whereas the applied pressure, p, is introduced simply as a multiplication term in the complete Burmister equations (see, for example, Equation 7). The major load attribute identified by theory is load geometry, which is reflected in the presence of load radius, a, in one of the two dimensionless ratios.

It is worth reflecting on the implications of the Burmister solution. Layered elastic theory states that the behavior of the pavement system is the result of the relative magnitude of the structural parameters of the component layers. The contribution of the thickness of the top layer can only be assessed after it has been compared with the size of the applied load, in the form of the h/a ratio. Similarly, the contribution of the elastic modulus of a given layer is of limited consequence when considered by itself. The dimensionless ratio E_1/E_s clearly illustrates that the governing system characteristic is the relative stiffness of the various layers, including the supporting foundation.

The same understanding and definition of a pavement system (as consisting of the interaction of the placed layers, of the natural supporting subgrade, and of the geometry of the applied load) is also evident in Westergaard's treatment of the slab-on-grade problem (*25*). Traditionally, comparisons of the layered elastic and plate theories have tended to highlight only the points on which Westergaard's analysis differs from that of Burmister. Yet, the congruence of these two pioneers on what constitutes a pavement is of much more fundamental significance.

According to plate theory, the governing dimensionless ratio is a/l, where l is the radius of relative stiffness of the slab-subgrade system (*26*). Thus, in a single ratio, a/l, all three fundamental components of a pavement system (overlying manmade layers, supporting natural medium, and geometry of applied load) are lumped in a manner that reflects their interaction and the significance of their relative (rather than absolute) magnitudes.

The understanding of the pavement system underlying the works of such respectable investigators as Westergaard and Burmister is based on a mechanistic evaluation of engineering behavior, not merely of materials. Consequently, structural response is a function of the relative (not absolute) magnitudes of the layer elastic characteristics and incorporates the effect of subgrade support and of the geometry of the applied load. This philosophy is therefore incompatible with the structural number concept.

CAN LAYER COEFFICIENTS BE DERIVED USING LAYERED ELASTIC THEORY?

A similar problem arises when layered elastic theory is used in determining layer coefficients for each of the in situ materials, as required by NDTM2. With the proliferation of a wide variety of construction materials and loading and support conditions in recent years, the adequacy of the original AASHO Road Test layer coefficients has been considerably decreased. This has created the need for the development of additional layer coefficients. The most common approach for this purpose has been the utilization of layered elastic theory (*27–31*). Thus, it is implicitly assumed that a layer coefficient can be uniquely defined if one knows the elastic characteristics (E_i, μ_i) of the material in that layer.

For the purposes of NDTM2, in particular, the following assertion is made at the start of the derivation presented in Appendix GPP: "If one views [two] pavement layers as having equal SN values, it can be concluded that the '[flexural] stiffness' $[E_i h_i^3/12(1 - \mu_i^2)]$ of both layers is also identical" (p. GPP-3). The weakness of this assertion lies in the following: The structural contribution of a layer is in a qualitative sense proportional to its flexural stiffness only if the material is linear, elastic, homogeneous, and isotropic. In practice, this translates to a bound material whose load-distributing capacity derives primarily from its ability to bend rather than to compress. Portland cement concrete slabs are comfortably within this category, but cement-treated or asphalt-treated materials may also be reasonable candidates.

Unbound granular materials, possessing no tensile strength, cannot be described adequately in terms of a "flexural stiffness." A case can be made that the concept is not even applicable to asphalt concrete. Although this material may possess significant tensile strength, it also exhibits a strongly viscoelastic, temperature- and loading-rate-dependent behavior, incompatible with the requirements of beam or slab bending theory. Furthermore, even for a system consisting of a series of layers each of which exhibits primarily flexural characteristics, total structural capacity is not merely proportional to the additive sum of the individual "flexural stiffness" of the layers. This is because the layers interact, each being influenced by and influencing every other layer. Thus, sequencing of layers, subgrade support provided, and geometry (if not the magnitude as well) of applied loadings need to be taken into account when structural system response is assessed.

A simple example suffices as illustration of the weakness of the basic assertion in Appendix GPP, quoted above, as well as the pitfalls of deriving layer coefficients using layered elastic theory. Consider a 4-in.-thick asphalt concrete (AC) layer whose modulus is 200 ksi. From Figure G2.5 (p. GII-19), $a_1 = 0.3$. The SN value for this layer is

$$SN_1 = h_1 a_1 = 4 \times 0.3 = 1.2$$

A granular layer with a modulus of 25 ksi and a layer coefficient, a_2, of 0.12 (from Figure G2.6, p. GII-20) will have the same structural number if its thickness, h_2, is 10 in. Then

$$SN_2 = h_2 a_2 = 10 \times 0.12 = 1.2 = SN_1$$

According to Appendix GPP, the following should be true:

$$\frac{E_1 h_1^3}{12(1 - \mu_1^2)} = \frac{E_2 h_2^3}{12(1 - \mu_2^2)} \qquad (36)$$

(cf. Equation GPP.5).

Yet, for the AC layer (ignoring the contribution of the μ-terms)

$$E_1 h_1^3 = 200{,}000 \times (4)^3 = 12{,}800{,}000 \text{ lb-in.}$$

whereas for the granular layer

$$E_2 h_2^3 = 25{,}000 \times (10)^3 = 25{,}000{,}000 \text{ lb-in.}$$

Thus, the two quantities assumed to be equal by Equation 36 (GPP.5) are really in the ratio of 1:2. If asphalt concrete is considered to be the reference material, a prediction with respect to the granular layer would be off by 100 percent, simply on account of Equation 36 (GPP.5). In a multiple-layer structure involving several transformations of the type described by Equations GPP.10 through GPP.13, discrepancies of a similar magnitude may be expected to be compounded, and not necessarily in an additive fashion.

Furthermore, the log-log relationship between maximum deflection, Δ_0, and the effective in situ structural number, SN_{xeff}, as a function of E_s derived in Appendix GPP for the Falling Weight Deflectometer (Figure GPP.2) conceals the extreme sensitivity of SN_{xeff} to Δ_0. When this figure is redrawn on an arithmetic plot, it becomes apparent that a very small error in Δ_0 can lead to a considerable error in SN_{xeff}, particularly for stiffer subgrades.

CONCLUSIONS AND RECOMMENDATIONS

A major problem facing researchers attempting to expand or improve the AASHTO design procedures is the incorporation of mechanistic concepts into a statistical/empirical framework. Recent investigations at the University of Illinois have provided a considerable amount of evidence that such a process rarely leads to reliable conclusions. The reverse approach, that is, the incorporation of statistical/empirical concepts in a mechanistic methodology, has been routinely adopted by practically all current design procedures. The most common statistical/empirical ingredients of current "mechanistic-based" procedures are the equivalent single axle load (ESAL) concept and the so-called "Miner's linear cumulative fatigue hypothesis." It is perhaps the proliferation of mechanistic-plus-statistical/empirical amalgams that has resulted in the assumption that statistical/empirical-plus-mechanistic mixtures are permissible, too. It must be recognized, however, that in the pursuit of developing a mechanistic procedure, use of statistical/empirical concepts is merely a matter of practical expediency. Concepts based exclusively on a statistical interpretation of accumulated information without any consideration of system behavior and factor interactions may provide the necessary bridges over gaps remaining in our ability to analyze mechanistically the complex problems associated with pavement design. As the state of the art in pavement modeling and analysis methods is improved, however, one by one these gaps are closed. Consequently, the statistical/empirical bridges are gradually being substituted by more reliable mechanistic planks, based on sound theoretical precepts, verified and calibrated—where necessary—by experimental field or laboratory observations. The verification and calibration process is by definition empirical, and is most effective when it employs statistics merely as a curve-fitting tool. Whenever possible, the fundamental cause-and-effect relations involved in the phenomena observed empirically should be interpreted in the light of mathematical formulations of basic laws of engineering mechanics, rather than heuristic rules of thumb that are valid only in a statistical sense. Efforts aimed at replacing statistical/empirical constructs (e.g. SN, ESAL, and Miner's fatigue concepts) by more mechanistic procedures should therefore be intensified, and attempts to define statistical/empirical parameters (e.g., layer coefficients, PSI, and load equivalency factors) using mechanistic theoretical tools should be abandoned.

Such considerations coupled with the theoretical weaknesses identified above suggest that use of NDTM2 should be discouraged. The justification for the original development of NDTM2, namely, the difficulty of conducting conventional deflection basin (backcalculation) analyses as required by NDT Method 1, can no longer be considered valid. Equipment for deflection basin testing is nowadays readily available, and comprehensive backcalculation programs have been developed for all types of pavements and are routinely used. Worth mentioning in particular is a backcalculation program developed recently (*32*, *33*) that evaluates directly the in situ subgrade modulus, k, thus eliminating the need for questionable correlations between E_s and k. If NDTM2 were to be used at all, it would be advisable to use the simpler, corrected equations derived in this paper, instead of those in the Guide.

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