Abridgment

Wave Model of Mat Foundation Movement on Expansive Clays

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Soil differential movement patterns from soil moisture changes cause considerable damages to all types of structures and pavements. Such damage may be assessed by modeling foundation movements with wave patterns. Angular distortion, an indicator of the degree of damage, may be readily evaluated from wave patterns of elevation profiles obtained with close spacings between measuring points, such as 1 ft (0.3 m). Preliminary data based on total deformations from elevation profiles taken in four facilities indicate that the performance of mat foundations for resisting wall cracks may be quantified by simple rating systems. The maximum relative thickness D_{relm} derived as part of this study was found consistent in rating the performance of these four facilities and also indicated the location of the most serious distortion in a given facility. The concept of D_{relm} also has potential as a design aid for foundations. Other potentially useful rating systems include the wave index.

Soil differential movement patterns cause considerable damages to all types of structures and pavements such as military facilities, commercial buildings, houses, pavements, and parking lots. Differential soil movement may accumulate almost immediately after construction and continue to increase over many years. Moisture changes in expansive clay soils are a common cause of destructive differential movements. These patterns are hypothesized as being representative of wave motion with relatively long wavelengths exceeding 4 ft.

DAMAGE ASSESSMENT BY SOIL AND FOUNDATION WAVE PATTERNS

Differential soil movement may be described with wave motion by using angular distortion, as shown in Figure 1.

$$\beta = \frac{2A}{l/2} = \frac{4A}{l} \tag{1}$$

where

β = average angular distortion bounded by crest and trough of wave;
A = amplitude of the wave pattern as restrained by the foundation, ft (m); and
l = wavelength of the movement pattern, ft (m).

Damage such as from cracks in walls, distorted floors, and misaligned door and window frames may be correlated with angular distortion (*1*,*2*). A slight degree of damage is expected if $\beta > 0.0015$ (*2*).

A reduction factor R_f relates restrained heave or amplitude A of soil beneath the mat with unrestrained amplitude A_u of the soil-foundation wave pattern.

$$R_f = \frac{A}{A_u} \tag{2}$$

The soil wavelength is assumed to remain unaltered when restrained by the foundation and the foundation is assumed to remain in complete contact with the soil. A_u is a measure of potential soil heave. R_f is analogous to a reduction factor that is the differential settlement beneath the mat restrained by mat stiffness divided by unrestrained differential settlement of a fully flexible mat; it is a function of relative stiffness K_s, shown in Figure 2, defined as (*3*,*4*)

$$K_s = \frac{E_c}{E_s} \cdot \frac{D_e^3}{\left(r \cdot \frac{l}{2}\right)^3} \cdot (1 - \nu_s^2) \tag{3}$$

where

E_c = modulus of concrete or foundation elasticity, ksf;
D_e = equivalent mat thickness, ft;
ν_s = soil Poisson's ratio, 0.4;
E_s = modulus of soil elasticity, ksf;
l = wavelength of soil movement, ft;
L = length of the mat, ft (m);
B = width of the mat, ft (m);
R = equivalent radius, $(LB/\pi)^{1/2}$ ft (m); and
r = ratio of mat diameter to the wavelength, $2R/l$.

The equivalent mat thickness D_e is the thickness of a flat mat or may be a measure of the thickness of ribbed mats when the stiffness contributed by stiffening beams is included.

The effective mat diameter $2R_c$ equals l when $2R$ exceeds the wavelength l because a mat with $2R > l$ can ride on top or span the wave, whereas a mat with $2R < l$ may tilt into the depression of the wave. The critical case occurs when D_e is maximum, i.e., $2R_c = l$ and $r = 1$.

A new term, relative thickness D_{rel}, may be defined from Equation 3

$$D_{rel} = D_e \left[\frac{E_c}{E_s} \cdot (1 - \nu_s^2)\right]^{1/3} = R_c(K_s)^{1/3} \tag{4}$$

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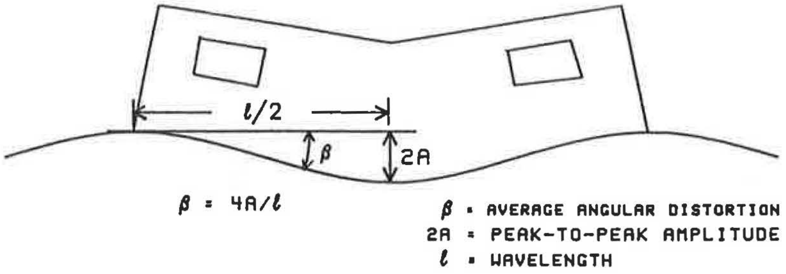

FIGURE 1 Angular distortion related to wave motion.

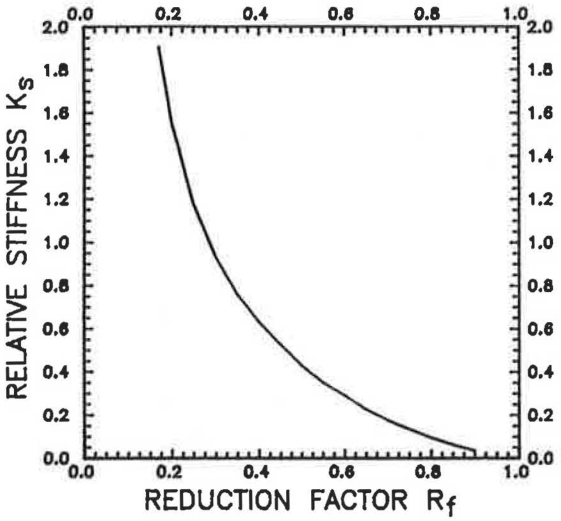

FIGURE 2 Relationship between reduction factor R_f and relative stiffness K_s (3,4).

where D_{rel} is in ft (m). D_{rel} is dependent only on R_c ($= l/2$) and K_s and may be calculated from given elevation profiles without knowledge of soil or foundation elastic parameters. D_{relm}, the maximum value of D_{rel} calculated for an elevation profile, is determined for a specific location and is expected to have the most potential for damage. D_{relm} may therefore be useful for rating performance of foundations while indicating the location of the most damaging distortion in the profile. D_{relm} is calculated from measured elevation profiles using the ADATG computer program.

For example, the full-loop elevation profile of line ATC1 of the Automated Technology Center, Figure 3, exhibits a large dip between 200 and 246 ft with a peak-to-peak (relatively) unrestrained amplitude $2A_u$ of about 1 in. and span length l of about 46 ft. The restrained amplitude $2A = \beta \cdot l/2$ from Equation 1 is $(0.0015)(46)(12/2) = 0.4$ in. Therefore, from Equation 2, $R_f = 2A/2A_u = 0.4$. For $R_f = 0.5$, K_s from Figure 2 is about 0.6. D_{rel} from Equation 4 is $(46/2)(0.4)^{1/3}$, or about 19 ft, which is the maximum value D_{relm} for this profile.

Relative thickness, calculated from Equation 4 using Equation 2, may be plotted versus $1/2R_c$ (or frequency $1/l$) for different angular distortions to determine how critical frequencies change depending on tolerable angular distortions β. The results of this type of analysis indicate that the relative thickness will be a maximum, D_{relmax}, at a particular frequency f_c, defined as follows:

$$D_{relmax} = \frac{0.066A_u}{\beta} \tag{5a}$$

$$f_c = \frac{4.8\beta}{A_u} \tag{5b}$$

in which A_u is in inches. If mat dimension $2R < 2R_c = l_c$, then relative thickness may be less than D_{relm} and may be calculated from Equation 4 setting $R_c = R$. Equations 5 may also be applied to pavements.

PERFORMANCE RATING SYSTEMS

Systems to quantify deformation patterns are useful to determine a way of rating foundation performance. The method of quantifying distortion should be independent of the length of the floor used to determine the elevation profile for a given spacing between measurement points Δx. This provision provides flexibility in making comparisons between elevation profile measurements obtained from different size structures. Three systems that are independent of the length of the elevation

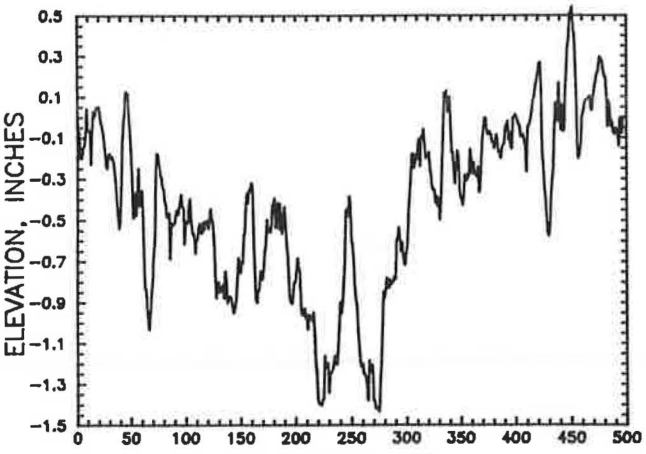

FIGURE 3 Elevation profile of line ATC1 of the Automated Technology Center for a full loop.

profile in addition to D_{relm} and may be useful to quantify damage to structures from foundation distortion are *F*-numbers FL and FF (ASTM E1155M-87), wave index WI (*5*), and a microrelief index MIF (*6*). FF is associated with floor flatness, whereas FL is associated with floor levelness. WI was evaluated over a spacing from 1 to 50 ft; therefore, WI = WI50. These systems provide a measure of average floor distortion over the length of the profile, whereas D_{relm} is a measure of extreme distortion in the profile.

The four rating systems are applied to four facilities briefly described in Table 1 with a description of damage. The Coastal Engineering Research Center (CERC) and Automated Technology Center (ATC) facilities are located in the Waterways Experiment Station, Vicksburg, Mississippi, whereas the Troop Medical Clinic (TMC) and Troop Dental Clinic (TDC) are located in Fort Sam Houston, Texas. These facilities are single-story, masonry structures constructed with initially horizontal floors. Several elevation full-loop profiles using an automated dipstick floor profiler with spacing $\Delta x = 1$ ft (0.3 m) were conducted along straight lines in hallways of each facility. These profiles include total deformations. Additional future surveys are required to deterine foundation distortion caused entirely by soil deformations; however, these preliminary surveys should indicate gross trends and provide a basis for analysis of future readings.

Analysis of the floor profiles using the rating systems are shown in Table 2 using the full loop. Walls adjacent to survey lines with the least *F*-numbers and the largest WI50, D_{relm}, and MIF values of a particular facility should be associated with the most damage. D_{relm} consistently provides the largest values associated with the most damage of all the survey lines of a given facility. WI50 is nearly consistent with the data.

CONCLUSIONS

Soil deformation may be modeled by waves. Angular distortion, an indicator of potential damage, is readily evaluated from wave patterns. The thickness of mat foundations required to accommodate a given unrestrained soil deformation may be calculated for a limiting angular distortion.

Preliminary data based on total deformations of some mat foundations indicate that the performance of structures relative to cracking in walls may be rated by simple systems. D_{relm} was consistent with cracking observed in the facilities, has the advantage of indicating the location of potential damage of walls from mat distortions, and may also assist in estimating the thickness of mat foundations required to reduce

TABLE 1 DESCRIPTION OF FACILITIES

Facility	Foundation	Performance
Coastal Engineering Research Center	4-inch slab-on-grade with spread footings	Large fractures 0.2 inch (5 mm) wide top of exterior walls at the middle adjacent and parallel with lines CERC4 and CERC5; performance near remaining survey lines satisfactory; elevation profiles vary in length from 100 to 200 ft (30 to 61 m); constructed in stages from 1965 to the present
Automated Technology Center (ATC)	8-inch slab-on-grade with spread footings	Jagged fracture in dry wall up to 0.2 inch (5 mm) wide and 8 ft (2.4 m) high adjacent and parallel with survey line ATC1 225 ft (67 m) from the initial point about 28 December 1989; constructed August 1989
Troop Medical Clinic (TMC)	Stiffened ribbed mat	Dry wall adjacent and parallel with line TMC5 contains several cracks near top of the wall up to 0.13 inch (3 mm) wide; elevation profiles vary from 80 to 200 ft (24 to 61 m); constructed during 1980 and 1981
Troop Dental Clinic (TDC)	Stiffened ribbed mat	Dry wall adjacent and parallel with line TDC7 contains minor cracking up to 0.06 inch (2 mm) wide; remaining walls less affected by distortions; overall performance satisfactory; elevation profiles varied from 150 to 200 ft (46 to 61 m); constructed during 1980 and 1981

TABLE 2 PERFORMANCE RATINGS OF FACILITIES

Facility	Date of Survey	Survey Line	F-Numbers FL	F-Numbers FF	Wave Index WI50, in.	Maximum Relative Thickness D_{relm}, ft	Microrelief Index MIF
CERC	07/20/89	CERC1	20.71	20.73	0.1061	5.7944	0.3131
		CERC2	16.82	17.64	0.1172	6.4958	0.2276
		CERC3	23.79	23.37	0.1020	6.8294	0.1630
		•CERC4	22.20	24.20	0.2261	7.7983	0.2604
		•CERC5	23.40	25.28	0.2160	13.7278	0.2103
ATC	12/12/89	• ATC1	14.75	14.25	0.2029	18.7418	0.5627
		ATC6	16.63	17.73	0.1289	2.8157	0.2413
		ATC7	15.22	14.34	0.2638	7.6883	0.1488
		ATC8	16.66	18.46	0.1257	8.6827	0.2144
		ATC9	16.30	17.94	0.1353	11.4439	0.2305
		ATC10A	19.51	19.05	0.1454	5.5272	0.2071
		ATC11	13.76	13.70	0.1499	9.5875	0.2319
TMC	02/01/90	TMC3	22.43	23.73	0.2027	12.0680	0.5148
		TMC4	27.19	26.11	0.0982	5.3678	0.1828
		• TMC5	18.91	21.73	0.2856	23.6370	0.4685
TDC	02/01/90	TDC6	17.43	18.64	0.1353	8.8534	0.2013
		• TDC7	16.42	18.07	0.2709	10.6759	0.3256
		TDC8	14.40	20.21	0.2610	8.6779	0.2069

Note: • indicates survey line adjacent to wall with most damage

a given soil deformation. WI50 was nearly consistent with all of the data.

ACKNOWLEDGMENT

This study was supported by the Office of the U.S. Army Chief of Engineers, Washington, D.C. Permission was granted by the Chief of Engineers to publish this information.

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The views of the authors do not purport to reflect the views of the Department of the Army or the Department of Defense.

Publication of this paper sponsored by Committee on Environmental Factors Except Frost.