Mathematical Model of Skid Resistance as a Function of Speed

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Skid resistance has been modeled as an exponential function of speed involving two parameters, zero-speed intercept and percent normalized gradient. Several inconsistencies associated with the percent normalized gradient are presented, and an alternative parameter called a "speed constant," which is free of those inconsistencies, is proposed. Two methods for identifying the model parameters are evaluated using test data collected on road sections in Pennsylvania. Both methods produce relatively accurate predictions of skid number at different speeds even though the estimates of the model parameters calculated in each method are different. The differences between the parameter estimates obtained from the two methods clearly demonstrate that great care must be exercised in relating the parameter estimates to pavement micro- and macrotexture characteristics. The normalized texture index, a new measure of combined micro- and macrotexture, is introduced. Finally, the effect of speed on the model parameters is investigated first analytically and then using experimental data.

Vehicle speed is one of the dominant factors affecting tire-pavement friction. The effect of speed is particularly strong when the pavement is wet. On a wet pavement surface, water trapped between the vehicle tire and the pavement surface can be expelled through the channels provided by the pavement macrotexture and by the tire tread. When vehicle speed increases, the time available for expelling water from the tire-pavement interface decreases and the lubricating effect of water becomes stronger even if the vehicle tires are new and the pavement macrotexture is good. As a result, pavement skid resistance decreases with increasing speed at a rate determined by the surface macrotexture. The larger the macrotexture, the smaller the rate of decline of skid resistance.

Because skid resistance depends on vehicle speed, the results of skid resistance measurements have to be adjusted when a pavement skid number at a speed other than the test speed is needed. Two methods have been developed for adjusting skid number to different speeds (*1*). The first method requires that measurements of skid resistance with the ASTM ribbed tire be performed at different speeds. The other method is based on skid tests at a single speed but with two different tires. Both methods produce estimates of two parameters, SN_0 and PNG, which are used in the mathematical model of the relationship between skid resistance and speed proposed by Leu and Henry (*2*). That model, known as the Penn State model, has the following form:

$$SN_v = SN_0 e^{-(PNG/100)v} \tag{1}$$

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where

v = vehicle speed,
SN_0 = skid number zero-speed intercept,
PNG = percent normalized gradient, and
SN_v = skid number at speed v.

The main objectives are to reevaluate the Penn State model with particular emphasis on the physical significance of the model parameters SN_0 and PNG, and to compare the two methods for adjusting skid number to a different speed. Skid resistance data collected during a recently completed study will be used to accomplish these objectives (*3*).

MODIFICATION OF THE PENN STATE MODEL

The first parameter of the Penn State model, SN_0, is related to the pavement microtexture. The physical significance of SN_0 is clear. It is the skid number for the sliding speed of the vehicle tire equal to zero. The second parameter, PNG, is defined by the following equation:

$$PNG = -100(dSN_v/dv)/SN_v \tag{2}$$

PNG is related to pavement macrotexture. The separation of the effects of micro- and macrotexture between the two model parameters is considered to be a significant advantage of the model. More important, Equation 1 represents an exponential relationship between skid resistance and speed, a relationship that proved to be in excellent agreement with experimental data (*1,2,4*).

In general, there is no doubt that the exponential form of the Penn State model is adequate for the physical process that the model describes. However, there are certain flaws associated with one of the model parameters, namely PNG. First, PNG as defined by Equation 2 is obtained by multiplying the normalized gradient of skid resistance versus speed by -100. The negative sign is used to make PNG a positive number. Although this may seem reasonable, in fact it is misleading because a positive gradient implies that the dependent variable (skid resistance in this case) increases when the independent variable (speed) increases, which is not true in this case. Second, multiplying the normalized gradient by 100 to make its values more reasonable does not quite justify the use of the word "percent" in PNG. The word "percent" implies that PNG equal to 100 percent has some special and clear significance, but it has none, except that it is totally unrealistic (typically, PNG values for existing pavements do not exceed

3 to 4 percent). Moreover, the meaning of a particular numerical value of PNG is not clear. For instance, what does it mean in terms of the pavement macrotexture or in terms of pavement skid resistance if PNG is found to be equal to, say, 1 percent? Furthermore, the use of the term "percent" is a source of inconsistency in the PNG units because the physical units of PNG are hours per mile or hours per kilometer and not percent.

These problems can be resolved by replacing PNG with a different parameter to represent the rate of reduction of skid resistance because of speed. Before this new parameter is introduced, first consider a mathematical function that is commonly used to describe a free response of a linear dynamic system. That function, $y(t)$, is given by

$$y(t) = y_0 e^{-t/\tau} \tag{3}$$

where y_0 is the initial value of $y(t)$ and τ is the system time constant. The significance of the time constant is shown in Figure 1. It can be interpreted as the time necessary for y to decrease from its initial value y_0 to zero if y were decreasing at a constant rate equal to the initial rate dy/dt at time $t = 0$. The system time constant is also a measure of the system speed of response. It takes a period of time approximately equal to five time constants to change the exponential function given by Equation 3 from its initial value to within 2 percent of a new steady-state value, zero in this case (*5*).

The relationship of skid number versus speed can be expressed in a form similar to that of Equation 3.

$$\mathrm{SN}(v) = \mathrm{SN}_0 e^{-v/v_0} \tag{4}$$

Two parameters, SN_0 and v_0, are used in this equation. SN_0 is the same zero-speed intercept used in the Penn State model. The other parameter, v_0, is analogous to τ in Equation 3 and can be called a "speed constant" rather than a time constant.

Equation 4 is the modified Penn State model. The difference between the modified model and the original model is the speed constant v_0 that replaces PNG. The speed constant v_0 is defined as the speed at which the skid number would reach zero if it were decreasing at a constant rate equal to the initial rate $d\mathrm{SN}/dv$ at $v = 0$. It is expressed in units of speed, miles per hour or kilometers per hour. Figure 2 shows the significance of the parameters of the modified model. From Equations 1 and 4, the relation between PNG and v_0 is

$$v_0 = 100/\mathrm{PNG} \tag{5}$$

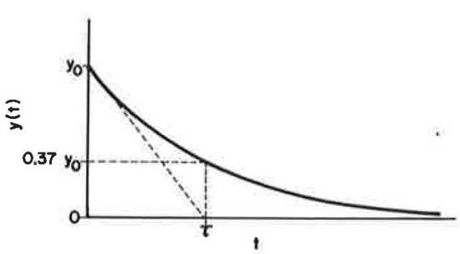

FIGURE 1 Free response of first-order linear system.

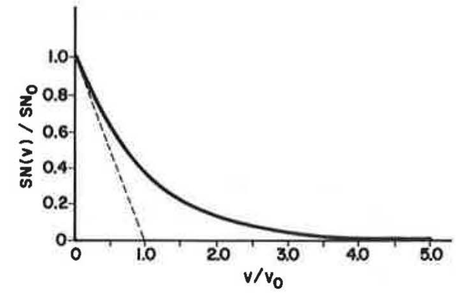

FIGURE 2 Modified model of skid resistance as a function of speed.

The speed constant is a measure of the rate of decline of skid number because of increasing speed, and, therefore, it is related to the surface macrotexture. A larger macrotexture is expected to result in a larger speed constant.

Although the proposed modification is minor (the general exponential form of the relationship is preserved), it clarifies the physical interpretation of the model and its parameters.

IDENTIFICATION OF MODEL PARAMETERS

In order to determine a skid number at a speed other than the speed of a skid test, the values of the parameters used in the skid resistance model (Equation 4) must be found. There are two methods for identifying the model parameters.

The first method, which will be called a direct method, can be applied when the pavement skid number is measured with the standard ASTM ribbed tire at two or more speeds (ASTM E501). When skid numbers at only two speeds, v_1 and v_2, are known, the model parameters can be calculated from the following equations:

$$v_0 = (v_2 - v_1)/\ln(\mathrm{SN}_{v1}/\mathrm{SN}_{v2}) \tag{6}$$

and

$$\mathrm{SN}_0 = \mathrm{SN}_{v1} e^{v_1/v_0} \tag{7}$$

or,

$$\mathrm{SN}_0 = \mathrm{SN}_{v2} e^{v_2/v_0} \tag{8}$$

Better estimates of SN_0 and v_0 can be obtained if skid numbers at more than just two speeds are measured. The estimates of the model parameters can then be found using the least squares technique (*6*).

The second method of determining SN_0 and v_0, called an indirect method, uses skid resistance measurements obtained with a ribbed tire, $\mathrm{SN^R}$, and a smooth tread (blank) tire, $\mathrm{SN^B}$, both at 40 mph (64 km/hr) (ASTM E524). The following statistical correlations for SN_0 and PNG have been developed from skid data collected in several states (*1*):

$$\mathrm{SN}_0 = 35.4 - 0.682\,\mathrm{SN^B} + 2.894\,\mathrm{SN^R} - 12.75\,(\mathrm{SN^R})^{1/2} + 24.7/(\mathrm{SN^B})^{1/2} \tag{9}$$

$$PNG = -0.49 - 0.01996\,SN^B + 0.0106\,SN^R + 0.113\,(SN^R)^{1/2} + 3.844/(SN^B)^{1/2} \quad (10)$$

The speed constant can then be found using Equations 5 and 10.

In order to evaluate the accuracy of the two methods for adjusting skid number to a different speed, results of skid tests with the ribbed tire at 30, 40, and 50 mph (48, 64, and 80 km/hr) and the blank tire at 40 mph (64 km/hr) conducted on 18 sites in Pennsylvania, including 11 bituminous and 7 concrete pavements, were used (*3*). The evaluation procedure was as follows. First, using the indirect method, the values of SN^R and SN^B measured at 40 mi/hr (64 km/hr) were used to calculate SN_0 and v_0 from Equations 9 and 10. Next, using the direct method, the SN^R measurements at 30 and 40 mph (48 and 64 km/hr) were used to calculate SN_0 and v_0 from Equations 6 and 7. The estimates of SN_0 and v_0 obtained from each method were then used to predict skid numbers at 50 mph (80 km/hr), which were compared with the measured values of SN_{50}. The numerical results are presented in Table 1. Figure 3 (*top*) shows the values of SN_{50} predicted using the indirect method versus the measured values of SN_{50}; Figure 3 (*bottom*) presents the same information for the direct method. The coefficient of correlation between the predicted and measured values of skid number at 50 mph (80 km/hr) is 0.95 using the indirect method and 0.94 using the direct method. On the basis of these results, both methods produce accurate results and can be recommended for adjusting skid numbers to different speeds. The accuracy of the direct method can also potentially be much better if measurements of skid resistance are conducted at more than just two speeds.

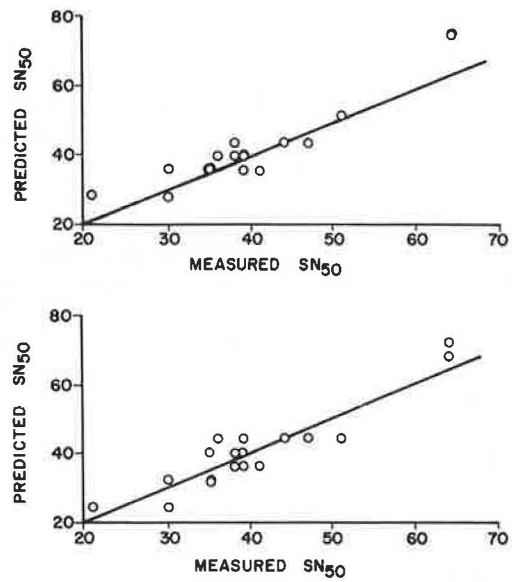

FIGURE 3 Predicted versus measured values of SN_{50} obtained (*top*) using the indirect method and (*bottom*) using the direct method.

MODEL PARAMETERS AND TEXTURE CHARACTERISTICS

As pointed out earlier, the two parameters of the skid number versus speed model—SN_0 and PNG in the original form given by Equation 1, and SN_0 and v_0 in the modified form given by Equation 4—represent two distinctly different characteristics

TABLE 1 PREDICTIONS OF SKID NUMBER AT 50 mi/hr (80 km/hr) PRODUCED BY DIRECT AND INDIRECT METHODS

		Predicted SN_{50}		Prediction Error	
Site No.	Measured SN_{50}	Direct	Indirect	Direct	Indirect
1	43.5	44.5	43.4	1.0	-0.1
2	37.9	38.3	38.4	0.4	0.5
3	35.1	39.4	36.6	4.3	1.5
4	46.7	43.7	44.1	-3.0	-2.6
5	35.1	32.1	34.2	-3.0	-0.9
6	30.3	31.7	35.9	1.4	5.6
7	36.3	43.0	40.8	6.7	4.5
8	21.3	24.4	27.2	3.1	5.9
9	34.5	33.6	34.3	-0.9	-0.2
10	50.6	45.0	52.0	-5.6	1.4
11	30.2	24.5	29.2	-5.7	-1.0
12	39.1	41.2	40.3	2.1	1.2
13	38.8	36.1	37.8	-2.7	-1.0
14	40.8	34.4	35.3	-6.4	-5.5
15	38.4	40.0	42.3	1.6	3.9
16	38.7	44.4	40.6	5.7	1.9
17	63.5	69.4	74.1	5.9	10.6
18	63.8	71.4	77.6	7.6	13.8
		Mean absolute error		3.7	3.5
		Root mean squared (RMS) error		2.3	3.8

of the pavement texture. The zero-speed intercept, SN_0, is related to the surface microtexture, defined as deviations from a planar surface with an amplitude smaller than 0.02 in. (0.5 mm). The model speed constant, v_0 (or the percent normalized gradient, PNG), is said to be related to the pavement macrotexture, defined as surface irregularities with an amplitude from 0.02 to 2 in. (0.5 to 50 mm). This interpretation of the model parameters is justified by the nature of the processes that take place between the tire and wet pavement surfaces. The level of skid resistance at low speeds, represented in the model by SN_0, is affected primarily by the microscale harshness of the pavement surface. As speed increases, the role of the microtexture is gradually reduced by the presence of a water film at the tire-pavement interface. Water is expelled from the tire footprint through the channels provided by tire tread and by surface macrotexture. If a standard tire is used, the rate of reduction of skid resistance because of increasing speed, represented by the model speed constant (or percent normalized gradient), is thus determined by the surface macrotexture.

From these considerations, it can be concluded that SN_0 and v_0 (or PNG) should provide unique, quantitative measures of surface micro- and macrotexture, respectively. In fact, Wambold et al. (*1*) and Leu and Henry (*2*), in separate studies, developed regression equations relating SN_0 to microtexture, represented by the British pendulum number (BPN), and equations relating PNG to macrotexture, measured by the sand patch method, which yields mean texture depth (MTD) (ASTM E965). Unfortunately, neither BPN nor MTD tests were performed in this study. However, if SN_0 and v_0 (or PNG) were indeed objective measures of surface texture characteristics, their values determined for the same sites with different methods would be close or, at least, would correlate well. Therefore, the existence of the relationship between the model parameters and the surface texture can be indirectly verified by evaluating the agreement between the values of SN_0 and v_0 obtained using the direct and indirect methods. Figure 4 shows the values of the zero-speed intercept calculated using the indirect and direct methods. The coefficient of correlation is 0.54. Figure 5 shows the values of the speed constant obtained using the two methods. The coefficient of correlation for these data is 0.46. All three sets of data collected at 30, 40, and 50 mi/hr (48, 64, and 80 km/hr) were used with the direct method. The considerable scatter of data and low coefficients of correlation obtained in this analysis raise serious doubts about whether the estimates of SN_0 and v_0 obtained from the direct and indirect methods may

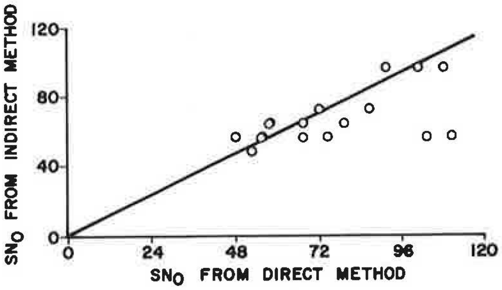

FIGURE 4 Comparison of zero-speed intercepts identified using indirect and direct methods.

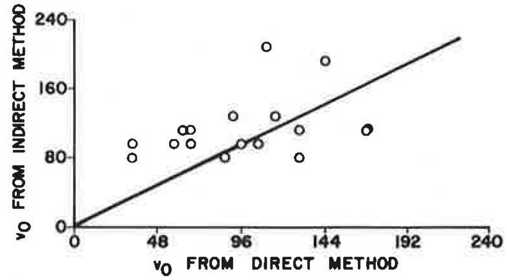

FIGURE 5 Comparison of speed constants identified using indirect and direct methods.

indeed be treated as reliable measures of pavement micro- and macrotexture. The results of statistical analyses of experimental data published by others do not alleviate those doubts completely. Leu and Henry (*2*) derived the following equation relating SN_0 and BPN:

$$SN_0 = -31 + 1.38BPN \tag{11}$$

It has a marginal coefficient of correlation, $R = 0.75$. Wambold et al. (*1*) found a different equation

$$SN_0 = 9.1 + 0.95BPN \tag{12}$$

with poor correlation, $R = 0.57$. The numerical results obtained from the above two equations are drastically different, e.g., for BPN = 50, Equation 11 produces $SN_0 = 38.0$, whereas Equation 12 gives $SN_0 = 56.6$.

Leu and Henry (*2*) obtained excellent correlation, $R = 0.96$, for the following relationship between PNG and MTD for 20 bituminous pavements in West Virginia.

$$PNG = 0.45MTD^{-0.47} \tag{13}$$

The units for MTD are millimeters and those for PNG are mph^{-1}. However, in the discussion section of the same paper, Chamberlin reported obtaining a very similar relationship from tests on concrete pavements in New York, but with the coefficient of correlation of only 0.61.

Wambold et al. (*1*) developed the following relationship between PNG and MTD:

$$PNG = -0.26 + 0.19/MTD \tag{14}$$

The coefficient of correlation is 0.96. However, this equation was derived from data collected on only seven sites. The significance of this correlation is further limited by the fact that it can only be applied for the range of mean texture depth from 0 to 0.53 mm (values of MTD greater than 0.53 mm produce negative values of PNG). It was also rather discouraging to observe that corresponding values of PNG calculated from Equations 13 and 14 differ dramatically.

It must be emphasized that, in spite of the rather negative observations regarding correlations between the model parameters and texture characteristics, the Penn State model has been quite accurate in predicting skid numbers at different speeds, as demonstrated in this as well as in other studies (*1,2*). Moreover, the Penn State model in the form of Equation 1 or 4 describes the relationship between skid resistance and speed adequately with model parameters that are related to the micro- and macrotexture of the pavement surface. However, what is calculated by the indirect or direct method is just least squares estimates of actual model parameters. Moreover, the estimates of both SN_0 and v_0 are identified simultaneously in the same process, and, as a result, they are not mathematically independent. The essence of the least squares technique is that it combines the effects of all of the model parameters together to best fit the experimental data. This is why a single-parameter estimate cannot quite stand alone, but it may still produce good results when combined with other parameters in the model equation.

In order to support the idea of the mutual dependence between the estimates of SN_0 and v_0, consider an index involving both estimates, defined by the following equation:

$$TI = SN_0 v_0 \quad (15)$$

This index will be called a texture index because its value is proportional to both microtexture (through SN_0) and macrotexture (through v_0). In order to avoid potential problems with measuring units, the texture index can be normalized with respect to the values of SN_0 and v_0 corresponding to the values of ribbed tire skid number $SN^R = 35$ and blank tire skid number $SN^B = 20$. These values are often considered to be the minimum acceptable values of skid resistance measured with the standard ASTM tires although they are not and should not, by any means, be treated as sole safety thresholds. They are used here only as a reference for normalization. A similar index based on laboratory measurements of microtexture and macrotexture was proposed by Forster (*7*).

The reference values of SN_0 and v_0, corresponding to $SN^R = 35$ and $SN^B = 20$, from Equations 5, 9, and 10 are

$$SN_{0ref} = 53.3 \quad (16a)$$

and

$$v_{0ref} = 100 \text{ mph (80 km/hr)} \quad (16b)$$

Thus, the normalized texture index, NTI, will be defined as

$$NTI = SN_0 v_0 / (SN_{0ref} v_{0ref}) \quad (17)$$

Using the numerical values for the reference zero-speed intercept and reference speed constant given in Equation 16b and rounding off their product, the expression for NTI takes the form

$$NTI = SN_0 v_0 / 5{,}300 \quad (18)$$

The values of NTI computed using the values of SN_0 and v_0 obtained from the indirect and direct methods are compared graphically in Figure 6. The coefficient of correlation is 0.85, which is considerably higher than the coefficients of correlation obtained for parameters SN_0 and v_0 separately, 0.54 and 0.46, respectively. The agreement between the NTI values is excellent, except for the two sites with the highest NTI values. Highway engineers are usually more concerned about the sites with texture in the lower range of NTI.

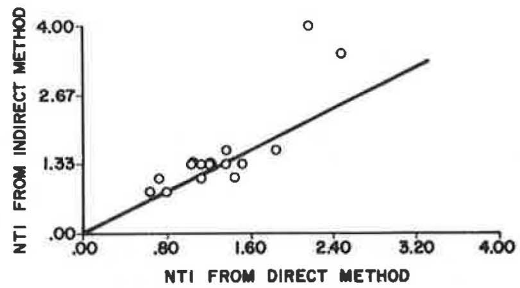

FIGURE 6 Comparison of NTI values obtained using indirect and direct methods.

IS THE PENN STATE MODEL SPEED-INVARIANT?

It has been frequently stated that PNG is independent of speed (*2,8*). Because SN_0 is obviously not affected by speed, the Penn State model is said to be speed-invariant. Of course, PNG as the model parameter is constant, but there seems to be no analytical nor empirical evidence to support the statement that the true normalized gradient of skid resistance with respect to speed, represented by PNG, is independent of speed.

The objective of the discussion presented in this section is to determine, first analytically and then using experimental data, whether the normalized speed gradient of skid resistance depends on speed. Note that the normalized speed gradient of skid resistance is represented in the Penn State model either by PNG or by v_0.

The analytical considerations will evolve from a simplified model of the tire–wet surface interaction shown in Figure 7. In this model, the tire sliding over the pavement surface with velocity v_t is pushing water into a chamber of the surface macrotexture with the volumetric rate Q_{in}. At the same time, water is flowing out of the chamber at the rate Q_{out}. The gauge pressure of water in the chamber is P_{wa}. The flow of water is described by the following basic equation (*5*):

$$C_f(dP_{wa}/dt) = Q_{in} - Q_{out} \quad (19)$$

where C_f is the hydraulic capacitance of the chamber and Q_{in} and Q_{out} are the flow rates of water into and out of the chamber, approximated by the following equations:

$$Q_{in} = A_t v_t \quad (20)$$

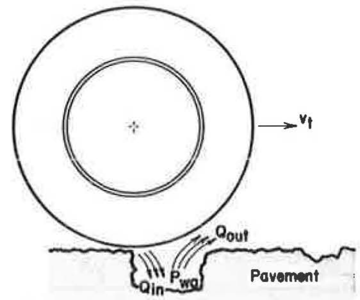

FIGURE 7 Simplified model of tire–wet pavement interaction.

$$Q_{out} = kP_{wa}^{0.5} \tag{21}$$

In these equations, A_t is the tire footprint area and k is a turbulent flow constant. Substituting Equations 20 and 21 into Equation 19 yields the water flow equation.

$$C_f(dP_{wa}/dt) = A_t v_t - kP_{wa}^{0.5} \tag{22}$$

If the tire velocity, v_t, is constant, the maximum steady-state pressure $\overline{P}_{wa}$ that can be found from Equation 22 by setting the derivative of P_{wa} equal to 0 is

$$\overline{P}_{wa} = A_t^2 v_t^2 / k^2 \tag{23}$$

In order to determine the form of the relationship between pressure $\overline{P}_{wa}$ and skid number, SN, consider first the two extreme cases of $\overline{P}_{wa}$ equal to zero and $\overline{P}_{wa}$ approaching a very high value. When $\overline{P}_{wa} = 0$, the surface is dry and thus $SN = SN_D$, where SN_D is a skid number of dry surface. Note also that because the skid resistance of a dry surface does not depend on speed, $SN_D = SN_0$. For the second extreme case, when the water pressure is high, the water film separates the tire from the pavement surface so that the resulting skid resistance is very, very small. Assuming that SN approaches zero when $\overline{P}_{wa}$ increases to infinity, the relationship between skid number and water pressure can be approximated by an exponential function of the form

$$SN(\overline{P}_{wa}) = SN_0 e^{-a\overline{P}_{wa}} \tag{24}$$

where a is a constant. Combining Equations 23 and 24, the normalized gradient of skid resistance with respect to speed, NG, is found to be

$$NG = (dSN/dv)/SN = -(2aA_t^2/k^2)v_t \tag{25}$$

Hence, the percent normalized gradient is

$$PNG = -100NG = (200aA_t^2/k^2)v_t \tag{26}$$

and the speed constant is

$$v_0 = k^2/2aA_t^2 v_t \tag{27}$$

On the basis of these results, the percent normalized gradient increases with speed, and, equivalently, the speed constant decreases with speed.

The experimental data available for validation of the analytical considerations include ribbed tire skid numbers at 30, 40, and 50 mph (48, 64, and 80 km/hr) for 16 sites in Pennsylvania. These data were used to calculate two sets of v_0 values, low-speed estimates $(v_0)_{30-40}$ from SN_{30} and SN_{40}, and high-speed estimates $(v_0)_{40-50}$ from SN_{40} and SN_{50} data. The results are plotted in Figure 8. The high-speed speed constants are smaller than the low-speed speed constants on 12 sites, a result that agrees with the analytical result given by Equation 27. The results for four sites do not agree with Equation 27.

Because each estimate of v_0 was calculated from just two skid numbers, great care is required in interpreting the experimental results. Certainly, a more extensive data base is necessary to fully validate Equation 27. Nevertheless, the combined analytical and experimental results presented here provide strong evidence that the speed gradient of skid resistance increases with speed. In spite of this observation, the Penn State model is a good approximation of the tire-pavement interaction, as indicated by accurate predictions of skid resistance obtained using this model.

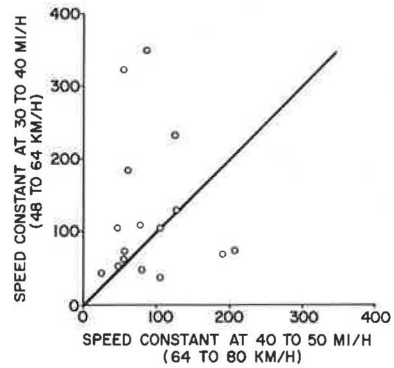

FIGURE 8 Comparison of low- and high-speed values of speed constants for 16 sites in Pennsylvania.

CONCLUSIONS

The Penn State model is an adequate representation of the effects of vehicle speed on tire-pavement friction. However, the model is only a mathematical approximation of a complex dynamic process and therefore suffers from inherent approximation errors. Also, even though the model parameters are related to certain surface characteristics, the individual least squares estimates must not be treated as quantitative measures of the surface micro- or macrotexture. The normalized texture index, incorporating estimates of both model parameters SN_0 and v_0 (or PNG), is potentially a more significant measure of texture characteristics than the two parameters taken separately. More experimental data are needed to verify the correlation between the texture index and actual surface characteristics.

ACKNOWLEDGMENT

Part of the work reported in this paper was conducted under the sponsorship of FHWA.

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Publication of this paper sponsored by Committee on Surface Properties–Vehicle Interaction.