Model for Determining the Optimum Rehabilitation Cycle for Concrete Bridge Decks

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The optimum rehabilitation cycle for concrete bridge decks is determined by calculating the average annual cost of patching the deck for a number of years and then rehabilitating it to extend life but not enhance the original functional characteristics. The model used to carry out the necessary calculations is based on the fact that every operation performed on the deck gives rise to a stream of future maintenance obligations. It relies on five user-defined inputs and makes specific provisions for the fact that patched areas fail in terms of a deterioration profile different from that exhibited by the original deck. A spreadsheet is developed to perform sensitivity analysis studies using different values for the user-defined inputs. The results are tabulated. It is found that physical life as defined by the deck deterioration profile and economic life as defined by the rehabilitation cycle must be balanced to achieve the best results. The limitations of the model are discussed. The user is cautioned about the use of relatively simple models in a complex environment.

The repair-or-replace decision must be taken at some point in time for every physical asset that deteriorates with age (*1,2*) Concrete bridge decks are no exception. Maintenance personnel are constantly seeking that elusive point in time when the increasing cost of patching a deteriorating deck becomes more expensive than rehabilitating the whole structure.

A solution to the problem is based on work performed under the Strategic Highway Research Program. The approach used is based on methodologies developed for quantifying the economic life of construction equipment (*3,4*). Modifications have been introduced to accommodate the special conditions found in bridge maintenance, and the model that has been developed is unique in this regard.

The average annual cost of patching the deck for a given number of years before rehabilitation and then rehabilitating it is used as the principal value function in the model. This function is defined as follows:

$$Ac_N = \left(\sum_{n=1}^{n=N-1} Pc_{n-1} + Rd_n \right) / N \qquad (1)$$

where

Ac_N = Average annual cost of patching the deck for N years and then rehabilitating it,

Pc_{n-1} = Cost of patching the deck in each period before rehabilitation,

Rd_n = Cost of rehabilitating the deck after n periods, and

N = Number of years for which the average annual cost of patching and rehabilitating the deck is to be calculated.

The value of Ac_N varies for assumed values of N; the optimum rehabilitation cycle is defined as the value of N that produces a minimum value for Ac_N.

DEFINITIONS

Replacement and maintenance have been defined as essentially the same process depending on the definition of the operating unit (*5,6*). In this case the operating unit is defined as a single bridge and thus patching and rehabilitating the deck become two alternatives in a spectrum of available bridge maintenance options. The terms are defined as follows:

1. Patching. Patching is the process whereby small localized areas of deterioration in the deck are repaired by removing the deteriorated concrete and replacing it with a substitute material. The objective is to reinstate the surface characteristics of the deck as far as possible. There is no intention of extending the life of the deck in any way and there is no enhancement of the original functional characteristics of the deck.
2. Rehabilitation. Rehabilitation is the process whereby large areas of chloride-contaminated or deteriorated concrete and reinforcing steel are systematically removed from the deck and replaced. The objective is to reinstate the deck to an as-new condition with regard to performance and life by removing chloride-contaminated concrete and arresting corrosion. Enhancing the original functional characteristics of the deck is not intended and the original geometry of the structure remains essentially unchanged.

The main considerations leading to replacement have been classified as excessive maintenance, declining efficiency, inadequacy, and obsolescence (*7*). Patching or rehabilitating all or part of a deck addresses the first two of these factors; the latter two require replacement or reconstruction and thus fall outside the scope of this paper.

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STRUCTURE OF THE MODEL

The model is based on the fact that every operation performed on the deck gives rise to a stream of future maintenance obligations. The amount of maintenance work that must be done in each period depends on the area of the deck, either new or patched, and the portion of this area that will deteriorate and become a candidate for maintenance in the period.

The situation for the original bridge deck may be described as follows:

$$Mo_N = A \times P(N) \tag{2}$$

where

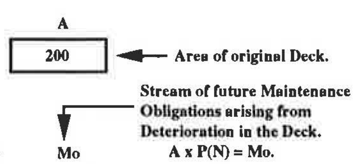
Mo_N = Maintenance obligation arising from the original deck for the period ending at Year N,
A = Area of the deteriorating deck, and
$P(N)$ = Portion of the original deck that deteriorates in the period ending at Year N and contributes to the stream of future maintenance obligations.

The series $P(1)$, $P(2)$, . . ., $P(N)$, which describes the deterioration profile for the deck, is defined such that

$$\sum_{n=1}^{n=N} P(n) = 1 \tag{3}$$

The deck deterioration profile plays an important part in determining the maintenance workload and is discussed again later. Figure 1 shows an example of how a stream of future maintenance obligations is generated from an assumed deck and deterioration profile. The fact that patched areas fail in terms of a deterioration profile different from that exhibited by the original deck means that patched areas must be identified and the stream of maintenance obligations arising from the need to repatch a previously patched area must be determined. This is done as follows:

$$Mpx_n = Apx \times Ppx(n) \qquad n = 1, 2, \ldots, N \tag{4}$$



N	P(N)	Mo
5	5%	10
10	10%	20
15	15%	30
20	20%	40
25	25%	40
30	30%	60

FIGURE 1 Maintenance obligations generated by deck. The stream of future obligations is the product of the area and the portion that deteriorates.

where

Mpx_n = Maintenance obligation arising from the need to repatch Patch x in the period ending at Year n,
Apx = Area of Patch x, and
$Ppx(n)$ = Portion of Patch x that deteriorates in the period ending at Year n.

The series $Ppx(1)$, $Ppx(2)$, . . ., $Ppx(N)$, which describes the deterioration profile for the patches, is defined such that

$$\sum_{n=1}^{n=N} Ppx(n) = 1 \tag{5}$$

The patch deterioration profile is different from the deterioration profile for the original deck but is common to all patches of a given type and age. It is discussed again later.

The total maintenance obligation in Period N arising from the need to patch the original deck and repatch the patches is given by

$$Mt_N = Mo_N + Mp_N \tag{6}$$

where

Mt_N = Total maintenance obligation for the period ending in Year N,
Mo_N = Maintenance obligation for the period ending in Year N arising from the original deck (see Equation 2), and
Mp_N = Total maintenance obligation for the period ending in Year N arising from all repatching.

$$Mp_N = \sum_{n=1}^{n=N} Mpx_n \tag{7}$$

Figure 2, which expands on Figure 1, shows how Mt_N may be calculated for an assumed deck area, deck deterioration profile, and patch deterioration profile.

The cost of patching the deck in each period before replacement was defined as Pc_{N-1} in Equation 1. This may now be written as

$$Pc_{N-1} = Mt_{N-1} \times Cp \tag{8}$$

where Cp is the unit cost of patching a portion of the deck.

The cost of rehabilitating all or part of the deck was defined as Rd_n in Equation 1. This term may now be written as

$$Rd_n = Ar \times Cr_n \tag{9}$$

where

Ar = Area of deck to be replaced (yd^2),
Cr_n = Unit cost of replacing the deck after n years ($\$/yd^2$).

If Pcr_n is defined as the ratio of the unit cost of patching to the unit cost of rehabilitating and put equal to Cp/Cr_n then Equation 1 may be written as follows:

$$Ac_N = Cr_N\left(\sum_{n=1}^{n=N-1} Mt_{n-1} \times Pcr_n + Ar\right)/N \tag{10}$$

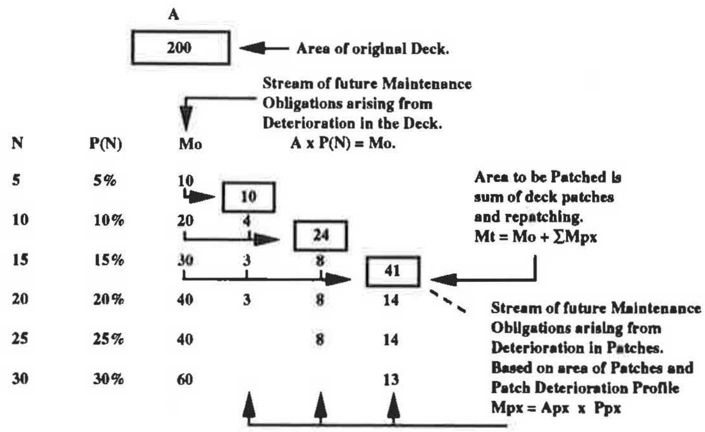


FIGURE 2 Maintenance obligations generated by deck and patches. The stream of future obligations arises from the deck and the need to repatch the patches.

This equation is used to calculate the optimum rehabilitation period in the sensitivity analysis performed later.

USER-DEFINED INPUTS

The model requires a number of user-defined inputs. These are discussed in this section to provide a full understanding of what is needed to use the model

1. Area of Deteriorating Deck, *A*. This parameter is relatively straightforward with the provision that areas not heavily trafficked or subject to chloride contamination should be eliminated.
2. Area of Deck to be Rehabilitated, Ar. This parameter equals the area of the original deck if a systematic milling hydrodemolition or deck rehabilitation option is selected as the deck replacement strategy. If removal of contaminated concrete and reconstruction to an as-new condition does not cover the full area, then Ar is less than *A*.
3. Deck Deterioration Profile, *P*(*N*). This parameter depends on a number of factors including the original design, quality of the original construction, past and future patterns of chloride contamination, location of the bridge, and traffic patterns. Three possible profiles based on prior research (*8*) are shown in Figure 3 where the ordinate represents the cumulative percentage of the deck that has deteriorated at the end of each 5-year period. The profiles may be classified as follows:

Profile	*Deterioration Commences (years)*	*Deterioration Reaches 30% (years)*	*Deterioration Reaches 80% (years)*
A	5	15	22
B	5	22	32
C	5	32	47

4. Patch Deterioration Profile, Ppx(*N*). As with the deck deterioration profile, this parameter depends on the methods and materials used for patching and the quality of workmanship. Little research is available on which to base values for the profiles and three different profiles are shown in Figure

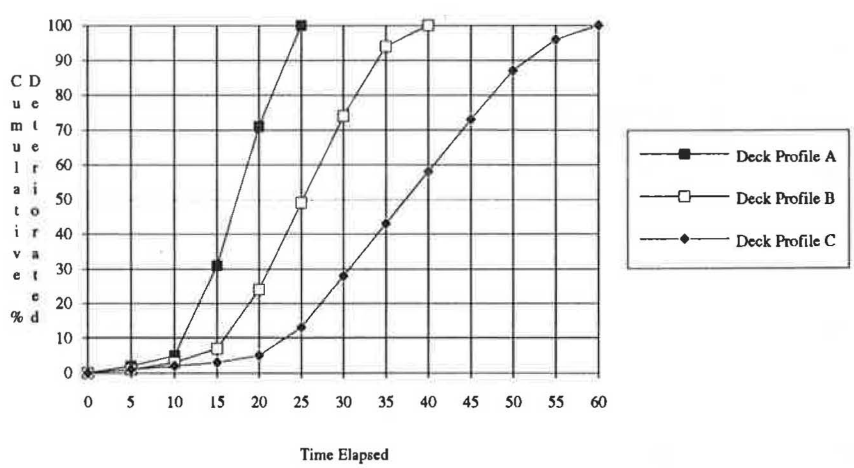


FIGURE 3 Deck deterioration profiles. Cumulative deterioration reaches 30 percent after 15, 22, and 32 years for Profiles A, B, and C, respectively.

4 where the ordinate again represents the cumulative percentage of patched area that needs to be repatched after the end of each 5-year period. The patch profiles may be summarized as follows:

Patch Type	*50% of Patches Deteriorated (years)*	*100% of Patches Deteriorated (years)*
1	3	5
2	4	10
3	13	20

5. Unit Cost Ratio, Pcr. This parameter reflects the ratio of the unit cost of patching to the unit cost of replacing. It is affected by the methods and materials used for patching as well as the cost of removal.

SENSITIVITY ANALYSIS

A spreadsheet was developed to perform the calculations needed to determine the value of Ac_N for various values of N under different conditions. Table 1 presents the nine studies conducted with the corresponding values assumed for the user-defined inputs.

The spreadsheet and graph from Study 5 is given in Figures 5 and 6 as examples of the results obtained. A full set of output graphs is shown in Figures 7–15.

The output graphs indicate how the average annual cost of patching the deck for N years and then replacing it varies with time to produce a minimum point that defines the optimum rehabilitation cycle. The cumulative deterioration profile for the deck is also plotted on the graph and thus it is possible to determine the following three important values from each of the sensitivity analysis studies:

1. The minimum value for the average annual cost, Ac*.
2. The optimum rehabilitation cycle N^*.
3. The cumulative percent deterioration in the deck when the optimum point D* occurs.

These values are presented in Table 2, which forms the basis of the analysis that follows.

ANALYSIS OF RESULTS

1. *Differences in Patch Cost Ratio.* The patch cost ratio (Pcr) for each patch type was varied by 20 percent above and below the median value assumed for each patch type (0.8, 1.0, and 1.2 for Types 1, 2, and 3, respectively.) Table 2 indicates that this choice had no effect on the timing and little effect on the magnitude of the minimum point. The optimum replacement cycle is thus not sensitive to minor variations in the cost of patching at or around the optimum cycle time.

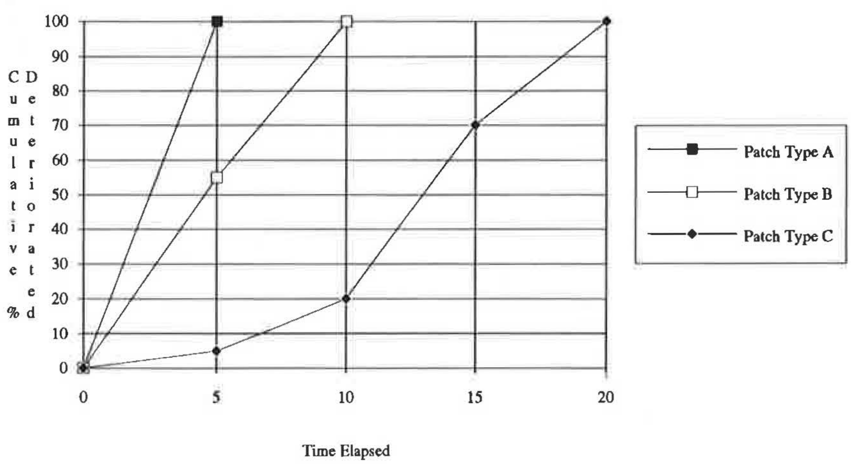


FIGURE 4 Patch deterioration profiles. Cumulative deterioration reaches 100 percent after 5, 10, and 20 years for Patch Types 1, 2, and 3, respectively.

TABLE 1 VALUES ASSUMED FOR USER-DEFINED INPUTS

Study Number	Deck Area (A)	Rehabilitated Area (Ar)	Deck Profile $P(N)$	Patch Type Ppx(N)	Cost Ratio Pcr From	To
1	100	100	A	1	0.64	0.96
2	100	100	A	2	0.80	1.20
3	100	100	A	3	0.96	1.44
4	100	100	B	1	0.64	0.96
5	100	100	B	2	0.80	1.20
6	100	100	B	3	0.96	1.44
7	100	100	C	1	0.64	0.96
8	100	100	C	2	0.80	1.20
9	100	100	C	3	0.96	1.44

User Defined Inputs														
A		100												
Ar		100												
P(N)	N	5	10	15	20	25	30	35	40	45	50	55	60	
	P	1	2	4	17	25	25	20	6					= Deck Profile B
Ppx(N)	N	5	10	15	20	25	30	35	40	45	50	55	60	
	Ppx	55	45	.	.	.	.	.	.	.	.	.	.	= Patch Type 2
Pcr		0.8	1	1.2										

N	DECK P(N)	DECK A Mo	PATCHES 1 Mt 1 Mpx	2 Mt 2 Mpx	3 Mt 3 Mpx	4 Mt 4 Mpx	5 Mt 5 Mpx	6 Mt 6 Mpx	7 Mt 7 Mpx	8 Mt 8 Mpx	9 Mt 9 Mpx	10 Mt 10 Mpx	11 Mt 11 Mpx	12 Mt 12 Mpx	Mt	ΣMt	AcN Pcr = 0.8	AcN Pcr = 1	AcN Pcr = 1.2
0		100																	
	1	1																	
5			1												1	1	20.0	20.0	20.0
	2	2	0.55																
10				2.55											2.55	3.55	10.1	10.1	10.1
	4	4	0.45	1.4025															
15					5.8525										5.8525	9.4025	6.9	6.9	7.0
	17	17	0	1.1475	3.2189														
20						21.366									21.366	30.769	5.4	5.5	5.6
	25	25	0	0	2.6336	11.752													
25							39.385								39.385	70.154	5.0	5.2	5.5
	25	25	0	0	0	9.6149	21.662												
30								56.277							56.277	126.43	5.2	5.7	6.1
	20	20		0	0	0	17.723	30.952											
35									68.675						68.675	195.11	5.7	6.5	7.2
	6	6			0	0	0	25.325	37.772										
40										69.096					69.096	264.2	6.4	7.4	8.4
	0	0				0	0	0	30.904	38.003									
45											68.907				68.907	333.11	6.9	8.1	9.3
	0	0					0	0	0	31.093	37.899								
50												68.992			68.992	402.1	7.3	8.7	10.0
	0	0						0	0	0	31.008	37.946							
55													68.954		68.954	471.05	7.7	9.1	10.6
	0	0							0	0	0	31.046	37.924						
60														68.971	68.971	540.03	7.9	9.5	11.1

FIGURE 5 Analysis of costs for Deck Profile B and Patch Type 2. The optimum rehabilitation cycle is 25 years.

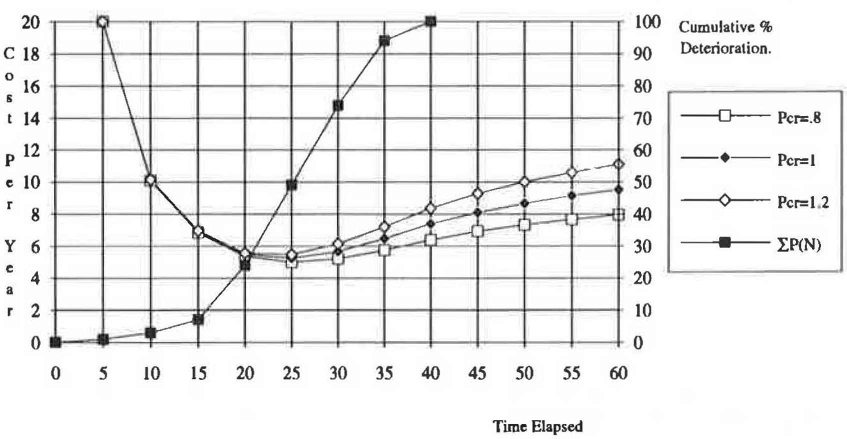


FIGURE 6 Average annual cost and cumulative deterioration profile for deck Profile B and Patch Type 2. The optimum rehabilitation cycle occurs after 25 years when the deck is 49 percent deteriorated.

The graphs do show that relative patch costs are important when the optimum period is exceeded.

2. *Differences in Patch Type.* The three different patch types assumed in the sensitivity analysis studies differed from one another in terms of both their deterioration profiles and their cost ratios, as summarized in the column headings of Table 2. The values obtained indicate that neither Ac* nor N^* changed when the patching type differed for a given deck specification. This result arises from the fact that the three patch types assumed all have the same benefit-to-cost ratio as defined by their deterioration profile [$Ppx(N)$] and patch cost ratio (Pcr). The results obtained thus illustrate the use of the model to quantify the elusive relationship between service life and cost for various alternative patching methods.

3. *Differences Between Deck Profiles.* The three different deck deterioration profiles assumed produced different values for Ac*, N^*, and D^*. For Profile A, the minimum cost is relatively high (6.2 to 7.3); it occurs at 20 years when deck deterioration has reached 70 percent. This case is clearly infeasible, as the physical life of the deck from a safety and ride

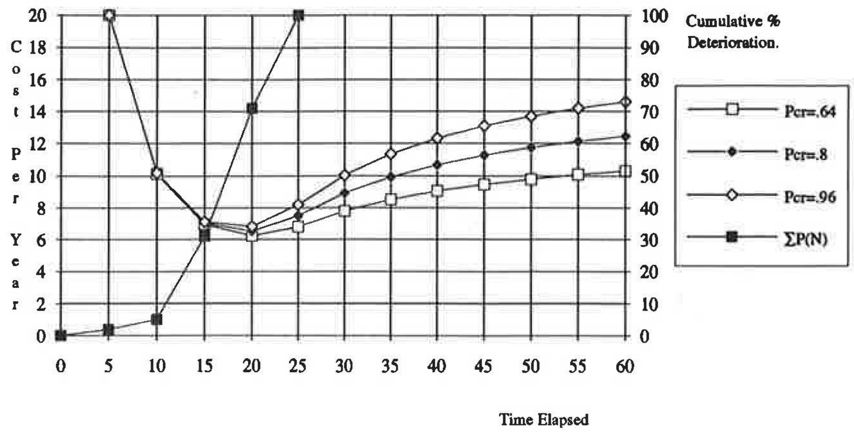


FIGURE 7 Average annual cost and cumulative deterioration profile for Deck Profile A and Patch Type 1. The optimum rehabilitation cycle occurs after 20 years when the deck is 71 percent deteriorated.

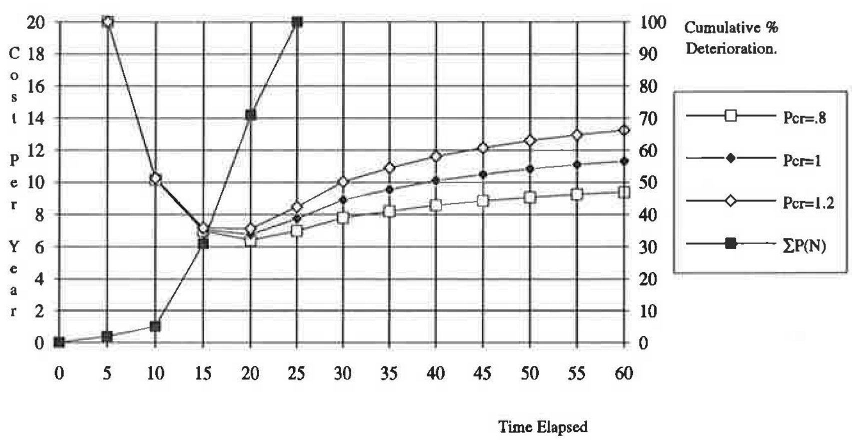


FIGURE 8 Average annual cost and cumulative deterioration profile for Deck Profile A and Patch Type 2. The optimum rehabilitation cycle occurs after 20 years when the deck is 71 percent deteriorated.

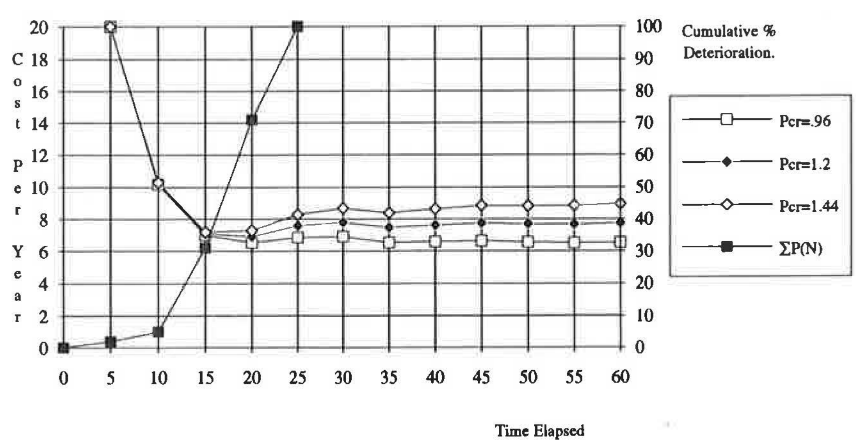


FIGURE 9 Average annual cost and cumulative deterioration profile for Deck Profile A and Patch Type 3. The optimum rehabilitation cycle occurs after 20 years when the deck is 71 percent deteriorated.

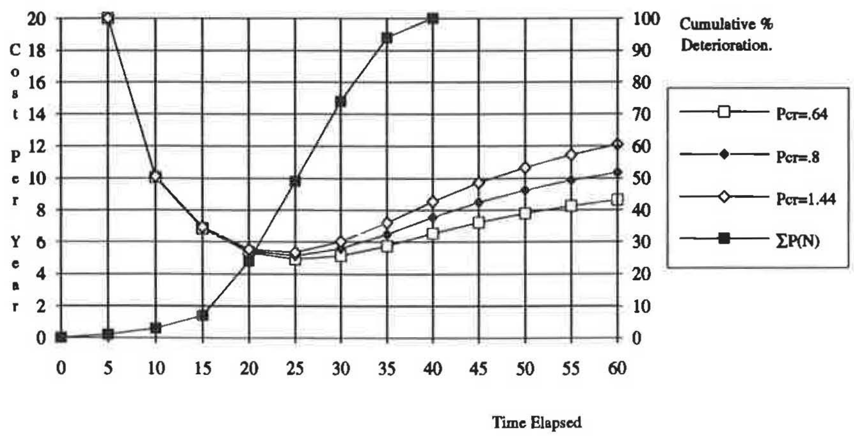


FIGURE 10 Average annual cost and cumulative deterioration profile for Deck Profile B and Patch Type 1. The optimum rehabilitation cycle occurs after 25 years when the deck is 49 percent deteriorated.

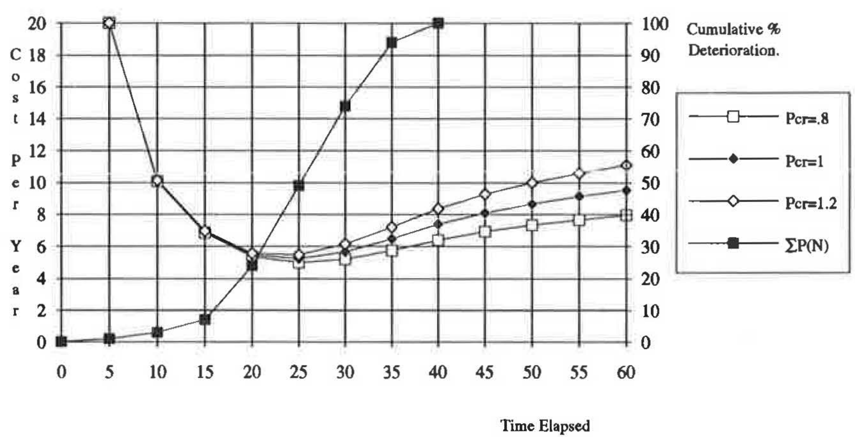


FIGURE 11 Average annual cost and cumulative deterioration profile for Deck Profile B and Patch Type 2. The optimum rehabilitation cycle occurs after 25 years when the deck is 49 percent deteriorated.

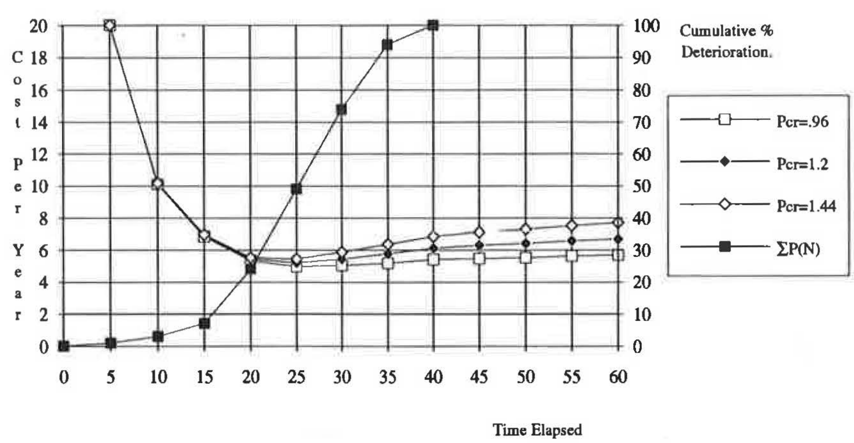


FIGURE 12 Average annual cost and cumulative deterioration profile for Deck Profile B and Patch Type 3. The optimum rehabilitation cycle occurs after 25 years when the deck is 49 percent deteriorated.

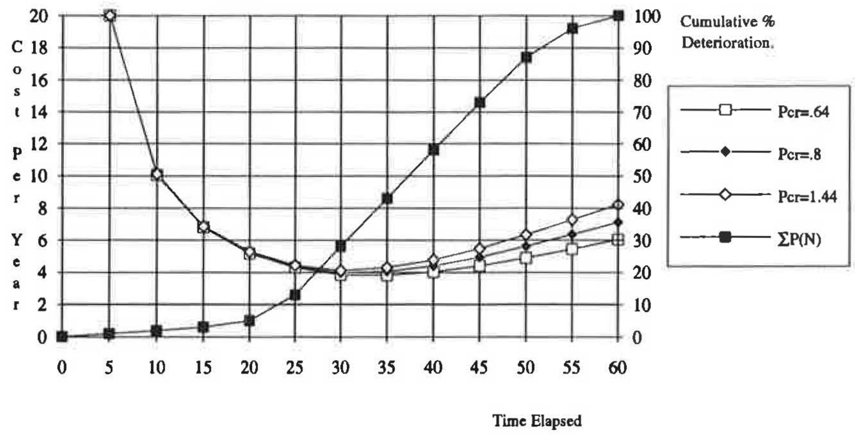


FIGURE 13 Average annual cost and cumulative deterioration profile for Deck Profile C and Patch Type 1. The optimum rehabilitation cycle occurs after 30 years when the deck is 28 percent deteriorated.

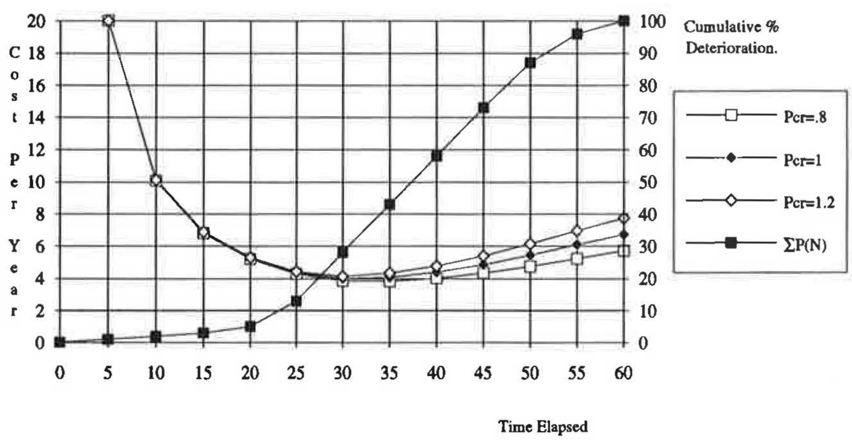


FIGURE 14 Average annual cost and cumulative deterioration profile for Deck Profile C and Patch Type 2. The optimum rehabilitation cycle occurs after 30 years when the deck is 28 percent deteriorated.

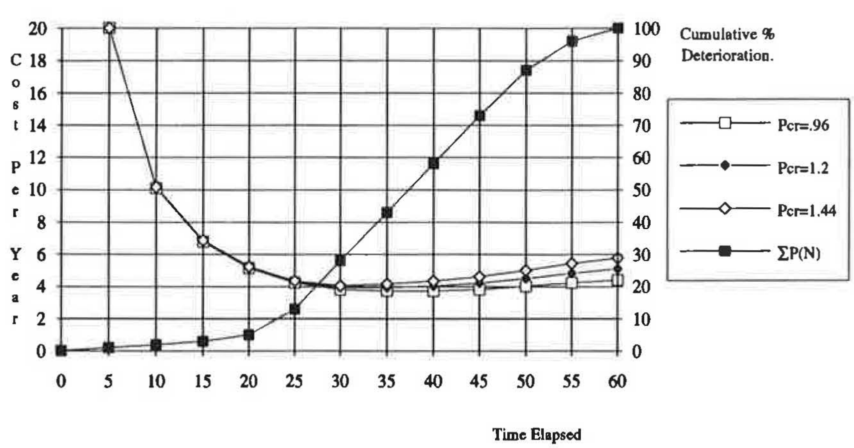


FIGURE 15 Average annual cost and cumulative deterioration profile for Deck Profile C and Patch Type 3. The optimum rehabilitation cycle occurs after 30 years when the deck is 28 percent deteriorated.

TABLE 2 TABULATION OF RESULTS

Deck specification.		Patch Type 1 100% deteriorated in 5 yrs Median cost ratio = .8			Patch Type 2 100% deteriorated in 10 yrs Median cost ratio = 1.0			Patch Type 3 100% deteriorated in 20 yrs Median cost ratio = 1.2		
		Ac*	N* yrs	D*%	Ac*	N* yrs	D*%	Ac*	N* yrs	D*%
Deck profile A 30% Deterioration in 15 yrs	Low Pcr	6.2	20	70	6.4	20	70	6.5	20	70
	Med Pcr	6.5	20	70	6.8	20	70	6.9	20	70
	High Pcr	6.8	20	70	7.1	20	70	7.3	20	70
Deck profile B 30% Deterioration in 22 yrs	Low Pcr	4.9	25	50	5.0	25	50	5.0	25	50
	Med Pcr	5.1	25	50	5.2	25	50	5.2	25	50
	High Pcr	5.3	25	50	5.5	25	50	5.5	25	50
Deck profile C 30% Deterioration in 32 yrs	Low Pcr	3.8	30	30	3.9	30	30	3.8	30	30
	Med Pcr	4	30	30	4.0	30	30	3.9	30	30
	High Pcr	4.1	30	30	4.1	30	30	4.1	30	30

Different deck profiles produce different values for Ac*, N* and D*

quality point of view would have passed by the time deterioration reaches 70 percent. The rehabilitation decisions would thus be driven by physical rather than economic factors, with the period being limited to approximately 15 years, at which time deterioration would have reached or exceeded 30 percent. The resulting cost would be well above the minimum cost.

For Profile B, the minimum cost is lower (4.9 to 5.5) and it occurs at 25 years when deterioration is about 50 percent. Physical life would again govern, with safety and ride quality dictating the rehabilitation decision. In this case, the cycle would be around 23 years at a cost slightly above the economic minimum.

Profile C presents a well-balanced picture. The costs are low (3.8 to 4.1) and the optimum point occurs at 30 years when deterioration is also about 30 percent. This case means that economic and physical life are essentially the same and the two work together to achieve a good result. A rehabilitation cycle of about 30 years would optimize both the physical and the economic aspects of the decision.

CONCLUSIONS

The model presented has been kept simple to develop the concepts and present the methodology. It can be expanded to include many more aspects but apparent quantitative precision in the model should not override the many nonquantifiable factors that affect bridge maintenance decisions. The following factors should be noted:

1. Reduced Ride Quality. No allowance has been made for the fact that the ride quality of a deteriorated or repeatedly patched deck declines. This is a legitimate user cost that could or should be factored into the cost of patching the deck.
2. Functional Obsolescence. No account has been taken of the fact that deck replacement often provides an opportunity to renovate or upgrade the functional aspects of the bridge. This is a complex analysis; suffice to say here that a knowledge of the optimum rehabilitation cycle assists in the timing and quality of the renovate decision.
3. Time Value of Money and Inflation. These factors have been omitted in the model so as not to clutter the computations. Their inclusion is a relatively simple process; the net present value of a stream of future costs can be calculated in the place of arithmetic totals; uniform annuities at the assumed interest rate can replace the average annual cost calculations. The replacement cycle is changed by a small amount, but the concept will not alter. Assumed increases in the cost of future patching and deck replacement can also be included to improve the quality of the answer.

These limitations should be seen in relation to problems associated with defining and quantifying the five user inputs described earlier. Experience leads to a better knowledge of both the inputs and the limitations.

The model has indicated that it is possible to determine the optimum rehabilitation cycle for concrete bridge decks under a set of conditions described by five user-defined inputs. This cycle determines the economic life of the deck, which can be compared with the physical life of the deck as determined by the cumulative deck deterioration profile. Two issues are important:

1. When physical life is less than economic life then costs are high compared with the situation found when physical and economic life are well balanced.
2. This balance can only be achieved when the quality of construction and maintenance on the deck is such that less than 30 percent of the deck requires patching in the first 30 years of its service life.

The relative cost of patching may not affect the situation before the optimum cost point. This conclusion must be tempered by the fact that all the patch types assumed in the sensitivity analysis studies had essentially the same benefit-to-cost ratios as defined by their deterioration profiles and patch cost ratios. This condition changes as the benefit-to-cost ratios change; the value of the model in quantifying these changes should be noted.

Three major points must be stressed:

1. Every operation performed on the deck gives rise to a stream of future maintenance obligations. These obligations cannot be denied and must be met timeously.
2. Quality in the design construction and maintenance of concrete bridge decks is required to balance physical and economic life and to achieve the best results.
3. Bridge maintenance decisions are extremely complex. Models such as the one presented should not limit the bounds of the decision; the results they produce must be seen as guidelines for improved field decisions. Analysis certainly has

a place in the process, but it must complement rather than replace experience.

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