Design Criteria for Right and Skew Slab-and-Girder Bridges

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Research on skew slab-and-girder bridges has had limited impact
on practical bridge design. A literature study indicates that there
is no information available that tells a designer exactly how to
take into account the effects of skew when designing a slab-and-
girder bridge. Thus, research on skew slab-and-girder bridges
with the goal of developing design criteria that include the effects
of skew is desirable. With this goal in mind, a parametric study
is done in which the behavior of simply supported, right and skew
slab-and-girder bridges is determined by varying the properties
of the structural members and the bridge geometry. The results
obtained from linear elastic finite element analyses on 108 bridges
are used to develop an accurate, simplified analysis procedure
for the maximum bending moments in the girders. Bridges with
five precast, prestressed, or steel I-beam girders subjected to two
AASHTO HS20–44 standard trucks are considered, but the anal-
ysis procedure is suitable for bridges with more than five girders
and with more than two traffic lanes. The proposed analysis pro-
cedure is based on improved AASHTO wheel load fractions for
right bridges, which are modified for skew bridges. It is found
that the AASHTO specifications for the distribution of wheel
loads in right slab-and-girder bridges are sometimes unsafe and
often too conservative.

A slab-and-girder bridge system is a favored structural choice
both on economic and on aesthetic grounds. It is so named
because it consists of two major structural elements. These
are a reinforced-concrete slab that serves as the roadway and
distribution medium for concentrated wheel loads, and a num-
ber of prefabricated girders that are parallel to traffic and
that carry these distributed wheel loads and dead weight to
the abutments.

The basic analysis problem is to determine the distribution
of wheel loads among the girders to obtain design loads that
the engineer can use to proportion the individual girders. This
problem has been studied for decades by many researchers
using different approaches. However, little research had been
done on skew slab-and-girder bridges until the advent of the
electronic digital computer, which made extensive numerical
solutions possible.

Research on skew slab-and-girder bridges has had limited
impact on practical bridge design. A literature survey indi-
cates that there is no information available that tells a designer
exactly how to take into account the effects of skew when
designing a slab-and-girder bridge. Even the current AASHTO
Standard Specifications for Highway Bridges (1) provides no

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guidance regarding the effects of skew on the behavior of a
bridge.

Previous researchers (2–4) were concerned mainly with the
development of analytical methods of analysis for skew bridges.
Skew bridge behavior has not been thoroughly investigated
and results have not been presented in a way that would be
helpful to the designer. Thus, research on skew slab-and-
girder bridges with the goal of developing design criteria that
include the effects of skew is desirable.

DESCRIPTION OF THE PROBLEM

With this goal in mind, a parametric study was done by ana-
lyzing different simply supported slab-and-girder bridges, us-
ing the finite element method of analysis. The data from these
analyses were used to determine wheel load distribution char-
acteristics of right and skew bridges, by plotting the maximum
girder bending moments that occur against the different pa-
rameters defining the geometric layout and structural prop-
erties of a bridge.

The changes in bridge behavior that result as a consequence
of varying individual parameters were further mulled over
and maximum girder bending moments were expressed as
functions of certain decisive combined parameters, use of
which led to small scatter in girder bending moment results.

An easy-to-use, reliable method of determining maximum
girder bending moments in simply supported right and skew
slab-and-girder bridges was developed in a form familiar to
practicing engineers (5, 6).

The purpose is to present the parameters that control the
behavior of a bridge and a proposed simplified method of
determining maximum girder bending moments in simply sup-
ported right and skew slab-and-girder bridges.

The typical skew bridge shown in Figure 1 consists of a
horizontal reinforced-concrete slab of uniform thickness, sup-
ported by five identical precast, prestressed concrete girders.
The slab edge and girder ends are simply supported at the
two parallel skew abutments. The girders are identical, pris-
matic, and equidistant from each other. The span a, of the
bridge, equals the length of the girders and varies from 12.19
to 24.38 m (40 to 80 ft). Only short-span bridges are consid-
ered for the reason discussed later. The girder spacing, b,
varies from 1.83 to 2.74 m (6 to 9 ft) and the angle of skew,
α, defined in Figure 1, varies from 0 to 60 degrees. The slab
thicknesses and girder properties used cover the practical ranges
for this type of bridge. A total of 108 two-lane slab-and-girder
bridges subjected to two AASHTO HS20–44 standard trucks
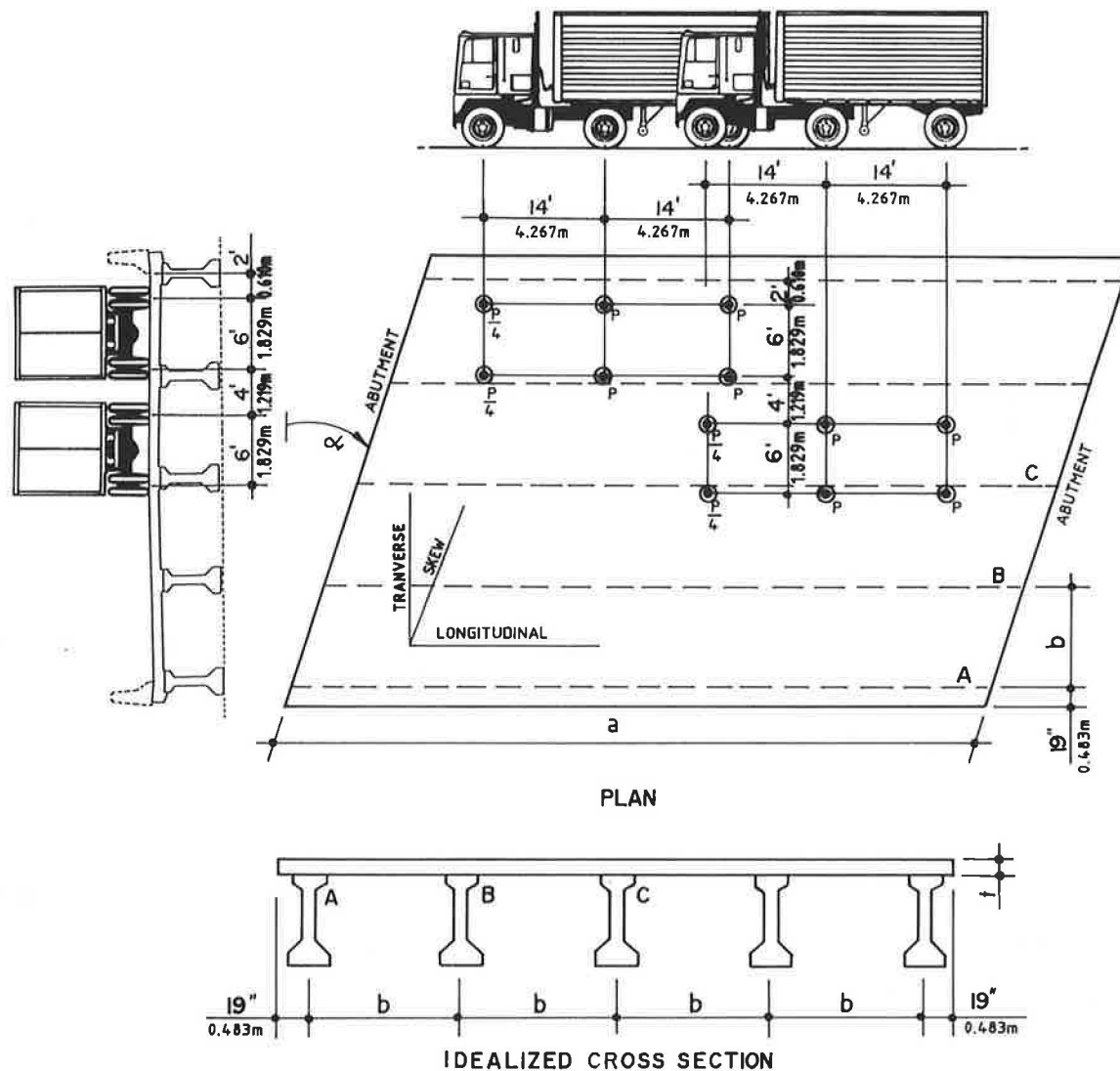
FIGURE 1 Typical bridge considered with AASHTO HS20–44 truck loads.

are analyzed principally to obtain the maximum girder bending moments. The results can also be used for bridges with steel I-beams when a minor modification is made. The following assumptions and limitations, which are justified and discussed in detail by Marx et al. (6), are applicable:

1. The material in the slab is homogeneous and isotropic; the slab and girders behave in a linearly elastic manner;
2. Full composite action occurs between the slab and eccentric girders;
3. The stiffening effect of the curbs and parapets is ignored;
4. The girder-slab interaction occurs along a line, that is, the girders have no width;
5. Except for rigid diaphragms at the abutments, no other diaphragms exist;
6. The width of the slab overhangs at the edge girders is 483 mm (19 in.);
7. The faces of the curbs are directly above the edge girders, i.e., no truck wheel can get closer than 0.610 m (2 ft) from an edge girder; and
8. Only I-shaped girders are considered.

It was found (6) that girder bending moment results obtained from a five-girder bridge closely and conservatively approximate the results for a bridge with more equidistant girders. Results for five-girder bridges can thus be used for bridges with more girders.

The width of the slab overhangs at the edge girders is not of much importance, but the position of the face of the curb relative to the edge girder has a significant influence on the edge girder bending moments. Illinois and some other states generally have not used span diaphragms in prestressed I-girder bridges for many years.

METHOD OF ANALYSIS USED

The finite element method was used to determine the linearly elastic behavior of a bridge under service loads. The girders were modeled with eccentric Lagrangian-type isoparametric beam elements with the St. Venant girder torsional stiffness taken into account. The bridge deck was modeled with nine-

node Lagrangian-type degenerated isoparametric thin-shell elements.

No closed-form exact solutions exist for skew slab-and-girder bridges with which results could be compared. First it was necessary, therefore, to determine whether the shell elements used to model the deck provided correct results when used in skew configuration. Furthermore, it was necessary to perform a convergence study on a typical bridge to determine to what extent the mesh had to be refined to ensure reliable results. For the purpose of comparing results, the finite element mesh selected was used to analyze certain slab-and-girder bridges for which other solutions existed.

Details on compatibility problems with eccentric beam modeling and on problems encountered with excessive stiffness when a rectangular shell element is distorted into a parallelogram that fits into a skew network have been provided by Marx et al. (5, 6).

INTRODUCTION OF THE PARAMETERS USED

Geometric Parameters

There are three parameters that determine the geometry of the bridge. They are the angle of skew, α, the bridge span, a, and the girder spacing, b. These three parameters have already been defined in Figure 1. Wherever convenient, a fourth dimensionless parameter, the girder spacing to span ratio, b/a, is used.

Structural Parameters

A large number of variables determine the structural properties of a bridge. The large amount of work involved in considering all of these variables in a parametric study would be prohibitive. It is necessary, therefore, to eliminate as many variables as possible without simplifying the structure to such an extent that the structural behavior would be altered. This procedure can be done by ignoring the unimportant variables and by combining others to bring about new ones that control the structural behavior.

The controlling parameters are determined by recognizing the major structural actions in a slab-and-girder bridge, as follows:

1. The slab distributes truck loads over the width of the bridge. To do this, it acts in flexure in the transverse direction, similar to a beam continuous over flexible supports. The transverse flexural rotation of the slab over a girder is resisted by the torsional rigidity of the girder.
2. The eccentric girders act together with the slab to form stiff composite T-section girders that carry the whole load to the abutments in flexure.

It is necessary, therefore, to combine the variables that determine the flexural slab stiffness and those that bring about the flexural composite girder stiffness and the torsional girder stiffness.

Flexural Slab Stiffness D

The flexural slab stiffness per unit width is given by

$$D = \frac{E_s t^3}{12(1 - \mu^2)} \tag{1}$$

where t is the slab thickness, E_s if Young's modulus of elasticity for the slab material, and μ is Poisson's ratio (taken as 0.2 for concrete). The thickness of the slab depends on the girder spacing and is normally between 152 and 254 mm (6 and 10 in.).

Flexural Composite Interior Girder Stiffness, $E_g I_{cg}$

A large number of unimportant structural variables can be eliminated by using the composite girder stiffness as a parameter. The composite moment of inertial of an interior girder, I_{cg}, can easily be determined by using the effective flange width recommendations in the AASHTO *Specifications for Highway Bridges* (1) and by transforming the slab concrete to equivalent girder concrete according to their modular ratio. The effect of this approximation is discussed later. E_g is Young's modulus of elasticity for the prefabricated girders, and also for the composite transformed girders.

Torsional Girder Stiffness

The transverse rotation of the slab over a supporting girder is resisted by the torsional rigidity of the girder. In effect, the slab is thereby stiffened in bending in the transverse direction. A stiffer slab distributes truck loads better, so that a more uniform cross distribution of load occurs.

The torsional stiffness of a steel I-beam is small and has negligible influence on the distribution of truck loads to the girders. It can, therefore, be ignored. Precast, prestressed concrete girders have considerably larger torsional stiffness. The influence of girder torsional stiffness becomes gradually larger as the angle of skew increases. However, the effect on the distribution of truck loads to the girders is only about 5 percent. On the other hand, the torsional stiffness of girders of box section, which are not considered in this study, has a significant influence.

Because the behavior of slab-and-girder bridges is so insensitive to girder torsional stiffness, the torsional stiffness is not used as a major parameter in this study. However, it is taken into account. The flexural and torsional properties of the girders used in the analyses are those of actual standard precast, pretensioned, prestressed concrete girders, which are used in practice for spans up to 24.38 m (80 ft).

Although the properties used in the analyses are those for prestressed concrete girders, the results can also be used for steel I-beams by increasing the design girder bending moments by 5 percent.

Dimensionless Stiffness Parameter, H

The behavior of a slab-and-girder bridge depends on the geometry of the bridge as well as on the structural properties

of the bridge members. Newmark (7), on the basis of analyses of bridges using girders having no eccentricity, found that the flexural stiffness of the slab and the flexural stiffness of the girders need not to be considered as two separate parameters. They can be combined to form a new, convenient dimensionless stiffness parameter. Newmark defined the parameter H as the ratio of the longitudinal bending stiffness of an isolated noncomposite girder, $E_g I_g$, to the transverse bending stiffness of a width of slab equal to the span of the bridge:

$$H = \frac{E_g I_g}{aD} \quad (2)$$

Because D is the flexural stiffness of the slab per unit width, it is necessary to multiply D by some width to make H dimensionless. The span, a, serves this purpose, although a is not the width of the slab effective in the transverse direction. H is simply a convenient dimensionless stiffness parameter. A large H value means that the bridge has large, stiff girders. Newmark (7) found that different bridges with noncomposite girders and the same H and b/a ratios always yield identical influence surfaces for girder bending moments.

A minor modification is necessary to apply H in this study. The moment of inertia of the isolated girders, I_g, should be replaced by the composite moment of inertia, I_{cg}, of an interior girder. The modified H used in this study is then

$$H = \frac{E_g I_{cg}}{aD} \quad (3)$$

The value of H is now a function of the effective flange width, because it depends on the composite girder moment of inertia. The effective flange width is an approximation to take into account the effect of shear lag in the slab, which depends, among other things, on the slab thickness, girder spacing, span, eccentricity of the girders, and loading condition. It is thus obvious that some differences in girder bending moments occur when two bridges with composite girders have the same H and b/a ratios but with different variables determining the H value. However, it is found that these expected differences are less than 2 percent for practical bridges subjected to truck loads, which means that the parameter H is adequate for bridges with composite girders.

Further uncertainties regard the real value of H. What are the real values of E_s and E_g? How much does the effect of slab cracking influence the average flexural stiffness of the slab? Fortunately, it is found that the girder bending moments are not particularly sensitive to moderate variations in H, especially for large values of H.

A particular bridge has only one H value, which is calculated using the flexural stiffness of an interior composite girder. Exterior girder bending moment results are also expressed in terms of this H value, despite the fact that their flexural composite stiffness is different from the interior girders as a result of the difference in effective flange widths. Bridge design manuals indicate that the H value of practical bridges falls between $H = 5$ and $H = 30$.

The value of H depends on both a and b since I_{cg} depends on the effective flange width, which equals the girder spacing, b, in most practical bridges. However, in the parametric study, the values of the parameters are changed one at a time. Thus if b or a is varied, the necessary changes are made to the slab thickness and cross-sectional properties of the prefabricated girders to keep the value of H the same.

Truck-Loading Parameter, P

In this study, the emphasis is on the distribution of truck loads among the girders in the bridge. The HS20–44 standard truck considered is a tractor truck with semitrailer and is in accordance with the AASHTO *Standard Specifications for Highway Bridges (1)*. It represents a large number and variety of actual truck types and loadings to which the bridge might be subjected under actual traffic conditions.

Figure 1 shows the locations of the wheel loads and the transverse location of one truck relative to another. Each truck occupies the central portion of a 3.05-m (10-ft)–wide load lane, one truck per lane. These load lanes can be placed anywhere in the entire roadway width of the bridge, which is the clear distance between the faces of the two curbs, to produce maximum moments in whichever girder is considered. This result means that no wheel centroid can act closer than 0.61 m (2 ft) from the face of a curb or edge girder, which is a greatly desired condition (6). As shown in Figure 1, it also means that the minimum transverse distance between the wheel centroids of two trucks in adjacent loading lanes 1.22 m (4 ft).

The AASHTO specifications make provision for the length of the semitrailer to vary such that the rear axle spacing is between 4.27 and 9.14 m (14 and 30 ft). Only simply supported bridges are dealt with, thus the minimum axle spacing of 4.27 m (14 ft) is used to obtain maximum girder bending moments.

Girder bending moment influence lines across the width of the bridge, many of which are reported in previous research (Figures 2 and 3), clearly indicate that the transverse truck spacing should be as small as possible to obtain the maximum moment in any one of the girders. Only the 1.22 m (4 ft) minimum distance between adjacent truck wheel centroids is thus used in the analyses.

Two of the three axles of an HS20–44 standard truck carry the same load. The front axle carries only one-quarter of the load carried by each of the other two axles. The truck loading parameter P is defined as half the load acting on one of the heavy axles of a truck. The total weight of a truck is thus $4.5P$. The value of P should be increased according to the AASHTO provision for impact. The trucks in adjacent loading lanes may travel in the same or in opposite directions. whichever case produces the maximum required effect.

If three or four of the traffic lanes on a bridge are occupied simultaneously, it may result in girder bending moments that are larger than the corresponding moments obtained if only two traffic lanes are loaded. In practice, however, it is unlikely that three or more lanes will be occupied in such a way that all trucks are producing their maximum contribution to the moment in the particular girder under consideration. It is also very unlikely that all of these trucks will be loaded to their maximum capacity. These considerations are recognized in Provision 1.2.9 of the AASHTO specifications, which allow for a reduction in girder design moments obtained from loading conditions in which three or more traffic lanes are loaded.
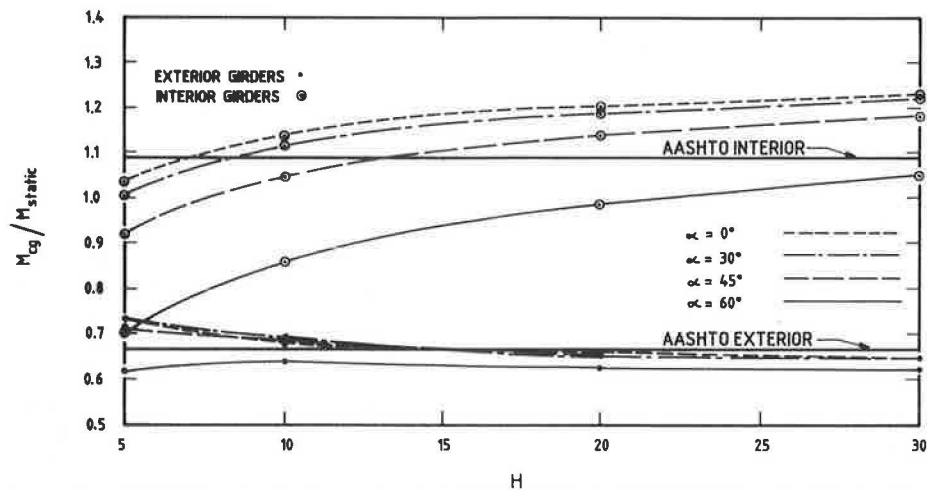
FIGURE 2 Midspan girder bending moment influence lines caused by a point load *P* moving transversely across a right bridge at midspan, *b/a* = 0.05.
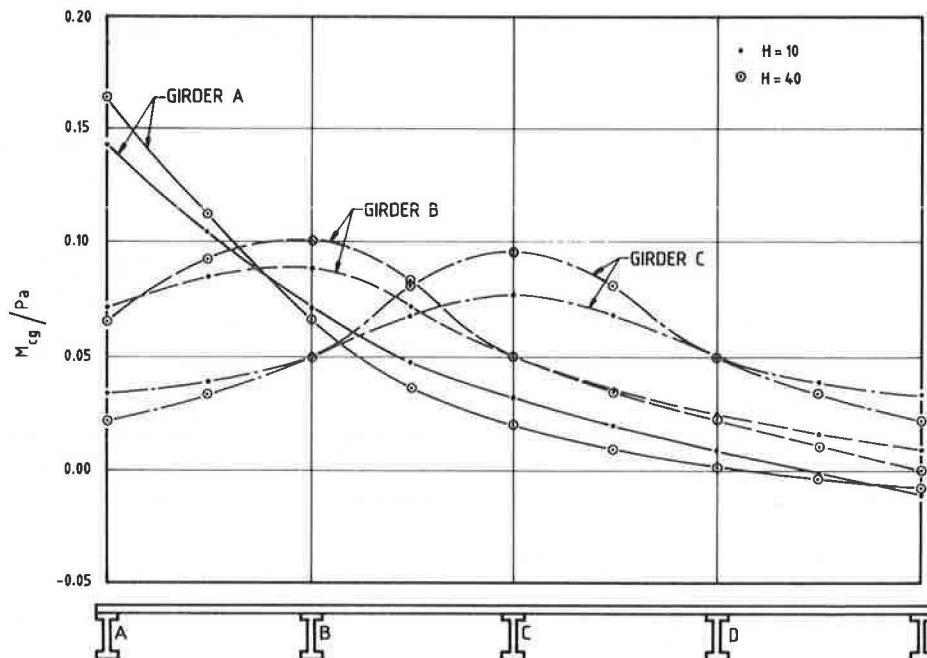

FIGURE 3 Midspan girder bending moment influence lines caused by a point load *P* moving transversely across a right bridge at midspan, *H* = 20.

If the girder moments obtained from load cases in which three or more traffic lanes are loaded are multiplied by their appropriate AASHTO reduction factors, it always results in design moments smaller than those obtained from load cases with two-lane loading. Thus, only two traffic lanes are loaded in this study.

The maximum bending moments in the composite girders, M_{cg}, are obtained from the bending moment envelope diagrams that result when the two trucks are moved progressively along the span. The directions of movement and transverse locations of the trucks, which produce maximum girder bending moments, are determined by trial and error.

The two other types of vehicle loading specified in the AASHTO specifications (*1*) are not of importance for the

range of spans considered. These are a lane loading, representing an approximation of a truck train that normally governs for spans longer than 44.8 m (147 ft), and two-axle military loading with axles spaced at 1.22 m (4 ft), which tends to govern in bridges with spans shorter than 11.28 m (37 ft).

BEHAVIOR OF SLAB-AND-GIRDER BRIDGES

Influence of the Vertical Stiffness Ratio, *R*

The behavior of a continuous slab over flexible girders is highly complex. In order to obtain some understanding of this behavior, it is useful, for the purpose of discussion, to degrade the complexity of the structure to something more familiar.

The vertical stiffness at any point along a beam is a function of

$$k_i EI/L^3$$

where

k_i = constant depending on the boundary conditions and the location of the point under consideration,
EI = bending stiffness of the beam, and
L = span of the beam.

The vertical stiffness of an interior composite girder in a slab-and-girder bridge is thus a function of $k_1 E_g I_{cg}/a^3$. Similarly, the vertical stiffness of a section of the slab that is effective in distributing load in the transverse direction is a function of $k_2(k_3 a)D/b^3$, where k_2 is a constant depending on the boundary conditions and $(k_3 a)$ is a fraction of the span, a. The parameters, a, b, and D have been previously defined.

The vertical stiffness ratio, R, is defined as the ratio of the vertical stiffness of an interior composite girder to the vertical stiffness of a section of the slab effective in the transverse direction and is thus proportional to

$$R \propto \frac{\left(\dfrac{E_g I_{cg}}{a^3}\right)}{\left(\dfrac{aD}{b^3}\right)} = \left(\frac{E_g I_{cg}}{aD}\right)\left(\frac{b}{a}\right)^3 = H(b/a)^3 \quad (4)$$

This vertical stiffness ratio, R, which determines the structural behavior of a slab-and-girder bridge, depends on two terms. The first term, which combines the flexural bending stiffness of the interior composite girders and that of the slab, is the flexural stiffness parameter H as previously defined. The second term, which is purely geometric, is the ratio between the girder spacing and span of the bridge.

These two terms have the following effects on the structural behavior. A bridge that has a large H value may either have stiff girders or a highly flexible slab. Consider the theoretical case where a slab-and-girder bridge has an extremely flexible

slab. A point load is applied directly above a girder. The particular girder deflects under the load, while the other girders deflect a negligible amount, because the slab is too flexible to transfer any significant loads to them. Thus, the loaded girder has to carry nearly all the load by itself and hardly any load sharing occurs.

On the other hand, a bridge that has a small H value can be thought of as one in which the slab is thick enough to distribute an applied point load, so that all the girders help to carry the load. A small H value thus corresponds to more uniform load distribution across the width of a bridge.

The effect of the second term, the b/a parameter, can be explained as follows. A small b/a ratio corresponds to a long-span bridge with girders at close spacing. The cross section of the bridge does not deform much and the bridge behaves like a single beam in which the load is distributed uniformly over the width. On the other hand, a large b/a ratio corresponds to a bridge with a short span and large girder spacing. The bridge behaves more like a wide slab in which the bending moments caused by a point load are nonuniformly distributed over the width.

Summarizing, a reduction in R caused by a reduction in H or b/a corresponds to an increase in the ability of the slab to distribute the load more uniformly. Because the b/a term is raised to the power three, it is obvious that a small change in its value has a more pronounced effect on the structural behavior than an equivalent change in the H value. The effects of these two terms are now more closely examined.

Effect of Varying the Stiffness Parameter, H

Figure 4 shows a typical graph for the maximum girder bending moment coefficient m_{cg}/M_{static} as a function of the stiffness parameter H for different angles of skew, when the bridge is subjected to two HS20–44 trucks. M_{static} is defined as the maximum static bending moment that results when one row of three wheels (say the left-front, left-middle, and left-rear wheel with loads $P/4$, P, and P, respectively) of one HS20–44 truck moves across a single isolated beam that has the same
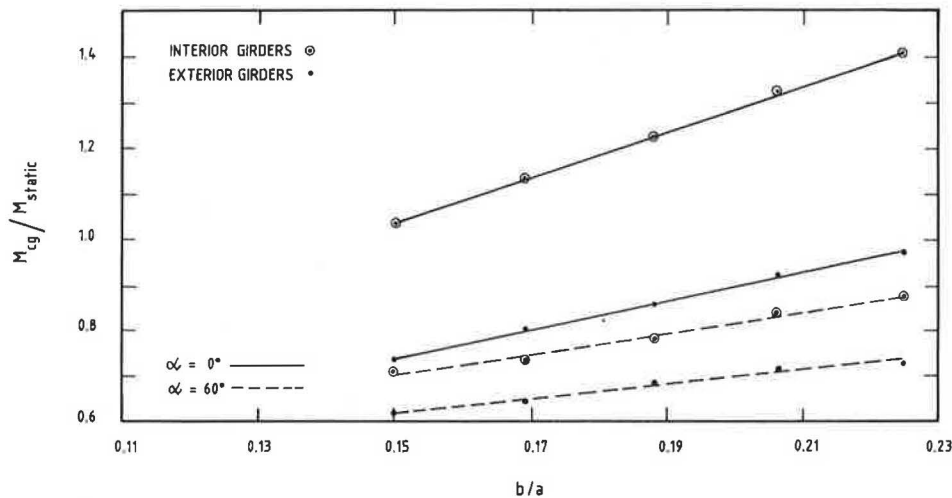

FIGURE 4 Maximum girder bending moment coefficient variation with H, $a = 12.19$ m (40 ft) and $b = 1.83$ m (6 ft).

span as the girders in the bridge. In this particular case, the girder spacing and span are 1.83 and 12.19 m (6 and 40 ft), respectively. No distinction is made between the interior girders. Because they are prefabricated, they are usually designed for the same moments, although small differences in maximum moments normally occur. The maximum bending moment in the interior girders always increases as H increases because the ability of the slab to distribute loads decreases.

The girder moments are more sensitive to changes in H when the H value is small. Figure 4 shows that, for $\alpha = 0$, an increase of 50 percent in the H value, from $H = 5$ to $H = 7.5$, results in a 5.8 percent increase in the maximum interior girder bending moment. An increase of 50 percent in the H value from $H = 20$ to $H = 30$ results in only a 2.4 percent increase in moment.

It is fortunate that the girder design moments are insensitive to moderate changes in the H value, because there are many uncertainties surrounding the true value of H. These uncertainties include the effect of cracks in the slab concrete, the true modulus of elasticity of the slab and girder concrete, and the approximation of the effect of shear lag by an effective flange width.

The exterior girder behaves differently. As a rule, an increase in H always results in a small decrease in the maximum exterior girder bending moment. However, it is found that when the angle of skew is 60 degrees, there is, in some cases, a slight increase in the maximum exterior girder moment when H is increased between $H = 5$ and $H \cong 15$, after which the moment decreases again. This is the case for the particular bridge shown in Figure 4. The maximum moment in the exterior girder is highly insensitive to changes in H over the whole range of H considered.

The difference in behavior of the interior and exterior girders can easily be explained with reference to Figure 2. The data were provided by Sithichaikasem (8). Figure 2 shows the midspan girder bending moment influence lines for a single point load P moving transversely across a right bridge at midspan. The b/a ratio of the bridge is 0.05. Assume that the girder spacing is 2.44 m (8 ft). To obtain the maximum bending moment in Girder C, Truck 1 is placed in Panel BC and Truck 2 in Panel CD, with the centroids of the nearest wheels 0.61 m (2 ft) away from Girder C on both sides. In Panels BC and CD, the values of the moment influence diagram for Girder C increase when H increases. Thus, the maximum bending moment in Girder C increases.

On the other hand, to obtain the maximum bending moment in the exterior girder, Girder A, the centroid of the nearest longitudinal row of wheels of Truck 1 is placed 0.61 m (2 ft) away from Girder A (face of the curb) according to the AASHTO requirement (1). The second row of wheels of Truck 1 falls directly on top of Girder B. All the wheels of Truck 2 fall in Panels BC and CD. The influence line for Girder A indicates that when H increases, only the first row of wheels of Truck 1 causes an increase in bending moment. All the other wheels of Trucks 1 and 2 cause a reduction in moment when H increases. The sum of reductions is slightly more than the moment increase produced by the first line of wheels of Truck 1. Thus, the maximum exterior girder bending moment decreases slightly when H is increased.

Because of the reciprocal law, the shape of a particular girder bending moment influence diagram also represents, to one or other scale, how a point load acting on the girder is distributed to the adjacent girders. Figure 2, also, therefore, demonstrates that a small H value corresponds to a more uniform distribution of load, because the influence lines are flatter for smaller H values.

Summarizing, a larger H value corresponds to a more flexible slab less capable of distributing load. Most of the wheel loads act on the deck in an area supported by the interior girders. Because of the diminished capability of the slab to transfer loads from this area to the edges of the bridge as H increases, the bending moments in the exterior girders decrease at the cost of an increase in moments in the interior girders.

Effect of Varying the Parameter b/a

Figure 5 shows a typical graph for the maximum girder bending moment coefficient M_{cg}/M_{static} as a function of b/a by varying only the girder spacing, b. The bridge is subjected to two HS20–44 trucks. In this particular case, the span is 12.19 m (40 ft) and $H = 5$. Because the span is kept constant, the static bending moment to be distributed remains the same.

Figure 5 shows that as the b/a ratio increases, the maximum girder bending moment coefficients increase approximately linearly for both interior and exterior girders. This linear variation with b/a also holds when skew is introduced.

Figure 3 shows influence lines for girder bending moments at midspan caused by a single point load P moving transversely across the bridge at midspan for $b/a = 0.1$ and 0.2 with $H = 20$. The data were provided by Sithichaikasem (8). The influence diagram for a particular girder has a larger peak in the vicinity of the girder for the larger b/a ratio. Because wheel loads are positioned as close as possible to a particular girder to obtain its maximum bending moment, the result is a larger bending moment if b/a is increased.

As b/a is increased, there is a decrease in influence values for wheels located more than approximately one girder spacing away from the girder under consideration. However, because b is increased and the wheels are kept as close as possible to the particular girder, there is a shift in the locations of wheels relative to the influence diagram in the direction of the girder (more wheels in the positive influence area), resulting in higher influence values.

The discussion has concerned a variation in the b/a ratio by changing b. The increase in girder bending moment with b is as a result of the larger slab area that each girder carries (more wheel loads can be applied), as well as the result of the larger b/a ratio, which decreases the ability of the slab to distribute the load in the transverse direction.

Figure 6 shows a typical graph for the maximum girder bending moment coefficient M_{cg}/M_{static} as a function of b/a for different angles of skew. The b/a ratio is now varied by changing only the span, a. The bridge is subjected to two HS20–44 trucks. The girder spacing is 2.74 m (9 ft) and $H = 5$.

Figure 6 shows that the variation in the maximum girder bending moment coefficients is approximately linear when b/a is varied for both right and skew bridges. For a right bridge, the maximum bending moment coefficient increases for the
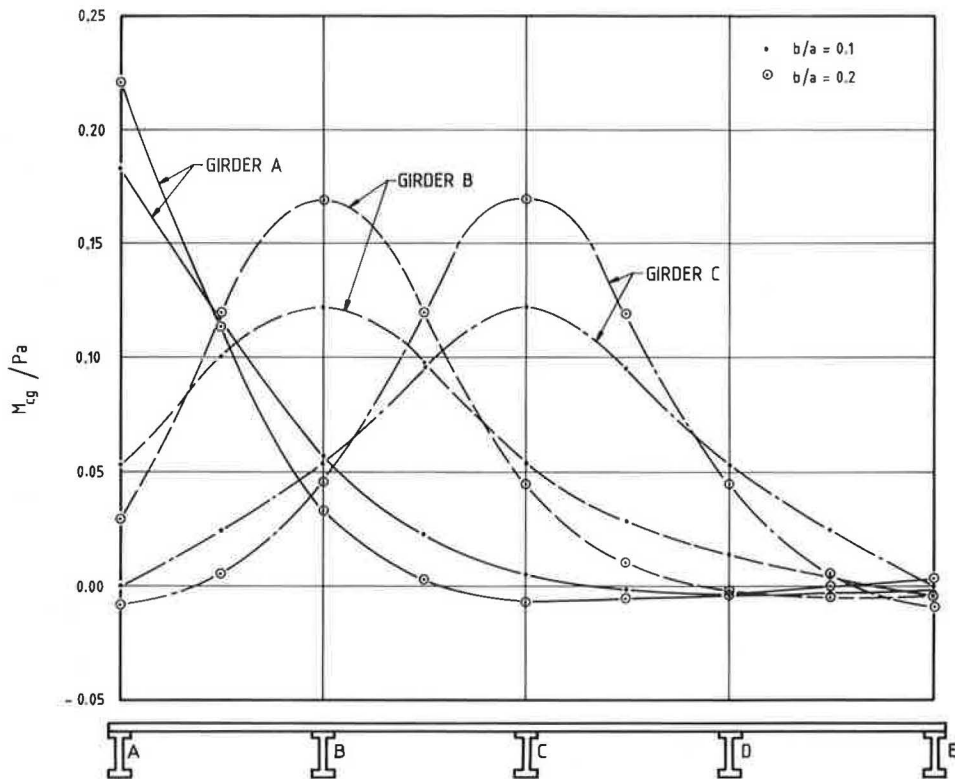
FIGURE 5 Maximum girder bending moment coefficient variation with *b/a* by changing only *b*, *H* = 5 and *a* = 12.19 m (40 ft).
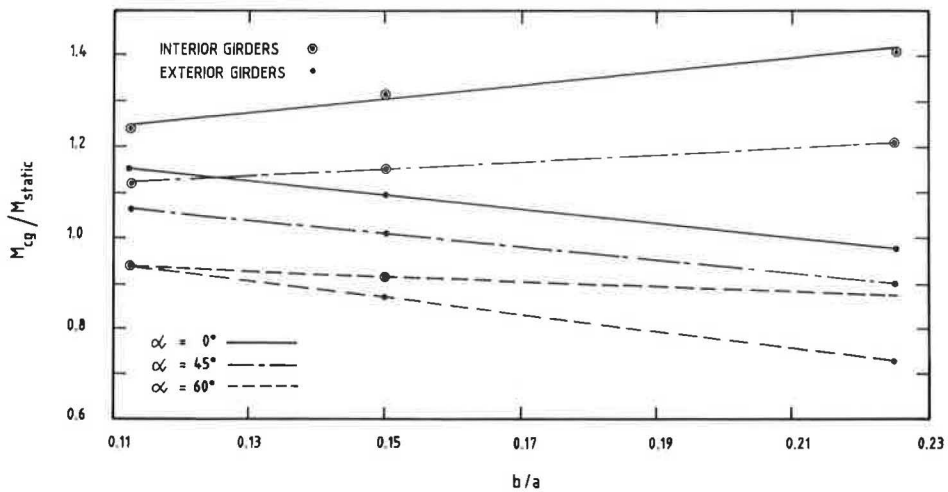

FIGURE 6 Maximum girder bending moment coefficient variation with *b/a* by changing only *a*, *H* = 5 and *b* = 2.74 m (9 ft).

interior girders and decreases for the exterior girders as *b/a* is increased by reducing the span. This behavior is similar to the effect of an increase in *H* as previously discussed.

The variation of the influence lines in Figure 2 caused by an increase in *H* is similar to the variation of the influence lines in Figure 3 caused by an increase in *b/a* ratio. Because *b* is constant in this case, similar effects are obtained because there is no shift in the locations of the wheels relative to the girders, as is the case in Figure 5.

Although the bending moment coefficient for the interior girders of the right bridge in Figure 6 increases with *b/a*, the bending moment is smaller because the static bending moment is smaller when the span is reduced. Figure 6 also shows that the interior girder bending moment coefficient decreases with a decrease in the span for large angles of skew, but this is dealt with in the next section.

The behavior of a slab-and-girder bridge is sensitive to changes in *b* and the *b/a* ratio. The importance of the *b* pa-

rameter and the linear behavior is reflected in the current AASHTO design specifications, because the interior girder bending moments may be calculated using the wheel load fraction $b/5.5$.

Effect of Varying the Angle of Skew α

The influence of the angle of skew on the distribution of wheel loads is the crux of this study. A bridge built on skew alignment always has smaller girder bending moments than its right counterpart with the same span. The larger the angle of skew becomes, the smaller the girder design moments obtained. This holds for all girders in the bridge.

The reduction in bending moments in the girders of skew bridges results as a consequence of the following two effects:

1. With the abutments not perpendicular to the girders, some of the wheels of the trucks are not on the bridge at all or are closer to the supports than in the corresponding right bridge. The total maximum static bending moment to be distributed between the girders is thus reduced.

2. In a short-span bridge with a large angle of skew, there is a tendency for the slab to span in the shortest diagonal direction. The slab transfers part of the load directly to the supports. This slab action decreases the loads that are normally carried by the girders in right bridges. There are corresponding changes in the magnitude of the bending moments in the slab. The effect of skew on the slab moments is not determined in this study.

Figure 7 shows a typical graph for the maximum girder bending moment coefficient M_{cg}/M_{static} as a function of the angle of skew for different H values. The girder spacing is 2.74 m (9 ft) and the span is 12.19 m (40 ft). Figure 7 indicates
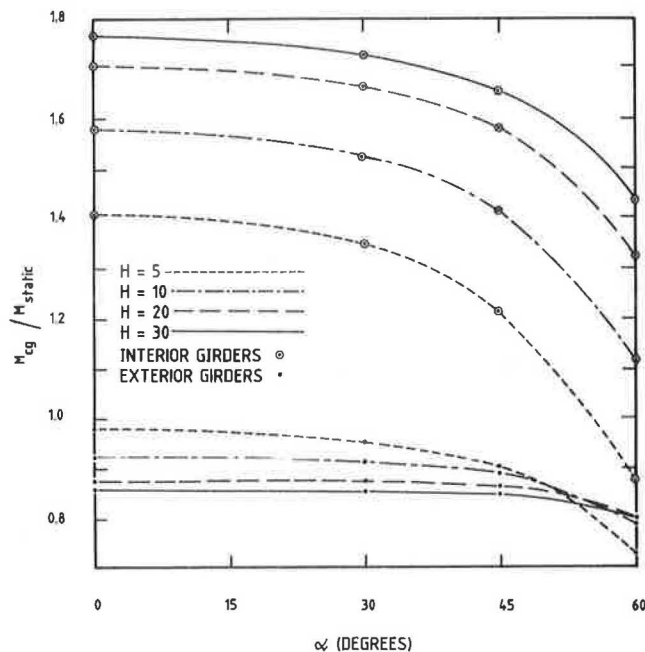

FIGURE 7 Maximum girder bending moment coefficient variation with α, $a = 12.19$ m (40 ft) and $b = 27.74$ m (9 ft).

that the exterior girders are highly insensitive to changes in the angle of skew for α between 0 and 45 degrees. The interior girders are also insensitive to changes in α between 0 and 30 degrees. Most of the reduction in girder bending moments occurs for angles of skew larger than 45 degrees. The effect of skew is more pronounced when the H value is small. This result can be explained by the tendency of stiff girders to oppose the action of the slab to span in the shortest diagonal direction. The reduction in girder bending moment because of skew is large for a combination of large angle of skew, large girder spacing, small span, and small H value.

The reduction in maximum interior girder bending moments because of skew is always less than 5 percent for angles of skew up to 30 degrees. When $\alpha = 60$ degrees, a reduction of as much as 38 percent is possible. The reduction in maximum exterior girder bending moments because of skew is always less than 8 percent for angles of skew up to 45 degrees. When $\alpha = 60$ degrees, the maximum possible reduction is 25 percent.

Figure 4 shows the typical variation in maximum girder bending moment coefficient with H for different angles of skew. The effect of skewness may only be a reduction in the girder moments, because the shape of the diagrams remains almost the same. This is especially true of the interior girders for which the largest bending moment reductions take place.

Figure 4 also shows that there is a tendency for an edge girder to become the controlling girder in a skew bridge, because the bending moments in the interior girders are reduced much more by skew than those in the exterior girders. This tendency becomes more pronounced for a combination of a large angle of skew, a small H value, a large span, and a small girder spacing. Cohen (9) made a similar observation. However, the edge girder controls in only 2 of the 108 bridges analyzed. In these two cases, the maximum exterior girder bending moment is only 0.3 and 1.0 percent larger than the maximum bending moment in the interior girders.

It is possible to avoid the undesired condition of having the controlling moment in an edge girder by keeping the truck wheels at least 0.61 m (2 ft) away from the edge girders. This conclusion is limited to bridges with spans not exceeding 24.38 m (80 ft).

COMPARISON WITH THE AASHTO DESIGN RECOMMENDATIONS FOR RIGHT BRIDGES

Although the actual distribution of load to the girders in a slab-and-girder bridge is highly complex, a fictitious load distribution, which is characterized by the well established concept of a wheel load fraction, can be used to account for the moments in the girders. The current AASHTO *Standard Specifications for Highway Bridges* (1) permits the use of wheel load fractions for the design of right slab-and-girder bridges subjected to standard truck loads.

The maximum bending moment coefficients for the interior and exterior girders obtained by using the AASHTO wheel load fractions are also shown in Figure 4 for the particular bridge. The bending moment coefficient for the exterior girders resulting from the wheel load fraction $b/(4 + b/4)$ for steel I-beams is not indicated. This fraction provides design moments that are too large by 30 to 60 percent.

A comparison between the present results for right bridges and the current AASHTO specifications indicates that the AASHTO provisions result in bending moments for the interior girders that are in many cases too small. This is especially so for a combination of a large H value, short span, and small girder spacing. For the range of parameters considered in this study, the AASHTO $b/5.5$ interior girder wheel load fraction is between 12 percent too low and 32 percent too high. Culham (*10*), who analyzed right bridges with intermediate diaphragms, also found that the $b/5.5$ fraction gives interior girder bending moment results that are too small for short spans and too large for large spans.

The current AASHTO provision for exterior girders, which is based on the assumption that the slab acts as if simply supported between adjacent girders, is unconservative in most of the cases considered. It is less safe when H is small and the span is large. The girder spacing does not have any significant effect. For the range of parameters considered in this study, this specified AASHTO method for the exterior girders is up to 23 percent of the unsafe side. However, the AASHTO requirement that edge girders must have at least the same load-carrying capacity as the interior girders governs in these cases. Culham (*10*) also found that this provision for the exterior girders underestimates the load carried by the exterior girders.

The AASHTO specification that requires the same load-carrying capacity for all the girders in the bridge leads to overconservative design of the exterior girders. For bridges with short spans, stiff girders, and large girder spacings, the design bending moments can be more than twice the actual values. However, it is practical to make all girders identical, which would then also allow for a possible future widening of the bridge. This study is based on bridges having identical interior and exterior girders.

SIMPLIFIED ANALYSIS PROCEDURE FOR RIGHT AND SKEW SLAB-AND-GIRDER BRIDGES

General

The comparison between the present girder bending moment results for right bridges and the current AASHTO wheel load fractions indicates that some improvements in the existing analysis method for right bridges are desirable. Furthermore, the need to have some sort of simplified analysis procedure for the girders of skew slab-and-girder bridges exists because the AASHTO specifications provide no design recommendations regarding this matter.

The use of a wheel load fraction to determine girder bending moments is now expanded to cover skew bridges as well. Instead of developing independent expressions for wheel load fractions in skew bridges for each angle of skew, it is more convenient to incorporate the effect of skew by multiplying improved wheel load fractions for right bridges by a skew reduction factor.

The maximum design bending moment for a composite girder can be expressed as

$$M_{cg} = (M_{static})(b/Q)(Z) \qquad (5)$$

where

$\quad b/Q$ = wheel load fraction;
$\quad\ \, Q$ = a variable that depends on the load distribution capability of the bridge, currently fixed as 1.68 m (5.5 ft) for interior girders according to AASHTO (*1*);
$\quad\ \, Z$ = skew reduction factor, defined as the maximum girder bending moment in a skew bridge divided by the maximum girder bending moment that results when the same bridge is made right; and
M_{static} = maximum static bending moment coefficient as defined before.

When $a > 10.06$ m (33 ft), the maximum static bending moment is

$$M_{static} = Pa(1.138/a^2 - 2.667/a + 9/16)$$

with a in meters $\hfill (6)$

or

$$M_{static} = Pa(12.25/a^2 - 8.75/a + 9/16)$$

with a in feet $\hfill (7)$

Because of the lack of torsional stiffness in steel I-beams, it is necessary to increase M_{cg} obtained from Equation 5 by 5 percent if steel I-beams are used as supporting girders.

This simplified analysis method has been developed to obtain the maximum bending moments in the girders. Bakht (*11*) has discussed the case of maximum shear forces in the girders.

Criteria for Interior Girders in Right Bridges

Figure 8 shows Q values for the interior girders in right bridges that should be used in Equation 5. The variable $a/(H)^{1/2}$ used by Newmark (*7*) yields less scatter of wheel load fractions than any other variable used in an attempt to find the best variable. This variable originates from the thought that the bending moments in the girders should depend in some way on the relative deflections of the girders that are proportional to the quantity $a^3/(E_g I_{cg})$. For a particular slab, the quantity a^2/H amounts to the same thing. If $a/(H)^{1/2}$ is used, a convenient linear relationship exists. Two well-defined Q value data bands can be distinguished. One for a group of bridges that has a girder spacing of 1.83 m (6 ft) and one for another group with girder spacing of 2.74 m (9 ft). The two straight lines indicated in Figure 8 as "present" are conservative estimates for Q when b is 1.83 and 2.74 m (6 and 9 ft).

The linearity of the maximum girder bending moments with b when H and a are kept constant (as shown in Figure 5) is recognized and applied to obtain a conservative expression for the Q values of interior girders in right bridges as follows:

$$Q = (0.01538 + b/45.72)[a/(H)^{1/2}]$$
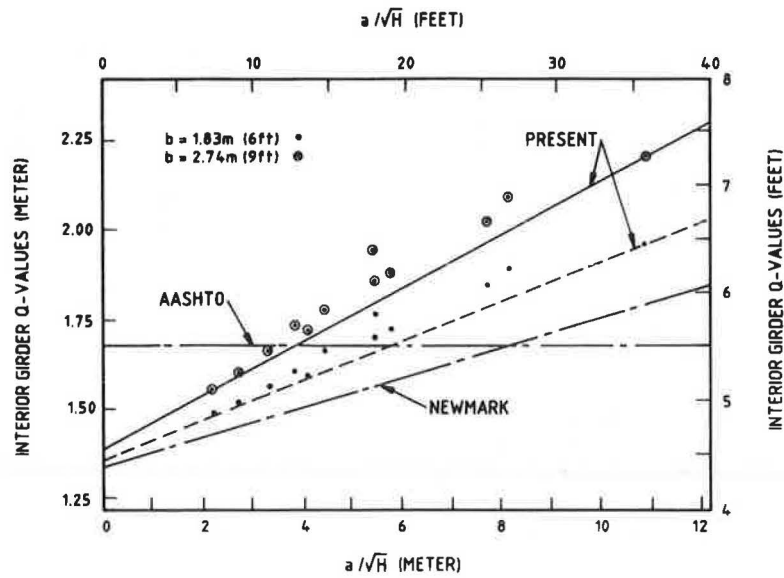$$+ 1.298 + b/30 \qquad \text{(meters)} \qquad (8)$$

FIGURE 8 *Q* values for maximum interior girder bending moments in right bridges.

or

$$Q = (0.01538 + b/150)[a/(H)]^{1/2}]$$
$$+ 4.26 + b/30 \quad \text{(feet)} \quad (9)$$

The AASHTO wheel load fraction, $b/5.5$, for the interior girders of right bridges is based on research done by Newmark (7) many years ago. The factor $b/5.5$ reflects the linear trend in b, which is observed in the present study, but it does not include directly the effects of H and b/a. This wheel load fraction is an oversimplification of the design equation proposed by Newmark, which includes all relevant parameters and is also indicated in Figure 8. Unlike Newmark's wheel load fractions, which are based on the distribution of load from only one axle of each truck, the current wheel load fractions are obtained directly from the maximum girder bending moments caused by two complete HS20–44 trucks.

Interior girder bending moments for right bridges obtained by using this equation for Q, and thus an improved varying b/Q wheel load factor, are conservative and within 8 percent of the finite element results, whereas, using the fixed AASHTO wheel load fraction, $b/5.5$ results in bending moments between 12 percent too small and 32 percent too large.

Criteria for Exterior Girders in Right Bridges

Figure 9 shows Q values for the exterior girders in right bridges that should be used in Equation 5. These Q values apply only when the minimum distance between the centroids of the edge girder and nearest truck wheels is 0.61 m (2 ft). Figure 9 shows a well-defined functional relationship between Q and $H(b/a)^3$. The quantity $H(b/a)^3$ is proportional to the vertical stiffness ratio, R, which has been discussed previously. The
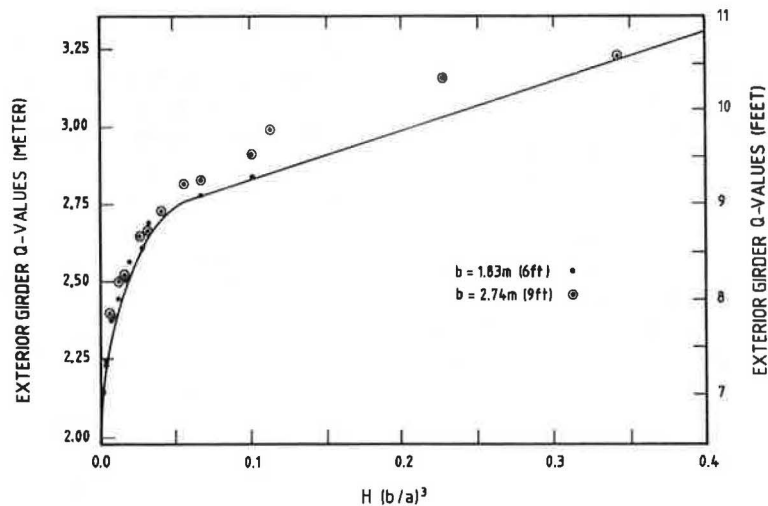

FIGURE 9 *Q* values for maximum exterior girder bending moments in right bridges.

following equations give conservative Q values for exterior girders in right bridges.

For $H(b/a)^3 < 0.0569$,

$$Q = 121.92H(b/a)^3 - 145.69[H(b/a)^3]^{1.1} + 2.042 \quad \text{(m)} \tag{10}$$

or

$$Q = 400H(b/a)^3 - 478[H(b/a)^3]^{1.1} + 6.7 \quad \text{(ft)} \tag{11}$$

For $H(b/a)^3 \geq 0.0569$,

$$Q = 1.597H(b/a)^3 + 2.664 \quad \text{(meters)} \tag{12}$$

or

$$Q = 5.24H(b/a)^3 + 8.74 \quad \text{(feet)} \tag{13}$$

Exterior girder bending moments for right bridges obtained by using these equations for Q and thus an improved varying b/Q wheel load factor are conservative and within 5 percent of the finite element results, whereas, the simply supported slab action AASHTO provision is unconservative in most cases by as much as 23 percent.

Criteria for Girders in Skew Bridges

Before 1985, Chen (9) was the only researcher who used his analytical results to develop practical design criteria for skew slab-and-girder bridges. He followed Newmark's (7) method

for right bridges to determine wheel load fractions for skew bridges by expressing Q values as a function of the variable $a/(H)^{1/2}$ for a particular angle of skew.

The variable $a/(H)^{1/2}$ is not a suitable parameter for skew bridges, as Chen's Q values exhibit large scatter, which increases with the angle of skew. At 60 degrees skew, the scatter is as much as 55 percent. As a result of this large scatter in Q values, Chen's conservative design equations do not effectively incorporate the beneficial effect of skew. Furthermore, Chen based his wheel load fractions on the distribution of load from only one axle of each truck and his solution accuracy was seriously impaired by the coarse finite difference network he used.

Recent research on skew slab-and-girder bridges by Bakht (11) and Khaleel (12) also provides practical analysis criteria for skew bridges.

Figures 10 and 11 show the skew reduction factor Z for interior and exterior girders that should be used in Equation 5. The parameter $b/(aH)$ is the logical choice of variable, because it is found that the reduction caused by skew is large in bridges with large girder spacing, small span, and small H value. See Figures 5, 6, and 7 for verification.

The Z value data points for the exterior girders for $\alpha = 30$ degrees are not indicated in Figure 11. It is obvious from Figure 7 that the reduction is insignificant. The conservative linear equations for Z shown in Figures 10 and 11 are presented in Table 1.

The parameter $b/(aH)$ becomes $bD/(E_g I_{cg})$ when H is substituted. As the bridge span increases, I_{cg} needs to increase drastically to satisfy the deflection limitations. The parameter $b/(aH)$ and thus the effect of skew becomes small for bridges with large spans. This is confirmed by Figure 6 provided by Bakht (11). Because the effect of skew is of primary concern
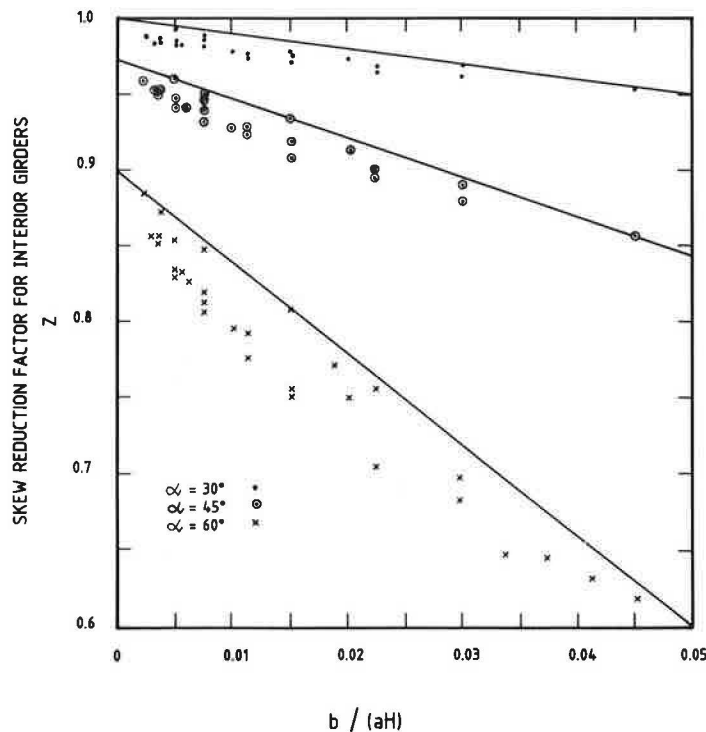

FIGURE 10 Skew reduction factors for maximum interior girder bending moments in skew bridges.
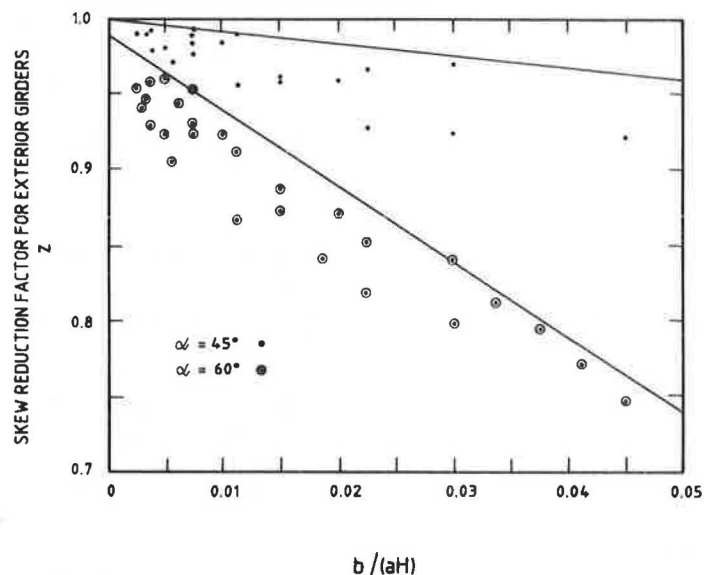
FIGURE 11 Skew reduction factors for maximum exterior girder bending moments in skew bridges.

TABLE 1 SKEW REDUCTION FACTORS, Z, AND MAXIMUM CONSERVATIVE ERRORS

α	INTERIOR GIRDERS		EXTERIOR GIRDERS	
degrees	Z	MAX. % ERROR	Z	MAX. % ERROR
0	1.0	8	1.0	5
30	1.0 - b/(aH)	8	1.0	8
45	.97 - 2.5 b/(aH)	8	1.0 - 0.8 b/(aH)	11
60	.90 - 6.0 b/(aH)	15	.99 - 5.0 b/(aH)	13

in this study, the span of the bridge considered is limited to the range in which the effect of skew is of importance. Therefore, the maximum span considered is 24.38 m (80 ft).

Accuracy of the Simplified Analysis Procedure

The maximum differences between the conservative girder bending moments from Equation 5 and the "correct" bending moments obtained from the finite element analyses are also presented in Table 1. The equations for Z suggested by Marx et al. (6) are less conservative, with the result that the error sizes are split in half, but to both sides of the "true" finite element results.

CONCLUSIONS

Proposed Simplified Analysis Method

The proposed simplified analysis procedure can be used to determine the maximum girder bending moments in simply supported right and skew slab-and-girder bridges subjected

to AASHTO HS20–44 truck loads. The method uses improved AASHTO wheel load fractions for right bridges, which are modified for skew bridges by using a skew reduction factor. The improved wheel load factor, b/Q, and the skew reduction factor, Z, depend not only on the girder spacing, b, but also on the stiffness parameter, H, and the span, a. The typical bridge considered is shown in Figure 1. The simplified analysis method is based on data obtained from finite element results on bridges with five identical girders subjected to two-lane truck traffic. However, it may be used for bridges with more girders and more load lanes. The accuracy of this method is indicated in Table 1.

Behavior of Slab-and-Girder Bridges

The behavior of slab-and-girder bridges is controlled by the following parameters:

1. Angle of skew, α;
2. Span, a, and girder spacing, b; and
3. Dimensionless stiffness ratio, H, which combines all the structural properties of the bridge members.

The length of the slab overhangs determines the location of the wheels closest to the exterior girders, which has a significant effect on the exterior girders, but a constant value is used in this study.

The effect of skew is a reduction in the girder bending moments. The larger the angle of skew and the ratio $b/(aH)$, the larger the resulting reductions. The maximum interior girder bending moment reduction as a consequence of skew is always less than 5 percent for angles of skew up to 30 degrees, but the reduction is as large as 38 percent when $\alpha = 60$ degrees. The exterior girders are less affected by skew. The maximum exterior girder bending moment reduction is always less than 8 percent for angles of skew up to 45 degrees, but the reduction is as large as 25 percent when $\alpha = 60$ degrees. For all girders, the most significant reductions occur when the angle of skew is more than 45 degrees.

Because the exterior girders are less affected by skew than the interior girders, there is a tendency for the edge girder to become the controlling girder in a skew bridge. This tendency is more pronounced in a bridge with large angle of skew, small H value, large span, and small girder spacing. However, by keeping the faces of the curbs directly above the edge girders, the maximum bending moment always occurs in an interior girder for spans up to 24.38 m (80 ft).

Current AASHTO Wheel Load Factors

For the range of parameters considered in this study, the AASHTO wheel load fraction for interior girders, $b/5.5$, yields results that are between 12 percent too small and 32 percent too large. It is likely that the interior girder bending moments will be underestimated for bridges with a large H value, small span, and small girder spacing. The AASHTO method to determine the maximum exterior girder bending moment by assuming that the slab acts as if simply supported between girders underestimates the actual exterior girder bending moments in most of the bridges considered. It gives bending moments that are up to 23 percent too small. The AASHTO exterior girder wheel load fraction $b/(4 + b/4)$ for steel I-beams yields results that are between 30 and 60 percent too large.

GLOSSARY

The following symbols are used in this paper:

A = identifier for the edge girder, as shown in Figure 1;
a = span of the bridge;
B = identifier for the first interior girder, as shown in Figure 1;
b = girder spacing;
b/a = ratio of the girder spacing to span;
b/Q = wheel load fraction;
C = identifier for the centre girder, as shown in Figure 1;
D = $E_s t^3/12(1 - \mu^2)$ flexural stiffness of the slab per unit width;
E_g = Young's modulus of elasticity for the prefabricated girders;

E_s = Young's modulus of elasticity for the slab;
H = $E_g L_{cg}/aD$ dimensionless stiffness parameter;
I_{cg} = bending moment of inertia of an interior composite girder;
I_g = bending moment of inertia of an isolated prefabricated girder;
M_{cg} = maximum bending moment acting in a composite girder;
M_{static} = maximum static bending moment in an isolated beam subjected to half the load of one AASHTO HS20–44 truck;
P = point load representing half the load of one heavy axle of an AASHTO HS20–44 truck;
Q = variable that depends on the load distribution capability of the bridge;
R = vertical stiffness ratio;
t = slab thickness;
Z = skew reduction factor;
α = angle of skew as defined in Figure 1; and
μ = Poisson's ratio, taken as 0.2 for concrete.

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