ALINEA: A Local Feedback Control Law for On-Ramp Metering

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ALINEA, a new local traffic-responsive strategy for ramp metering, is presented. The new control strategy is based on a feedback structure and is derived by use of classical automatic control methods. ALINEA is a simple, robust, flexible, and effective local strategy for ramp metering. Real-life results from application of the new strategy to a single on-ramp of the Boulevard Périphérique in Paris are provided.

Traffic-responsive on-ramp metering systems are currently operating at several freeways in the United States. Furthermore, on-ramp metering has been tested or introduced in some freeways in France, Italy, West Germany, New Zealand, United Kingdom, and the Netherlands.

More or less sophisticated coordinated on-ramp control strategies have been proposed and tested by simulation, and some installations have been reported. Nevertheless, operating ramp metering systems are mainly of a local, traffic-responsive character. Real-time measurements used in local control strategies are provided by loop detectors that are located in the vicinity of the corresponding on-ramps.

An extensive review of local traffic-responsive strategies was provided by Mashier et al. (1). Strangely enough, many local strategies presented or implemented so far are based on a disturbance compensation feedforward philosophy. However, direct disturbance compensation is known to lead to fairly sensitive control strategies. In fact, in the last 40 years, an independent scientific discipline, automatic control, has evolved from the need to eliminate disturbances in a robust and efficient way by use of the basic notion of feedback.

A new local, traffic-responsive strategy for ramp metering applying a feedback control structure is presented. The feedback law is derived by use of classical methods of automatic control theory and is characterized by remarkable simplicity, high efficiency, and robustness. The main aim is to thoroughly present the theoretical background for the strategy's development and to discuss qualitative features such as simplicity, robustness, and flexibility. On the other hand, the new strategy for ramp metering has been tested and compared to five known strategies during an experimentation period of several months on the ramp of the Coen Tunnel near Amsterdam. Results of this experimentation were reported by Middelham and Smulders (13).

SOME GENERAL CONTROL STRUCTURES

Consider a process under control (e.g., house heating), and a selected output (e.g., inner temperature), shown schematically in Figure 1. The process is affected by some process inputs. Process inputs that can be manipulated are called controllable inputs, whereas process inputs that cannot be manipulated are called disturbances (e.g., outer temperatures). Disturbances may be predictable or unpredictable, measurable or nonmeasurable, etc. The control problem is to appropriately select the controllable inputs so as to achieve—despite the impact of disturbances—a desired process output value called the "set value" (e.g., 20°C).

Assume that a mathematical model of the process is available, and furthermore that all essential disturbances are time variant but measurable. Then, at any instant of time, inputs can be calculated on the basis of the process model using disturbance measurements to achieve a given set value (Figure 1a). This feedforward control procedure is broadly known as disturbance compensation. Because of inevitable inaccuracies of mathematical models and occurrence of other unexpected, nonmeasurable disturbances, disturbance compensation is known to be a particularly sensitive control structure.

In the heating example, disturbance compensation would correspond to measuring the outer temperature and controlling the heating valves to achieve a constant inner temperature, say, 20°C.

Consider a traffic flow process around an on-ramp as shown in Figure 2. Assuming absence of congestion, the downstream traffic volume \( q_{in} \) may be declared as the process output with a set value \( \bar{q} \) (e.g., equal to capacity), whereas the on-ramp volume \( r \) is identified as the controllable input and the upstream traffic volume \( q_{up} \) as a measurable disturbance. In order to keep \( q_{in} \) near \( \bar{q} \), an intuitive way to do this is to calculate

\[
r = \bar{q} - q_{in}
\]

using current measurements of the disturbance \( q_{in} \). Obviously, this feedforward procedure, which is essentially applied by many known local methods for ramp metering, corresponds to the disturbance compensation of Figure 1a.

The quality of results depends on the accuracy of the applied process model, but complicated models lead to complicated control algorithms. However, even for highly complex strategies, sensitivity with respect to existing inaccuracies and unexpected disturbances remains a structural drawback.

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sumed to be constant during the time interval \([(k - 1)T, kT]\). The constant parameter \(\beta\) results from the discretization procedure to be \(\beta = \exp(-\alpha T/8)\).

The parameter \(\beta\) may be neglected if the ratio \(8/T\) is sufficiently small. This will be the case if traffic volume entering the freeway reaches Site 2 during the time interval \(T\). In this case, the effect of entering traffic \(r(k)\) will be visible at Site 2 by the end of the corresponding time interval. Setting \(\beta = 0\) in Equation 5,

\[
\Delta o_{\text{out}}(k + 1) = [\Delta q_{\text{in}}(k) + \Delta r(k)]/Q' \tag{6}
\]

In the next section, a feedback law is developed under the assumption \(\beta = 0\). An extension applying to the case of higher ratios \(8/T\) will be derived later. Finally, a complementary disturbance compensation mechanism will be presented.

Regulator Design

An appropriate feedback law for the process of Equation 6 is given by the following integral regulator

\[
r(k) = r(k - 1) - K_R[\dot{o} - o_{\text{out}}(k)] \tag{7}
\]

where \(K_R\) is a constant and positive regulator parameter. After applying this regulator to Equation 6, the following z-transfer function is obtained for the closed-loop system:

\[
H(z) = o_{\text{out}}(z)/\dot{o} = \frac{K_R/Q'}{z - 1 + (K_R/Q')} \tag{8}
\]

A time-optimal deadbeat regulator is obtained by choosing \(K_R = Q'\). Because of the sign of \(Q'\), the linearization of Equation 6 is strictly valid only on the left-hand side of the fundamental diagram. However, even for congested traffic the feedback law of Equation 7 leads to traffic occupancy reduction and can thus be applied in the same way.

Extension for Bottleneck Further Downstream

If Site 2 is located at a bottleneck further downstream and if \(T\) is chosen accordingly short, the entering traffic may not reach Site 2 at the end of each time interval, in which case \(\beta\) cannot be neglected. Applying the regulator of Equation 7 to the original process model of Equation 5, the closed-loop z-transfer function becomes

\[
H(z) = o_{\text{out}}(z)/\dot{o} = \frac{z(1 - \beta)}{z^2 - 2\beta z + \beta} \tag{9}
\]

with the eigenvalues \(\lambda_{1,2} = \beta \pm j[\beta(1 - \beta)]^{1/2}\). Although the closed-loop system remains stable for any \(\beta < 1\), the transient behavior may be slow. An amelioration may be achieved by application of the following proportional-plus-integral feedback law

\[
r(k) = r(k - 1) - K_R[\dot{o} - o_{\text{out}}(k)]
- K_p[o_{\text{out}}(k) - o_{\text{out}}(k - 1)] \tag{10}
\]

where \(K_p\) is a further constant and positive regulator parameter. A time-optimal deadbeat regulation is achieved by the choice \(K_R = Q', K_p = \beta Q'/1 - \beta\), as can be readily demonstrated. In summary, a further term has been added in Equation 9 as compared with Equation 7 for the particular case of, say, \(8/T > \alpha Q'\).

Elimination of Constant Disturbances

The regulator of Equation 7 is capable of eliminating constant disturbances. The z-transfer function of the feedback law is

\[
r(z) - \Delta o_{\text{out}}(z) = \frac{K_R z}{z - 1} \tag{11}
\]

Considering the process model of Equation 6 with \(K = 1/Q'\), the corresponding closed-loop linearized system is shown in Figure 3. Assume a disturbance \(d\) as indicated in Figure 3. The z-transfer function \(o_{\text{out}}/d\) is given by

\[
o_{\text{out}}(z)/d(z) = \frac{K(z - 1)}{z(z - 1 + K K_R)} \tag{12}
\]

If \(d\) is constant, this equation yields in the steady-state (i.e., for \(z = 1\)) \(o_{\text{out}}/d = 0\), i.e., any constant disturbance is eliminated in the steady state by the control system.

Because the upstream traffic volume \(q_{\text{in}}\) acts as the disturbance \(d\) in Figure 3, this statement holds true for constant (or slowly varying) upstream traffic volumes.

Elimination of Biased On-Ramp Volume Realization

What happens if the implemented on-ramp volume is biased as compared with the on-ramp volume ordered by the regulator? A bias in the on-ramp volume realization corresponds exactly to the disturbance \(d\) of Figure 3. Hence, the bias is automatically eliminated by the control system.

This statement does not hold true if the implemented on-ramp volume is used for the retarded value \(r(k - 1)\) in the feedback law of Equation 7. Figure 4 shows the signal flow diagram of the closed-loop system in this case. The z-transfer function is now given by

\[
o_{\text{out}}(z)/d(z) = \frac{K}{z - 1 + K K_R} \tag{13}
\]
Moreover, adjustment of threshold parameters during implementation becomes a difficult task for complicated strategies.

A much more elegant, robust, simple, and efficient way of solving the control problem is to introduce a feedback structure (Figure 1b). The measurable output is fed back, and the controllable input is permanently modified by an appropriate regulator to keep the output near its set value despite the influence of time-variant disturbances. Design of the regulator may be performed by use of well-known automatic control methods. Because of its feedback structure, a control system of this kind is much more precise and much less sensitive with respect to model inaccuracies and unexpected disturbances as compared with disturbance compensation.

In the heating example, feedback control corresponds to measuring the inner temperature and modifying the heating valves accordingly to achieve the desired set value despite the variations of the outer temperature.

The feedback methodology can now be transmitted to the problem of ramp metering shown in Figure 2. Because the same feedback law is to be applied both for congested and for free flowing traffic, it is preferable to consider occupancy \( o_{\text{out}} \) as an output variable instead of \( q_{\text{out}} \). This is because traffic volume may have the same values both for light and congested traffic because of the characteristic form of the fundamental diagram. The corresponding set value \( \bar{o}_{\text{out}} \) for traffic occupancy may be easily found on the basis of the fundamental diagram at the output line; alternatively, a desired downstream occupancy value may be provided directly.

An additional advantage of choosing \( o_{\text{out}} \) rather than \( q_{\text{out}} \) as an output variable arises from the fact that the critical occupancy \( o_c \) seems to be less sensitive with respect to weather conditions and other operational influences compared with the capacity \( q_{\text{cap}} \) of a freeway stretch. This statement is supported by data material provided by Keen et al. (14). As a consequence, considering the set value \( \bar{o} = o_c \) is a more robust way of achieving capacity flow than considering \( q = q_{\text{cap}} \) because variations of \( q_{\text{cap}} \) caused by environmental or other conditions are stronger compared with variations of \( o_c \).

Thus, the next step is to derive a feedback control law \( r = R(\bar{o}, o_{\text{out}}) \) according to Figure 1b to keep \( o_{\text{out}} \) near \( \bar{o} \). Derivation of this feedback law is the subject of the next section.

**DERIVATION OF THE FEEDBACK LAW**

**Modeling**

Consider the traffic flow process shown in Figure 2. Site 1 is assumed to be situated just upstream of the on-ramp. Site 2 is situated downstream of the on-ramp, at a distance 8 from Site 1. Assume that the on-ramp volume \( r \) is updated every \( T \) time units, where \( T = 10 \sec, \ldots, 1 \text{ min} \), or more. The conservation equation for the freeway stretch between Sites 1 and 2 (Figure 2) is

\[
\dot{q}(t) = [q_{\text{in}}(t) + \bar{r}(t) - q_{\text{out}}(t)]/8
\]

where the traffic density \( \rho \) (veh/km) is defined as the number of cars included in the stretch, divided by length \( \delta \), \( \bar{r} \) being the time argument. Because traffic density is not readily measurable, it is convenient to replace \( \rho(t) \) in Equation 1 by the occupancy \( o_{\text{out}}(t) \) using the approximate relationship \( \rho = \alpha o_{\text{out}} \), where \( \alpha = \mu/100\lambda \), \( \mu \) being the number of lanes of the mainstream and \( \lambda \) being the mean effective vehicle length in kilometers.

Assuming \( q_{\text{out}}(t) \) is given as a nonlinear function of \( o_{\text{out}}(t) \) (fundamental diagram)

\[
q_{\text{out}}(t) = Q[o_{\text{out}}(t)]
\]

and substituting Equation 2 into Equation 1, a nonlinear first-order dynamic system model is obtained. This model may be linearized around a nominal steady state \((\bar{o}_{\text{out}}, \bar{q}_{\text{in}}, \bar{r})\) such that

\[
\bar{o}_{\text{out}} = \delta; \bar{q}_{\text{out}} = Q(\bar{o}_{\text{out}}); \text{and } \bar{r} = \bar{q}_{\text{in}} - \bar{q}_{\text{out}}
\]

With the notation \( \Delta o_{\text{out}}(t) = o_{\text{out}}(t) - \bar{o}_{\text{out}} \) used analogously for all variables, the linearization yields

\[
\Delta \dot{o}_{\text{out}}(t) = [\Delta q_{\text{in}}(t) + \Delta r(t) - \dot{Q}' \Delta o_{\text{out}}(t)]/(8\alpha)
\]

where \( \dot{Q}' = dQ(\delta)/d\omega_{\text{out}}, \) i.e., \( \dot{Q}' \) is the slope of the tangent of the fundamental diagram at \( \delta \) and hence its value is proportional to the speed of the corresponding kinematic traffic wave (15). Clearly, \( \dot{Q}' \) is positive on the left-hand side of the fundamental diagram.

Because the control input \( r \) is updated every \( T \) time units, time discretization of Equation 4 with sample time interval \( T \) yields

\[
\Delta o_{\text{out}}(k + 1) = \beta \Delta o_{\text{out}}(k) + [(1 - \beta)\dot{Q}']
\]

\[
\times [\Delta q_{\text{in}}(k) - \Delta r(k)]
\]

where \( k = 0, 1, \ldots \) is the sample time index. Thus \( o_{\text{out}}(k) \) is the occupancy at time \( kT \), and \( \Delta q_{\text{in}}(k) \) and \( \Delta r(k) \) are as-
If \( d \) is constant, this equation yields in the steady state \( \dot{o}_{\text{out}}/d = 1/K_R \), i.e., a bias \( d \) leads to a steady-state error (offset) of \( d/K_R \).

**DISCUSSION OF THE FEEDBACK CONTROL LAW**

**The Regulator**

The proposed feedback control law to be applied at the time instants \( kT, k = 0, 1, 2, \ldots \), for any sample interval \( T \) (e.g., \( T = 60 \) sec), is

\[
r(k) = r(k-1) + K_R[\bar{o} - o_{\text{out}}(k)]
\]  

where \( K_R \) is a positive, constant regulator parameter and \( o_{\text{out}}(k) \) is the current occupancy measurement. The feedback law of Equation 10, which is much simpler than any other local metering strategy, was given the name asservissement linéaire d’entrée autoroutière (ALINEA).

**Heuristic Interpretation**

Equation 10 suggests a fairly plausible control behavior. If the measured occupancy \( o_{\text{out}}(k) \) at time \( k \) is found to be lower (higher) than the desired occupancy \( \bar{o} \), the second term of the right-hand side of Equation 10 becomes positive (negative) and the ordered on-ramp volume \( r(k) \) is increased (decreased) as compared to its last value \( r(k-1) \). Clearly, the feedback law of Equation 10 acts in the same way both for congested and for light traffic (no switchings are necessary).

Note that some occupancy control strategies (1) react to excessive occupancies only after congestion is created and has reached an upstream measurement location, whereas ALINEA reacts smoothly even to slight differences \( \bar{o} - o_{\text{out}}(k) \) and may thus prevent congestion in an elegant way.

On the other hand, some demand-capacity strategies react to excessive downstream occupancies only after a threshold value is exceeded. Typically, and in contrast to ALINEA, the reaction of these control strategies to excessive occupancies is rather crude: on-ramp volumes are set equal to their minimal values. In this way, a nonnecessary underload of the freeway may occur. On the contrary, the essential effect of ALINEA is to stabilize traffic flow at a high throughput level and eventually to reduce the risk of a breakdown without underloading the freeway.

**The Value of \( K_R \)**

A value of \( K_R = 70 \) veh/hr was found to yield good results in real-life experiments. \( K_R \) is the only parameter to be adjusted in the implementation phase because no thresholds or other constants are included in Equation 10. Moreover, from theoretical considerations,

- Results are insensitive for a wide range of \( K_R \) values;
- Increasing (decreasing) \( K_R \) values lead to stronger (smoother) reactions of the regulator, and regulation times get shorter (longer);
- For extremely high values of \( K_R \), the regulator may have an oscillatory, unstable behavior.

In view of these statements, real-life calibration of the unique free parameter \( K_R \) is—if at all necessary—particularly easy.

**Measurements**

ALINEA requires only one detector station that measures occupancy \( o_{\text{out}} \) downstream of the merge area. This is equal or less than the measurement requirements of other local strategies. The measurement location should be such that a congestion originating from excessive on-ramp volumes be visible in the measurements. A distance of 40 m downstream of the on-ramp nose was found to be adequate at the Boulevard Périphérique in Paris. The strategy was also found to work adequately with a distance of 400 m in Amsterdam (13).

**Disturbance Reduction**

If upstream traffic volume \( q_{\text{in}} \) is constant, then the feedback law of Equation 10 is easily demonstrated to lead to \( o_{\text{out}} = \bar{o} \) in the steady state. In other words, whatever the value of a constant (and not measured) upstream traffic volume might be, the feedback law leads occupancy to its desired value.

Similarly, if upstream traffic volume \( q_{\text{in}} \) is perturbed around a constant or slowly varying average, the feedback law of Equation 10 keeps downstream occupancy \( o_{\text{out}} \) close to \( \bar{o} \) in the average. On the other hand, rapid oscillations of \( q_{\text{in}} \) around an average value are only slightly reduced by the control system.

**Embedding in a Coordinated Control System**

Realization of the feedback law of Equation 10 requires a set value \( \bar{o} \) to be provided by the user. The set value may be changed any time it is needed and hence, the feedback law may be embedded in a simple and natural way into a hierarchical control system with set values of the individual sections being specified (and changed in real time) by a superior coordination level or by an operator.

**Restrictions, Override Tactics**

The on-ramp volume values resulting from Equation 10 may be limited if some maximum or minimum values are exceeded. Moreover, override tactics (e.g., for preventing interference of the on-ramp queue with surface traffic) may be applied. When either a limitation or override tactic becomes active, the last on-ramp volume \( r(k-1) \) required in the feedback law of Equation 10 for calculating \( r(k) \) should correspond to the actual number of cars that entered the freeway (because of the limitation or override) and not to the calculated but
Realization of On-Ramp Volumes

ALINEA is compatible with any kind of realization of the required on-ramp volumes (one-by-one, platoon, traffic cycle, etc.). Obviously, the implemented on-ramp volumes should be equal to the on-ramp volumes ordered by the control law of Equation 10 if the value of \( r(k-1) \) required in Equation 10 should be equal to the on-ramp volume calculated by Equation 10 in the last time interval and not equal to the on-ramp volume that actually entered the freeway, unless an override or a constraint has become active, as already mentioned.

Efficacy

A preliminary version of ALINEA and some popular previous control strategies have been implemented and tested on an on-ramp of the Boulevard Périphérique in Paris during an experimentation period of 6 months. Results of this lengthy experimentation period showed a clear superiority of ALINEA in preventing congestion and increasing traffic throughput as compared to other local traffic responsive strategies. Details were reported by Hadj-Salem et al. (2,12). More recent field results from the Netherlands confirm the superiority of ALINEA as compared with the demand-capacity type of strategies (13).

Extensions

Two possible extensions may be envisaged if the ratio \( b/T \) is relatively high. This condition may hold if the output occupancy \( o_{out} \) is measured at a site far downstream of the on-ramp nose or the sample time interval is short (e.g., \( T = 10 \) sec). As a result of the theoretical development, these extensions are recommended if \( b/T > \alpha K_R \), where \( \alpha \) was defined earlier.

The first extension is to use the feedback law of Equation 9 rather than that of Equation 10 with the parameter value \( K_R = 0 \). Thus, even with this extension, \( K_R \) remains the only parameter to be calibrated during implementation.

The second extension for the case \( b/T > \alpha K_R \) is to add to the feedback laws of Equations 9 or 10 the term \( \gamma [q_{in}(k) - q_{in}(k - 1)] \) if the value \( q_{in}(k) \) can be predicted accurately enough by upstream measurements. This second extension corresponds to a disturbance compensation aiming at improving the feedback efficiency. The positive smoothing parameter \( \gamma \leq 1 \) should be appropriately chosen to avoid excessive oscillations of the on-ramp volumes caused by noise of the \( q_{in} \) measurements.

However, if the output Site 2 is being located far downstream of the on-ramp (at a downstream bottleneck) without any intermediary measurements, there is a risk of congestion being built up (because of incidents or other disturbances) in the interior of the stretch (1,2) without being visible at the output measurement Site 2. In such cases, the feedback (or any other) control system may be of little help for eliminating the congestion if no additional measurement stations are added.

REAL-LIFE APPLICATION RESULTS

ALINEA was first implemented and tested at the on-ramp Brançon of the Boulevard Périphérique in Paris. On-ramp volumes are realized on the basis of a traffic cycle of 40 sec with a minimum green phase of 10 sec. The set value \( \delta \) for downstream occupancy is 29 percent, which corresponds to capacity flow. Override tactics are applied to avoid interference of the on-ramp queue with the surface street traffic.

The on-ramp volumes ordered by the feedback law of Equation 10 are transformed into corresponding green-phase durations by use of a saturation flow value that is estimated in real time. The particular algorithm used for estimation of the saturation flow leads to a biased implementation of the ordered on-ramp volumes. The average difference (bias) between ordered and implemented on-ramp volumes is 2 vch/ cycle or \(-180 \) veh/hr (which is completely independent of ALINEA). The closed-loop system was implemented as indicated in earlier sections (see also Figure 3) with \( K_R = 70 \) veh/hr. The bias of \(-180 \) veh/hr acts as an additional disturbance that is automatically eliminated by the feedback system.

Typical results of a 30-min time period are shown on Figures 5-7. Figure 5 shows the upstream and downstream occupancies and the periods of active minimum (10 sec) or maximum (40 sec) green-phase constraints. Figure 6 shows the corresponding upstream and downstream traffic volumes. Figure 7 shows the ordered and implemented on-ramp volumes, and the corresponding green phase duration.

Results of Figure 5 indicate a proper functioning of ALINEA: on-ramp volumes ordered by the feedback law of Equation 10 keep the downstream occupancy \( o_{out} \) near its set value in the average, despite the variation of upstream traffic volume and despite the bias in the on-ramp volume realization.

Clearly, when constraints become active because of lacking demand (green phase equal to 40 sec) or because of traffic

![FIGURE 5 Occupancies and constraints activation.](image-url)
shock waves arriving from downstream (green-phase duration equal to 10 sec), some deviations from the set value may be observed. Figure 6 shows that the downstream traffic volume attains values around the capacity (6,300 veh/hr) of the three-lane freeway stretch. Thus the feedback law acts elegantly to avoid breakdowns and to fully utilize the freeway capacity on the basis of one single equation (Equation 10). Extended long-term results are reported elsewhere (2,12,13).

ALINEA is currently operational at the on-ramps Brancion, Chatillon, and Italie of the Boulevard Périphérique in Paris. Its implementation on a number of further freeway on-ramps of the Paris region is currently under way. Moreover, ALINEA has been tested at the on-ramp of the Coen Tunnel in the Netherlands.

CONCLUSION

ALINEA, a new local traffic-responsive strategy for ramp metering has been presented. The new control strategy is based on a feedback structure, which was derived by classical automatic control methods. ALINEA

- Is simpler than other known algorithms,
- Requires a minimal amount of real time measurements (detectors),
- Is easily adjustable to particular traffic conditions because only one parameter is to be adjusted in a prescribed way,
- Has proven in real life experiments to be more efficient in preventing congestion and preserving capacity flow compared to some known algorithms,
- Can be embedded in a coordinated on-ramp control system in a natural way,
- Can be easily modified in case of changing operational requirements,
- Is highly robust with respect to inaccuracies and different kinds of disturbances, and
- Is theoretically supported by automatic control theory.

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