# **Integrated System of Freeway Corridor Simulation Models**

LANNON LEIMAN, MOURAD BOUAOUINA, AND ADOLF D. MAY

The newly developed FREQ10 integrated system of freeway corridor simulation models is described with emphasis on traffic simulation, freeway improvement strategies, measures of effectiveness, and traveler responses. The major accomplishments have been to combine the previously developed entry control model, an extensively modified on-freeway priority model, and a newly developed common menu-driven interactive interface into an integrated system of models to extend the types of traffic management strategies that can be evaluated for a freeway corridor. The FREQ10 system of models enables the user to analyze design improvements, implementation of an HOV facility, implementation of normal and priority entry control, or implementation of time-varying capacity reduction situations such as reconstruction activities or freeway incidents. The extensive modifications that have been made to the on-freeway priority model include refining the procedure for traffic simulation, adding many new features such as user-supplied emission and fuel rates, and developing a new method for modeling the arterial and spatial response to reflect current policies that HOV facilities be considered as lanes added to the freeway. This system of models has received extensive testing and has been applied to freeway corridors in several urban areas in California.

During the past 20 years, the Institute of Transportation Studies at the University of California at Berkeley has developed and applied a sequence of freeway simulation models for the evaluation of various design and operational improvements. These models have been used by researchers and professionals both in the United States and abroad. Interactions with model users and experience in educational programs have led to continuous improvements in these models. Numerous reports are available describing these earlier developments (1-20).

A significant advancement in this modeling effort has been made as a result of a recently completed 2-year research and educational program sponsored by the California Department of Transportation and the FHWA. The major accomplishments have been to combine the previously developed entry control model, the extensively modified on-freeway priority lane model, and a newly developed common menu-driven interactive interface into an integrated system of models to extend the types of traffic management strategies that can be evaluated for a freeway corridor.

The purpose is to describe the newly developed FREQ10 integrated system of freeway corridor simulation models with emphasis on traffic simulation, freeway improvement strategies, measures of effectiveness, and traveler responses. A comprehensive final report of the research program (21) and two previous research reports (9,17) are available for in-depth coverage of each element of this newly developed system of

Institute of Transportation Studies, University of California, 109 McLaughlin Hall, Berkeley, Calif. 94720.

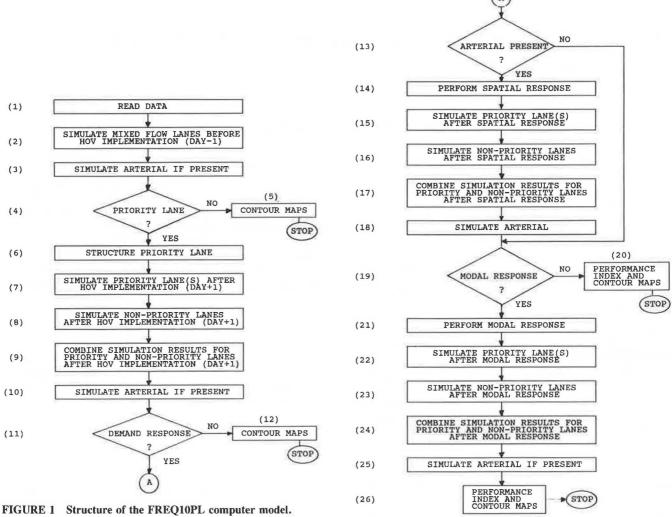
models. Although this paper will not cover programming aspects nor present actual applications, these issues were of special importance and are covered in the final report. The computerized system of programs includes a menu-driven interactive graphical interface with comprehensive input checking, carefully selected default values, and user-selected output options including traffic performance contour maps. The extensive modifications that have been made to the on-freeway priority lane model include refining the procedure for traffic simulation, adding many new features such as user-supplied emission and fuel rates, and developing a new method for modeling the parallel arterial routes and spatial response to reflect current policies that HOV facilities be considered as lanes added to the freeway. Through a 6-month educational program for Caltrans professionals, 11 teams of engineers used FREQ10 to model existing freeway corridors in metropolitan areas throughout California.

# MODEL OVERVIEW

The FREQ10 system contains two models: FREQ10PE, an entry control model for analyzing ramp metering; and FREQ10PL, an on-freeway priority model for analyzing HOV facilities. On entering the FREQ10 interface, the user may select either of the two models. Once the basic data have been entered for either FREQ10PE or FREQ10PL, the interface creates a data set that it can later access for either model. However, the models themselves do not interact; thus, entry control and on-freeway HOV facilities must be modeled independently. The two programs are similar in structure and use. The FREQ10PL model will be emphasized, but where major differences occur between the two models, the unique features in FREQ10PE will be identified.

The structure of the FREQ10PL priority lane model is shown in Figure 1. In the following general discussion, step numbers refer to numbers in Figure 1.

- Step 1 The data are read from the input file. It includes freeway and parallel arterial design features, freeway ramp counts, arterial flows, and proposed design of freeway priority lanes.
- Step 2 The model simulates the peak-period traffic operations for the (Day -1) conditions before implementation of the priority lane. The results of this simulation serve as the basis of comparison for later simulations.
- Step 3 If the freeway corridor has a parallel arterial route, the model simulates it for the (Day -1) conditions.
- Steps 4-5 If the user has requested only simulation of the (Day -1) conditions before implementation, the model



constructs any selected contour maps, provides the requested output, and stops.

Step 6 The model structures the HOV facility by adding the priority lanes to the freeway, changing the freeway capacities, splitting the origin-destination (O-D) tables into different occupancies, and making other manipulations necessary before the (Day + 1) short-term conditions after HOV implementation can be simulated. Note that all these changes are done by the program and are transparent to the user.

Steps 7–8 The model simulates the (Day + 1) conditions after implementation of the HOV facility in an effort to duplicate the first day of operations, before drivers have changed their driving behavior, i.e., all vehicles have the same time, space, and occupancy patterns as before. To do this, the model splits the freeway into two roadways, the priority lanes and the nonpriority lanes (including the upstream and downstream mixed-flow lanes); the model then simulates each roadway separately.

Step 9 The model combines the results of the simulations of the priority lanes and the nonpriority lanes for (Day + 1) to determine the performance of the entire freeway for (Day + 1). The results of this combined (Day + 1) sim-

ulation are used for comparison with the (Day -1) simulation.

Step 10 If the freeway corridor has a parallel arterial route, the model simulates it for the (Day + 1) conditions. The results of simulation of the arterial for (Day - 1) and (Day + 1) are identical, because no spatial shift has occurred yet.

Steps 11–12 If the user has not requested any demand response, the model constructs any selected contour maps, provides the requested output, and stops.

Step 13 If there is no parallel arterial, the model will jump to Step 19. If there is an arterial, the model will proceed to Step 14.

Step 14 The model performs spatial response by shifting from the parallel arterial to the freeway those automobiles that save sufficient time. This includes priority as well as nonpriority automobiles.

Steps 15-16 The model simulates the corridor after spatial response in an effort to duplicate operations, after drivers have changed their driving behavior by changing routes. To do this, the model splits the freeway into two roadways, the priority lanes, and the nonpriority lanes (including the up-

stream and downstream mixed-flow lanes); the model then simulates each roadway separately, adding the vehicles that have shifted from the arterial to the appropriate roadway.

Step 17 The model combines the results of the simulations of the priority lanes and the nonpriority lanes after spatial response to determine the performance of the entire freeway after spatial response. The results of this combined simulation after spatial response are used for comparison with the (Day - 1) simulation.

Step 18 The model simulates the arterial for the conditions after spatial shift.

Steps 19–20 If modal response has not been requested, the model calculates the performance index, constructs any contour maps that have been selected, provides the requested output, and stops.

Step 21 The model performs modal response by shifting the passengers who would save sufficient time from nonpriority vehicles to priority vehicles. The actual number of passengers shifted is based on either program- or user-supplied elasticities.

Steps 22–23 The model simulates the corridor after modal response in an effort to duplicate operations, after drivers have changed their driving behavior by changing their mode of travel. To do this, the model again splits the freeway into two roadways, the priority lanes and the nonpriority lanes (including the upstream and downstream mixed-flow lanes); the model then simulates each roadway separately, adjusting the demand with respect to the passengers that have shifted from nonpriority vehicles to priority vehicles.

Step 24 The model combines the results of the simulations of the priority lanes and the nonpriority lanes after modal response to determine the performance of the entire freeway after modal response. The results of this combined simulation after modal response are used for comparison with the (Day - 1) simulation.

Step 25 If the freeway corridor has a parallel arterial route, the model simulates it for conditions after modal response. The results of simulation of the arterial for conditions after spatial response and after modal response are identical, because no additional spatial response has occurred.

Step 26 The model calculates the performance index, constructs any selected contour maps, provides requested output, and then stops.

The FREQ8PE research report (9) contains a description of the structure of the FREQ10PE model.

# TRAFFIC SIMULATION

The same simulation module is the core of both the FREQ10PL model and the FREQ10PE model. The freeway corridor is simulated for existing conditions and after each change in those conditions. The simulation module analyzes the freeway corridor for each of a possible 24 time slices beginning with the first and continuing through the last. Within a time slice, each of a possible 38 subsections is processed sequentially from upstream to downstream. The simulation module consists of two parts: one for simulating conditions on the freeway, the other for simulating conditions on the parallel arterial.

The input for the simulation module initially consists of the freeway and arterial designs, the freeway demand in the form

of synthetic O-D trip tables generated by the models from user-supplied ramp counts for each time slice, freeway and arterial occupancy distributions, and freeway and arterial subsection vehicle and passenger flows. Later in the run, additional input, consisting of those vehicles whose drivers have changed routes and those vehicles whose passengers have changed mode, is provided to the simulation module by the spatial response and modal response modules. Traffic demands, and subsection vehicle and passenger flows, can then be modified by the simulation module to reflect the conditions after traveler response in either the FREQ10PL or FREQ10PE model. Metered vehicles in the FREQ10PE model are handled similarly.

Simulation of conditions on the freeway can be divided into two parts. The first involves ramp queuing, ramp merging, and weaving analysis. The second includes mainline travel time and queuing analysis.

On- and off-ramp queues, which are modeled as vertical queues, form whenever ramp demand exceeds ramp capacity. Ramp merging analysis, at each on-ramp and at the merge point on the freeway, and weaving analysis, are performed in accordance with the 1965 Highway Capacity Manual (22). The model considers only simple and two-part compound weaving.

The average speed is calculated for each subsection as a function of the flow and the presence or absence of queuing by using speed versus volume-to-capacity ratio curves. A mainline queue develops whenever demand in any given subsection exceeds the subsection capacity. Shockwave analysis is used for determining queue evolution, queue collisions, queue splits, and queue discharge. In addition, the simulation module computes travel time, travel distance, fuel consumption, and vehicle emissions for each subsection during each time slice.

The simulation of conditions on the arterial are based on subsection flows. Travel times, travel distance, average speed, fuel consumption, and vehicle emissions are calculated for each subsection during each time slice.

The simulation module produces results for each time slice as well as summaries over all time slices. Time slice output includes O-D tables as modified by the simulation module; O-D tables of freeway travel times, transferred, and diverted vehicles; freeway and arterial performance tables displaying the subsection characteristics and impacts; and detailed ramp queuing information. After all time slices have been processed, simulation summary tables are produced for the freeway and the arterial that display the impacts during each time slice and the totals over all time slices. For multiple simulation runs, a differential effects table is produced that compares the impact for the corridor totals from the current simulation with the corridor totals from the initial simulation. The final output can include distance-time contour maps of the first and last simulations for 10 different variables. The user selects which of these many outputs are to be provided for any particular run. Figure 2 shows an example of a freeway performance table for one time slice; Figure 3 shows an example of a freeway summary table.

Recent modifications have been made to the FREQ10PL simulation module to incorporate features of the more advanced FREQ10PE model. These include adding user-supplied speed versus volume-to-capacity ratio curves, user-supplied fuel con-

# SIMULATION BEFORE IMPLEMENTATION OF PRIORITY LANE TIME SLICE 4 OF 11

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FIGURE 2 Sample performance table.

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2	*	254.	320.*	0.	0.*	254.	220 +	11680.	14717.*	46.0		650.	*	33.	*	173.	*	35.
3	*	254.	320.*	٥.	*	234.	320."	11000.	14/1/."	40.0	-	650.	ï	33.	-	1/3.	-	35.
4	*	307.	387.*	0.	0.*	307.	387.*	11729.	14779.*	38.2	*	646.	*	36.	*	202.	*	34.
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5	*	318.	400.*	0.	0.*	318.	400.*	11471.	14454.*	36.1	*	640.	*	36.	*	209.	*	33.
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6	*	302.	380.*	0.	0.*	302.	380.*	11347.	14297.*	37.6	*	649.	*	35.	*	201.	*	33.
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,	*	299.	376.*	0.	0.*	299.	376.*	10704.	13487.*	35.9		628.		34.	*	198.		31.
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·	*	511.	*	٠.	*	511.	*	10002.	*	34.7	*	020.	*	34.	*	200.	*	31.
9	*	300.	378.*	0.	0.*	300.	378.*	10567.	13314.*	35.3	*	608.	*	33.	*	198.	*	31.
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FIGURE 3 Sample summary table.

sumption rates, user-supplied vehicle emission rates, and detailed user output specifications. In addition, FREQ10PL has been modified so that bus O-D tables for the freeway can be generated by the program based on user-supplied occupancy distributions. Several refinements have been made to the queuing analysis and the differential effects tables have been added. Finally, noise contour maps, a new fuel consumption module, a new vehicle emissions module, and a new flow-dependent simulation module for the arterial have been added to the FREQ10PL model. These modifications and additions to the FREQ10PL model ensure that both the FREQ10PL and the FREQ10PE models now produce identical results for simulation of existing conditions.

A more complete description of the simulation module can be found in the final report (21).

#### FREEWAY IMPROVEMENT STRATEGIES

Improvement strategies that can be modeled by the FREQ10 system of models fall into four categories: design improvements, implementation of an HOV facility, implementation of normal and priority entry control, and implementation of time-varying capacity reduction situations.

Design improvements can be evaluated by using either the FREQ10PL or FREQ10PE model. If a simulation run for existing conditions reveals congestion on the freeway that appears to be caused by isolated bottlenecks, the user should first try to improve the design of the freeway at the congested areas before considering other options. Such improvements might include adding an extra lane in one or two subsections, adding a lane to an on- or off-ramp, or making other improvements to the freeway design that would increase capacity at the congested areas. These design changes can be easily made to the original data set, and the new conditions simulated by either model. The user may discover after trying several possible design improvements, that the freeway congestion can be eliminated by making design improvements alone. However, if the elimination of the original bottlenecks merely results in new problems downstream, a more comprehensive solution such as the implementation of a priority lane, or the implementation of normal or priority entry control, should be explored.

The FREQ10PL model enables the user to evaluate the implementation of exclusive priority lanes. The user specifies the number of lanes in the HOV facility, where it begins, and where it ends. The occupancy cutoff level for vehicles that can use the HOV facility can be two or more passengers plus buses, three or more passengers plus buses, or buses only.

A major conceptual change needed to be made in the FREQ10PL model to reflect the policy that an HOV facility will always represent lanes added to the freeway. Because existing conditions are always simulated first during any run, the model needed the added capability of automatically modeling the additional HOV lanes after simulating existing conditions.

It is assumed that the HOV facility is continuous, that it is on the left side of the roadway, and that there is no barrier between it and the other freeway lanes. In addition, it is assumed that all HOV vehicles use the HOV facility, enter it immediately downstream of the freeway on-ramps, and exit it immediately upstream of the freeway off-ramps.

The effects of a particular HOV operational design can be evaluated for the day after implementation of the HOV lanes, after spatial response, which in the new FREQ10PL occurs from the parallel arterials to the freeway because of improved conditions on the freeway, and after modal response. The contour maps help the user visualize the effects of the improvement strategy. Figure 4 shows queue length contour maps for the existing conditions and for conditions after modal response for both the HOV facility and the non-HOV lanes.

The FREQ10PE model enables the user to consider improving freeway conditions by modeling normal or priority entry control at the on-ramps. The model can analyze a user supplied metering plan or provide an optimum computergenerated metering plan. The effects of either metering plan can be simulated after implementation, after spatial response (which in FREQ10PE is from the freeway to the arterial), and after modal response.

Although evaluating normal and priority entry control is the main function of the FREQ10PE model, it can also be used to simulate the operational effects of freeway maintenance and reconstruction activities by allowing the user to vary the subsection capacities over time (17). In this way, a schedule for maintenance or reconstruction can be developed that will cause the least disruption to traffic on the freeway. Another application of the time-varying capacity reduction capability of the FREQ10PE model would be to simulate incidents on the freeway.

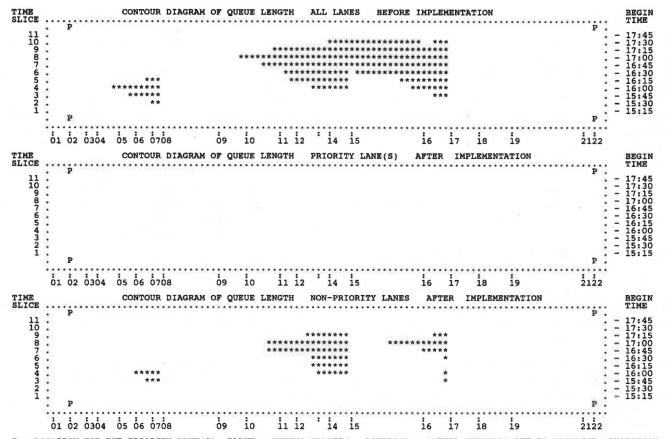
The FREQ10 interactive interface enables the user to model any of the many different improvement strategies with minimum effort. The data set is structured in such a way that the interface can access it for either a FREQ10PL or FREQ10PE simulation. Having entered the interface, the user selects the desired model, makes the necessary changes to the original data set, selects the desired output, and executes a run that simulates the improvement strategies under consideration.

# MEASURES OF EFFECTIVENESS

In determining the effectiveness of any freeway improvement, it is important to evaluate traffic impacts on the freeway and parallel arterial routes. The FREQ10PL and FREQ10PE models provide information to help in this evaluation by calculating travel times, fuel consumption, vehicle emissions, traffic noise, and a cost-effectiveness performance index.

#### **Travel Times**

The travel times are computed for each subsection on the freeway and the arterial for each time slice. The freeway travel time for a given subsection and time slice is a function of the length of the subsection; the length of the time slice; and the speed, flow, and density of the subsection during the time slice. If the density of the subsection is not uniform throughout the time slice, travel time becomes the sum of the travel time during the congested period and the travel time during the noncongested period of the time slice. Ramp delays are added to the freeway travel times. The arterial travel time for a given subsection is a function of the flow, capacity, and signal properties of the arterial subsection.



P = LOCATION FOR THE PRIORITY LANE(S) BLANK = MOVING TRAFFIC. ASTERISK = QUEUED VEHICLES DUE TO MAINLINE CONGESTION.

FIGURE 4 Sample queue length contour maps.

## **Fuel Consumption**

The fuel is calculated for each subsection during the mainline and arterial simulations for each time slice. Fuel consumption is a function of the vehicle-miles traveled on the subsection during the time slice, the class of vehicle, the fuel consumption rate of the class of vehicle traveling at the average speed of the subsection, the grade correction factor of the subsection, and the fraction of vehicles of each class on the subsection.

The vehicle fractions are supplied by the user. These fractions are constant over the entire simulation period. For the arterial, the vehicle mix is assumed to be the same throughout the entire length of the corridor section, while the vehicle mix on the freeway will usually vary from subsection to subsection because the vehicle distribution can be entered for each origin. The three vehicle classes that are simulated are automobiles, which include all of the four-wheeled vehicles; gasolinepowered trucks; and diesel-powered trucks. The grade correction factor is a function of the average speed, which is calculated by the program, and the mean subsection grade, which is provided by the user. Fuel consumption rate tables stored within the models give the rates at various average speeds for the three vehicle classes and the two facility types, freeway and arterial. A complete description of these and other tables mentioned in this section can be found in the final report (21). The user may supply substitute fuel consumption rate tables, if conditions are different from those assumed in the tables that are stored in the models.

## Vehicle Emissions

The FREQ10PL and FREQ10PE models calculate the amounts of hydrocarbons (HC), carbon monoxide (CO), and oxides of nitrogen ( $NO_x$ ) emitted by all vehicles in the modeled freeway corridor. The total emissions produced during the simulation are made up of two components: (a) emissions from vehicles traveling on the freeway and the arterial, and (b) emissions from vehicles delayed at on- and off-ramps. These calculations are performed during each simulation for each subsection and for each time slice.

The emissions generated from vehicles traveling on the freeway and on the arterial during a given time slice for a given subsection are a function of the type of pollutant, the class of vehicle, the vehicle emissions of the pollutant, the emission rate of the pollutant of the class of vehicle traveling at the average speed of the subsection, and the fraction of vehicles in each class.

The vehicle fractions, which are supplied by the user, and the vehicle classes are the same as those described for fuel consumption. Two emission rates tables are stored in the models, one for California vehicles and one for low-altitude non-California vehicles. These emission rate tables were obtained from the Environmental Protection Agency's MOBILE1 computer program (23) and give the emission rates at various average speeds for the three pollutants and the three classes of vehicles. The same tables are used for the freeway and the arterial. In addition, the models can read user-supplied emis-

sion rates that can then be used in place of the programsupplied rates.

The emissions generated by vehicles delayed at all freeway ramps during a given time slice are a function of the type of pollutant, the class of vehicle, the vehicle emission from delayed vehicles for the pollutant, the total vehicle delay at the ramps, the fraction of vehicles in each class, and the idling emission rate. The idling emission rates stored in the models were obtained from the MOBILE1 program (23). The user may supply different idling emission rates, which the model can then use instead of the program-supplied rates.

#### **Traffic Noise**

The level of noise generated by a freeway subsection during a time slice is computed for two different measurements. The  $L_{10}$  index expresses the noise level (in dBA) exceeded 10 percent of the time. This measure has been popular in the United States for many years. The equivalent noise level ( $L_{\rm eq}$ ) is an estimate of total noise. This measurement has been widely used in Europe. The output for both of these noise measurements consists of distance-time contour maps.

#### **Cost-Effectiveness Performance Index**

A new cost-effectiveness performance index has been added to the FREQ10PL model to aid the user in comparing the impacts of different HOV operational designs on a freeway corridor. The index is based on program- or user-supplied cost-benefit coefficients and the differential effects between

the initial (Day -1) conditions and the conditions after the last demand response. The differential effects that are considered are savings in travel time, savings in travel distance, savings in fuel consumption, and savings in vehicle emissions. The performance index is expressed as the savings in cost per peak period per mile of HOV lane added. It is recommended that the cost-effectiveness performance index only be used to compare different HOV operational designs for the same freeway, using identical cost-benefit coefficients. There is no cost-effectiveness performance index in the FREQ10PE model.

Figure 5 shows an example of a differential effects table that compares simulation of conditions after modal response to the initial conditions of the corridor. It contains most of the measures of effectiveness discussed in this section. Figure 5 also displays the cost-effectiveness performance index for the same run.

#### MODELING THE PARALLEL ARTERIAL ROUTES

The parallel arterial routes, if present, are aggregated and modeled as one. The freeway corridor can be modeled as having one of the following three arterial configurations:

- 1. No parallel arterial,
- 2. A parallel arterial that is continuous along all of the freeway, and
- 3. A parallel arterial that is discontinuous along the freeway, i.e., present at some locations and not at others.

The parallel arterial is incorporated into the analysis of the freeway corridor by a series of simplifying assumptions. The

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CHANGE	PER	TOTAL *	ART.RTE *	FREEWAY *	TOTAL *	ALT.RTE *	FREEWAY *	TOTAL *	ALT.RTE *	FREEWAY *	*1	
*****	***	*	*******	********	*******	******	********	*******	********	********	*****	******
19.20 *		-971.*	-697 *	-274.*	4089.*	1421.*	2668.*	5060.*	2118.*	2942.*	H-HR *	TRAVEL *
19.60		-1250.*		-371.*	5126.*	1790.*	3336.*	6375.*	2668.*	3707.*	n-nk *	
		*	*	*	*	*	3330.**	03/3.**	2000."	3707.	3-nk -	. IIME -
-0.68		-1058.*	-5716.*	4658.*	155507.*	31149.*	124358.*	156565.*	36865 *	119700.*	H-MI *	TRAVEL *
-0.48			-7201.*	6254.*	196324.*		157075.*	197272.*	the second second	150822.*		
		*	*	*	*	*	*	*	*	*	*	*
22.92		7.09*	4.52*	5.93*	38.03*	21.93*	46.61*	30.94*	17.41*	40.68*	. *	AVG.SPD.*
,		*	*	*	*	*	*	*	*	*	*	*
-3.98		-372.*	-591.*	219.*	8977.*	1934.*	7043.*	9349.*	2525.*	6824.*	LONS *	GASOLINE*
		*	*	*	*	*	*	*	*	*	*	*
-8.37 *		-43.*	-37.*	-5.*	469.*	118.*	351.*	511.*	155.*	356.*	OGRAMS*	HYD-CARB*
		*	*	*	*		*		*	*	*	
16.99		-535.*	-368.*	-167.*	2616.*	811.*	1805.*	3152.*	1179.*	1973.*	OGRAMS*	CARB-MON*
		*	*	*	*	*	*	*	*	*	*	
3.84		16.*	-7.*	23.*	444.*	61.*	383.*	428.*	68.*	359.*	OGRAMS*	NITOX.*
		*	*	*	*	*	*	*	*	*	*	
13.73 *		-562.*	-413.*	-149.*	3529.*	990.*	2539.*	4091.*	1403.*	2688.*	OGRAMS*	ALL-EMIS*
		*	*	*	*	*	*			*	*	*

PERFORMANCE INDEX COMPUTED AFTER MODAL SHIFT = \$ 1275. SAVINGS PER PEAK PERIOD PER MILE OF HOV LANE

PERFORMANCE INDEX SHOULD BE USED TO COMPARE DIFFERENT HOV OPERATIONAL DESIGNS IN RUNS USING IDENTICAL COST COEFFICIENTS

FIGURE 5 Sample differential effects table and cost-effectiveness index.

arterial is divided into subsections that correspond to those of the freeway. The boundaries of these arterial subsections are thus defined by the boundaries of the freeway subsections. Figure 6 shows an example of a modeled freeway corridor.

FREQ10PL is the first model in the FREQ family of programs that diverts vehicles from the arterial to the freeway. In order for the model to keep track of traffic patterns and the occupancy of vehicles shifting from the arterial to the freeway, FREQ10PL, unlike previous FREQ programs, must model the arterial in such a way that arterial O-D tables can be produced. Such tables are not needed in FREQ10PE, because in that model it is the freeway vehicles that change their routes to the arterial.

Because the arterial users of interest in FREQ10PL are those who could potentially change their routes to use the freeway if they save time, the origins and destinations on the parallel arterial are assumed to be located where there is a connection to the freeway (one-way or two-way traffic). This assumption is also shown in Figure 6.

The directional capacity of each arterial subsection is constant throughout the simulation run. The demand information for each arterial subsection consists of two variables: the average arterial flow and the percentage of vehicles leaving the arterial at the end of the subsection. The number of vehicles leaving the arterial at any destination is calculated by multiplying the percent turning off at the end of the subsection by the average subsection flow. The number of vehicles entering the arterial at any origin is calculated from the difference in flow volumes of the two consecutive arterial subsections. After using this method to calculate the demand at each arterial origin and destination, the model uses these demands to generate a synthetic arterial origin-destination trip table. This process is repeated for each of the time slices of the simulation period. The model then splits these arterial O-D matrices into priority and nonpriority vehicle matrices. The ability of the FREQ10PL model to generate synthetic O-D tables for the arterial from the origin and destination demands computed from arterial subsection flows and percent turning values eliminates the need for the user to collect detailed arterial O-D survey data.

Another important arterial modeling feature is the flow dependency of arterial travel times. Arterial travel times are calculated as a function of the characteristics of the subsections: flow, capacity, and control (arterial subsection with or without signalized intersections). For subsections with no signalized intersections, arterial vehicles are assumed to travel at a constant speed. For subsections with signalized intersections, an equation developed by Davidson (24) is used. This equation adjusts the speed according to the quality of traffic signal progression, the flow, and the capacity along the arterial subsection. This procedure has been added to the FREQ10PL model so that travel times on the arterial are now computed the same way in both the FREQ10PL and FREQ10PE models.

#### SPATIAL RESPONSE MODELING

In order to conform to current policies, FREQ10PL has been modified to model the on-freeway HOV facility as one or more add-on lanes. As such, the LOS on the freeway is always expected to improve, because additional capacity is provided. Therefore, spatial response, for the first time, is defined as a change in the travelers' route choice from the parallel directional arterials, to the freeway in response to the perceived changes in the freeway travel conditions. This change in direction of the spatial response required the development of a completely new spatial response module. The arterial vehicles that change their route to the freeway are assigned to the on-freeway HOV facility or general-purpose freeway lanes depending on the occupancy of the arterial vehicles. The new

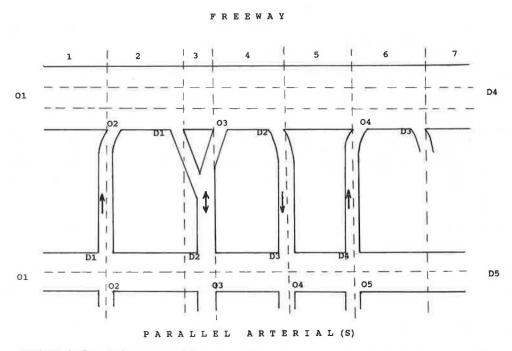


FIGURE 6 Sample freeway corridor.

model-generated arterial O-D tables, described in the previous section, enable the spatial response module to keep track of the traffic patterns and occupancy of vehicles that change their route from the arterial to the freeway after the addition of the HOV facility.

Spatial response is expected to occur within a few months of the implementation of the HOV lanes and before any significant modal response occurs. It is assumed that parallel arterial drivers reconsider their route choice (spatial response), at least to the extent of using or not using the freeway, before passengers in the nonpriority vehicles on the freeway consider changing modes (modal response). The reasons for making this assumption are as follows: mode choice generally represents a much more difficult decision than route choice, because drivers would need to adjust their trips to meet outside constraints, such as forming a carpool or adhering to a bus schedule. Route choice, on the other hand, typically has travel time or some other form of generalized cost as its only criterion. Therefore, the argument is that the modal split, existing before the freeway exclusive lanes implementation, reflects the difficult mode choice decision already made by commuters, and that changing mode would require more decision time than changing routes.

The two assumptions discussed earlier, that spatial shift occurs from the arterial to the freeway and spatial response occurs before modal response, are the most important ones in the development of the new spatial response algorithm. Additional assumptions and limitations are listed in the final report of the research program (21).

The basic rationale of the new FREQ10PL spatial response module is to set a travel time savings criterion for spatial response to occur, to calculate the freeway alternative route travel times for arterial travelers, and to divert to the freeway those arterial vehicles that meet the criterion of saving sufficient travel time. The minimum criterion for spatial diversion to occur is a travel time savings equal to a time penalty. This penalty is the sum of the average access time between the parallel arterial and the freeway, and a minimum perceived travel time savings.

The freeway corridor is assumed to be in equilibrium before the HOV facility is added; travelers use the arterial or the freeway depending on which better suits their needs. After the HOV facility is added and spatial response has occurred, the freeway corridor should once again be in equilibrium. It is extremely important to prevent the shifting of too many vehicles from the arterial to the freeway during spatial response, so that equilibrium can be achieved without the need to shift vehicles back to the arterial. In order to prevent over shifting from the arterial to the freeway and to better simulate the actual diversion process, the shifts are made in increments following a newly developed iterative scheme. Those arterial travelers saving the most time shift first, the arterial priority vehicles using the HOV facility wherever possible and the nonpriority vehicles using the general purpose lanes. The corridor is then resimulated to calculate the new travel costs before the next incremental group of vehicles is shifted from the arterial to the freeway. The user controls the number of incremental shifts that are performed by specifying the number of iterations. The model computes the criterion of travel time savings required for spatial response to occur during each iteration beginning with the maximum time that could be saved given the conditions in the corridor and decreasing the criterion after each iteration.

The structure of the new FREQ10PL spatial response module is shown in Figure 7. The following discussion refers to Figure 7.

- Step 1. The maximum criterion for spatial diversion is calculated. This criterion defines the minimum travel time savings for spatial shift to occur during the first iteration.
- Step 2. The potential travel time savings should arterial autos use the freeway to make their trips is calculated. Arterial vehicles are assumed to travel on the arterial until the first possible freeway on-ramp and come back to the arterial using the last off-ramp before their destination. Once on the freeway, arterial HOV vehicles are assumed to use the on-freeway HOV facility wherever possible and arterial non-HOV vehicles are assumed to use the general purpose lanes.
- Step 3. Arterial priority automobiles meeting the criterion are diverted. For each arterial priority O-D pair for which vehicles have a travel time savings greater or equal to the criterion set for the current iteration, all of the vehicles are shifted to the appropriate corresponding freeway subsections.
- Step 4. Arterial nonpriority automobiles meeting the criterion are diverted as in the previous step, except that vehicles are assigned to the general-purpose lanes of the freeway.
- Step 5. After each arterial O-D pair for the current time slice is examined for possible diversion to the freeway, the program either proceeds with the next time slice and repeats Steps 2 to 4 above, or if the current time slice is the last one, it proceeds to the next step.
- Step 6. After all of the time slices are analyzed for this iteration and potential shifters meeting the spatial shift criterion are diverted, the corridor is resimulated using the revised freeway and arterial demands that reflect the vehicles that have changed their routes during this iteration. The new travel times that are computed during this resimulation of the corridor are used for the next iteration.
- Step 7. If all iterations are not completed, the model reduces the spatial response criterion and repeats Steps 2 to 6. After the last iteration, there are no more vehicles that would save time by changing their routes to the freeway and, thus, the corridor is once again in equilibrium. Control is returned to the main program that simulates the entire corridor after spatial response and provides any requested output for this simulation.

For more details on the FREQ10PL spatial response module, please see the final report (21).

Spatial response in FREQ10PE occurs from the freeway to the arterial. To avoid ramp delays caused by metering, vehicles divert to the arterial to enter the freeway at on-ramps downstream of their originally designated entry locations. For more information on the FREQ10PE spatial response module, please see the FREQ8PE research report (9).

#### MODAL RESPONSE MODELING

After spatial response has occurred, the arterial is in equilibrium with respect to the freeway, but the freeway itself is no

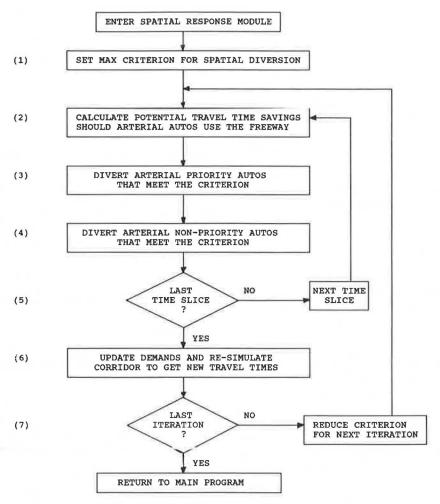


FIGURE 7 Overview of the spatial response module.

longer in equilibrium. The general-purpose lanes of the freeway are even more congested relative to the priority lanes than they were before spatial response because there are fewer HOV vehicles than non-HOV vehicles on the arterial that could potentially change their routes to the freeway. Thus, there could be an even stronger incentive for commuters to change their mode of travel to either buses or carpools to enjoy the increased savings in travel time offered by the HOV facility.

However, not all travelers who would save time change their mode of travel. The level of modal response depends on many variables. Some of these variables include socioeconomic characteristics of the commuters, perceived convenience, and attributes of the modes under investigation. The model predicts how many travelers change mode by applying elasticities that were derived from a multinomial logit model (25). These elasticities are expressed as a proportion of passengers shifting per minute of travel time saved. They are used within the program to predict how many freeway non-HOV passengers who would save time would actually change their mode of transportation to carpools or to buses. The values of elasticities depend on the priority cut-off level and the level of bus service. The user has the option to override these values by supplying the user's own elasticities. These user-supplied elasticities are also expressed as a proportion of passengers shifting per minute of on-freeway travel

The basic rationale of the modal response module is to set a criterion for sufficient time saved in order for modal response to occur, to calculate the difference in on-freeway travel times between priority and nonpriority vehicles, and to then shift to carpools and buses a percentage of those non-priority passengers saving sufficient travel time as specified by the criterion. The percent shifted depends on the amount of time saved, the investigated priority strategy, and the LOS of the bus system. The priority strategy could be one of the following: buses only or buses and carpools (2+ or 3+).

In order to prevent overestimating the total number of passengers that change to carpools or buses and to better model the actual modal response, a new iterative scheme similar to that used in modeling spatial response was developed. Those nonpriority passengers saving the most time consider shifting first. The freeway is then resimulated before the next iteration to calculate the new travel costs. The number of iterations is provided by the user. The model computes the criterion of travel time savings required for modal response to occur for each iteration beginning with the maximum time that could be saved given the conditions on the freeway and decreasing the criterion appropriately for each successive iteration.

An overview of the structure of the modal response module along with its new iterative process is shown in Figure 8. The following discussion refers to Figure 8.

- Step 1. The maximum criterion for modal shift is calculated. This criterion defines the minimum travel time savings required for modal response to occur during the first iteration.
- Step 2. The potential travel time savings should passengers in nonpriority automobiles form carpools or ride buses is calculated.
- Step 3. Passengers in nonpriority vehicles meeting the criterion are shifted to carpools and buses. For each nonpriority O-D pair for which vehicles have a travel time savings greater than or equal to the criterion set for the current iteration, a percentage of the passengers is shifted to the appropriate priority mode and thus to the priority lane subsections. The percent shift is determined by the elasticities provided by the program or by the user. Once an O-D pair for a given time slice has been partially shifted, it is never reconsidered during a subsequent iteration for the same time slice. This restriction prevents compounding the application of the elasticity values.
- Step 4. After each nonpriority O-D pair for the current time slice is examined for possible modal response, the program either proceeds with the next time slice and repeats Steps 2 and 3, or if the current time slice is the last one, it proceeds to the next step.
- Step 5. After all of the time slices are analyzed for this iteration and a portion of potential priority passengers meeting the time savings criterion are shifted, the corridor is resimulated using the revised freeway demands which reflect the reduction in nonpriority vehicles and the increase in priority vehicles because of modal response. The new travel times that are computed during this resimulation of the freeway are used for the next iteration.
- Step 6. If all iterations are not completed, the model reduces the modal response criterion and repeats Steps 2 to 5. After the last iteration, no nonpriority O-D pairs exist that would save time with a modal response that has not already been considered. Control is then returned to the main program, which simulates the entire corridor after spatial response and provides any requested output for this simulation.

For further details on the modal response module, please refer to the final report (21).

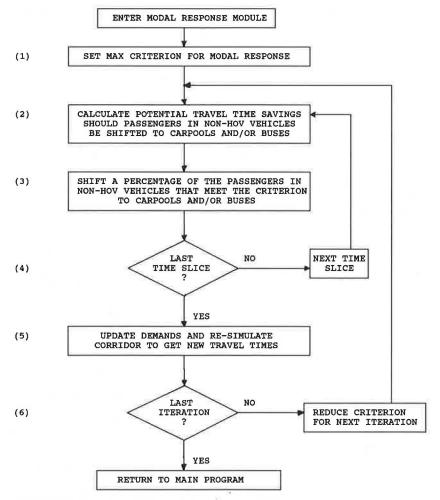


FIGURE 8 Overview of the modal response module.

#### **SUMMARY**

This paper has described the newly developed FREQ10 integrated system of freeway corridor simulation models with emphasis on traffic simulation, freeway improvement strategies, measures of effectiveness, and traveler responses. The major accomplishments have been to combine the previously developed entry control model, the extensively modified onfreeway priority lane model, and a newly developed common menu-driven interactive interface into an integrated system of models to extend the types of traffic management strategies that can be evaluated for a freeway corridor. This system of models has received extensive testing and, with the exception of the spatial response module, has been applied to freeway corridors in several urban areas in California. Application of the model to corridors with parallel arterials is currently in progress. Although outside the scope of this paper, a hypothetical case study of a freeway corridor analysis using various management strategies within the FREQ10 system of models is presented in the final report (21). An application of the FREQ10PE model in a reconstruction study is also available (17).

Research in general, and research related to simulation models in particular, is never complete. There is always further research to be undertaken and related enhancements to be incorporated into the model. The next paragraphs identify such possible enhancements in regard to traffic simulation, freeway improvement strategies, measures of effectiveness, and traveler responses.

The traffic simulation portion of the model is currently limited to a corridor length of some 12 to 16 mi. Increasing the number of subsections and the number of on- and offramps would permit application to longer freeway corridors. Currently on-ramp, merge, and off-ramp queues are handled separately from freeway mainline queues. When these queues interact, the traffic performance results are approximate and there is a need to integrate them. In addition, a possible parallel freeway cannot be simulated.

Currently, the only improvement strategy that can be automatically optimized by the model is entry control. Incorporating optimization techniques within the model for onfreeway priority lanes and combined strategies such as design improvements with entry control would be desirable. The onfreeway priority lane improvement strategy in the model has several limitations that could be relaxed with further research. For example, no buffer is simulated to be present between the priority lanes and nonpriority lanes, and it is assumed that priority vehicles will enter the HOV lanes as soon as possible and stay in the HOV lanes as long as possible regardless of length of trip and with no regard to possible buffers or barriers.

The fuel, noise, and particularly the emission measures of effectiveness need to be updated to represent current and future vehicle fleet performance. The current cost-effectiveness calculations are in an early stage of development and a number of enhancements would be desirable.

The spatial and modal traveler response algorithms need to be tested against present real-life situations and calibrated when needed. The current model does not handle temporal or longer-term traveler responses. After 20 years of almost continuous research and development, an integrated system of freeway corridor simulation models has emerged. Much has been accomplished and a comprehensive tool is available to researchers and professionals. Yet with the complexities of today's traffic situations, the comprehensiveness of available traffic management strategies, the wide variety of measures of effectiveness that require collective assessment, and the requirement for a timestream system evaluation as travelers respond to system changes, further research is needed. Much has been accomplished—much has yet to be done.

#### **ACKNOWLEDGMENTS**

Many individuals assisted the project staff in this multiyear project. From the Berkeley campus, we wish to thank Dimitris Scapinakis for his significant contributions throughout this project. In addition, we appreciate the many contributions made by Joe Oliveira and Eric Freitag during the early phases of the project. We thankfully acknowledge that this research was sponsored by the California Department of Transportation and the FHWA. In particular, we wish to thank the following individuals who contributed to this research: Phil Jang and Mike Auslam from CALTRANS—we appreciate their support and cooperation throughout the life of this project—and Jeffrey Lindley and Frank Cechini from FHWA. In addition, we appreciate all the helpful feedback from members of our HOV professional panel.

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Publication of this paper sponsored by Committee on High-Occupancy Vehicle Systems.