Scheme To Optimize Circular Phasing Sequences

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The provision of traffic progression along an arterial street has long been accepted as a desirable traffic control objective to improve the level of service. Bandwidth maximization is often used to optimize progression. A new scheme optimizes two circular phasing sequences in addition to those available in existing bandwidth maximizing programs. The new circular sequences, having the form main1-cross1-main2-cross2, can be clockwise or counterclockwise. The MAXBAND 89T program uses this enhanced optimization capability to find maximum progression bands on an arterial. This scheme expands the arterial formulation used in MAXBAND 86. It uses the mixed-integer linear programming method for optimizing progression bandwidth. Experimental results for some cases show that the new formulation can produce wider progression bands on an arterial than those of the MAXBAND 86 formulation.

The problem of finding signal timings that produce the maximum sum of progression bands along a two-way arterial was first modeled by Little (*1*) as a mixed-integer linear program (MILP). This formulation found the cycle length and offsets for a two-phase signal system. The basic MILP formulation was later enhanced by Little et al. (*2*) for optimizing National Electrical Manufacturers Association (NEMA) left-turn phasing sequences. Little et al. also developed the MAXBAND computer program (*2*) to optimize signal timing on signalized arterials and triangular networks. In 1986, MAXBAND was upgraded to MAXBAND 86 (*3*). The enhanced program was capable of optimizing signal timing on suburban arterials and urban grid networks. In recent years, Tsay et al. (*4*) and Gartner et al. (*5*) have produced further enhancements to the arterial bandwidth formulation used in MAXBAND; both of these enhancements deal with the shape of the progression bands. In addition, Chaudhary et al. (*6*; see also companion paper in this Record) and Mireault (*7,8*) have recently developed more efficient schemes to optimize arterial and network signal timing problems.

We present another enhancement to Little's basic MILP formulation for the arterial signal timing optimization problem (*2*). This enhancement provides the capability to optimize two new circular phasing sequences in addition to the existing NEMA phasing sequence. The form of these four-phase sequences is main1-cross1-main2-cross2 (i.e., main lead–cross lead–main lag–cross lag), as opposed to the conventional four-phase sequences for which both green phases on one arterial are provided in a single contiguous block as main-cross (i.e., main lead–main lag–cross lead–cross lag). Figure 1 shows the circular phasing, which can be either clockwise or counterclockwise. Evaluation of the new formulation shows that it can produce wider bands than the original formulation and improve arterial performance in some cases.

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ENHANCED MATHEMATICAL FORMULATION

In this section we develop the enhanced arterial formulation. Readers not interested in the mathematical details may wish to scan this section. First, the original MILP arterial formulation is reproduced. A formulation that has the capability to optimize only circular phasing sequences is then shown. Finally, we show how these two formulations can be combined to produce a comprehensive formulation capable of optimizing both NEMA and circular phasing sequences.

Original MILP Arterial Formulation

The arterial formulation for maximizing progression bandwidth is obtained by using the basic progression bandwidth geometry shown in Figure 2. The variable are defined as follows:

b ($\bar{b}$) = outbound (inbound) bandwidth, cycles;
z = signal cycle, inverse of cycle length;
S_i = signal i, $i = 1, \ldots, n$;
w_i ($\bar{w}_i$) = time from right (left) side of red at S_i to left (right) edge of outbound (inbound) green band, cycles;
t_i ($\bar{t}_i$) = outbound (inbound) travel time from S_i to S_{i+1} (S_{i+1} to S_i), cycles;
Δ_i = time in cycles from the center of $\bar{r}_i$ to the nearest center of r_i: positive if the center of r_i is to the right of the center of $\bar{r}_i$;
δ_i ($\bar{\delta}_i$) = 0-1 variables for phasing sequence selection; and
m_i = an integer number.

The constants are defined as follows:

r_i ($\bar{r}_i$) = outbound (inbound) red time at S_i, cycles;
g_i ($\bar{g}_i$) = outbound (inbound) green time for through traffic at S_i, cycles;
ℓ_i ($\bar{\ell}_i$) = time allocated for outbound (inbound) left-turn green at S_i, cycles;
τ_i ($\bar{\tau}_i$) = queue clearance time, in cycles, an advance of the outbound (inbound) bandwidth upon leaving S_i;
c ($\bar{c}$) = outbound (inbound) objective function weight;
k = inbound to outbound target bandwidth ratio;
T_1 = lower limit on signal cycle length;
T_2 = upper limit on signal cycle length;

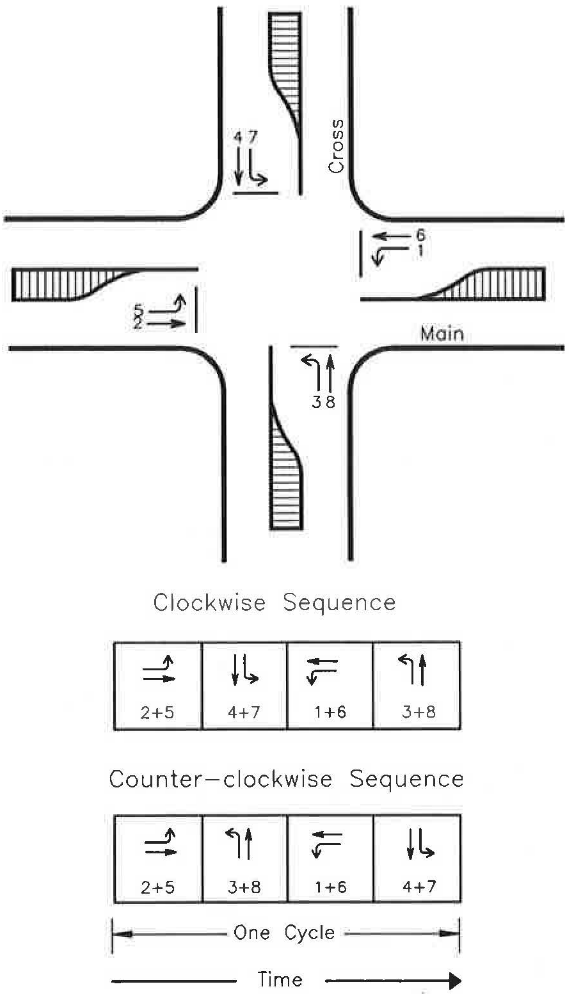

FIGURE 1 Circular phasing sequences.

$d_i\ (\bar{d}_i)$ = outbound (inbound) distance between S_i and S_{i+1};
$e_i\ (\bar{e}_i)$ = lower limit on outbound (inbound) speed on link between S_i and S_{i+1};
$f_i\ (\bar{f}_i)$ = upper limit on outbound (inbound) speed on link between S_i and S_{i+1};
$\frac{1}{h_i}\left(\frac{1}{\bar{h}_i}\right)$ = lower limit on outbound (inbound) reciprocal change between two adjacent links; and
$\frac{1}{g_i}\left(\frac{1}{\bar{g}_i}\right)$ = lower limit on outbound (inbound) reciprocal change between two adjacent links.

The fundamental loop equation for formulating the arterial problem obtained from Figure 2 is as follows:

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \Delta_i - \Delta_{i+1} - m_i$$
$$= -\frac{1}{2}(r_i + \bar{r}_i) + \frac{1}{2}(r_{i+1} + \bar{r}_{i+1}) + (\bar{\tau}_i + \tau_{i+1}) \qquad (1)$$

Figure 3 shows the four conventional left-turn green phases. The time from the center of $\bar{r}_i$ to the next center of r_i in terms of ℓ_i and $\bar{\ell}_i$ for each case, (Δ_i), can be expressed as a single equation having two binary variables as

$$\Delta_i = \frac{1}{2}\left[(2\delta_i - 1)\ell_i - (2\bar{\delta}_i - 1)\bar{\ell}_i\right] \qquad (2)$$

Each of the four possible left-turn patterns can be determined by the following combinations of binary decision variables:

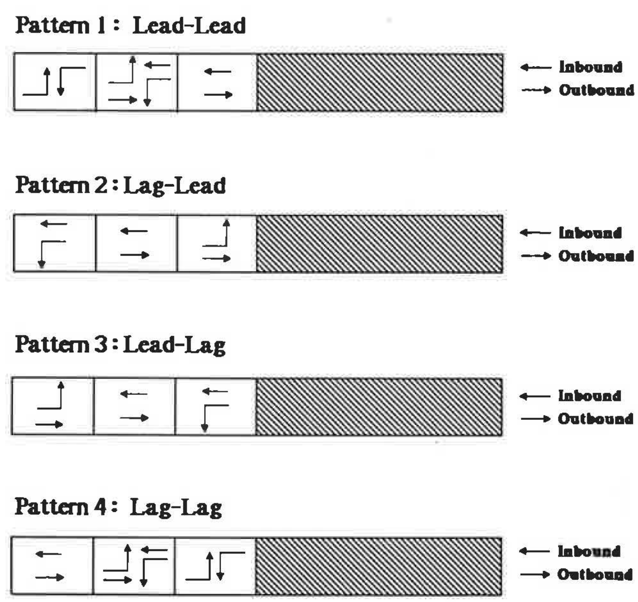

FIGURE 3 Four conventional phasing sequences.

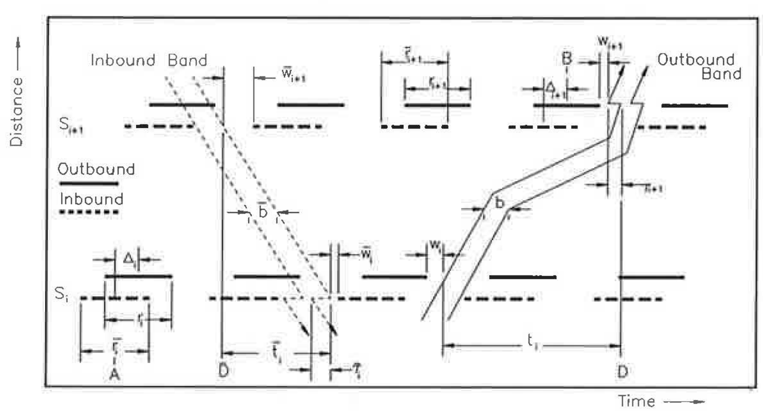

FIGURE 2 Basic progression bandwidth geometry.

Pattern	Signal Phase Sequence	δ_i	$\bar{\delta}_i$
1	Outbound leads–inbound leads	0	1
2	Outbound lags–inbound leads	1	0
3	Outbound leads–inbound lags	0	0
4	Outbound lags–inbound lags	1	1

Substituting Equation 2 in Equation 1 eliminates Δ_i and Δ_{i+1} and results in a more simplified equation. However, for ease in presenting this material, we use the two equations as they are. Following is the complete MILP arterial formulation:

MILP 1 Find $b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, m_i$ and Δ_i to maximize $cb + \overline{cb}$ subject to

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \Delta_i - \Delta_{i+1} - m_i = -\frac{1}{2}(r_i + \bar{r}_i) + \frac{1}{2}(r_{i+1} + \bar{r}_{i+1}) + (\bar{\tau}_i + \tau_{i+1})$$

$$i = 1, \ldots, n - 1 \quad (1)$$

$$\Delta_i = \frac{1}{2}[(2\delta_i - 1)\ell_i - (2\bar{\delta}_i - 1)\bar{\ell}_i] \qquad i = 1, \ldots, n \quad (2)$$

$$-kb + \bar{b}\begin{cases} = 0 & \text{if } k = 1 \\ \geq 0 & \text{if } k < 1 \\ \leq 0 & \text{if } k > 1 \end{cases} \quad (3)$$

$$\frac{1}{T_2} \leq z \leq \frac{1}{T_1} \quad (4)$$

$$w_i + b \leq 1 - r_i \qquad i = 1, \ldots, n \quad (5a)$$

$$\bar{w}_i + \bar{b} \leq 1 - \bar{r}_i \qquad i = 1, \ldots, n \quad (5b)$$

$$\left(\frac{d_i}{f_i}\right)z \leq t_i \leq \left(\frac{d_i}{e_i}\right)z \qquad i = 1, \ldots, n - 1 \quad (6a)$$

$$\left(\frac{\bar{d}_i}{\bar{f}_i}\right)z \leq \bar{t}_i \leq \left(\frac{\bar{d}_i}{\bar{e}_i}\right)z \qquad i = 1, \ldots, n - 1 \quad (6b)$$

$$\left(\frac{d_i}{h_i}\right)z \leq \left(\frac{d_i}{d_{i+1}}\right)t_{i+1} - t_i \leq \left(\frac{d_i}{g_i}\right)z$$

$$i = 1, \ldots, n - 2 \quad (7a)$$

$$\left(\frac{\bar{d}_i}{\bar{h}_i}\right)z \leq \left(\frac{\bar{d}_i}{\bar{d}_{i+1}}\right)\bar{t}_{i+1} - \bar{t}_i \leq \left(\frac{\bar{d}_i}{\bar{g}_i}\right)z$$

$$i = 1, \ldots, n - 2 \quad (7b)$$

Optional Left-Turn Choice Constraints on δ_i and $\bar{\delta}_i$

$$i = 1, \ldots, n$$

$b, \bar{b}, z, w_i, \bar{w}_i, t_j, \bar{t}_j$, and $m_j \geq 0$

$$i = 1, \ldots, n, j = 1, \ldots, n - 1$$

Δ_i unrestricted continuous variables

$$i = 1, \ldots, n$$

m_i general integer variables

$$i = 1, \ldots, n - 1$$

δ_i and $\bar{\delta}_i$ binary variables

$$i = 1, \ldots, n \quad (8)$$

Circular Phasing Sequence Optimization Capability

In this section we present details on an arterial formulation that selects the cycle length, offsets, and only circular phasing sequences to maximize the sum of progression bands in both arterial directions. There are two reasons for devoting a section to this development. The first is to document completely this major step in the production of the complete mathematical formulation derived in the next section. The second is to emphasize that circular phasing sequence is not conventionally used in progression-based signal timings, and that reprogramming conventional signal controllers may be needed to implement a circular phasing sequence. However, this is not a difficult procedure with microprocessor-based systems.

The circular phasing sequences shown in Figure 1 have four signal phases, each of which displays a green indication to both through and left-turn movements on a signalized approach. These phasing sequences are different from the conventional NEMA sequences in two ways. First, the green splits for circular phasing sequences, in general, are not the same as those for NEMA phases. Second, the time between centers of red for inbound and outbound directions is different. Therefore, there is a need to calculate new green splits for all phases, and to develop a new equation representing time between the centers of red for inbound and outbound movements on an arterial.

Green Split Calculation

As opposed to Webster's method (*9*) of computing green splits for the NEMA sequences used in the original formulation, the green splits for the circular phasing sequences are calculated as follows:

$$\max(g_{1,i}, g_{6,i}) + \max(g_{2,i}, g_{5,i}) + \max(g_{4,i}, g_{7,i}) + \max(g_{3,i}, g_{8,i}) = 1$$

$$G_{1,i} = G_{6,i} = \max(g_{1,i}, g_{6,i})$$

= main-street outbound movement

$$G_{2,i} = G_{5,i} = \max(g_{2,i}, g_{5,i})$$

= main-street inbound movement

$$G_{4,i} = G_{7,i} = \max(g_{4,i}, g_{7,i})$$

= cross-street outbound movement

$$G_{3,i} = G_{8,i} = \max(g_{3,i}, g_{8,i})$$

$$= \text{cross-street inbound movement}$$

where

$g_{m,i}$ = calculated green based on volume to saturation flow ratio for movement m of signal i, and
$G_{m,i}$ = green split for movement m of signal i for a circular phasing sequence.

The labels "inbound" and "outbound" are assigned to the movements for consistency with the original MILP formulation. The split calculation given above requires the following modifications to Constraints 1, 5a, and 5b given previously:

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) + \Delta_i - \Delta_{i+1} - m_i$$
$$= -\frac{1}{2}(R_{6,i} + R_{2,i}) + \frac{1}{2}(R_{6,i+1} + R_{2,i+1})$$
$$+ (\bar{\tau}_i + \tau_{i+1}) \qquad i = 1, \ldots, n-1 \qquad (9)$$

$$w_i + b \le 1 - R_{6,i} \qquad i = 1, \ldots, n \qquad (10a)$$

$$\bar{w}_i + \bar{b} \le 1 - R_{2,i} \qquad i = 1, \ldots, n \qquad (10b)$$

Equation for Selecting Best Circular Sequence

Following is the derivation of a single equation describing the time between the centers of red for the two arterial phases in a circular phasing sequence.

For the clockwise phasing sequence:

$$\Delta_{6-2,i} = \frac{1}{2} - \frac{1}{2}(G_{8,i} - G_{4,i}) = \frac{1}{2} - \frac{1}{2}(R_{4,i} - R_{8,i})$$

$$\Delta_{4-8,i} = \frac{1}{2} - \frac{1}{2}(G_{6,i} - G_{2,i}) = \frac{1}{2} - \frac{1}{2}(R_{2,i} - R_{6,i})$$

For the counterclockwise phasing sequence:

$$\Delta_{6-2,i} = \frac{1}{2} + \frac{1}{2}(G_{8,i} - G_{4,i}) = \frac{1}{2} + \frac{1}{2}(R_{4,i} - R_{8,i})$$

$$\Delta_{4-8,i} = \frac{1}{2} + \frac{1}{2}(G_{6,i} - G_{2,i}) = \frac{1}{2} + \frac{1}{2}(R_{2,i} - R_{6,i})$$

where

$\Delta_{j-k,i}$ = difference between the centers of red for outbound movement j and inbound movement k on signal i of the artery, and
$R_{m,i}$ = $1 - G_{m,i}$ is the red split for movement m of signal i.

Subscripts 6 and 4 represent outbound movements on the main and cross arteries, respectively. Subscripts 2 and 8 represent inbound movement on the main and cross arteries, respectively.

As opposed to the original formulation, the Δ values derived here for the main artery contain red splits for the cross artery, and vice versa. Combining the above equations to obtain a single set of equations for both clockwise and counterclockwise phasing sequences, we have

$$\Delta_{6-2,i} = \Delta_i = \frac{1}{2} + \left(\frac{1}{2} - \beta_i\right)(R_{4,i} - R_{8,i}) \qquad (11)$$

for the main artery, and

$$\Delta_{4-8,i} = \Delta_{c,i} = \frac{1}{2} + \left(\frac{1}{2} - \beta_i\right)(R_{2,i} - R_{6,i}) \qquad (12)$$

for the cross artery.

The binary decision variable (β_i) selects clockwise phasing for the ith signal when its value is 1 and selects counterclockwise phasing for the ith signal when its value is 0. Equation 11 is a replacement for Equation 2 of the original formulation. Equation 12 is needed only when one also desires to simultaneously optimize bands on the cross artery at signal i (i.e., in a multiarterial network problem).

Substituting Equations 1, 2, 5a, and 5b in the original arterial formulation with Equations 9, 11, 10a, and 10b, we obtain a new formulation that is capable of optimizing only circular phasing sequences.

The new formulation was manually tested on several real-world test problems. These problems were optimized using the LINDO optimization package *(10)* on a personal computer. Figure 4 shows the time-space diagram for a five-intersection test problem using only circular phasing sequences. The intersections at 906 ft and 1,965 ft have only three phases. This illustrates the fact that for a three-legged intersection, a circular phasing sequence reduces to a conventional three-phase sequence with either lead-lag or lag-lead phasing for the two-way street.

Combined NEMA and Circular Phasing Optimization Capability

Next, we develop a comprehensive formulation having the capability to select either NEMA or circular phasing sequences that produce the maximum total progression bands on an arterial. We accomplish this by combining the original and new formulations described previously. The process of combining these two formulations is slightly more complicated because Constraints 1, 2, 5a, and 5b in the original formulation and corresponding Constraints 9, 11, 10a, and 10b in the new formulation are mutually exclusive. We combine these constraints by introducing binary variables into the problem formulation. The purpose of these variables is to provide a systematic way of selecting either NEMA or circular phasing sequences. Additional variables are defined as follows:

$R_{6,i}$ = main-street outbound approach (Movements 1 + 6) red split for signal i in a circular phasing;
$R_{2,i}$ = main-street inbound approach (Movements 2 + 5) red split for signal i in a circular phasing;
$R_{8,i}$ = circular phase red split for cross-street Movements 3 + 8 at node i;
$R_{4,i}$ = circular phase red split for cross-street Movements 4 + 7 for node i;

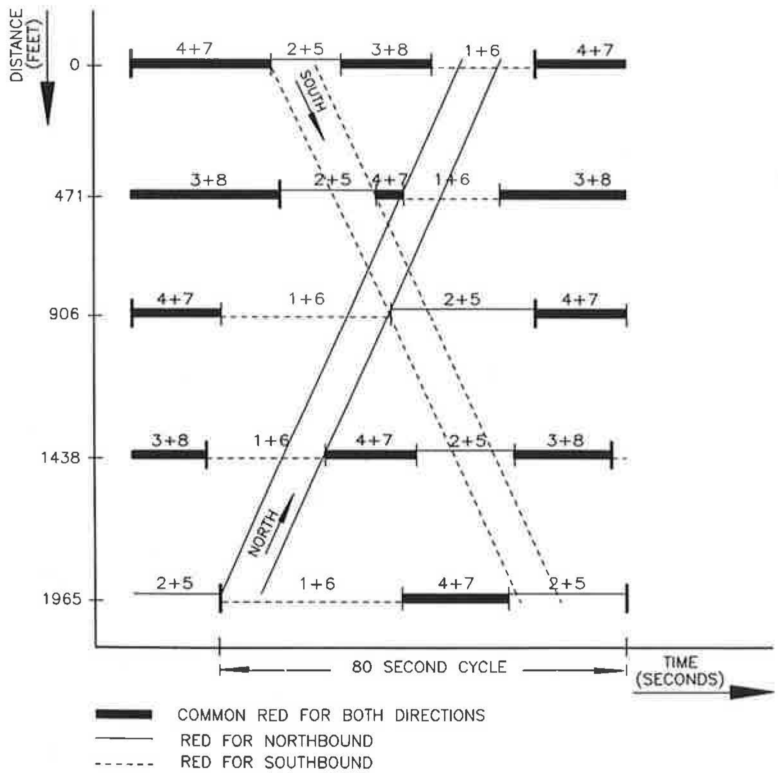

FIGURE 4 Time-space diagram for solution with only circular phases.

β_i = binary variable that selects one of the two circular phasing sequences: a value of 0 selects the counter-clockwise sequence, and a value of 1 selects the clockwise sequence; and

α_i = binary variable that selects between the NEMA and the circular phasing sequences: a value of 1 selects the NEMA sequence, and a value of 0 selects the circular sequence.

Constraints on Progression Bands

Constraints 5a and 5b are combined with Constraints 10a and 10b, respectively, to form the following two constraints:

$$w_i + b + (r_i - R_{6,i})\alpha_i \leq 1 - R_{6,i} \quad (13a)$$

$$\bar{w}_i + \bar{b} + (\bar{r}_i - R_{2,i})\alpha_i \leq 1 - R_{2,i} \quad (13b)$$

Loop Constraints

The Loop Constraints 1 and 9 are combined using the binary variable α_i, defined above, as follows:

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) - m_i + \Delta_i$$

$$+ \frac{1}{2}(r_i + \bar{r}_i - R_{6,i} - R_{2,i})\alpha_i - \Delta_{i+1}$$

$$- \frac{1}{2}(r_{i+1} + \bar{r}_{i+1} - R_{6,i+1} - R_{2,i+1})\alpha_{i+1} =$$

$$-\frac{1}{2}(R_{6,i} + R_{2,i} - R_{6,i+1} - R_{2,i+1}) + (\bar{\tau}_i + \tau_{i+1}) \quad (14)$$

Phasing Sequence Selection Equations

During optimization, the proper values of Δ_i used in Equation 14 depend on whether NEMA or circular phasing is selected by the optimization program. This means that for a signal, either Constraint 2 or Constraint 11 is active. In order to implement these either/or (disjunctive) constraints, each of them is replaced by two inequality constraints. Because the Δ_i variables are unrestricted (i.e., their values can also be negative), we add a value of 2 to these variables to ensure that they are able to achieve a lower bound of -2. However, this transformation does not change the formulation, because the added values are canceled when Δ_i and Δ_{i+1} are substituted in Loop Constraint 4. Finally, using the binary variable α_i defined earlier, we obtain the following set of constraints:

$$\Delta_i + 2\alpha_i - \delta_i\ell_i + \bar{\delta}_i\bar{\ell}_i \leq \frac{1}{2}(-\ell_i + \bar{\ell}_i) + 4 \quad (15a)$$

$$\Delta_i - 2\alpha_i - \delta_i\ell_i + \bar{\delta}_i\bar{\ell}_i \geq \frac{1}{2}(-\ell_i + \bar{\ell}_i) \quad (15b)$$

$$\Delta_i - 2\alpha_i + (R_{4,i} - R_{8,i})\beta_i \leq \frac{1}{2}(R_{4,i} - R_{8,i}) + \frac{5}{2} \quad (16a)$$

$$\Delta_i + 2\alpha_i + (R_{4,i} - R_{8,i})\beta_i \geq \frac{1}{2}(R_{4,i} - R_{8,i}) + \frac{5}{2} \quad (16b)$$

Note that for simplicity, Movements 1 + 6 are assumed to be outbound movements on the main artery and Movements 4 + 7 are assumed to be outbound movements on the cross artery. Further, this notation is used for consistency with that used in the original MILP formulation presented earlier.

The final comprehensive formulation is as follows:

MILP 2 Find $b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, \alpha_i, \beta_i, m_i$ and Δ_i to maximize $cb + c\bar{b}$ subject to

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) - m_i + \Delta_i + \frac{1}{2}(r_i + \bar{r}_i - R_{6,i} - R_{2,i})\alpha_i - \Delta_{i+1} - \frac{1}{2}(r_{i+1} + \bar{r}_{i+1} - R_{6,i+1} - R_{2,i+1})\alpha_{i+1} = -\frac{1}{2}(R_{6,i} + R_{2,i} - R_{6,i+1} - R_{2,i+1}) + (\bar{\tau}_i + \tau_{i+1}) \quad i = 1, \ldots, n-1 \tag{14}$$

$$\Delta_i + 2\alpha_i - \delta_i\ell_i + \bar{\delta}_i\bar{\ell}_i \le \frac{1}{2}(-\ell_i + \bar{\ell}_i) + 4 \quad i = 1, \ldots, n \tag{15a}$$

$$\Delta_i - 2\alpha_i - \delta_i\ell_i + \bar{\delta}_i\bar{\ell}_i \ge \frac{1}{2}(-\ell_i + \bar{\ell}_i) \quad i = 1, \ldots, n \tag{15b}$$

$$\Delta_i - 2\alpha_i + (R_{4,i} - R_{8,i})\beta_i \le \frac{1}{2}(R_{4,i} - R_{8,i}) + \frac{5}{2} \quad i = 1, \ldots, n \tag{16a}$$

$$\Delta_i + 2\alpha_i + (R_{4,i} - R_{8,i})\beta_i \ge \frac{1}{2}(R_{4,i} - R_{8,i}) + \frac{5}{2} \quad i = 1, \ldots, n \tag{16b}$$

$$-kb + \bar{b} \begin{cases} = 0 & \text{if } k = 1 \\ \ge 0 & \text{if } k < 1 \\ \le 0 & \text{if } k > 1 \end{cases} \tag{3}$$

$$\frac{1}{T_2} \le z \le \frac{1}{T_1} \tag{4}$$

$$w_i + b + (r_i - R_{6,i})\alpha_i \le 1 - R_{6,i} \quad i = 1, \ldots, n \tag{13a}$$

$$\bar{w}_i + \bar{b} + (\bar{r}_i - R_{2,i})\alpha_i \le 1 - R_{2,i} \quad i = 1, \ldots, n \tag{13b}$$

$$\left(\frac{d_i}{f_i}\right)z \le t_i \le \left(\frac{d_i}{e_i}\right)z \quad i = 1, \ldots, n-1 \tag{6a}$$

$$\left(\frac{\bar{d}_i}{\bar{f}_i}\right)z \le \bar{t}_i \le \left(\frac{\bar{d}_i}{\bar{e}_i}\right)z \quad i = 1, \ldots, n-1 \tag{6b}$$

$$\left(\frac{d_i}{h_i}\right)z \le \left(\frac{d_i}{d_{i+1}}\right)t_{i+1} - t_i \le \left(\frac{d_i}{g_i}\right)z \quad i = 1, \ldots, n-2 \tag{7a}$$

$$\left(\frac{\bar{d}_i}{\bar{h}_i}\right)z \le \left(\frac{\bar{d}_i}{\bar{d}_{i+1}}\right)\bar{t}_{i+1} - \bar{t}_i \le \left(\frac{\bar{d}_i}{\bar{g}_i}\right)z \quad i = 1, \ldots, n-2 \tag{7b}$$

NEMA Left-Turn Choice Constraints on δ_i and $\bar{\delta}_i$

$i = 1, \ldots, n$

$b, \bar{b}, z, w_i, \bar{w}_i, t_j, \bar{t}_j$, and $m_j \ge 0$

$i = 1, \ldots, n, j = 1, \ldots, n-1$

m_i general integer variables

$i = 1, \ldots, n-1$

$\delta_i, \bar{\delta}_i, \alpha_i$ and β_i binary variables

$$i = 1, \ldots, n \tag{8}$$

EXPERIMENTS WITH ENHANCED FORMULATION

The comprehensive formulation presented in the previous section was manually tested using four real-world arterial test problems with 3, 5, 6, and 12 intersections. Constraint 3, which enforces a user-desired relationship between the inbound and outbound bandwidths, was relaxed for this set of experiments. All optimizations were performed on a personal computer using the LINDO optimization package. The optimum time-space diagram for the 12-intersection problem, obtained using the original formulation, is shown in Figure 5. The optimum time-space diagram for the same test problem, obtained using the enhanced formulation, is shown in Figure 6. Table 1 summarizes the results of these two optimization runs on the 12-intersection problem. Following is a description of results for this problem.

1. The enhanced formulation resulted in a cycle length of 78 sec, compared to a value of 71 sec with the original formulation.
2. The enhanced formulation produced larger progression bands with a total increase of 15.04 sec in the total bandwidth. This increase can also be verified by a comparison of bands in terms of percent of cycle length.
3. The enhanced formulation resulted in different phasing sequences on intersections at 0, 3,480, 4,520, 11,050, 12,145, and 12,950 ft. A counterclockwise circular phasing sequence was selected at the intersection at 12,145 ft. For the other intersections, different NEMA sequences were selected.

Optimizing combined NEMA and circular phasing sequences (using MAXBAND 89T) for the problems with three, five, and six intersections produced the same total bandwidths as the optimization of NEMA phasing sequences alone

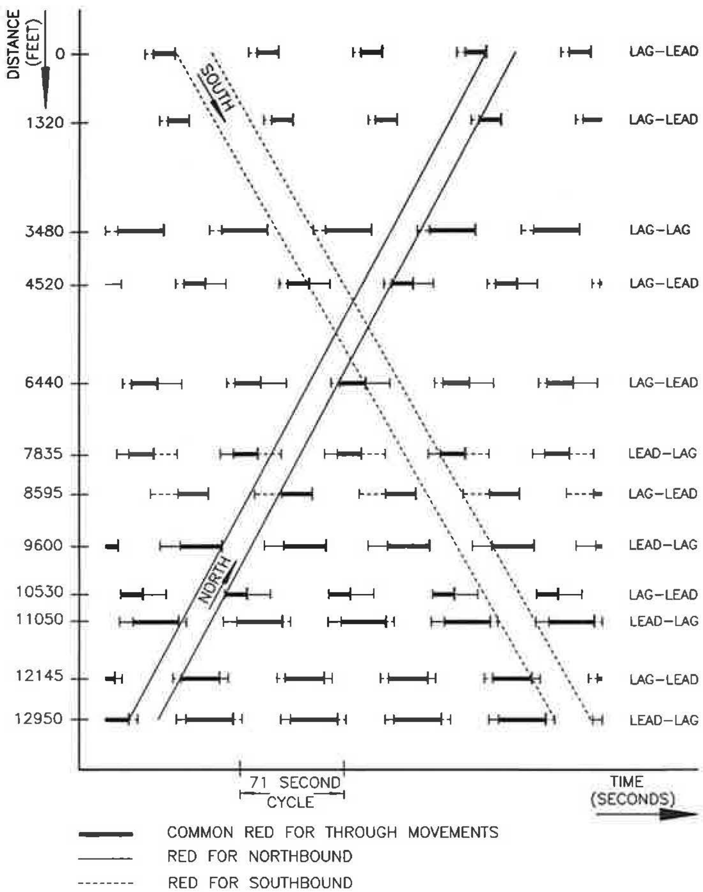

FIGURE 5 NEMA-only phasing sequence optimization for 12-intersection problem.

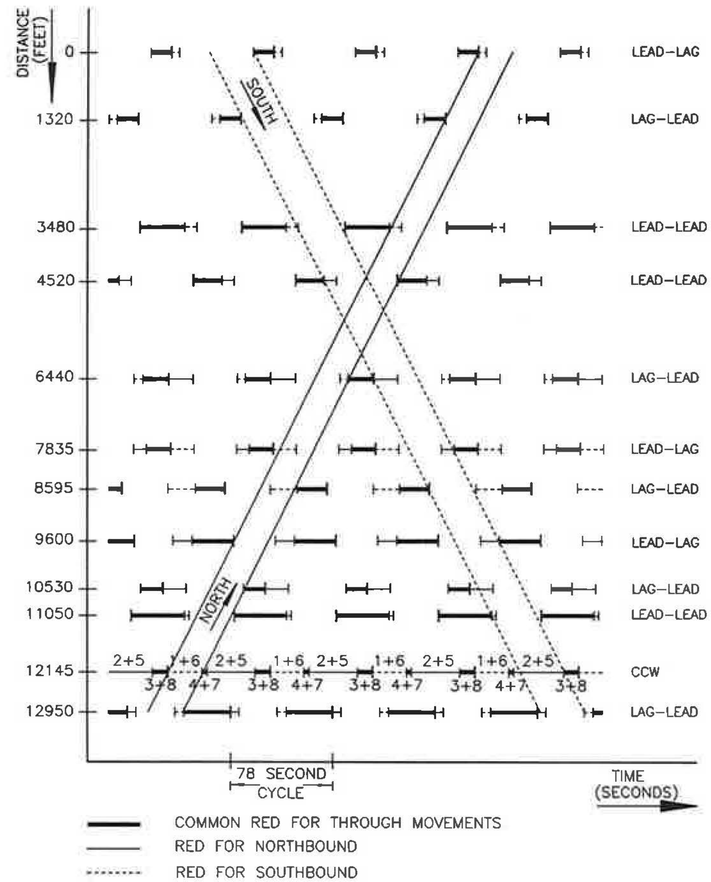

FIGURE 6 Combined NEMA and circular phasing sequence optimization for 12-intersection problem.

TABLE 1 MAXBAND 86 AND MAXBAND 89T RESULTS FOR 12-INTERSECTION PROBLEM

	MAXBAND 86	MAXBAND 89T
Cycle length (Sec)	71	78
Northbound Band		
Seconds (% of Cycle)	19.0 (28.1)	26.0 (33.3)
Southbound Band		
Seconds (% of Cycle)	24.6 (34.5)	33.6 (43.0)
Phasing Sequence		
Signal 1	Lag-Lead	Lead-Lag
Signal 2	Lag-Lead	Lag-Lead
Signal 3	Lag-Lag	Lead-Lead
Signal 4	Lag-Lead	Lead-Lead
Signal 5	Lag-Lead	Lag-Lead
Signal 6	Lead-Lag	Lead-Lag
Signal 7	Lag-Lead	Lag-Lead
Signal 8	Lead-Lag	Lead-Lag
Signal 9	Lag-Lead	Lag-Lead
Signal 10	Lead-Lag	Lead-Lead
Signal 11	Lag-Lead	Counter-Clock Circular
Signal 12	Lead-Lag	Lag-Lead

(MAXBAND 86). These results were not unexpected for the following reasons:

1. The maximum progression bandwidth cannot be greater than the minimum through green time. Thus, the upper limit on total bandwidth is the sum of minimum inbound green and minimum outbound green. For many arterial problems, only NEMA phasing optimization produces this maximum. For these problems, the enhanced formulation will result in the same total bandwidth, even when a circular phasing is selected.
2. Even if the total band produced by NEMA- only optimization is less than the upper limit, the signal spacings, combined with practical travel speeds, may not allow full utilization of the additional green time windows provided by circular phasing sequences.

DEVELOPMENT OF MAXBAND 89T

Because the manual procedure was too tedious and time-consuming, an automated method was developed to perform this task. We accomplished this by modifying the arterial signal-timing optimization capability of MAXBAND 86. The modified program, MAXBAND 89T, is capable of optimizing arterial signal-timing problems only, and allows the selection of best values for cycle length, offset, link travel speeds, and either NEMA or combined NEMA and circular phasing sequences. The following phasing restrictions are programmed in MAXBAND 89T:

1. Circular phasing sequence optimization is not allowed for a three-legged intersection because this is a special case of the conventional NEMA phasing sequence.
2. If circular phasing optimization is desired in addition to the NEMA phasing, at least one of the two signalized approaches on an arterial must have left-turn demand.

The time-space diagram of MAXBAND 89T prints characters 6666, 2222, 4444, and 8888, to indicate Signal Phases 1 + 6, 2 + 5, 4 + 7, and 3 + 8, respectively. The length of a character string indicates the duration of corresponding phase.

EXPERIMENTS WITH ENHANCED FORMULATION USING MAXBAND 89T

Eight real-world arterial problems were used to test MAXBAND 89T on a DecStation 3100 computer. This computer is about two times faster than a Compaq 386/25 personal computer with a math coprocessor (*11*). For these test problems, we used fixed cycle lengths to ensure proper comparison with the original arterial formulation. In addition, we forced the inbound and outbound progression bandwidths to be the same.

Figures 7 and 8 show optimal time-space diagrams for Problem 2 (Ridgewood Avenue) obtained from the two programs. The optimal MAXBAND solution produced bands equal to 28.9 sec in each direction. The travel speeds selected for northbound and southbound directions were equal to 34.5 mph. The phasing sequences selected for Signals 1 through 4 were lag-lead, lead-lag, lead-lag, and lag-lead, respectively. In comparison, MAXBAND 89T produced 37.5-sec bands in each direction, an increase of over 17 sec in the total band. As compared to the solutions from MAXBAND 86, the enhanced program selected higher travel speeds of 37.7 and 37.8 mph for northbound and southbound directions, respectively. The phasing sequences selected by MAXBAND 89T were also different from those selected by MAXBAND 86. It selected counterclockwise circular phasing for the first intersection, and selected lead-lead phasing for the second and third intersections as compared to lag-lead phasing selected by MAXBAND 86.

Table 2 compares the MAXBAND 86 and MAXBAND 89T optimization results for all eight test problems. A description of the overall test results follows:

1. Combined NEMA and circular optimization produced wider progression bands for four out of eight problems. The improvement in bandwidth was from 15.35 to 29.55 percent.
2. Bandwidth improvement in terms of actual time was 3.5, 5.73, 6.51, and 17.09 sec for Problems 2, 5, 7, and 8, respectively.
3. As expected, the computational time required to optimize the enhanced formulation (MAXBAND 89T) increased slightly.

The TRANSYT-7F (*12*) program was used to further analyze the two solutions for Problems 2, 5, 7, and 8. For each optimal solution obtained from MAXBAND 86 and MAXBAND 89T, we performed (a) evaluation of delay, (b) delay optimization without constraining the bands, and (c) bandwidth-constrained delay minimization. For delay op-

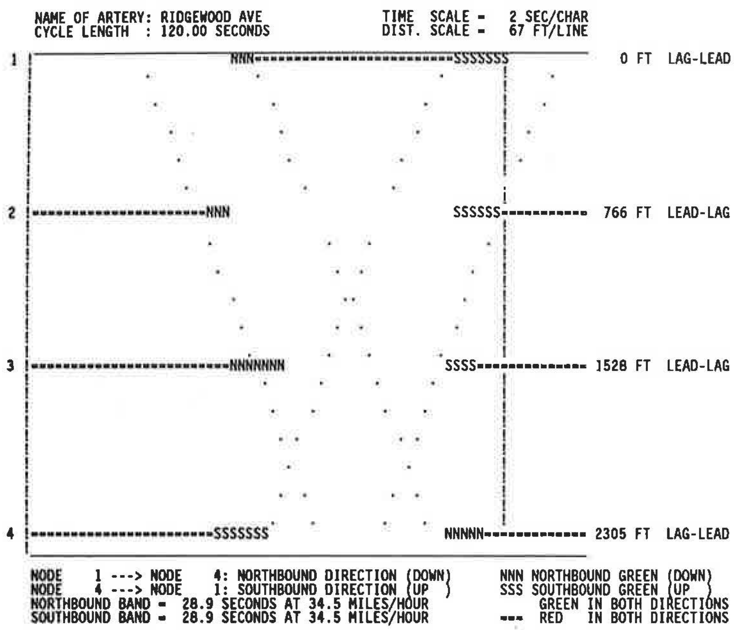

FIGURE 7 MAXBAND 86 time-space diagram for Ridgewood Avenue problem.

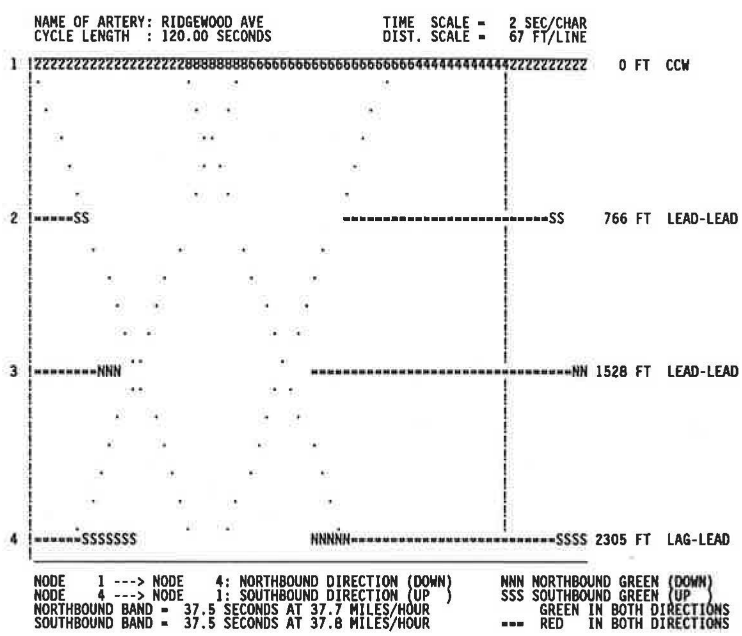

FIGURE 8 MAXBAND 89T time-space diagram for Ridgewood Avenue problem.

timization, we requested a stop penalty that minimized fuel consumption. The results are shown in Table 3. Following is a summary of these additional TRANSYT-7F studies:

1. For Problem 8 (Fannin), combined NEMA and circular phasing optimization (MAXBAND 89T) resulted in smaller values of delay, stops, and fuel consumption. For the other three problems, these measures were worse than those produced by optimizing NEMA sequences alone (MAXBAND 86).

2. For all cases, the use of an optimal bandwidth solution as a starting point for TRANSYT optimization resulted in reduced delay, stops, and fuel consumption. However, minimization without constraining the bands was always better than the constrained bandwidth option.

SUMMARY

We present an enhanced MILP arterial formulation that produces maximal progression bands by finding the best cycle

TABLE 2 COMPARISON OF MAXBAND 86 AND MAXBAND 89T OPTIMIZATION RESULTS

No	Problem Name	No. of Signals	Cycle Length (Sec)	Original $b = \bar{b}$ (Sec)	Original CPU Time (Sec)	Original+Circular $b = \bar{b}$ (Sec)	Original+Circular CPU Time (Sec)	Bandwidth Increase (Sec)	Bandwidth Increase (%)
1.	Washington	4	80	29.71	1.4	29.71	2.5	--	--
2.	Ridgewood	4	120	28.92	1.6	37.47	2.0	17.09	29.55
3.	Fourth Street	5	100	11.19	1.1	11.19	1.6	--	--
4.	M Street	8	80	29.30	1.6	29.30	2.1	--	--
5.	N 33rd	9	75	10.07	25.4	11.82	25.3	3.50	17.38
6.	Nicholasville	12	80	27.06	88.8	27.06	47.8	--	--
7.	N Michigan	13	90	18.68	22.5	21.54	25.8	5.73	15.35
8.	Fannin	15	80	20.87	30.7	24.13	165.9	6.51	15.59

-- indicate no difference in the total bandwidth

TABLE 3 DELAY COMPARISON OF SOLUTIONS FROM MAXBAND 86 AND MAXBAND 89T

Test Problem	TRANSYT Option	Total Delay (Veh-hr/hr)	Ave. Delay (sec/veh)	Stops (veh/hr)	Fuel Cons. (gal/hr)
Ridgewood (N)	1	79.00	25.40	7426.0	144.00
	2	75.00	24.10	7101.0	139.00
	3	75.00	24.10	7101.0	139.00
Ridgewood (N+C)	1	84.00	27.10	6694.0	145.00
	2	80.00	26.00	6466.0	140.00
	3	81.00	26.10	6459.0	141.00
N 33rd (N)	1	243.27	26.01	24119.4	552.92
	2	221.90	23.72	23621.0	532.90
	3	225.66	24.12	23314.4	533.21
N 33rd (N+C)	1	289.44	30.94	22337.8	571.24
	2	259.53	27.74	22613.0	551.46
	3	263.97	28.22	22293.9	552.26
N Michigan (N)	1	182.98	15.15	16687.6	362.28
	2	152.04	12.58	15172.7	331.00
	3	154.43	12.78	15766.4	336.15
N Michigan (N+C)	1	249.60	20.66	16289.5	408.83
	2	186.78	15.46	15015.0	355.00
	3	258.00	21.40	15135.0	409.00
Fannin (N)	1	150.95	12.47	19224.2	458.56
	2	140.78	11.63	16984.3	433.48
	3	140.10	11.57	17429.4	436.34
Fannin (N+C)	1	146.76	12.12	18842.5	452.27
	2	137.86	11.39	17434.6	434.70
	3	137.60	11.36	18018.1	439.03

(N) - NEMA optimization using MAXBAND 86

(N+C) - NEMA+Circular optimization using MAXBAND 89T

TRANSYT Options (1) Evaluation, (2) Unconstrained Optimization, (3) Bandwidth Constrained Optimization

length, offsets, and either NEMA or circular phasing sequences. We also present results of computational experience with MAXBAND 89T, which allows easy use of the enhanced formulation. In addition, the results from MAXBAND 86 and MAXBAND 89T were further analyzed using TRANSYT-7F. Although experiments were performed using a limited number of test problems, the results demonstrate that, in some cases, combined NEMA and circular phasing sequence optimization may produce wider progression bands than NEMA phasing optimization alone. In addition, arterial performance may improve as a result of the improved bandwidth solutions. The results can be summarized as follows:

1. Combined NEMA and circular phasing sequence optimization can produce wider arterial progression bands in some cases.
2. In some cases, selection of circular phasing may produce lower total vehicular delay, depending on factors such as demand on approaches.
3. As expected, the CPU time for combined NEMA and circular phasing optimization is, in general, slightly more than that for NEMA phasing optimization only.

The results demonstrate that for some cases, MAXBAND 89T can produce solutions with improved progression bands and arterial performance.

CONCLUSIONS AND RECOMMENDATIONS

These results are based on a small set of test problems. However, the results show that combined NEMA and circular phasing sequences can produce improved signal timings; therefore, optimization using the circular phasing sequence should not be excluded from the choices examined.

Further research is needed to more fully understand the advantages and disadvantages of the circular phasing optimization. Some of the questions that need to be answered are

1. What is the effect of circular phasing selection on the cross street? On pedestrian traffic?

2. What causes the total delay to be increased or decreased?
3. Under what signal conditions (i.e., demand, signal spacings) can maximum benefits be obtained from circular phasing?
4. How would drivers react when they encounter a signal with the nontraditional circular phasing sequence?
5. What type of results would be obtained from using this capability in the other recent nonuniform arterial bandwidth models (*4,5*)?

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