Proposed Enhancements to MAXBAND 86 Program

NADEEM A. CHAUDHARY, ANULARK PINNOI, AND CARROLL J. MESSER

MAXBAND 86 is the only operational traffic signal program that allows progression bandwidth optimization in multiarterial, closed-loop traffic signal networks. The program formulates the problem as a mixed integer linear program and is capable of optimizing network-wide cycle length, signal offsets, and signal phasing sequences. However, hours of computer time may be required to optimize a medium-sized network problem, even on a mainframe computer. This computational inefficiency of MAXBAND 86 makes it impractical for use by the traffic engineering community. However, two heuristic methods efficiently optimize network signal timing problems modeled by MAXBAND 86. The experimental results demonstrate that these heuristic methods produce tremendous savings in the computer time required to solve optimization problems in traffic network signal timing. In addition, computational benefits are achieved by explicitly modeling one-way arterials in a network rather than as two-way arterials, as used in MAXBAND 86.

Traffic signal synchronization for maximum progression bandwidth is widely used because progression bands can be easily visualized and understood by traffic engineers as well as drivers. The capabilities of two existing computer programs, PASSER II (*1*) and MAXBAND 86 (*2,3*), allow traffic engineers to obtain progression bandwidth solutions to signal synchronization problems. The advantage that bandwidth optimization programs have over delay-based programs, such as TRANSYT-7F (*4*) and SIGOP (*5*), is their capability to select the best signal phasing sequences from the available set of possibilities.

PASSER II uses an efficient heuristic optimization technique based on the concept of minimizing interference to progression bands (*6*). However, the drawback of this technique is its inability to handle multiarterial networks with closed loops.

MAXBAND 86, on the other hand, is based on mathematical programming and therefore is capable of optimizing signal timing in networks having several arterials and closed loops. In spite of its mathematical elegance, MAXBAND 86 has two problems. First, its traffic flow model is extremely simplistic, and second, the program is computationally inefficient. For these reasons, MAXBAND 86 has not gained acceptance in the traffic engineering community. Research by Cohen (*7*) and Liu (*8*) has demonstrated that the concurrent use of MAXBAND and TRANSYT produces better signal timings than those produced by either program alone. Thus, an efficient MAXBAND program can provide practical technology for solving existing complex urban traffic congestion problems.

Texas Transportation Institute, Texas A&M University, College Station, Tex. 77843-3135.

Almost all recent research related to the bandwidth maximizing approach has dealt with either some sort of enhancement to the arterial model used in MAXBAND 86 (*9,10*; see companion paper in this Record by Chaudhary et al.) or to the computational efficiency of the arterial problems (*11,12*). Little attention has been paid to the computational efficiency of MAXBAND 86 for optimizing multiarterial problems. We present two heuristic algorithms that efficiently solve signal synchronization problems in multiarterial networks. In addition, we demonstrate that explicitly formulating one-way arterials in a network problem produces better, quicker results than the method of treating one-way arterials as two-way arterials used in MAXBAND 86.

BACKGROUND

The mixed integer linear programming (MILP) formulation for maximizing the sum of progression bandwidths on a two-way arterial was originally formulated by Little (*13*). Little et al. (*14*) later expanded the arterial formulation for a triangular network with three arterials and a single closed loop, and developed MAXBAND, a computer program based on this formulation. MAXBAND had the capability to automatically set up and solve an MILP for a given set of traffic data. MPCODE, the optimization package used in MAXBAND, is a set of computer routines developed by Land and Powell (*15*). Messer et al. enhanced MAXBAND for multiarterial, multiple closed-loop signal network problems (*2*). The enhanced program was named MAXBAND 86. Recent experience with the application of MAXBAND 86 to networks with multiple closed loops indicates that the central processing time (CPU) on a mainframe computer may be in hours, even for medium-sized network problems (*16,17*). These researchers thought that the MPCODE optimization package was inefficient and that replacing it with a more efficient package would solve the MAXBAND inefficiency problem (*2*).

Most research dealing with the computational efficiency of the progression bandwidth maximizing approach has dealt with single arterial problems (*6,11,12*). Mireault successfully applied Benders's decomposition technique to arterial and network problems (*11,18*). This research indicates that Benders's decomposition approach for optimizing network problems with fixed cycle length and fixed travel speeds is up to 10 times faster than the branch and bound method (*18*, pp. 161, 274). However, problems become difficult to solve in a practical amount of time (3 hr) when cycle length and travel speeds are made variable (*17*, p. 275). Recently, Chaudhary et al. (*12*) developed a two-step heuristic method for solving arterial problems efficiently. Chaudhary et al. also compared the performance of MPCODE with that of LINDO (*19*), an efficient commercial optimization package. The results indicate that, at least for bandwidth optimization problems, MPCODE is as efficient as LINDO, and that it is the nature of the problem, rather than any weakness of the MPCODE optimization package, that makes it difficult to solve. Thus, the need remains to develop heuristic methods to enhance the efficiency of MAXBAND 86 for optimizing network signal synchronization problems using MPCODE.

MATHEMATICAL FORMULATION

The mathematical formulation used in MAXBAND 86 for optimizing progression bandwidth in arterial and network problems is given in several publications (*2,3,14,17*). This formulation determines cycle length, offsets, and signal phasing sequences that maximize the sum of progression bandwidths for all arterials. The green splits remain constant during the optimization process. The formulation for a multiarterial, closed-loop network problem consists of three types of integer variables, shown in Figure 1. The following statements describe these integer variables:

1. General integer variable m_i ensures that the sum of offsets around a loop formed by two one-way links connecting a pair of adjacent traffic signals is an integer multiple of the signal cycle length.

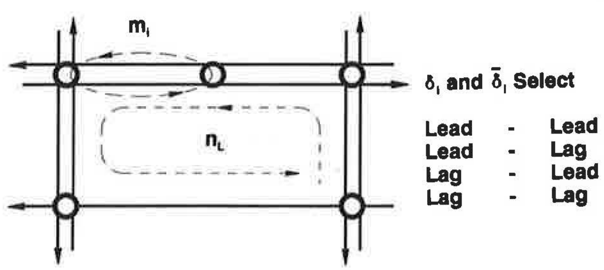

FIGURE 1 Description of integer variables.

2. General integer variable n_L ensures that the sum of offsets around a closed loop formed by three or more arterials is an integer multiple of the cycle length.
3. Binary variables δ_i and $\bar{\delta}_i$ select signal phasing sequences that produce maximum progression bandwidth.

TEST PROBLEMS

Thirteen real-world network problems were used to compare the efficiency of the basic simultaneous optimization procedure and the heuristic optimization methods. Detailed information about most of these networks is given by Cohen (*16*). Table 1 describes the test problems. A constant cycle length was used for each test problem.

The optimization runs were performed on a DecStation 3100 computer. This computer is considered about twice as fast (15,284 Khornerstones/sec) as a Compaq 386/25 80386 computer (7,417 Khornerstones/sec) (*20*). For practical reasons, an upper bound of 2 million branch and bound (BB) iterations was placed on all optimization runs.

OPTIMIZATION USING EXISTING MAXBAND 86 PROGRAM

The simultaneous optimization of test problems was run using MAXBAND 86. The purpose of this set of optimization runs was twofold: to gain insight into the nature of network optimization problems and to develop a basis for determining the effectiveness of the heuristic methods developed later.

Table 2 summarizes the results of simultaneous optimization using the existing MAXBAND 86 program. The following statements describe some of the results:

1. For five problems, the upper limit of 2 million BB iterations was not enough to complete the search. This means that the solutions obtained for these problems are not guaranteed to be the absolute best.
2. These results support previous research findings and show that large amounts of CPU time may be required to optimize

TABLE 1 DESCRIPTION OF NETWORK PROBLEMS

			NETWORK GEOMETRY				MILP SIZE					
								VARIABLES				
NO.	DATA NAME	NETWORK NAME	ARTERIALS	SIGNALS	LINKS	LOOPS	TOTAL CONSTRAINTS	CONTINUOUS	INTEGER	BINARY	TOTAL	NON ZERO ELEMENTS
1.	W311	University/Canyon/12th Street	3	11	11	1	121	57	12	21	90	372
2.	W315	Wisconsin/Massachusetts/Garfield	3	15	15	1	168	73	16	3	92	472
3.	W317	Pennsylvania/Connecticut/K Street	3	17	17	1	190	81	18	7	106	494
4.	W509	Hawthorne Blvd. mini network, California	5	9	10	2	109	61	12	16	89	346
5.	W613	Walnut Creek Network, California	6	13	15	3	164	85	18	33	136	544
6.	W712	Daytona Beach Network, Florida	7	12	17	6	188	97	23	36	156	658
7.	W813B	Post Oak Network, Houston, Texas	8	13	18	6	198	105	24	29	158	654
8.	W813C	Ogden Network, Utah	8	13	18	6	198	105	24	18	147	632
9.	W814	Ann Arbor Michigan	8	14	20	7	221	113	27	31	171	751
10.	W815	Los Angeles, California	8	15	21	7	232	117	28	33	178	777
11.	W816A	Owosso, Michigan	8	16	18	3	195	105	21	27	153	602
12.	W816B	Bay City, Michigan	8	16	20	5	219	113	25	30	168	719
13.	W817	Downtown Memphis Network, Tennessee	8	17	22	6	242	121	28	15	164	771

TABLE 2 RESULTS FROM MAXBAND 86 SIMULTANEOUS OPTIMIZATION

		BEST INTEGER SOLUTIONS					SOLUTIONS WITHIN 95% OF BEST		
NO.	DATA	SOLNS. FOUND	TOTAL BAND(SEC)	FOUND AT ITERATION	BB SEARCH STOPPED AT	CPU TIME HRS:MIN:SEC	SOLUTIONS FOUND	FIRST SOLUTION	FOUND AT ITERATION
1.	W311	17	104.32	6129	12548	1:38	7	100.96	3421
2.	W315	41	95.97	23741	28306	5:18	4	95.97	21246
3.	W317	24	84.08	42809	46212	9:45	4	84.08	42112
4.	W509	16	359.64	4365	4571	0:36	14	342.54	550
5.	W613	72	277.00	857591	955959	3:34:23	10	265.20	458325
6.	W712	47	298.32[a]	1306120	1999818	9:51:44	16	285.48	217712
7.	W813B	42	281.60	1254322	1359574	7:10:07	8	273.60	802663
8.	W813C	29	246.88	1579052	1892872	9:21:51	4	242.48	369606
9.	W814	36	337.20[a]	1287992	1999828	11:59:56	4	328.10	1248071
10.	W815	41	288.50[a]	1767302	1999803	13:49:08	17	275.80	243536
11.	W816A	96	335.60	1159536	1257239	5:47:47	13	327.60	1070847
12.	W816B	56	357.60[a]	1798311	1999809	11:17:23	11	342.60	1473867
13.	W817	62	335.70[a]	1982661	1999759	12:31:45	4	323.50	1888652

[a] Suboptimal solution due to imposed upper limit of 2,000,000 on BB iterations.

a medium-sized problem. For our test problems, CPU time varied from 36 sec to 13 hr and 50 min.

3. All test problems have multiple-integer solutions. Multiple solutions within 95 percent of the best solution were also found; some of these were close to or the same as the best solution.
4. For this set of test problems, an upper limit of 1,888,652 BB iterations would have been required to guarantee that the solutions obtained were within 95 percent of the best solution.
5. Optimal values of the loop variables (m_i) were always in the interval [0,2].

A close examination of the intermediate results from the MPCODE optimization package revealed that several sets of values for loop variables (m_i) may satisfy the loop-closure constraints. Also, for a given set of values for the loop variables (m_i), several different phasing sequence combinations produced the same value of progression bands. These properties of the network problems are proposed as the basic cause of MAXBAND 86's computational inefficiency. Two heuristic algorithms were developed to exploit the properties described. The following sections describe the heuristic algorithms and present the results of optimization using these algorithms. These experiments used lower and upper bounds of 0 and 2 on the arterial loop variables.

TWO-STEP HEURISTIC OPTIMIZATION METHOD

Let 2SMP1 be a problem obtained by relaxing the integrality constraints on the phasing sequence selection variables (δ_i and $\bar{\delta}_i$) in the network optimization problem, and let 2SMP2 be another problem obtained by setting the arterial loop variables (m_i) and network loop variables (n_L) in the network problem to a specific set of values. Then, the two-step heuristic method is as follows:

- Step 1. Optimize Problem 2SMP1 and save the set of values of variables m_i and n_L corresponding to the last six best solutions.
- Step 2. For each set of values of loop variables saved in Step 1, solve Problem 2SMP2. Select the best of these as the optimal solution.

The two-step method was used to optimize the 13 test problems described previously. MPCODE was used to optimize all subproblems. Table 3 summarizes the results of the two-step method on the test problems and compares the results to those obtained from the prior simultaneous optimization (MAXBAND 86). The following statements describe the results:

1. Progression bands obtained are almost the same (within 99.5 percent) or better than those obtained by MAXBAND 86. On the average, the bands are 4.4 percent better than those obtained by MAXBAND 86. This result occurred because MAXBAND 86 did not finish several problems because of the upper limit of 2 million BB iterations.
2. The number of BB iterations required varied from 4.11 to 42.86 percent of that required by MAXBAND 86. In other words, a savings of up to 95.89 percent in BB iterations was achieved. The savings in BB iterations for the nine largest problems was at least 80.12 percent. The average savings in BB iterations was 81.15 percent.
3. The maximum CPU time was about 2.5 hr, compared to the maximum CPU time of 13.82 hr for MAXBAND 86 optimization.
4. Step 2 (phasing sequence selection process) of the heuristic algorithm required the least amount of computational effort.

In summary, the two-step heuristic algorithm is far superior computationally to the simultaneous optimization method. The results are better than those obtained by the simultaneous

TABLE 3 RESULTS FROM TWO-STEP HEURISTIC OPTIMIZATION

NO.	DATA NAME	BEST SOLUTION BAND(SEC)	% OF MAX 86	BRANCH AND BOUND ITERATIONS (STEP 1 +	STEP 2 =	TOTAL)	% OF MAX 86	CPU TIME HRS:MIN:SEC
1.	W311	104.32	100.00	4301	1077	5378	42.86	0:00:42
2.	W315	95.97	100.00	10984	749	11733	41.45	0:02:07
3.	W317	84.08	100.00	15753	780	16533	35.78	0:03:11
4.	W509	359.64	100.00	1009	342	1351	29.56	0:00:10
5.	W613	275.30	99.39	44331	3722	48053	5.03	0:11:00
6.	W712	349.20	117.01	209043	1200	210243	10.51	1:02:26
7.	W813B	281.60	100.00	124366	5238	129604	9.53	0:39:41
8.	W813C	246.88	100.00	374836	1464	376300	19.88	1:51:50
9.	W814	370.60	109.91	141119	2112	143231	7.16	0:49:55
10.	W815	360.90	125.10	382868	2508	385376	19.27	2:29:09
11.	W816A	335.60	100.00	50552	1064	51616	4.11	0:14:24
12.	W816B	373.20	104.36	153819	2766	156585	7.83	0:53:15
13.	W817	340.80	101.52	239965	1612	241577	12.08	1:31:32
MEAN			104.41				18.85	

optimization method completed in a feasible amount of time. However, the CPU time required may still be more than that desired for practical applications. The following section presents a three-step heuristic method designed to be more efficient than the two-step algorithm.

THREE-STEP HEURISTIC OPTIMIZATION METHOD

Let 3SMP1 be the original network problem obtained by relaxing the integrality constraints on the network loop variables (n_L) and phasing sequence selection variables (δ_i and $\bar{\delta}_i$). Let 3SMP2 be another network problem obtained by fixing arterial loop variables (m_i) to a specific set of values and relaxing the integrality constraints on the phasing sequence selection variables. Finally, let 3SMP3 be the original network problem with loop variables (m_i, and n_L) fixed at specific values. Then, the three-step heuristic method is as follows:

- Step 1. Optimize Problem 3SMP1 and save the set of values of the variables m_i corresponding to the last six best solutions.
- Step 2. For each set of values of arterial loop variables saved in Step 1, solve Problem 2SMP2. Save the values of network loop variables corresponding to the six best solutions.
- Step 3. For each of the six sets of values of loop variables (arterial and network) saved at the second step, optimize 3SMP3. Select the best of these as the optimal solution.

MPCODE was used to test the effectiveness of the three-step method on the same set of 13 problems. Table 4 summarizes the results of these optimization runs and compares the results with those from the simultaneous optimization. The following statements describe the results:

1. The sums of progression bands obtained were, on the average, as good as those obtained by MAXBAND 86. Except for one problem, the solutions were within 90 percent of those obtained by MAXBAND 86. For one problem, the total band was 25 percent more; for this problem, MAXBAND 86 did not finish because of the upper limit on BB iterations.
2. The average number of BB iterations required to optimize the test problems was less than 9 percent of that required by the simultaneous optimization. The savings in BB iterations was from 65.6 percent to 99.41 percent. For the nine largest problems, the savings in BB iterations was at least 98.13 percent.
3. The CPU time required to solve these problems varied from 10.8 sec to 13.1 min.
4. The third step (phasing sequence selection process) of the heuristic method required the least amount of computational effort.
5. For seven problems, total bands obtained were the same as those produced by the two-step method. For five problems, the bands were within 90.8 percent of those from the two-step method.
6. Except for one small problem (118 more BB iterations), the three-step method optimized the problems much more efficiently than the two-step method. Compared to the two-step method, the reduction in BB iterations ranged from 37 to 91 percent. Reduction in BB iterations for the nine largest problems was at least 72 percent.

In summary, the three-step method is much more efficient than the two-step heuristic method. The three-step method solved in less than 13.1 min problems for which the two-step method required up to 2.5 hr of CPU time. The total progression bandwidths produced by the three-step method were generally less than those produced by the two-step method. The reduction in bandwidth, as compared to the two-step

TABLE 4 RESULTS FROM THREE-STEP HEURISTIC OPTIMIZATION

NO.	DATA NAME	BEST SOLUTION BAND(SEC)	BEST SOLUTION % OF MAX 86	BRANCH AND BOUND ITERATIONS (STEP 1 +	STEP 2 +	STEP 3 =	TOTAL)	% OF MAX 86	CPU TIME MIN:SEC
1.	W311	104.32	100.00	1454	808	1127	3389	27.01	0:25
2.	W315	95.97	100.00	4155	1024	751	5930	20.95	0:56
3.	W317	84.08	100.00	6856	832	574	8262	17.88	1:34
4.	W509	359.64	100.00	337	575	557	1469	32.14	0:11
5.	W613	272.90	98.52	5987	2798	4498	13283	1.39	2:36
6.	W712	259.20	86.89	21629	11194	1548	34371	1.72	8:37
7.	W813B	274.00	97.30	6940	13662	3808	24410	1.80	6:24
8.	W813C	229.76	93.07	24084	8142	1279	33505	1.77	8:22
9.	W814	336.60	99.82	2357	26932	1725	31014	1.55	9:34
10.	W815	360.90	125.10	13847	21142	2375	37364	1.87	13:06
11.	W816A	335.60	100.00	3816	2357	1241	7414	0.59	1:39
12.	W816B	357.60	100.00	11217	7926	1700	20843	1.04	5:44
13.	W817	340.80	101.52	17141	13010	1492	31643	1.58	9:34
MEAN			100.17					8.56	

method, may not be significant in view of the computational savings achieved by the three-step method. The total bands produced by the three-step method were, on the average, close to those obtained from MAXBAND 86.

EXPLICIT MODELING OF ONE-WAY ARTERIALS IN NETWORK PROBLEMS

In multiarterial network problems, MAXBAND 86 deals with one-way arterials as two-way arterials. This flaw results in the addition of unnecessary variables and constraints and affects CPU time as well as the quality of the solution. This flaw can be removed by using the linear programming formulation of a one-way arterial instead of the MILP formulation used in MAXBAND 86. For an arterial with n intersections and k left-turn movements, this formulation has $(3n + k - 1)$ to $(2n + 1)$ fewer variables, and a $(6n - 6)$ to $(5n - 4)$ reduction in the number of constraints as compared to the original formulation. This reduction, especially the elimination of $(n - 1)$ general integer variables, is quite significant.

This section shows the results of experimentation using the correct network formulation. The effectiveness of this formulation was tested for Problems W813C and W814. Problem W813C has two one-way arterials, and Problem W814 has one. Table 5 compares the original and corrected problem formulations in terms of the number of variables and constraints that each problem contains. We optimized the corrected formulations for these two problems using simultaneous and heuristic optimization methods. Table 6 compares these results with those given previously. The following statements summarize the results:

1. The size of the new problem formulation is reduced considerably.
2. Simultaneous optimization of the new formulation for Problem W813C resulted in a reduction of almost 72 percent in BB iterations. The quality of the best solution also improved slightly. Simultaneous optimization of Problem W814 produced a better solution; however, like the original problem, the BB search failed to terminate within the specified limit of 2 million BB iterations.

TABLE 5 COMPARISON OF ORIGINAL AND CORRECTED FORMULATIONS

DATA NAME AND DESCRIPTION	VARIABLES CONTINUOUS	INTEGER	BINARY	TOTAL	TOTAL CONSTRAINTS	NON ZERO ELEMENTS
W813C (MAXBAND 86)	105	24	18	147	198	632
W813C (Corrected)	91	19	18	128	168	543
Reduction	14	5	0	19	30	89
W814 (MAXBAND 86)	113	27	31	171	221	751
W814 (Corrected)	103	23	31	157	197	678
Reduction	10	4	0	14	24	73

TABLE 6 COMPARISON OF OPTIMIZATION RESULTS

DATA NAME AND DESCRIPTION	MAX 86 OPTIMIZATION BAND(SEC)	MAX 86 OPTIMIZATION ITERATIONS	TWO STEP HEURISTIC BAND(SEC)	TWO STEP HEURISTIC ITERATIONS	THREE STEP HEURISTIC BAND(SEC)	THREE STEP HEURISTIC ITERATIONS
W813C (MAXBAND 86)	246.88	1,892,872	246.88	376,300	229.76	33,505
W813C (Corrected)	253.28	408,408	253.28	114,610	242.24	15,945
Difference	6.40	-1,484,464	6.40	-261,690	13.48	-17,560
	2.6%	-78.4%	2.6%	-69.5%	5.9%	-52.4%
W814 (MAXBAND 86)	337.20	1,999,828	370.60	143,231	336.60	31,014
W814 (Corrected)	357.20	1,999,859	392.30	71,726	382.90	11,475
Difference	20.00*	*	21.70	-71,505	46.30	-19,539
	5.9%*	*	5.9%	-49.9%	13.8%	-63.0%

* Upper limit of 2 million BB iterations reached. These are sub-optimal solutions.

3. Compared to the two-step optimization of the original formulations for the two problems, the new formulation required 69.54 and 49.92 percent less effort, respectively. In addition, the quality of solutions (total bandwidth) was better.
4. The three-step method also produced quicker results on the new problems. The savings in BB iterations for the test problems were 52.41 and 63 percent, respectively, as compared to optimization of the original formulation.

In summary, the correct formulation is easier to solve using all three optimization methods. In addition, a larger total progression bandwidth is obtained, compared to the original formulation used in MAXBAND 86. This means that the new formulation can optimize signal timings in larger networks than those that could be optimized with the existing MAXBAND 86, especially downtown grid networks in which most of the arterials are one-way. Further, arterial networks can now include one-way frontage roads. This may make it easy to combine freeway ramp metering optimization with optimization of signal timing on the surface street system.

SUMMARY AND RECOMMENDATIONS

We present two ways to increase the computational efficiency of the MAXBAND 86 program. First, we demonstrate that traffic signal network problems can be efficiently optimized using the proposed heuristic methods, without sacrificing the quality of solutions. Second, we demonstrate that the explicit formulation of one-way arterials reduces the network problem size, and produces better progression bands as compared to the network formulation used in MAXBAND 86.

We recommend that the heuristic optimization methods be incorporated in the MAXBAND 86 program for three reasons. First, implementation of these methods is straightforward; second, these methods are robust, that is, any enhancement or modification in the problem formulation will not affect these procedures; and third, they provide more increase in the problem-solving computational efficiency than any other available method. It is recommended that the two-step and three-step optimization methods be added to the simultaneous optimization capability of MAXBAND 86. The choice of the optimization method to be used can then be based on the problem size. It is also recommended that the MAXBAND 86 formulation for one-way arterials be corrected. This will allow signal timing optimization in larger networks than is currently achievable. In addition, wider progression bands can be obtained.

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REFERENCES

1. E. C. Chang, B. G. Marsden, and B. R. Derr. The PASSER II-84 Microcomputer Environment System—A Practical Signal Timing Tool. *Journal of Transportation Engineering*, Vol. 113, Nov. 1987, pp. 625–641.
2. C. J. Messer, G. L. Hogg, N. A. Chaudhary, and E. C. P. Chang. *Optimization of Left Turn Phase Sequence in Signalized Networks Using MAXBAND 86, Volume 1: Summary Report.* Report FHWA/RD-84/082. FHWA, U.S. Department of Transportation, Jan. 1986.
3. E. C. Chang, S. L. Cohen, C. Liu, C. J. Messer, and N. A. Chaudhary. MAXBAND-86: Program for Optimizing Left-Turn Phase Sequence in Multiarterial Closed Networks. In *Transportation Research Record 1181*, TRB, National Research Council, Washington, D.C., 1989, pp. 61–67.
4. C. E. Wallace, K. G. Courage, D. P. Reaves, G. W. Schoene, G. W. Euler, and A. Wilbur. *TRANSYT-7F User's Manual.* FHWA, U.S. Department of Transportation, 1988.
5. E. B. Lieberman and J. L. Woo. SIGOP II: A New Computer Program for Calculating Optimal Signal Patterns. In *Transportation Research Record 596*, TRB, National Research Council, Washington, D.C., 1976, pp. 16–21.

6. C. J. Messer, R. H. Whitson, C. L. Dudek, and E. J. Romano. A Variable-Sequence Multiphase Progression Optimization Program. In *Highway Research Record 445*, HRB, National Research Council, Washington, D.C., 1973, pp. 24–33.
7. S. L. Cohen. Concurrent Use of MAXBAND and TRANSYT Signal Timing Programs for Arterial Signal Optimization. In *Transportation Research Record 906*, TRB, National Research Council, Washington, D.C., 1983, pp. 81–84.
8. C. C. Liu. Bandwidth-Constrained Delay Optimization for Signal Systems. *ITE Journal*, Vol. 58, No. 12, Dec. 1988, pp. 21–26.
9. N. H. Gartner, S. F. Assmann, F. Lasaga, and D. L. Hou. MULTIBAND—A Variable-Bandwidth Arterial Progression Scheme. In *Transportation Research Record 1287*, TRB, National Research Council, Washington, D.C., 1990.
10. H. S. Tsay and L. J. Lin. A New Algorithm for Solving the Maximum Progression Bandwidth. Presented at the 67th Annual Meeting of the Transportation Research Board, Washington D.C., Jan. 1988.
11. P. Mireault. Solving the Single Artery Traffic Signal Synchronization with Benders Decomposition. Presented at CORS/ORSA/TIMS Joint National Meeting, Vancouver, Canada, May 8–10, 1990.
12. N. A. Chaudhary, C. J. Messer, and A. Pinnoi. Efficiency of Mixed Integer Linear Programs for Traffic Signal Synchronization Problems. *Proc., 25th Annual SE TIMS Meeting*, Oct. 1989, pp. 155–157.
13. J. D. C. Little. The Synchronization of Traffic Signals by Mixed-Integer Linear Programming. *Operations Research*, Vol. 14, 1966, pp. 568–594.
14. J. D. C. Little, M. D. Kelson, and N. H. Gartner. MAXBAND: A Program for Setting Signals on Arteries and Triangular Networks. In *Transportation Research Record 795*, TRB, National Research Council, Washington, D.C., 1981, pp. 40–46.
15. A. Land and S. Powell. *Fortran Codes for Mathematical Programming*. John Wiley & Sons, Ltd., London, England, 1973.
16. S. L. Cohen. *Optimization of Left Turn Phase Sequence in Signalized Closed Networks*. Final Report. FHWA, U.S. Department of Transportation, 1988.
17. N. A. Chaudhary. *A Mixed Integer Linear Programming Approach For Obtaining an Optimal Signal Timing Plan in General Traffic Networks*. Ph.D. dissertation. Texas A&M University, College Station, Aug. 1987.
18. P. Mireault. *An Integer Programming Approach to The Traffic Signal Synchronization Problem*. Ph.D. dissertation. Massachusetts Institute of Technology, Cambridge, Feb. 1988.
19. L. Schrage. *User's Manual for LINDO*, 3rd ed. The Scientific Press, Redwood City, Calif., 1987.
20. P. Magney. DECstation 3100: A Leader. *Computer Reseller News*, Sept. 4, 1989, pp. 57–58.

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