Route-Specific and Time-of-Day Demand Elasticities

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In assessing user response to cost and service changes, demand elasticities are useful tools. Current compilations of demand elasticities, however, are not helpful to scheduled transportation operators. The reason is that the range of an elasticity is too wide, and there is no practical guideline for picking an appropriate value within the range. Furthermore, they are often compiled for systemwide and average-day operations, whereas most analyses need to be performed on a route-by-route basis during peak or off-peak periods. A methodology to address this problem is presented with the objective of providing demand elasticities that are practical for patronage analyses in an operating agency. Through statistical analyses of spatial and temporal data aggregation, the methodology explains the differences among elasticity tabulations, and in so doing, provides insights into the variations among elasticity values. The results of this research include (*a*) guidelines for selecting an appropriate value of elasticity among the broad range of values (instead of simply taking the average or midpoint of the range) and (*b*) a method for converting the most commonly available elasticities (which are usually in aggregate form) to a more useful form, such as route-specific and time-of-day elasticities. The results have been demonstrated in a case study of the transit system in York, Pennsylvania.

A common concern expressed by scheduled-transportation operators is how a system can become more attuned to user preferences by innovation in service changes (*1*). Clearly, addressing such concerns requires knowledge of travelers' (shippers') response to changes in such items as user charges, schedules, and route coverage. For example, would a user still patronize the system if the fare is raised or the frequency of service is cut back (*2*)? Users' decisions clearly affect the revenue or well-being of the operator, which in turn leads to either success or failure of the scheduled transportation system.

Increasingly, systems are scrutinized on a route-by-route basis (*3*). Schedules and user charges are becoming more and more differentiated between peak and off-peak periods because user responses are widely different among routes and time of day. For example, peak-hour travel is typically inelastic inasmuch as it is made up largely of work trips on routes from suburb to center city. This contrasts with off-peak travel, which typically consists of discretionary trips. Unless distinctions are made between routes and time of day, fare and schedule changes cannot be planned judiciously to cater to the user's preferences (*4*).

A common way to address this issue is to examine demand elasticities, which by definition measure patronage responses to changes in attributes such as fares and service. Although several tabulations of urban demand elasticities exist (*5*), the use of these tabulations has been limited because of the tremendous variations in reported elasticity values—even for the same city or scenario (*6*). For example, within a given commuter rail authority, the time elasticity can vary from -0.31 to -0.87—a threefold difference. This is mainly due to the different levels of aggregation used in the derivation of these elasticities. Whereas there is tremendous need, operators rarely have usable guidelines available for selecting an appropriate elasticity for day-to-day operations—from route-level to peak versus off-peak analyses. There is no reason to believe that the midpoint (between -0.31 and -0.87) is the number to use (*5*) or that a uniform scale factor can be applied indiscriminately (between -0.31 and -0.87) to arrive at the appropriate value for a specific route or time of day.

Thus there is a knowledge gap to be filled in establishing probable variations in elasticities among routes and peak versus off-peak hours. The provision of these guidelines would allow operators to fine-tune elasticities by route and time of day on the basis of associated activity-system and level-of-service characteristics. Thus user responses to service changes can be accurately estimated for operational planning purposes.

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LITERATURE REVIEW

Among the various approaches to estimating demand on a route-specific level, the elasticity method performs extremely well (*7,8*). In fact it can be argued that as a general method, it is most satisfying. Recent work on chain pivot-point analysis is merely a variation of the elasticity method (*9*). The same can be said for areawide-equilibration approaches (*10*).

Various fare-policy evaluation techniques have been built around demand elasticities. Ballou and Mohan (*11*) gave an example of the use of elasticities to analyze differentiated fare change on selected routes. Their analyses indicate the diverse effects of fare changes among routes and according to time of day, trip purpose, trip duration, and schedule frequency. Because demand elasticities are the basis of such fare-policy evaluations, the need for a reliable set of elasticities becomes obvious. To attain this accuracy, we need elasticities disaggregated by route, time of day, service level, and the socioeconomic composition of the potential riders (*12,13*). This in turn requires knowledge about how elasticities are derived, which is the only way to arrive at such a disaggregation procedure.

Calibrated elasticities are of two types, depending on the methodology and data used for their computation. Disaggregate elasticities are estimated using detailed information (such as household data), whereas aggregate elasticities are based on coarser data (such as zonal averages). Aggregate elasticities implicitly assume that travel behavior is homogeneous within a geographic subdivision, such as a zone or a route. Use of aggregate elasticities therefore ignores the high levels of variation in travel behavior (particularly modal choice) that may exist among users of a route. The use of such aggregate data fails to extract much of the variational information from the constituent observations used to obtain zonal aggregates. In contrast, disaggregate elasticities, which use households as the unit of analysis, are able to tap an extremely rich source of variation.

In other words, one potential problem of aggregate-level analysis is the risk of ecological fallacy, in which aggregate-level correlations are mistakenly attributed to individuals. Another problem is the loss of variability in the data used for estimation. Because a model's coefficients are determined by explaining variations in observed travel behavior, the less the variation to be explained, the less reliable the model will be. The reduced variability in aggregate data also results in a high level of collinearity between variables at the aggregate level, which does not exist at the disaggregate level. These observation errors tend to impart downward bias to estimated coefficients. For example, the results of a study *(14)* show that by using disaggregate demand models, disaggregate elasticities—for a given level of service—are usually lower than the corresponding aggregate elasticities in algebraic value. Thus, a route-specific fare elasticity may be -0.4, whereas a systemwide figure may be -0.3.

THEORY

To be able to use the tabulated elasticities meaningfully, the relationship between aggregate elasticities and disaggregate elasticities needs to be reviewed as a first step. The most disaggregate level of demand elasticities is at the household level. These elasticities are usually derived from logit models of the following form *(15)*:

$$p(m, t) = \frac{\exp(R_m X_{mt})}{\sum_{j=1}^{M} \exp(R_j X_{jt})} \qquad m = 1, 2, \ldots, M \tag{1}$$

where

$p(m, t)$ = probability of Mode m being taken by Household t ($t = 1, 2, \ldots, T$),
X_{mt} = vector of level-of-service and activity-system characteristics of Mode m for Household t, and
R_m = estimated vector of coefficients for the level-of-service and activity-system variables of Mode m.

From this model form, elasticities can be derived as

$$\eta(m, x_m, t) = [1 - p(m, t)]r_x x_{mt} \tag{2}$$

where

$\eta(m, x_m, t)$ = elasticity for Mode m with respect to attribute x_m for Household t given a destination and given that a trip will be made;
r_x = estimated coefficient for level-of-service variable x_m; and
x_{mt} = level-of-service attribute of Mode m, by Household t for a given destination.

There are mathematical relationships between disaggregate and aggregate elasticities. Recall that the elasticity for Household t, $\eta(m, x_m, t)$, is given in Equation 2. The demand elasticity aggregated over T households in a geographic subdivision such as a route is correspondingly

$$\eta(m, x_m) = \frac{\sum_{t=1}^{T} \eta(m, x_m, t)}{\sum_{t=1}^{T} p(m, t)} \tag{3}$$

Equation 3 can also be derived in a different way *(16)*. Define $\overline{p}_m$ as the average market share of Mode m for a subdivision with a population of T households:

$$\overline{p}_m = \frac{1}{T} \sum_{t=1}^{T} p(m, t) \tag{4}$$

We proceed to differentiate $\overline{p}_m$ with respect to x_m:

$$\begin{aligned} \frac{\partial \overline{p}_m}{\partial x_m} &= \frac{\partial}{\partial x_m} \left[\sum_{t=1}^{T} p(m, t)/T \right] \\ &= \frac{1}{T} \sum_{t=1}^{T} p(m, t)[1 - p(m, t)]r_x \end{aligned} \tag{5}$$

The aggregate elasticity then becomes

$$\begin{aligned} \eta(m, x_m) &= \frac{\partial \overline{p}_m}{\partial x_m} \cdot \frac{x_m}{\overline{p}_m} \\ &= \left\{ \frac{1}{T} \sum_{t=1}^{T} p(m, t)[1 - p(m, t)] \right\} r_x \frac{x_m}{\overline{p}_m} \end{aligned} \tag{6}$$

By substituting $\overline{p}_m(1 - \overline{p}_m)$ for the terms in brackets { }, the following is obtained:

$$\begin{aligned} \overline{\eta}(m, x_m) &= \frac{\overline{p}_m(1 - \overline{p}_m)}{\overline{p}_m} r_x x_m \\ &= (1 - \overline{p}_m) r_x x_m \end{aligned} \tag{7}$$

This is analogous to the disaggregate elasticity Equation 2.

The substitution above, however, assumes that

$$\overline{p}_m^2 = \frac{1}{T} \sum_{t=1}^{T} p(m, t)^2 \tag{8}$$

This assumption, when interpreted together with the definition of Equation 4, implies that individual mode-choice behavior is homogeneous within the subdivision. Because the individual travel decisions are not likely to be homogeneous in actuality, Equation 7 can only be an approximation of Equation 6.

Systemwide transit elasticities—being another level of aggregation—can also be derived from disaggregate elasticities calibrated for households *(16)*. Let us write T as N_k to specifically denote the number of households on the kth route and to show that there is more than one route systemwide. Now let $\bar{p}(m, k)$ be the average probability of choosing Mode m in the kth route (where $k = 1, 2, 3, \ldots, K$). By definition,

$$N = \sum_{k=1}^{K} N_k \tag{9}$$

For the study area, using the new notation, and with the substitution

$$N_k\bar{p}(m, k)[1 - \bar{p}(m, k)] \leftarrow$$

$$\sum_{t=1}^{N_k} p(m, k, t)[1 - p(m, k, t)] \tag{10}$$

systemwide elasticity can be approximated as

$$\begin{aligned}\eta(m, x_m, k) &= \frac{\partial\bar{p}(m, k)}{\partial x_{mk}} \cdot \frac{x_{mk}}{\bar{p}(m, k)} \\ &= \left\{\sum_{k=1}^{K} N_k\bar{p}(m, k)[1 - \bar{p}(m, k)]r_x\right\}\frac{x_{mk}}{\sum_{k=1}^{K} p(m, k)}\end{aligned} \tag{11}$$

This relationship is obtained on the basis of the assumption of homogeneous household behavior within a route. The variability in the data is among routes (rather than households) in this case.

IMPLICATIONS

Now that the relationship between household, route, and systemwide elasticities has been derived, their applications in the field can be discussed. Specifically, the numerical values of these elasticities when they are applied in scheduled transportation analysis can be compared. In a study consisting of K routes, for example, the difference between the approximation of Equations 7 and 11 and the theoretical elasticity (Equations 6 and its route equivalent shown by substitution Equation 10) can be computed. The resulting difference between an aggregate and disaggregate elasticity on Route k is

$$\begin{aligned}\eta(m,x_m,k) - \eta(m,x_m,t) &= \frac{r_x x_{mk}}{N\bar{p}_m}\left\{N_k\bar{p}(m,k)[1 - \bar{p}(m,k)]\right. \\ &\qquad \left. - \sum_{t=1}^{N_k} p(m,k,t)[1 - p(m,k,t)]\right\} \\ &= \frac{r_x x_{mk}}{N\bar{p}_m}\left[\sum_{t=1}^{N_k} p(m,k,t)^2 - N_k\bar{p}(m,k)^2\right] \\ &= \frac{r_x x_{mk}}{N\bar{p}_m} N_k\sigma_k^2\end{aligned} \tag{12}$$

where σ_k^2 is the variance of $p(m, k, t)$ in Route k. The difference between an aggregate elasticity and a disaggregate elasticity for the kth route is therefore

$$\frac{r_x x_{mk}}{N\bar{p}_m} N_k\sigma_k^2 \tag{13}$$

If $\hat{\sigma}^2$ is defined as the maximum σ_k over Route $k = 1, 2, 3, \ldots, K$ (hence $\sigma_k^2 \leq \hat{\sigma}^2$), the inaccuracy of an elasticity estimated for a route is bounded by the inequality

$$\sum_{k=1}^{K} \frac{N_k}{N\bar{p}_m} \sigma_k^2 r_x x_{mk} \leq \frac{\hat{\sigma}^2}{\bar{p}_m} r_x x_{mk} \tag{14}$$

Because σ^2 is the squared deviation of $p(m, t)$'s from $\bar{p}_m$, in general, and both $p(m, t)$ and $\bar{p}_m$ are between 0 and 1, it appears that σ should be very small. However, in a case study using a 1955 data sample from Chicago, Warner (*16*) found that σ^2 was about 0.04 (or $\sigma = 0.2$).

What can be said about the estimation error associated with an elasticity? Let us compare the estimated value using Equation 7 and the following value, which has been corrected by such error terms as Equations 13 and 14:

$$\bar{\eta}(m, x_m) = \left[(1 - \bar{p}_m) - \frac{\sigma^2}{\bar{p}_m}\right] r_x x_m \tag{15}$$

By using values of $\bar{p}_m = .824$ and $\sigma^2 = 0.04$ from Warner's experiment in Chicago, the following is obtained from Equation 7:

$$\bar{\eta}(m, x_m) = 0.176 r_x x_m \tag{16}$$

and

$$\bar{\eta}(m, x_m) = 0.127 r_x x_m \tag{17}$$

according to Equation 15. The ratio between the two values is

$$\frac{\text{direct estimation}}{\text{corrected estimation}} = \frac{0.176 r_x x_m}{0.127 r_x x_m} = 1.386 \tag{18}$$

This indicates that a route elasticity, for example, could be overestimated by as much as 40 percent by an aggregate figure (which assumes homogeneous travel behavior among all users in the route).

Similarly, any individual route elasticity can be overestimated by the ratio

$$\frac{1 - \bar{p}_m - \sigma_k^2/\bar{p}_m}{1 - \bar{p}_m - \hat{\sigma}^2/\bar{p}_m} \geq 1 \tag{19}$$

even after corrections of the estimated values are made according to Equation 15.

SITE TESTING

The theory described in the earlier section was applied to a transit line in a medium-sized city: York, Pennsylvania. The bus line, called the W. Market/E. Market line, runs between York's central city and two suburbs—one to the west and the other to the east. The line covers these zones en route: 15 →

$14 \rightarrow 5 \rightarrow 1 \rightarrow 2 \rightarrow 10 \rightarrow 32$, with 15 denoting the western suburb and 32 the eastern suburb.

We can obtain the travel times for our model from the published time schedule of the transit line. The waiting time (usually approximated by half of the headway) has to be added to the line-haul times to obtain the user's actual travel time. The activity-system and level-of-service data were collected for calibrating a logit model. The model specification includes the following explanatory variables for the zones the transit line goes through:

- Interzonal travel times by automobile at posted speed;
- Transit travel times, including wait time and walk time (at 5 ft/sec from zone centroid to bus station);
- Automobile travel costs expressed in perceived costs (at 5 cents per mile); and
- Transit travel costs at a uniform fare of 65 cents per trip.

The data are summarized in Table 1.

A logit model was calibrated by Chan (*17*) for the city of York, yielding a disutility function (or impedance function) of

$$R_m X_{mt} = -290.2c_{mt} - 70.5\tau_{mt} + 1.51 \quad (20)$$

Only level-of-service variables such as cost (c) and time (τ) are explicitly modeled; activity-system variables were all lumped under the calibration constant of 1.51, and there are only two modes (m = automobile or transit).

From logit Equations 1, a table can be generated consisting of the probability of a household t taking bus transit B, $p(B, t)$, among the seven zones ($i = 1, 2, \ldots, 7; j = 1, 2, \ldots, 7$) covered in the transit line. From the 42 (or 7×6) nonzero values in the table, we can obtain the average market share of the bus mode for Route k, $\bar{p}_B$, as

$$\frac{1}{42}\sum_{t=1}^{42} p(B, t) = 0.0064 \quad (21)$$

and

$$\sigma_k^2 = 0.0007 \quad (22)$$

Using Equation 7 for Route k, we have

$$\bar{\eta}(m, x_m) = (1 - .0064)r_x x_m = 0.994 r_x x_m \quad (23)$$

Similarly, through Equation 15, we have

$$\tilde{\eta}(m, x_m) = \left[(1 - 0.0064) - \frac{0.0007}{0.0064}\right] r_x x_m$$

$$= 0.8842 r_x x_m \quad (24)$$

The ratio of the two values is

$$\frac{\bar{\eta}}{\tilde{\eta}} = 1.124 \quad (25)$$

which means that the route elasticities can be overestimated by as much as 12 percent. Notice that we have not ascertained the other routes, and hence $\hat{\sigma}$, the maximum over σ_k's, is not known. For this reason, it is possible to be off by significantly more than 12 percent in certain routes.

Even though this is a site testing of limited scope, we have obtained a few useful observations. It is clear that care should be exercised in the use of elasticities in route-specific transit analysis in view of the Chicago and York experiences. If any aggregate elasticity is used, adjustments should be made using Equation 15.

Whereas the preceding study illustrates elasticity adjustment for route-specific applications, the same procedure can be applied to time-of-day variations. In the latter application, the subscript k in Equations 12 through 14 simply takes on the meaning "peak period," "off-peak period," and so forth. The rest of the procedure follows in a manner similar to the disaggregation by route, keeping in mind that we are now performing temporal instead of spatial aggregation.

TABLE 1 INTERZONAL TRAVEL TIME AND COST

	To Zone j						
From zone i	1	2	5	10	14	15	32
1	0	2.7/68 (4.1)	3.8/28 (6.0)	6.1/22 (10)	5.6/24 (10.7)	6.1/24 (10.1)	8.3/35 (14.9)
2	27/29 (4.1)	0	5.2/35 (9.2)	4.6/44 (7.5)	7.1/31 (13.8)	7.6/31 (13.3)	6.8/57 (12.3)
5	3.8/31 (6.0)	5.2/99 (9.2)	0	8.6/53 (15.2)	3.7/4 (7.0)	6.2/8 (10.1)	10.8/66 (19.9)
10	6.1/29 (10.1)	4.6/22 (7.5)	8.6/42 (15.2)	0	10.5/38 (19.8)	11.0/38 (19.3)	3.2/28 (5.8)
14	5.6/27 (10.7)	7.1/95 (13.8)	3.7/4 (7.0)	10.5/49 (19.8)	0	4.8/8 (6.9)	13.7/62 (24.5)
15	6.1/27 (10.1)	7.6/95 (13.3)	6.2/12 (10.1)	11.0/49 (19.3)	4.8/8 (6.9)	0	14.2/62 (25.4)
32	8.3/55 (14.9)	6.8/48 (12.3)	10.8/68 (19.9)	3.2/41 (5.8)	13.7/64 (24.5)	14.2/64 (25.4)	0

KEY (for each entry): auto-time-in-min/transit-time-in-min
(auto-cost-in-cents)

A FORMULA FOR PRACTITIONERS

The foregoing represents a formal development of the elasticity adjustment procedure. The development is necessarily predicated on the availability of a disaggregate logit model for the study area. In real-world applications, however, a calibrated logit model is seldom available. The only available information is often a set of demand elasticities—if one is fortunate enough to have them. Frequently, elasticity figures have to be borrowed from similar cities in the form of a range of numbers that are disparate in value (*18*). Care should be exercised in the definition of a "similar" city, which should include both size and urban structure to guarantee transferability (*5,10*). The problem now is to choose an approximate value within this range for the application at hand.

Suppose maximum and minimum algebraic values are both available, representing the two extremes of the range. It is possible to work backwards to get an appropriate value through the use of an adjustment factor. From Equations 7 and 15, it can be assumed that

$$\frac{\eta_{\min}(m, x_m)}{\eta_{\max}(m, x_m)} \approx \frac{(1 - \bar{p}_m)}{\left[(1 - \bar{p}_m) - \dfrac{\hat{\sigma}^2}{\bar{p}_m}\right]} \tag{26}$$

inasmuch as the two elasticity estimates come from the two extreme levels of aggregation with the $\eta_{\min}$ having the most aggregation bias and $\eta_{\max}$ the least. Assume further that the average modal split $\bar{p}_m$ can be approximated by empirical data obtainable locally (meaning the logit model replicates observed data reasonably well). The term σ^2, being the only unknown in Equation 26, can now be estimated.

If more than two elasticities are available, we can have even better information on the variability of elasticities among routes or time of day. Let us say we have a third elasticity for the study area. It is clear that a third level of aggregation was used in model calibration, different from the previous two. Again, we take the ratio according to Equation 19:

$$\frac{\eta_k(m, x_m)}{\eta_{\max}(m, x_m)} \approx \frac{\left[(1 - \bar{p}_m) - \dfrac{\sigma_k^2}{\bar{p}_m}\right]}{\left[(1 - \bar{p}_m) - \dfrac{\hat{\sigma}^2}{\bar{p}_m}\right]} \tag{27}$$

which allows σ_k, the only unknown in the equation, to be estimated.

In general, the availability of more than one calibrated elasticity, rather than being confusing, is now an asset. The more elasticity tabulations available, the more we can reconstruct the elasticity variability among routes and time of day. Suppose these σ's are obtained:

$$\sigma_{\min} < \sigma_1 < \cdot \cdot \cdot < \alpha_{K-2} < \hat{\sigma} \tag{28}$$

We can now match each route k (or time of day k) against one of the σ's. A rule of thumb may be that the shorter the route (or the shorter the time period) the less the σ value. A shorter route has less variability in such explanatory variables as travel time, travel cost, and car ownership, and hence less variability in mode choice:

$$\frac{\sigma_k}{\hat{\sigma}} = \frac{n_k - n_{\min}}{n_{\max} - n_{\min}} = s \tag{29}$$

where n_k is the number of stops on the route under consideration, and $n_{\min}$ and $n_{\max}$ are the number of stops on the shortest and longest routes. Such a linear scale, s, will place the mode-choice variability of route k at

$$\sigma_k = \hat{\sigma} \cdot s \tag{30}$$

Other rules of thumb can be used if additional information on local travel behavior becomes available, thus allowing a better match between routes and σ's in Equation 28.

As an example, suppose the disaggregate fare elasticity values are available (*19*) as a range of values characterized by a mean and a standard deviation: -0.35 ± 0.14. In the absence of the actual constituent figures that make up the range, we take two standard deviations (i.e., 0.28) from the mean of -0.35 as the maximum and minimum elasticity value. For a normal distribution this would cover 95 percent of the 12 data points in the sample: $\eta_{\max} = -0.07$ and $\eta_{\min} = -0.63$.

Suppose further that the observed modal split in York is 88.83 percent automobile trips and 11.17 percent transit trips. From Equation 26, $\hat{\sigma}^2$ can be estimated as 0.0882 (or $\hat{\sigma}$ = 0.297), which is a higher variance than the data sample collected in Chicago by Warner.

The W. Market/E. Market line was 1 of 10 bus routes operating in York. With its 17 stops, it was among the longer routes in the York Area Transit Authority. Only two routes were longer, made up of 18 stops, and the remaining seven were 14-stop routes. If route length can be a proxy for mode-choice variability, we can scale the W. Market/E. Market route as somewhere between $\hat{\sigma}$ and $\sigma_{\min}$. If a linear scale s is applied as shown in Equations 29 and 30, the variability associated with the route concerned is $\sigma_k = 0.222$, corresponding to $s = 3/4$. According to Equation 27, this translates to a route elasticity of -0.314. Compared with the average, -0.35, a 10 percent difference is observed in this case. Again, other routes can have biases much larger than 10 percent, considering that the maximum variance $\hat{\sigma}$ is close to 0.3.

SUMMARY AND CONCLUSIONS

In assessing user response to cost and service changes, demand elasticities are useful tools. Current compilations of demand elasticities, however, are not helpful to scheduled transportation operators. The range of an elasticity is too wide, and there is no practical guideline for picking an appropriate value in the range. Furthermore, they are often calibrated on the basis of different levels of aggregation, rendering them incompatible with route-by-route or peak versus off-peak analyses, which are critical to current operational concerns.

A methodology providing elasticities that are practical for patronage analyses in an operating agency was presented to address this problem. Through statistical analyses of spatial/temporal data aggregation, the methodology explains the differences among elasticity tabulations, and in so doing, provides insights into the variations that exist among elasticity values.

The results of this research include (*a*) guidelines for scaling an appropriate value of elasticity among its broad range of values (instead of simply taking the average or midpoint of the range) and (*b*) a method for converting the most commonly available elasticities, which are usually calibrated in different levels of aggregation, to a more useful form, such as route-specific and time-of-day elasticities.

Rigorous yet simple statistical developments were followed in deriving the adjustment procedure. The results were demonstrated in a case study of a transit line in York, Pennsylvania, including a step-by-step calculation procedure for practitioners.

Additional work can obviously be carried out to extend this research. It is recommended that more case studies be performed, particularly studies geared toward time-of-day rather than route-specific applications. One such case study could include before-and-after validation (*20*).

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