Multicriteria Decision Making in Location Modeling

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Public- and private-sector location decisions are generally made in a multiobjective planning environment. Examples include location of emergency medical service facilities and warehouses. Various criteria considered in making these decisions may include minimization of costs, maximization of demands that are satisfied within a prespecified time or distance, and minimization of average distance from demand points to the nearest facility. Generally, location problems are formulated and solved as mathematical optimization problems. Attempts have been made to solve location problems through multiobjective optimization. Generally, these models account for more than one objective by using a variety of weighting schemes or by concentrating on one objective and incorporating the others into the model as constraints. An alternative approach to consider several objectives simultaneously is presented. The approach is based on the analytical hierarchy process, which is a useful tool in multicriteria decision making. The approach is demonstrated by a numerical example.

Engineers and planners in both the public and the private sectors are frequently faced with deciding where to locate facilities and how to allocate the resources for these facilities among competing demands. Public-sector examples of such decisions include where to locate emergency medical service (EMS) vehicles to serve a community, how such vehicles should be dispatched to incidents, how many fire stations are needed so that all points in an urban area may be reached within a prespecified time, and where public schools should be located. Private-sector examples include where to locate maintenance facilities over an airline or railroad network and how many warehouses are needed in a distribution system and where they should be located. In all cases, the location of the facilities significantly affects both the operating costs of the system and the ability of the system to satisfy the demands placed on it.

Facility location decisions in both public and private sectors are generally multiobjective in nature. For example, in locating emergency medical service facilities, the objectives are to minimize the average travel time to an incident and the number of vehicles deployed, and hence the operating cost of the system. There may also be a need to consider many other objectives, such as balancing the work load, increasing the number of people served by the system, and equity. Some objectives may be conflicting in nature. For example, minimizing the number of deployed vehicles conflicts with maximizing the number of people served.

Generally, location-allocation problems have been formulated and solved as mathematical optimization problems. Different formulation approaches and the use of different objective functions have resulted in a variety of location models.

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Although attempts have been made to deal with location decisions from a multiobjective point of view (*1–5*), most location models are formulated and solved in a single-objective framework. Among the single-objective models, a few studies attempt to deal with the location-allocation problem in a hierarchical modeling approach, and mostly from a dual objective point of view (*6–9*). This paper presents an alternative approach for making multiobjective location decisions. Theoretically, the approach enables analysts and decision makers to simultaneously consider as many objectives as they wish without defining any a priori weights for the objectives or setting any constraints on the level of achievement of those objectives.

PROBLEM STATEMENT

Location problems are frequently confronted in both the public and the private sectors. The most common location problems in the public domain include choosing fire station sites and determining the number and location of EMS vehicles. Private-sector location problems include determining the number of maintenance facilities needed on an airline or railroad network and the optimal location of these facilities and identifying optimal warehousing locations in product distribution networks.

The allocation problem is closely related to the location problem. The allocation problem deals with optimal assignment of the demands to service centers. The location-allocation problem derives its importance from two sources. First, in many cases, the location of service centers in a network and the allocation of demands to service facilities has a direct effect on system operating costs. This is particularly true in private-sector applications. The second reason for interest in the location-allocation problem is that, in some cases, the nature of the demands and of the service depends dramatically on the ability of the customers to obtain service quickly. An example is medical emergencies, in which timely response is clearly needed.

Research on the location-allocation problem must recognize that location decisions are generally made in a multiobjective planning framework. For example, in locating EMS vehicles, the objectives are to minimize the average travel time to an incident, the maximum travel time to an incident, and service differences between geographic areas of the city; maximize the number of people or potential demands that can be served by the system within a given time limit; and minimize the number of vehicles deployed. In fact, these are only representative of the many objectives that public decision makers must consider in locating and in choosing response districts for EMS vehicles. Similarly, private location-allocation decisions are also multiobjective in nature. In choosing repair centers for an airline network, for example, the objectives are to minimize the amount of deadheading required to reach the facilities and the construction and operating costs of the facilities.

One important difference between private- and public-sector location problems is that it is generally easier to collapse the different objectives into a single objective in private-sector location problems. For example, deadheading may be converted into an operating cost. By using appropriate weighting and discounting factors, the deadheading, construction, and operating costs in the airline example may be collapsed into a single cost figure. In the EMS vehicle location example, however, it would be virtually impossible to collapse measures of service inequities across geographic areas and measures of vehicle work loads into a single value to be optimized. Even if it were possible, it would require the analyst to implicitly weight the two objectives. This task is better left to the public decision makers. This, however, demands that the analyst develop techniques capable of clearly displaying the trade-offs between objectives.

This paper presents an approach that, theoretically, enables simultaneous consideration of as many objectives as desired in making location decisions. This approach is based on the analytical hierarchy process (AHP). AHP is selected as a vehicle for evaluation and ranking of alternative sites (or combinations of sites) on the basis of multiple objectives. It has proven to be a valuable tool in multicriteria decision making. In this paper we deal with objectives that are generally used in public-sector planning. However, the methodology can easily be generalized to include as many and as different objectives as needed. The paper focuses on discussion of issues involved in implementation of AHP in multiobjective location modeling.

DETERMINISTIC LOCATION MODELS

Facility location is one of the most important long-term logistical decisions faced in the public or private sector. Most facility location models are concerned with network locations rather than plane locations. The location models can be categorized on the basis of such important criteria as type of objective function and the deterministic versus stochastic models. In this section we focus on deterministic location models and distinguish them by their objective function. Stochastic models are beyond the scope of this study.

Three general types of objective functions have been used the most in the literature concerning public-sector location decisions. Covering models locate facilities on a network such that the demands are covered within a prespecified critical time or distance. Median or minisum models locate the facilities such that the weighted average distance between the facilities and the demands served by those facilities is minimized. Center or minimax models locate facilities to minimize the weighted maximum distance from the facilities to the demands served by them. Median, covering, and center models encompass a large portion of the location models, and therefore we focus on these models. The interested reader is referred to Handler and Mirchandani (*10*), Tansel et al. (*11*), or Brandeau and Chiu (*12*) for a more complete review. In what follows, we describe covering, median, and center location models and then briefly review some of the multiobjective approaches to location modeling.

Covering Models

The inputs to location models include a network with a set of N nodes and A links. Associated with each node is a demand to be served by the facilities. Demands are assumed to be generated at nodes only. The travel time (or travel distance) between Nodes i and j is represented by a shortest path matrix. Also, a prespecified critical time (or distance) is defined, which is the maximum time (or distance) limit.

The simplest covering model is the location set covering model, which was originally formulated by Toregas et al. (*13*). This formulation attempts to minimize the number of service centers on a network such that all demands are covered within the critical time or distance. This model has been used in locating ambulances (*14*) and fire stations (*15,16*). Plane and Hendrick (*17*) studied a dual objective formulation of the set covering problem. They minimize the number of fire stations needed and maximize the number of existing stations in the solution. Daskin and Stern (*6*) proposed an alternative dual objective formulation that minimizes the number of service centers and maximizes the amount of backup coverage. Demand-weighted backup coverage has also been formulated (*18,19*).

The set covering model fails to recognize that demands are generated at the nodes at different rates. The maximum covering location model formulated by Church and ReVelle (*20*) accounts for this by trying to maximize the number of demands that are within the critical time (or distance) of one of the P facilities that are to be located. Maximum covering models have been used in practice to locate service facilities (*21–23*).

Minisum or Median Models

The P-median or minisum problem was originally formulated by Hakimi (*24*). This model minimizes the weighted average distance from a demand node i to the facility to which it is assigned. In the absence of capacity constraints or other complications, demands are assigned to the nearest facilities. In using this model for an EMS facility location problem, the objective is to locate facilities so that the best average behavior of the system is obtained.

Hakimi (*24*) has shown that at least one optimal solution to this problem consists of locating only on the nodes of the network. Hakimi's result has been generalized to a number of extensions of the P-median problem (*25,26*). A variety of heuristic methods have been proposed for solving the P-median problem (*10,27*).

Minimax or Center Models

These models minimize the weighted maximum distance from the demand points to the nearest facilities. When facilities are to be located on nodes of a network, minimax models are referred to as vertex-center models. Center models attempt to locate facilities over the network so that the level of service in the worst possible case is as good as possible (*10*).

The objectives incorporated in the covering, median, and center models represent three of the most important objectives that can be considered in location of public facilities—in particular, EMS facilities such as ambulance and fire stations. It is clear that the best solution under one modeling approach and for one of the foregoing objectives may not be the best or even a good solution for the other objectives.

Furthermore, although these objectives are among the most important, they are not necessarily the only ones to be considered. In public-sector facility location many other objectives may be equally important. Balancing the work load among facilities, providing equitable service, providing as much backup service as possible, and political considerations are other objectives that may be considered. Some objectives may not even be quantifiable and cannot be incorporated into a mathematical model. It is desirable, therefore, to approach location decisions in a multiobjective framework that addresses these issues.

Multiobjective Approaches

One of the important aspects of location decisions is their long-term effects on the service provided to the public. Several models have been formulated to deal with the public facility location problem in a multiobjective framework. Multiobjective approaches have been used to locate energy facilities (*28–30*) and fast food restaurants (*31*). Cohon et al. (*29*) formulated a multiobjective linear programming model for locating energy facilities. Mladineo et al. (*30*) propose a multicriteria approach for ranking the alternative sites. Min's model (*31*) considers the behavioral and spatial aspects of location scenarios. Fortenberry et al. (*32*) and Heller et al. (*33*) propose models to locate emergency medical service facilities in a multiobjective environment. Fortenberry et al. (*32*) use linear programming to determine optimal locations, and Heller et al. (*33*) propose a model that minimizes the mean response time and balances the facility work load. The results of the latter model are validated by simulation techniques.

Ross and Soland (*34*) present an interactive algorithm for multicriteria optimization that solves a finite sequence of generalized assignment problems for location of public facilities. Schilling (*35*) proposed a dynamic location model to locate public facilities. His approach uses multiobjective analysis to plan public-sector facility systems that operate in a dynamic environment.

Goal programming is used as a technique for approaching location decisions in a multicriteria environment (*36–38*). Some researchers have developed interactive models for locating facilities (*39,40*). Multicriteria approaches have also been used in locating private-sector facilities (*36,41–43*). Buhl (*44*) presents several theorems characterizing single-objective reductions of multiobjective problems and shows that the objective functions used in location theory contain both implicit and explicit value judgments.

In general, all but a few of the multiobjective approaches to location modeling use an optimization framework. Either all of the objectives to be considered are collapsed into a single objective with a weighting scheme or thresholds are defined for all but one objective and the problem is approached by optimizing that objective while the others are constrained within their predefined thresholds (as in goal programming). In these approaches, nonquantifiable objectives are either ignored or dealt with exogenously.

Although efforts have been made to formulate and solve location models that deal with more than one objective, this area of research is still promising. In particular, a multiobjective approach that can incorporate several objectives, provide a systematic evaluation and ranking of alternatives, and, particularly, handle nonquantifiable objectives would be desirable.

In contrast to other multiobjective approaches, optimization is proposed as an alternative generation tool while at the same time a framework is devised enabling us to implement a multicriteria decision-making tool such as AHP to select the best alternative from those generated by the optimization approach. This approach theoretically enables us to consider as many objectives as desired simultaneously with no prior weighting schemes or threshold definitions.

MULTIOBJECTIVE APPROACH TO LOCATION DECISIONS

In this section a multiobjective approach to location modeling incorporating the maximum covering, median, and center is presented. We also incorporate cost considerations as the fourth criterion to be considered in the location of facilities. The approach is based on AHP (*45–50*). The procedure has proven useful in multicriteria decision making (*51*).

AHP

AHP provides a way to organize complex decision-making problems in a manner that allows for interaction and interdependence among factors influencing the decisions and still allows the analyst to think about these factors in a simple way. It enables the analyst to make effective decisions on complex issues by simplifying and expediting the natural decision-making process.

AHP is based on three principles: decomposition, comparative judgments, and synthesis of priorities. A complex, unstructured problem is decomposed into its component parts, which are further arranged into a hierarchic order. The elements in the hierarchy define the problem. A matrix of pairwise comparisons of the relative importance of the elements in a level of hierarchy with respect to the elements in the level immediately above it is then set up. Finally, the global or composite priorities of elements at the lowest level of the hierarchy (alternative solutions) are synthesized.

AHP also provides an effective structure for group decision making. It enables decision makers to represent the simultaneous interactions of many factors in complex, unstructured situations and helps to identify and set priorities on the basis of various objectives. A detailed description of AHP is beyond the scope of this paper. The interested reader is referred to Saaty (*45,48*). However, the main steps of the process can be summarized as follows (*46*):

1. Define the problem and specify the alternative solutions.
2. Break the problem down into a system of components at different levels of hierarchies. Each component in a higher level of hierarchy encompasses some or all of the components in the next lower level.
3. Construct a pairwise comparison matrix of the relevant contributions or impacts of each component on each governing component (criterion) in the next higher level. In this matrix, pairs of elements are compared with respect to a criterion in the superior level. The numbers in the cells of this matrix represent the superiority (inferiority) of each component compared with the others, with respect to their contribution to the governing component. The numbers can be based on either subjective judgments or available numerical data that measure the performance of the alternatives with respect to the criteria under consideration.
4. Obtain all judgments required to develop the set of matrices in the third step.
5. Establish the priorities of components at each level of hierarchy with respect to the criterion or component in the higher level encompassing those components.
6. Repeat Steps 3 through 5 until the priorities are established for all levels of hierarchy.
7. Vectors of priorities are then weighted by the weight of the criteria of each level, and this process is repeated until a priority vector for the lowest level of hierarchy (the alternative solutions) is obtained.
8. The final decision or outcome depends on the vector of priorities for the lowest level of hierarchy and can be evaluated on the basis of consistency measures.

AHP in Location Analysis

The facility location problem can be approached as a multiobjective decision-making process using AHP. The problem can be decomposed to three levels of hierarchy. The first, finding the best sites among the candidates for locating the facilities, is the focus of the problem. The second level of hierarchy consists of the factors or objectives affecting this decision. Finally, the third level of hierarchy includes alternative sites or combinations of sites. According to AHP, matrices of pairwise comparisons between various candidate sites can be established on the basis of individual objectives. The matrices allow the analyst to rank the alternative sites according to the individual objectives. Then an overall vector of priorities (or ranking of alternatives) can be obtained by weighting the priorities according to the individual objectives by the relative weights of the objectives. Note that a vector of priorities for the different objectives can also be obtained through construction of a pairwise comparison matrix for the objectives themselves. The priorities can then be used as weights for the corresponding objectives. Construction of this pairwise comparison matrix requires judgment by the decision maker or the analyst on the relative importance of the objectives. The important issue, however, is that the value judgments can be made by comparing the relative merits of only two objectives at a time, a much easier task than comparing all objectives simultaneously. When these pairwise value judgments are obtained and the comparison matrix is completed, the overall weights for all of the objectives can be obtained. Therefore no a priori weights for the objectives need to be known and no threshold on the level of achievements of the objectives needs to be defined.

There are three major issues in implementation of a tool like AHP in location decision making, in particular when we are dealing with multiple facility location. The first is the hierarchical structure to be used. In a single-family location case, a hierarchical structure like the one mentioned earlier can easily be used. However, in a more complex multifacility location problem it is not clear that such a structure is the best. The second major issue, and perhaps the most important, is to generate the set of alternatives to be evaluated using AHP. In cases where a single facility is to be located, candidate sites can be used as individual alternatives. When we are dealing with large networks on which multiple facilities must be located, we have a combinatorial problem, and identification of the alternatives is not a simple task. Construction of the pairwise comparison matrices is the third issue. If these issues are successfully resolved, AHP will offer tremendous advantages in its simplicity of application, its ability to deal with qualitative objectives, and its theoretical ability to deal simultaneously with as many objectives as desired.

In this paper, we try to deal with these issues in a simple example. Further research is required to address them in a more general case, along with development of other multiobjective approaches that could present similar advantages. In the next section we present the implementation of a multiobjective approach to locating facilities over a small network based on AHP. The example, although simple, provides useful insights into the multiobjective nature of location decisions.

A Simple Example

Consider the problem of locating a single facility or two facilities on the nodes of the network shown in Figure 1. The facilities must respond to the demands generated at the nodes of the network. The shortest path distance between the nodes of the network, along with the demand for service at each node and the cost of operation and maintenance of a facility located at each of the nodes, is given in Table 1.

Assume that in this location decision we need to consider four different criteria: (*a*) achievement of lowest operation and maintenance costs, (*b*) coverage of as much demand as possible within a critical distance of 10 units, (*c*) provision of service such that the average distance from the located facilities to the demand locations is minimized, and (*d*) provision of service such that the level of service in the worst possible case is as good as possible. As mentioned before, consideration of these criteria individually generally results in location decisions that are drastically different. In fact, locations that might be optimal under one criterion may not even be good locations under another. Therefore, we must consider the trade-offs among the criteria. All nodes are considered to be candidate sites for the location of the facilities.

First, we explore the results of different approaches to the single-facility location problem. Using the individual criteria, this simple location problem can be formulated as either a cost minimization, a maximum covering, a 1-median, or a vertex-1 center problem. Table 2 gives the value of each of

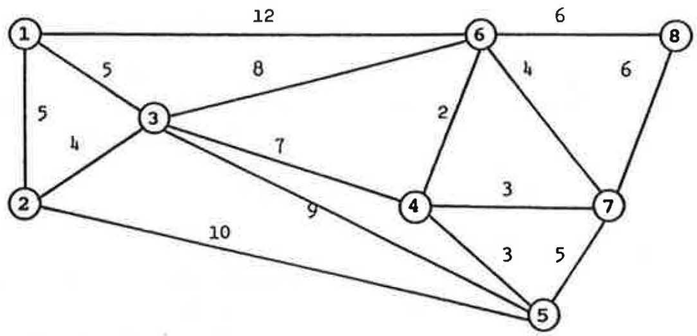

FIGURE 1 Example network.

TABLE 1 DISTANCES, DEMANDS, AND COSTS

Nodes	1	2	3	4	5	6	7	8
1	-	5	5	12	14	12	15	18
2	5	-	4	11	10	12	14	18
3	5	4	-	7	9	8	10	14
4	12	11	7	-	3	2	3	8
5	14	10	9	3	-	5	5	11
6	12	12	8	2	5	-	4	6
7	15	14	10	3	5	4	-	6
8	18	18	14	8	11	6	6	-
Demands	9	13	6	3	8	5	7	10
Costs	9	10	13	12	12	8	9	7

TABLE 2 OBJECTIVE FUNCTION EVALUATION

Location	Vertex-1 Center	1-Median	Maximum Covering	Cost
1	180	588	28	9
2	180	520	36	10
3	140	440	51	13
4	143	428	39	12
5	130	489	42	12
6	156	446	39	8
7	182	506	39	9
8	234	664	25	7
Optimal Location	5	4	3	8

the individual objectives when the facility is located at each of the nodes. The solution of the single-facility location problem is trivial and can be determined by inspection in this case. Table 2 also gives the best location on the basis of the individual criteria.

Note that on the basis of each of these criteria, the optimal location of the facility is a different node. It is clear that each location, optimal on the basis of an individual objective, may not be optimal when all four objectives are considered simultaneously.

We now present the results of an AHP-based approach. For this simple problem, the hierarchies can be structured as shown in Figure 2.

The next step is to set up the pairwise comparison matrices for the alternative sites on the basis of the individual objectives. The cells of each matrix indicate whether or not the site represented by the row is superior or inferior to the site represented by the column on the basis of the objective represented by that matrix. A cell value greater than (less than) 1.0 indicates that the site represented by the row in the matrix is superior (inferior) to the site represented by the column. A cell value of 1.0 indicates that the two sites are equivalent with respect to that objective.

In general, to fill the comparison matrices, Saaty (*46*) suggests using values from 1 to 9 in comparing two alternatives. The value 1 indicates that the two alternatives are equivalent with respect to the criterion under consideration, and the value 9 indicates that one alternative has the highest possible priority over the other. The diagonal elements of these matrices are all 1's, and when the value for Cell (i, j) is determined, the value for Cell (j, i) is the reciprocal of the value for Cell (i, j).

An important issue in setting up the comparison matrices is consistency in judgment when comparing alternatives. Consistency means that if Alternative i is preferred twice as much as Alternative j, and Alternative j is preferred twice as much as Alternative k, then Alternative i should be preferred four times as much as Alternative k. In many cases in the real world enforcing perfect consistency in judgment is not possible; however, relative consistency is desirable so that the judgments do not appear to be random. This is particularly true in dealing with qualitative judgments rather than judgments based on quantitative measures. Fortunately, AHP provides a measure to determine the overall consistency of

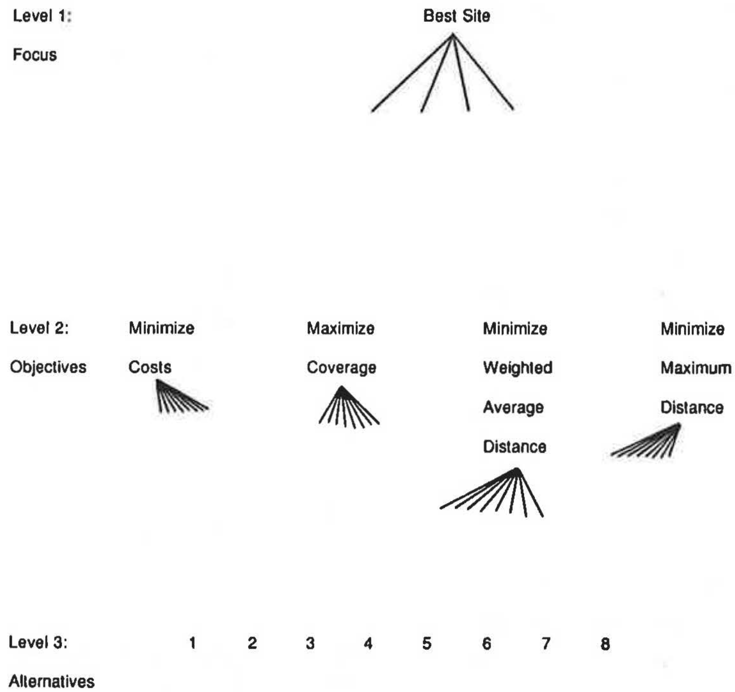

FIGURE 2 Levels of hierarchy for the location problem.

judgments by means of a consistency ratio. The details of calculation of this ratio are given in Saaty (*46*), and a consistency ratio of 0.10 or less is deemed to represent good consistency (a consistent matrix has a consistency ratio of 0).

To construct the comparison matrices, our preference among alternative sites with respect to the individual criterion is based on the value of the mathematical objective function that represents the criterion under consideration. The criteria we are considering in our problem are represented by cost minimization, maximum covering, 1-median, and vertex-1 center objective functions. Therefore, the level of preference of one site over the other with respect to a particular criterion is determined by the ratio of the values of the objective function that represents that criterion when those sites are chosen for locating the facility. For example, when comparing Site 3 with Site 1 with respect to the center objective function, we use the ratio 180/140 in Cell (3, 1), which indicates that Site 3 is preferred 1.286 times as much as Site 1 for this objective, and we use the reciprocal of this value (0.778) in Cell (1, 3). Note that we are implicitly assuming that our preference among the alternatives is directly proportional to their degree of attainment of the objective under consideration. In this case, where the objectives are readily quantifiable, such an assumption may be appropriate. However, in a general case when we are dealing with nonquantifiable objectives, the preference structure may be complex, and such an assumption cannot be made. In that case, obtaining numerical values for the cells of the comparison matrices depends on comparing the values of the objectives among the alternatives, which by itself is an important task. Even when the objectives are quantifiable, it does not necessarily mean that an alternative with twice the objective function value of another alternative will be preferred twice as much. These are important issues that must be addressed in future research. In any event, this assumption can easily be relaxed by incorporating other preference structures. The advantage of AHP is that, as long as a preference structure is agreed upon by the decision makers, it provides an excellent framework for further analysis.

This approach has the advantage that the comparison matrices are perfectly consistent because all of the cells represent the ratios of the objective function values. In cases where the criteria are not easily quantifiable or calculation of numerical values is difficult, Saaty's general procedure could be used. These matrices are shown in Table 3, and the vectors of priorities synthesized on the basis of these comparison matrices are shown in Table 4 [details on how the vectors of priorities are synthesized are given by Saaty (*46*)]. The underlined cell in each column indicates the most attractive alternative according to the objective represented by that column. The most attractive alternatives identified by AHP on the basis of the individual objectives correspond to the optimal locations identified in Table 2.

The overall ranking of the alternatives is obtained by multiplying the priority of each alternative based on each of the criteria by the weight of that criterion and summing the results for each alternative over all criteria.

As long as a simple pairwise comparison of alternatives can be made, there is no need to know the weights of the different criteria a priori. The weights can be determined in the same way as the vectors of priorities for individual criteria, by setting up a comparison matrix for the criteria themselves. The cells of this matrix determine whether a particular criterion is superior or inferior to another and show the preference structure among the criteria.

In this example, if all criteria have the same level of importance, they will have equal weights of 0.25 each, and the vector of overall priorities is (0.112, 0.118, 0.138, 0.130, 0.131, 0.138, 0.125, 0.109). This means that both Nodes 3 and 6 are

TABLE 3 COMPARISON MATRICES BASED ON DIFFERENT OBJECTIVES

Nodes	1	2	3	4	5	6	7	8
1	1.00	1.00	0.78	0.79	0.72	0.87	1.01	1.30
2	1.00	1.00	0.78	0.79	0.72	0.86	1.01	1.30
3	1.29	1.29	1.00	1.02	0.93	1.11	1.30	1.67
4	1.26	1.26	0.98	1.00	0.91	1.09	1.27	1.64
5	1.39	1.39	1.08	1.10	1.00	1.20	1.40	1.80
6	1.15	1.15	0.90	0.92	0.83	1.00	1.17	1.50
7	0.99	0.99	0.77	0.79	0.71	0.86	1.00	1.29
8	0.77	0.77	0.60	0.61	0.56	0.67	0.78	1.00

(a) Comparison Based on Center Objective Function

Nodes	1	2	3	4	5	6	7	8
1	1.00	0.88	0.75	0.73	0.83	0.76	0.86	1.12
2	1.13	1.00	0.85	0.82	0.94	0.86	0.97	1.28
3	1.34	1.18	1.00	0.97	1.11	1.01	1.15	1.51
4	1.37	1.22	1.03	1.00	1.14	1.04	1.18	1.55
5	1.20	1.06	0.90	0.88	1.00	0.91	1.04	1.36
6	1.32	1.17	0.99	0.96	1.10	1.00	1.14	1.49
7	1.16	1.03	0.87	0.85	0.97	0.88	1.00	1.31
8	0.89	0.78	0.66	0.65	0.74	0.67	0.76	1.00

(b) Comparison Based on Median Objective Function

Nodes	1	2	3	4	5	6	7	8
1	1.00	0.79	0.55	0.72	0.67	0.72	0.72	1.12
2	1.29	1.00	0.71	0.92	0.86	0.92	0.92	1.44
3	1.82	1.42	1.00	1.31	1.21	1.31	1.31	2.04
4	1.39	1.08	0.77	1.00	0.93	1.00	1.00	1.56
5	1.50	1.17	0.82	1.08	1.00	1.08	1.08	1.68
6	1.39	1.08	0.77	1.00	0.93	1.00	1.00	1.56
7	1.39	1.08	0.77	1.00	0.93	1.00	1.00	1.56
8	0.89	0.69	0.49	0.64	0.60	0.64	0.64	1.00

(c) Comparison Based on Covering Objective Function

Nodes	1	2	3	4	5	6	7	8
1	1.00	1.11	1.44	1.33	1.33	0.89	1.00	0.78
2	0.90	1.00	1.30	1.20	1.20	0.80	0.90	0.70
3	0.69	0.77	1.00	0.92	0.92	0.62	0.69	0.54
4	0.75	0.83	1.08	1.00	1.00	0.67	0.75	0.58
5	0.75	0.83	1.08	1.00	1.00	0.67	0.75	0.58
6	1.13	1.25	1.63	1.50	1.50	1.00	1.13	0.88
7	1.00	1.11	1.44	1.33	1.33	0.89	1.00	0.78
8	1.29	1.43	1.86	1.71	1.71	1.14	1.29	1.00

(d) Comparison Based on Cost Minimization Objective Function

TABLE 4 VECTORS OF PRIORITIES ACCORDING TO OBJECTIVES

	Objective Function			
Nodes	Cost	Covering	Median	Center
1	0.133	0.094	0.106	0.113
2	0.120	0.120	0.120	0.113
3	0.092	<u>0.171</u>	0.142	0.145
4	0.100	0.130	<u>0.146</u>	0.142
5	0.100	0.140	0.128	<u>0.157</u>
6	0.150	0.130	0.140	0.131
7	0.133	0.130	0.123	0.112
8	<u>0.171</u>	0.084	0.094	0.087

the preferred sites if all of the criteria are of equal importance. However, if the cost considerations are twice as important as the other criteria (i.e., have a weight of 0.4 while the other three have a weight of 0.2), the overall priority vector becomes (0.116, 0.119, 0.129, 0.124, 0.125, 0.140, 0.126, 0.121), which suggests that Node 6 is the preferred node on the basis of this preference among the criteria. Note that this site is different from those selected on the basis of consideration of the individual objectives. Also note that although Node 6 is not the optimal site when we consider the individual objectives, it performs well compared with the other sites; therefore, the AHP approach has resulted in a logical choice considering all of the criteria.

To see the effects of inconsistency in judgments, we changed some of the cells in the comparison matrices and introduced some inconsistency in those matrices. However, the changes were made only to the extent that the overall consistency ratio for the matrices remained under 0.1, so that we still had relatively consistent matrices. With these new matrices, the priorities of some of the sites changed. However, the preferred sites remained the same. This suggests that AHP results are not sensitive with respect to consistency in judgments, and if the comparison matrices are relatively consistent, the AHP provides overall results similar to those provided when perfect consistency exists.

The single-facility location problem was an excellent tool to show the implementation of AHP. However, when we are dealing with multifacility location problems, the task is not so easy. In multifacility location problems we are dealing with combinatorial problems that may have numerous alternative solutions. One must note that AHP is not a tool for generating alternative good or optimal solutions; rather, it provides a framework for evaluating and ranking alternative solutions on the basis of multiple criteria. Therefore, in a multifacility location problem, the major issue is how to generate good alternative solutions that can later be evaluated using AHP.

A naive approach to the multifacility location problem can be as follows. Assume for the moment that we want to locate P facilities. We can first rank all of the individual sites according to all of the criteria (as we did in the single-facility location problem); then the P sites with the highest ranks could be selected for location of facilities. This method is a type of greedy adding heuristic which will result in a solution quickly; however, there is no guarantee that we have the best possible set of sites. To obtain better results, the alternative solutions that are input to AHP must be combinations of P out of N sites that perform well with respect to the individual criteria.

To generate good alternative solutions that can be input to AHP, we can use single-objective mathematical optimization. This can be done in two ways. A simple approach is to find a number of optimal and near-optimal sites on the basis of the individual objectives. This can easily be achieved by solving a sequence of uniobjective mathematical programming problems. When a number of good solutions based on each objective are identified, they are combined in a global set of alternatives, which can then be evaluated and ranked using AHP as discussed previously.

For example, assume that we want to locate three facilities in a 15-node network using the same criteria as in the single-facility location problem. We first identify a set of optimal or near-optimal alternatives on the basis of the individual criteria. This can be achieved through an iterative process of solving the individual optimization problems, recording the optimal solution, and forcing it out of the solution space in the next iteration. Assume that the following sets of alternatives were identified:

- Cost considerations: {1, 4, 7}, {2, 4, 8}, and {4, 7, 8};
- Coverage criterion: {5, 6, 9}, {6, 8, 4}, and {1, 2, 5};
- Median criterion: {6, 7, 9}, {2, 4, 8}, and {1, 8, 11}; and
- Center criterion: {5, 6, 8}, {7, 8, 11}, and {4, 8, 9}.

Then the global set of alternatives to be considered would be the combination of all 12 alternative sets of sites.

A more involved approach is to identify the individual sets of alternatives as discussed. However, to find the global set of alternatives, consider all of the sites providing the individual alternatives and examine all possible combinations of these sites. For example, in the previous problem, the sites providing the individual alternatives are {1, 2, 4, 5, 6, 7, 8, 9, 11}. We can therefore consider all possible combinations of three of these sites. This results in 84 alternatives. This approach identifies many more alternatives, which in turn results in a more thorough evaluation of the alternative space, but if the set of sites is large, the number of alternatives to be considered becomes impractical. This approach is particularly useful when a relatively small number of sites provide all individual alternatives.

To do a preliminary evaluation of these approaches, we considered a two-facility location problem on the network of Figure 2, with all data and criteria being the same.

On the basis of the vector of overall priorities, the naive approach suggests that the two highest-ranked sites are Nodes 6 and 3 if all criteria are of equal importance. To implement the first alternative generation approach, we identified several alternatives under each criterion. These alternatives are as follows:

- Cost: {1, 6}, {1, 7}, {2, 6}, {2, 7}, and {6, 7};
- Coverage: {1, 5}, {1, 6}, {1, 7}, {2, 5}, {2, 6}, {2, 7}, {3, 5}, {3, 6}, and {3, 7};
- Median: {1, 7}, {2, 6}, {2, 7}, and {3, 7}; and
- Center: {1, 6}, {1, 7}, {2, 6}, {2, 7}, {3, 6}, and {3, 7}.

This results in 10 distinct alternatives, which were evaluated using AHP. On the basis of equal preference among the criteria, Nodes 2 and 6 were identified as the best locations.

To implement the second alternative generation approach, we used the set of sites providing all of the preceding alternatives. This set is {1, 2, 3, 5, 6, 7}. All combinations of two sites out of these six were identified and considered as possible alternatives. This resulted in 15 alternatives, which were examined using AHP. Again on the basis of equal preference among criteria, Nodes 2 and 6 were the best sites. Nodes 2 and 6 are one set of the optimal locations based on center and covering objectives. The optimal locations based on median and cost objectives are {2, 7} and {1, 6} or {6, 7}, respectively.

Although this problem is a small one and does not fully serve the purpose of evaluating these approaches, it provides useful insights into the applicability of the alternative generation approaches and the AHP. The power of AHP in multicriteria decision making is further realized where the presence of qualitative objectives complicates the application of traditional multiobjective optimization techniques, such as weighting and constraint methods.

CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this paper we presented an approach to dealing with location decisions in a multiobjective planning framework. The approach is based on AHP, which is a useful tool for multicriteria decision making. The approach was implemented in the context of a single- and a two-facility location problem. The example, although simple, provides useful insights into the multiobjective nature of location decisions and clearly shows that when location decisions are made in view of a single objective, the selected sites are likely to be inferior with respect to other objectives. In some cases, as the example indicates, the preferred location when considering all objectives may not even be the optimal location when considering any of those objectives individually.

The paper indicates that AHP is a promising approach in dealing with location decisions in a multiobjective planning framework. We presented a brief sensitivity analysis of the AHP results with respect to the numerical values representing the comparative judgments among alternative sites (the cells of the comparison matrices). More detailed sensitivity analyses and exploration of methods of generating these numerical values other than those presented in this paper are important areas of research. In particular, the implicit assumption regarding the preference structure among the various criteria must be examined, and the sensitivity of AHP results with respect to changes in this preference structure should be analyzed.

Alternative generation techniques that could provide better results should also be explored. The results of this approach should be further tested on a larger network and in a multi-facility location problem context. These results should also be compared with those obtained from implementation of other multiobjective optimization techniques, such as weighting and bounding schemes. Finally, the results of this paper clearly indicate the multiobjective nature of location decisions. Development of other tools enabling analysts to approach location problems from the point of view of several simultaneous objectives is yet another promising research area.

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