Dynamic Method to Assess the Stiffness of Soil Underlying Spread Foundations

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A simple and easily implemented experimental method, the WAK test (wave activated stiffness K test), was proposed by Briaud and Lepert (*1*) to estimate, on site, the stiffness of the soil underlying a rigid foundation. The theoretical background of the method is summarized, and the interpretation of the results is explained. Case histories are presented to illustrate the method, its performances, and its potential applications.

Many theoretical and experimental studies have been devoted in the last few years to spread foundations. Nevertheless, there is still a lack of quick nondestructive testing methods to check the design of such foundations. In situ static load tests can be performed, but they are expensive.

An easily implemented method, called the WAK test (wave activated stiffness K test) was proposed by Briaud and Lepert (*1*). It enables the measurement of some basic parameters of the foundation, mainly the elastic stiffness of the soil underlying the foundation, in the small strain range (10^{-4} to 5.10^{-3} percent) a useful parameter to predict the short-term behavior of the foundation under design loads. The method also enables an estimate of the actual mass of the foundation and the equivalent damping of the soil. This was rigorously proven on several scaled foundation models resting on a layer of sand (*1,2*).

Recently, other experiments were performed on full-scale foundations. The results of these experiments, which are reported here, confirm the reliability of the WAK test. Furthermore, these case histories illustrate several potential applications of the method.

METHOD

In the proposed method, the "soil + foundation" system is considered as a single degree of freedom (d.o.f.) system (Figure 1). The WAK test is aimed at identifying the equivalent dynamic parameters of this system: M, K, and C. The static parameters of the foundation, m and k, can then be derived. Finally, the shear modulus of the soil in the small strain range, G_o, can be estimated from the latter values.

Theoretical Background

The equilibrium equation of the single d.o.f. system of Figure 1 can be written as

$$Mx'' + Cx' + Kx = F_o e^{j\omega t} \quad (1)$$

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where F_o is the amplitude of the applied harmonic force and ω is its frequency in radians per second. The steady state solution of this equation is as follows [see work by Brebbia et al. (*3*) for more details]:

$$x(t) = x_o \cdot e^{j(\omega t + \phi)} \quad (2)$$

with

$$x_o = F_o/[(K - M\omega^2)^2 + C^2\omega^2]^{1/2} \quad (3)$$

$$\tan(\phi) = C\omega/(K - M\omega^2) \quad (4)$$

The ratio x_o/F_o is a function of called the "displacement versus force transfer function" or "compliance" of the single d.o.f. system. The modulus of this transfer function is thus expressed as

$$|x_o/F_o| = 1/[(K - M\omega^2)^2 + C^2\omega^2]^{1/2} \quad (5)$$

whereas its phase is given by Equation 4. The "velocity versus force transfer function" or "mobility" of the same system can be derived by multiplying Equation 5 by ω and shifting the phase angle ϕ by a value of $\pi/2$. This function is shown in Figure 2. An important feature of the curve is a peak that appears on the modulus function at a frequency (ω_n) close to the natural resonance (ω_o) of the system:

$$\omega_n \sim \omega_o = (K/M)^{1/2} \quad (6)$$

Application to the "Foundation + Soil" System

Because the dynamic analysis is limited to the low frequency range (<100 Hz), the foundation can be considered a rigid block. Furthermore, the test induces only small strains in the soil, which may thus be regarded as an elastic medium radiating energy. Following work by Barkan (*4*), the "soil + foundation" system is assumed to behave as a single d.o.f. system.

The dynamic parameters of this system are related to the characteristics of the foundation and of the half-space through the following relationships:

$$M = \beta \cdot m \quad (7)$$

$$K = 2\pi^{1/2} G r_o c_s/(1 - \nu) \quad (8)$$

$$C = 2M\omega_o\xi \quad (9)$$

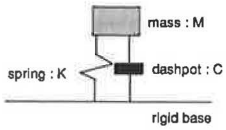

FIGURE 1 Single d.o.f. system.

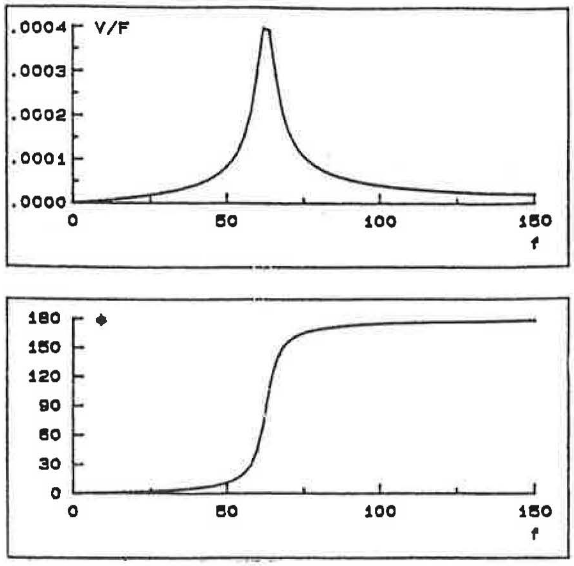

FIGURE 2 Typical transfer function of single d.o.f. system.

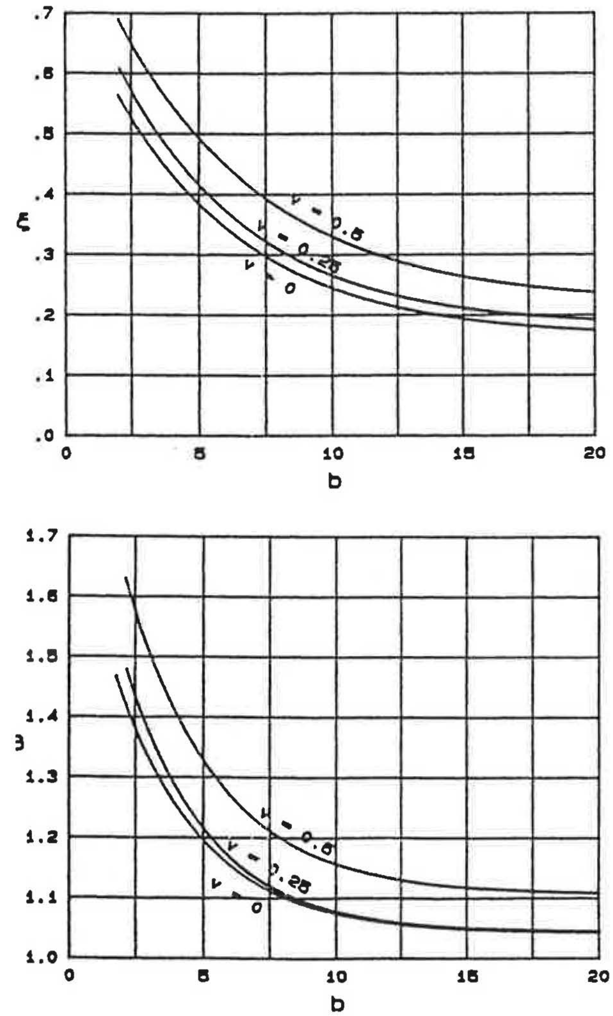

FIGURE 3 Auxiliary diagrams for determination of reduced damping coefficient ξ (top) and added mass coefficient β (bottom) (3).

where

$$\omega_o = (K/M)^{1/2}, \tag{10}$$

m = mass of the foundation,
r_o = equivalent radius of foundation [i.e., $(S/\pi)^{1/2}$],
S = horizontal area of foundation,
c_s = a factor depending on shape of foundation [see Table 1 in work by Barkan (4)],
G = shear modulus of soil, and
ν = Poisson's ratio.

β and ξ are two factors that depend on the dimensionless mass factor b according to Figure 3. This mass factor is defined as

$$b = m/\gamma r_o^3 \tag{11}$$

where γ is the soil density. The following table presents auxiliary values for the shape factor c_s (4):

α	c_s
1.0	1.08
1.5	1.09
2.0	1.10
3.0	1.15
5.0	1.24
10.0	1.41

where α is the length-width ratio of the foundation.

Other authors, such as Bycroft (5) and Lysmer (6), confirmed that the "soil + foundation" system could be approximated by a single d.o.f. system. For instance, Lysmer and Richart (7) derived, for usual cases of circular foundations, the following relationships:

$$M = m \tag{12}$$

$$K = 4\ Gr/(1 - \nu) \tag{13}$$

$$C = 3.4r\ (G\rho)^{1/2}/(1 - \nu) \tag{14}$$

Although this approach seems somewhat different from Barkan's, both lead to similar results in most engineering applications. The difference in K values is generally within a few percent. The approach from Barkan is usually retained because it seems more straightforward and yields results closer to those obtained experimentally.

Test Procedure

A small vertical impact is applied to the foundation along its gravity axis by means of a sledgehammer (see Figure 4). This

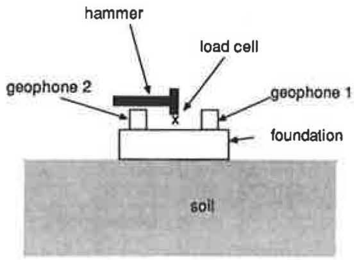

FIGURE 4 Experimental set-up and instrumentation.

hammer is instrumented with a load cell which provides the force versus time signal $f(t)$. The vertical response of the foundation is measured by two geophones that are fixed, symmetrical with respect to the impact location, on the upper face of the foundation.

The vertical velocity of the center of gravity $v(t)$ of the foundation is derived by averaging the two velocity versus time signals:

$$v(t) = 0.5[v_1(t) + v_2(t)] \tag{15}$$

A Fast Fourier Transform analyzer is used to compute the transfer function $T(\omega)$ between the velocity $v(t)$ and the force $f(t)$.

Figure 5 shows the mobility measured on the "foundation + soil" system shown in Figure 6. This function is similar to the one in Figure 2. Equation 5 (multiplied by ω because velocity is used instead of displacement) is adjusted to the experimental curve in Figure 5. A set of dynamic parameters (M, K, and C) is derived from this adjustment. The shear modulus (G) can be calculated from these dynamic parameters through equations 7 to 11 and Figure 3.

LABORATORY INVESTIGATIONS

To validate the method, some laboratory investigations were first conducted on a scaled model—a cubic concrete mass resting on a layer of loose coarse sand (Figure 6). The mobility measured on this model is displayed in Figure 5 (solid line).

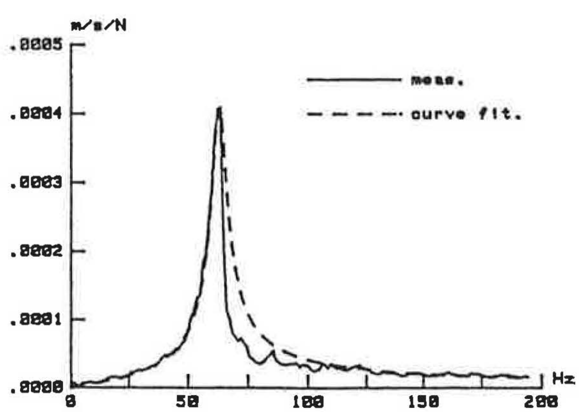

FIGURE 5 Mobility measured on the "foundation + soil" system shown in Figure 6.

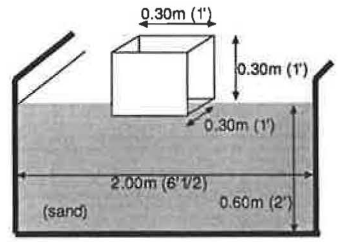

FIGURE 6 Model of concrete foundation used for first laboratory tests.

The soil stiffness resulting from these tests using a curve fitting procedure and equations 7 to 11 compared quite well with the value estimated from static loading tests. The results from these investigations, and the related conclusions, were reported in detail by Briaud and Lepert (*1*).

Other tests were performed on steel scaled models resting on Fontainebleau sand. They demonstrated that the soil stiffness derived from the WAK test was representative of the mechanical characteristics of a significant thickness of soil under the foundation. For more details about these investigations and the related conclusions, see work by Lepert and Briaud (*2*).

CASE HISTORIES

Case 1: Site Correlation Between WAK and Static Loading Tests

Static load tests on small spread footings were conducted at the FHWA Research Center. The footings, ranging in size from 0.3 m × 0.3 m × 0.15 m (1 ft × 1 ft × 6 in.) to 0.6 m × 0.6 m × 0.2 m (2 ft × 2 ft × 7.5 in.), were set in a test pit 5.5 m × 7 m × 6.1 m (18 ft × 23 ft × 20 ft). The properties of the sandy soil are as follows: density, 1475 kg/m^3 (92.4 lb/ft^3); SPT, 4 to 7 blows/ft; CPT (cone b.), 2 MPa (20 tons/ft^2); and friction resistance, 9.5 kPa (200 lb/ft^2).

Before each static load test was performed, the WAK test was used in an attempt to predict the static stiffness of the soil-footing system. The test procedure consisted of four steps:

1. Place the footing on smoothed level sand and seat it by rotating back and forth about a gravity axis while pushing down.
2. Fix two geophones at the extremities of a diagonal of the footing (see section on test procedure).
3. Impact the footing at center; record and process the data (WAK test).
4. Load the footing to failure.

Step 3 was repeated about 10 times to obtain a significant set of dynamic results. Figure 7 (top) shows the soil-footing stiffness derived from 9 successive WAK tests on the 1.5-ft × 1.5-ft × 7.5-in. footing. Figure 7 (bottom) shows the average loading curve (average of four dial gauges: one at each corner of the footing) obtained on the same footing.

The stiffnesses shown, K33 and K50, were obtained from an intersection of the straight lines with the one-third and

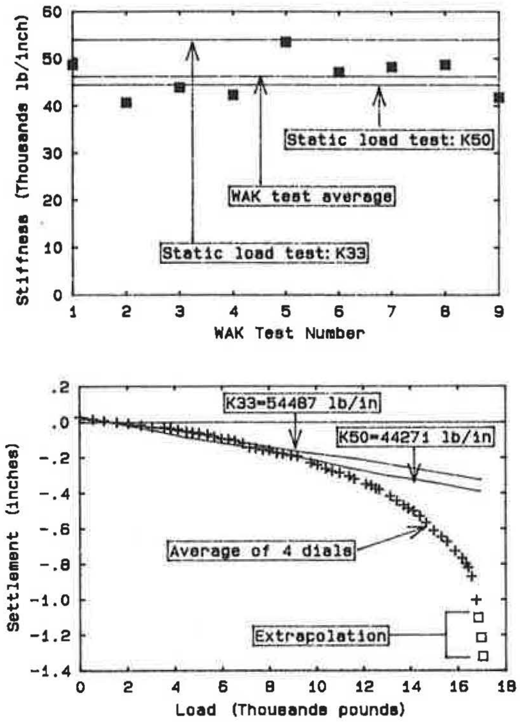

FIGURE 7 Soil-footing stiffness derived from nine successive WAK tests (*top*) and average loading curve obtained (*bottom*) on 1.5-ft × 1.5-ft × 7.5-in. footing.

one-half points of the curve, respectively. The points fall at one-third and one-half of the ultimate load, which was obtained through extrapolation. (Difficulties were encountered in obtaining measurements with settlements greater than 1 in.). This extrapolation is not results sensitive. The comparison indicates a good consistency between both tests.

Case 2: Application of WAK Test to Evaluation of Soil Strength after Dynamic Compaction

A landfill of rubbish, including everything from lumber to scrap metal, was covered with 22 in. of course gravel. Dynamic compaction was then used to improve the stiffness of the landfill in an attempt to allow building construction. The characteristics of the dynamic compaction mass are as follows: weight, 15 metric tons (16.5 tons); size, 1.5 × 1.5 × 0.9 m (5 × 5 × 3 ft); height of fall, 20 m (65.6 ft); and crater depth, 1 to 2 m (3 to 6 ft). A study was conducted to determine whether the WAK test would be a suitable means to monitor the progress of dynamic compaction. The test procedure involved nine steps:

1. Set the dynamic compaction mass on a new grid (the landfill was divided into grids for bookkeeping), and slack the crane's cable.
2. Place geophones at the opposite extremities of a diagonal of the mass.
3. Impact the mass with the test hammer, the mass of which is 5.4 kg (12 lb), and record data.
4. Remove the dynamic compaction mass.
5. Place a small concrete footing (1 ft × 1 ft × 6 in.) on the surface where the mass was set.
6. Fix the geophones at the extremities of a diagonal of the small footing.
7. Impact the footing with the test hammer, and record data.
8. Perform dynamic compaction.
9. Repeat steps 2 to 7, making sure that a good contact exists between the small footing and the bottom of the crater during steps 5 to 7.

Steps 5 to 7 were added to the intended procedure because of expected high noise-signal ratios when the compaction mass was struck directly. The data acquired from striking the small footing were used instead.

Figure 8 (top) shows that the measured soil stiffness is increasing with the number of drops of the dynamic compaction mass.

Thirty-one plate load tests were performed at the site with a 30-in.-diameter plate in accordance with ASTM D1194. The stiffness value (KPLATE) was determined from the beginning of the load settlement curve. Figure 8 (bottom) displays a comparison between these values and the corresponding ones derived from the WAK test (KWAK). The results match quite well.

Case 3: Application of WAK Test to Check Embedded Foundations

Even when an embedded foundation is correctly designed, its ability to support the design loads may be dangerously re-

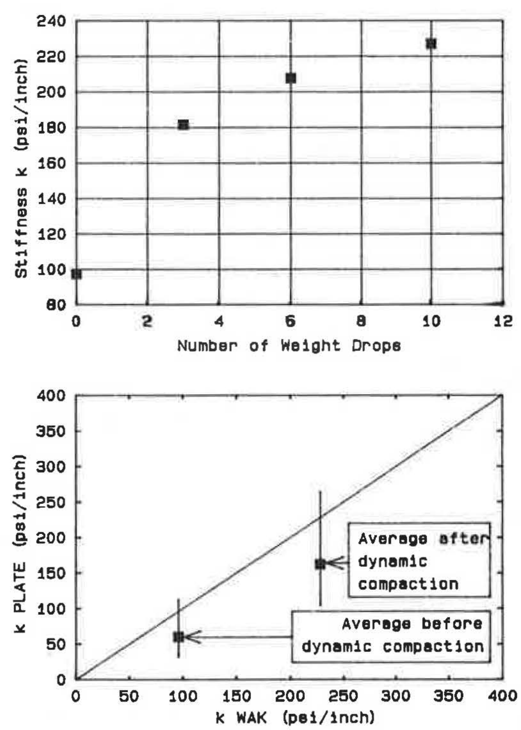

FIGURE 8 Soil stiffness versus number of drops (from WAK tests) (*top*) and stiffness values from plate tests versus WAK tests (*bottom*).

duced by errors made during construction. These may include an excavation that is too small, irrespective of the design dimensions, or an excavation that has not been properly cleared of excess rubble before casting. This observation especially applies to standardized foundations, such as foundations of pylons. Such faults are not detected through visual inspection. The WAK test could serve as a means to monitor the nonconformities to foundation design.

Investigations were conducted on the foundations of two identical concrete pylons. The first pylon was founded on a large (10 m^3) concrete block properly embedded in the soil, whereas the foundation of the second pylon was intentionally faulty: the excavation was too small (4 m^3) and uncleaned (Figure 9).

The WAK test was performed on both foundations and led to the following dynamic parameters: for the sound foundation,

$K_s = 1\ 10^9$ N/m
$M_s = 21\ 920$ kg

for the faulty foundation,

$K_f = 0.47\ 10^9$ N/m
$M_f = 9\ 370$ kg

The dynamic mass of the sound foundation (M_s) is approximately 2.3 times the mass of the faulty one (M_f), which is consistent with the known size of the excavations (10 and 4 m^3, respectively) and thus, with the actual mass of cast-in-place concrete. This difference in size can also be used to partly explain the difference between the dynamic stiffnesses (K_s and K_f) because this parameter is proportional to the size of the foundation (see Equation 8). The latter parameters were used to calculate the shear modulus of the soil (G_o) by using Equation 8. Although this equation takes into account the actual size of the foundation, the result of the calculation exhibits a significant gap between the value derived from the test performed on the sound foundation (G_o = 150 MPa) and the one derived from the test performed on the faulty foundation (G_o = 96 MPa). This difference can only be explained by the presence of disaggregated material between the concrete foundation and the surrounding soil (see Figure 9).

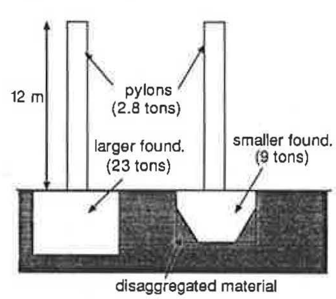

FIGURE 9 Sketch of two pylons with foundation.

Therefore, the WAK test appears as a discriminatory method to check standard foundations. Reliable information is given about the actual size of the foundations and the quality of their embedment in the surrounding soil.

CONCLUSIONS

A quick and inexpensive method was proposed by Briaud and Lepert (*2*) to measure the dynamic parameters of a spread foundation. Static characteristics (soil-foundation system stiffness, mass of the foundation) could be derived from these dynamic parameters. The first results, from laboratory tests, were encouraging.

The in situ experiments discussed here confirm these results. Furthermore, they show that the method can be useful in various contexts of geotechnical engineering: as a design control tool in the field for spread footings, a monitoring tool during dynamic compaction, and a construction control tool for embedded foundations.

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