Abridgment

Analytical Expressions for Uniaxial Tensile Strength of Concrete in Terms of Uniaxial Compressive Strength

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For the purpose of including reasonable values of tensile strengths in the available failure criterion (for frictional materials with effective cohesion), it may be necessary to include the uniaxial tensile strength in the parameter determination. A simple expression for evaluation of the uniaxial tensile strength on the basis of the uniaxial compressive strength is proposed. Different types of geological materials were included in the study. Also, on the basis of available experimental data on flexural and splitting tensile strengths of concrete, simple expressions for evaluation of these strengths as a function of uniaxial comprehensive strength are given. The correlation between the experimental results and analytical predictions are good and provide a simple approach for developing tensile strength models for plain concrete.

The uniaxial tensile strength (f_t), split cylinder tensile strength (f'_{sp}), and flexural tensile strength (f_r) have often been expressed as a fraction of the uniaxial cylindrical compressive strength (f'_c) and uniaxial cubical compressive strength (f_c). Mitchell (*1*) indicates that f_t (given as the flexural strength, which may be higher than the true value of f_t) for cemented soils is about 1/5 to 1/3 of f'_c, whereas data compiled by Hannant (*2*) show that f_t for concrete varies between 5 and 13 percent of f'_c. However, Bortolotti (*3,4*) shows from experimental results conducted by Shah and Ahmad (*5*) that linear relationships exist among the ratios of f'_c/f_t, $f_c c/f_t$, f'_{sp}/f'_c, and f_r/f_t as follows:

$$\frac{f'_c}{f_t} = 5.12 + 0.000844\, f'_c \quad (1)$$

$$\frac{f_c}{f_t} = 6.4 + 0.000844\, f_c \quad (2)$$

$$\frac{f'_{sp}}{f'_c} = 5.0 + 0.001\, f'_c \quad (3)$$

$$\frac{f_r}{f_t} = 1.2 + 0.0000167\, f'_c \quad (4)$$

Therefore, from Equations 1–4, the uniaxial strength tensile strength, split cylindrical tensile strength (direct tensile strength), and beam flexural tensile strength of concrete can be determined as a function of cylindrical compressive strength (f'_c) or as a function of cubical compressive strength (f_c), assuming that f'_c is equal to 0.8 f_c as follows:

$$f_t = \frac{f'_c}{(5.12 + 0.000844 f'_c)} \quad (5)$$

$$f_t = \frac{f_c}{(6.4 + 0.000844 f_c)} \quad (6)$$

$$f'_{sp} = \frac{f'_c}{(5.0 + 0.001 f'_c)} \quad (7)$$

$$f_r = (1.2 + 0.0000167\, f'_c)\, f_t \quad (8)$$

where f_t, f'_{sp}, f_r, f_c, *and* f'_c are given in psi. The value of f_t differs from f'_{sp} by about 10 percent, as shown in Figure 1.

TEST DATA AND EXPERIMENTAL RELATIONSHIPS BETWEEN CONCRETE STRENGTH PARAMETERS

Figure 1 (*4,5*) shows the plot of the experimental data and equations for predicting the split cylinder strength of concrete as a function of its cylindrical compressive strength. Figure 2 (*4,5*) shows the plot of the experimental data and equations for predicting the beam flexural tensile strength of concrete as a function of its cylindrical compressive strength. Figure 3 shows a comparison of the relationships between uniaxial ten-

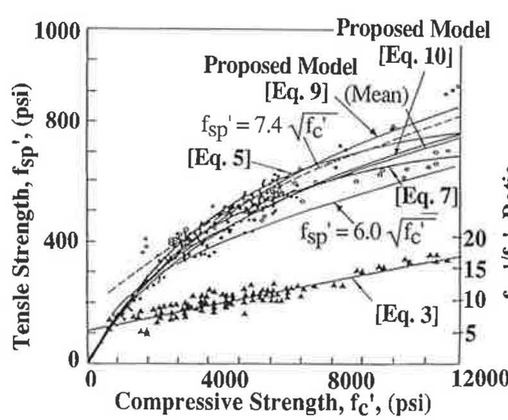

FIGURE 1 Split cylinder tensile strength (*4,5*).

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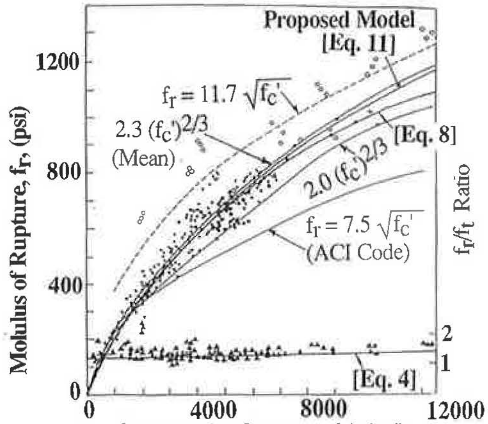

FIGURE 2 Beam flexural tensile strength (*4,5*).

sile and uniaxial compressive strengths for materials presented in Table 1.

PROPOSED TENSILE STRENGTH EXPRESSION

The proposed analytical expression of uniaxial strength (f_t) as a function of cylindrical compressive strength for concrete, on the basis of experimental results obtained by Shah and Ahmad (*5*), Salami (*6*), Salami and Desai (*7*), Egging (*8*), and Lade (*9*), is given as follows:

$$f_t = -mP_a\left(\frac{f_c'}{P_a}\right)^n \quad \text{(compression positive)} \qquad (9)$$

where m and n are dimensionless numbers (for plain concrete $m = 0.62$ and $n = 0.68$) and P_a is atmospheric pressure in the same units as f_t and f_c'. The proposed equation is shown in Figure 1. It is evident from the figure that the prediction from the proposed model compares well with experimental results and the model proposed by Bortolotti (*4*), which is also shown in Figure 1. Values of m and n have been determined for several frictional materials, which are presented in Table 1. These values represent the best fit between experimental data and the simple expression in Equation 9. A comparison of the relationships between uniaxial compressive strengths for the materials in Table 1 is shown in Figure 3. The straight lines shown on the log-log diagram span over the ranges of uniaxial compressive strengths indicated by available data. Both relatively weak and very strong frictional materials are presented in Figure 3. Note that the lines are clustered and slope away from the line representing equal uniaxial tensile and uniaxial compressive strengths. Thus, the weak materials have relatively higher uniaxial tensile strengths than the strong materials.

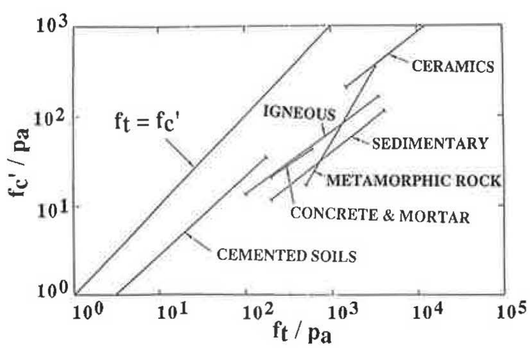

FIGURE 3 Relationships between uniaxial tensile strength and uniaxial compressive strengths for various types of frictional materials.

TABLE 1 VALUES OF PARAMETERS *m* AND *n* FOR VARIOUS TYPES OF FRICTIONAL MATERIALS

Material	*m*	*n*
Cemented soils (*1*)	0.37	0.88
Plain concrete (for f_t)	0.62	0.68
Plain concrete (for f'_{sp})	$\lambda = 0.56$	0.68
Plain concrete (for f_r)	$\alpha = 0.68$	$\beta = 0.71$
Igneous rocks (*9*)	0.53	0.70
Metamorphic rocks (*9*)	0.00082	1.60
Sedimentary rocks (*9*)	0.22	0.75
Ceramics (*9*)	1.0	0.73
Mortar (*10*)	0.61	0.67

PROPOSED EXPRESSION FOR SPLIT TENSILE STRENGTH

On the basis of experimental results obtained by Shah and Ahmad (*5*), Salami (*6*), and Salami and Desai (*7*) and Equation 9, the split tensile strength of concrete is given as follows:

$$f'_{sp} = -\lambda P_a\left(\frac{f_c'}{P_a}\right)^n \quad \text{(compression positive)} \qquad (10)$$

where λ and n are dimensionless numbers (for plain concrete $\lambda = 0.56$ and $n = 0.68$) and P_a is atmospheric pressure in the same units as those of f_t and f_c'. The proposed equation is shown in Figure 1. It is evident from the figure that the prediction from the proposed model compares well with experimental results and the model proposed by Bortolotti (*4*), which is also shown in Figure 1.

PROPOSED BEAM FLEXURAL TENSILE STRENGTH

Based on experimental results obtained by Shah and Ahmad (*5*), Salami (*6*), and Salami and Desai (*7*) and Equation 9 the flexural tensile strength is given as follows:

$$f_r = \alpha P_a\left(\frac{f_c'}{P_a}\right)^\beta \quad \text{(compression positive)} \qquad (11)$$

where α and β are dimensionless numbers (for plain concrete $\alpha = 0.69$ and $\beta = 0.68$) and P_a is atmospheric pressure in the same units as f_t and f_c'. The proposed equation is shown to afford a satisfactory fit of the experimental data plotted in

Figure 2. In addition, the proposed model compares well with the model proposed by Bortolotti (*4*), which is also shown in Figure 2.

CONCLUSION

For the purpose of including reasonable values of tensile strengths in the available failure criterion (for frictional materials with effective cohesion), it may be necessary to include the uniaxial tensile strength in the parameter determination. Simple expressions for evaluation of the uniaxial tensile strengths on the basis of the uniaxial compressive strength are given.

Uniaxial compressive and tensile tests were performed on plain concrete. The purpose of these tests was to acquire some understanding of the strength behavior of plain concrete subjected to compressive and tensile load histories, and the results of these tests were used to calibrate the proposed tensile strength model for predicting the tensile strength of plain concrete on the basis of experimental uniaxial compressive loading.

The proposed model has two material constants. Laboratory tests were performed to determine them. The proposed model predictions are shown in Figures 1 and 2. The correlation between the experimental results and analytical predictions are good and provide a simple approach for developing tensile strength models for plain concrete.

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NOTATION

The following symbols are used in this paper:

f'_c = uniaxial cylindrical compressive strength,
f_c = uniaxial cubical compressive strength,
f_t = direct uniaxial tensile strength,
f'_{sp} = split cylinder tensile strength,
f_r = beam flexural tensile strength,
m, n = dimensionless constants,
P_a = atmospheric pressure,
λ = dimensionless constant, and
α, β = dimensionless constants.

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