Dynamic Centrifuge Modeling of Geotechnical Structures

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The basic principles of dynamic centrifuge model testing are explained and some advantages and disadvantages of centrifuge modeling are described. Two examples of centrifuge model tests relevant to the performance of transportation structures during earthquakes are described: (*a*) a study of Struve Slough Bridge, which collapsed in the Loma Prieta Earthquake, and (*b*) mechanisms of liquefaction and development of sand boils. Two themes emerged from the examples cited. First, the results from centrifuge testing often provide an improved understanding of the deformation and failure mechanisms. Second, the improved understanding provides a basis for the development of simplified but adequate methods of analyzing full-scale geotechnical structures.

The similitude of the scale model testing is significantly enhanced in a centrifuge because the increased self weight produces identical stresses in model and prototype. Schofield (*1*) summarizes the principles of dynamic centrifuge modeling. Two recent volumes (*2,3*) containing about 80 papers indicate the broad scope of applications of centrifuge modeling.

The deformation of an element of soil depends on stress, strain, and time. The behavior of elements of soil under three-dimensional stress states and under cyclic loading is not fully understood. Additional questions arise regarding prediction capabilities for complex boundary value problems (e.g., embankments, bridge abutments, dams, pile foundations, retaining walls, consolidation, and seepage through aquifers) under complex loading conditions such as an earthquake. We have little data to show that our existing design procedures result in safe and economical designs. Large earthquakes such as the 1906 San Francisco earthquake occur so infrequently that it is difficult to obtain full-scale data to study them.

Direct modeling, in which researchers attempt to exactly simulate a particular prototype, has not often been the chosen approach of physical modelers. The model tests are usually treated as real events in themselves, and the results are interpreted accordingly. Using this approach, the centrifuge can provide data to directly observe failure mechanisms, calibrate design or analysis procedures, and conduct parametric studies. Physical models can be subjected to extreme loading conditions to study the response of structures during major earthquakes. Model tests are repeatable and economical, unlike the failures caused by real earthquakes.

Comparisons with full-scale field data are undoubtedly the most direct means of verification of a design or analysis procedure. No assumptions regarding particle size effects, strain rate effects, or the effects of confining pressures are needed if the actual prototype is tested. Full-scale data may be obtained in controlled field tests or by back analysis of the behavior or failure of an uncontrolled event. Difficulties with full-scale data are their cost and nonrepeatability. In the case of earthquake loading, the earthquakes studied are usually smaller than the design earthquake.

The high cost of full-scale tests precludes the possibility of conducting many experiments that cover the full range of variation of all important parameters. For example, laterally loaded piles may be in groups with different geometry, they may penetrate to various depths through layered soils, and they may be loaded with inclined eccentric loads. The matrix of possible parameters is very large compared with the number of full-scale tests that may be conducted.

In a sense, obtaining data for verification of analysis procedures by back analysis of the failure of a prototype is even more expensive. The failures usually involve significant property damage and loss of lives. Furthermore, the data obtained from unplanned failures are often difficult to interpret because of the uncertainty in determining the exact conditions before failure, the precise nature of the loading causing failure, and the absence of sufficient instrumentation to provide detailed data regarding the sequence of important events leading to the failure.

Model tests provide the luxury of repeatability. The generality of findings based on full-scale data is unknown. Changes in structure dimensions, soil profiles, and earthquake motion characteristics have a significant impact on response, and the impact cannot be adequately assessed by analysis of a few full-scale events.

Soils have stress-dependent stiffness, strength, and dilatancy. Geotechnical models are often tested on a centrifuge to obtain stresses in a small model identical to those that occur in a large prototype. Testing models on a centrifuge accounts for the stress dependency, improving the similarity between model and prototype. This makes extrapolation of data to field situations more accurate than is possible for scale model tests conducted in earth's gravity.

The centrifuge also permits certain gravity-driven phenomena to be accelerated in time. For example, consolidation of a clay layer that takes 1 year is modeled in about a 1-hr test at a centrifugal acceleration of 100 g.

CENTRIFUGE MODELING LAWS

The scale factor for length may be expressed as $L^* = 1/N$. The asterisk on a quantity refers to the scale factor for that

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quantity. Hence, L^* is the ratio of length in the model to length in the prototype. N is an arbitrary scale factor. In geotechnical centrifuge modeling, the vertical and horizontal length scale factors are identical.

When dealing with coarse-grained soils, it is sometimes suggested that the size of the particles should be scaled. As pointed out by Bolton and Lau (*4*), however, fine material at a similar density is likely to be stronger and more dilatant than coarse material. Partly for this reason, the same soil (at the same density and water content) is used in model and prototype. This also ensures that intergranular contact forces will be the same in model and prototype (since both are subject to the same stresses), helping to ensure that the soil properties will be the same in model and prototype. If the same soils are used in model and prototype, the scale factor for density is $\rho^* = 1$.

The scale factor for gravity is $g^* = N$. That is, gravity is N times larger in the model than in the prototype. If a model is made 100 times smaller than the prototype (i.e., $N = 100$), and it is tested in a gravity field that is 100 times greater than earth's gravity, the stresses due to gravity loading would be identical in model and prototype. Of course, it is not really feasible to produce a large gravitational field, but a centrifuge can be used to provide a large acceleration field. The inertia forces produced by spinning a model around an axis are similar to the gravitational forces that develop in a large prototype.

From the scale factors for length, gravity, and density, the scaling relationships for other physical quantities such as mass, force, stress, strain, and time can be derived. For example, the scale factor for mass follows from the relation that a density times a volume must equal a mass ($m = \rho L^3$):

$$m^* = \rho^* L^{*3} = (1)(N^{-1})^3 = N^{-3} \tag{1}$$

From Newton's law of gravitation, the scale law for force is

$$F^* = m^* g^* = (N^{-3})(N) = N^{-2} \tag{2}$$

The scale factor for stress must then be

$$\sigma^* = F^*/L^{*2} = (N^{-2})(N)^2 = 1 \tag{3}$$

This confirms that if the same materials are used in model and prototype, and if gravity is increased in the same proportion that length dimensions are reduced, the stresses obtained in model and prototype will be identical.

If strains within the model are only a function of the stresses, it follows that the strains will also be identical in model and prototype:

$$\varepsilon^* = 1 \tag{4}$$

Of course, the strength and stiffness of a soil are not only a function of the current stresses in the soil; they are also a function of the stress history. In the development of a model, then, it is necessary to simulate the stress history. This may be accomplished by appropriately preconsolidating a soil layer in the laboratory and attempting to simulate the complete construction sequence during the testing of a centrifuge model.

If the relationship between stress and strain is time dependent, the scaling of stresses and stains is more difficult. The assumption of rate-independent mechanical properties is embedded in the preceding derivation of scale factors.

Also, embedded in terms such as stress and strain are the assumptions of continuum mechanics; particle size effects are not considered. It seems plausible that as long as the ratio of model dimensions to the particle dimensions is "very large," the soil may be assumed to be a continuum. But how large is "very large"? The answer to this question depends on the type of problem being studied; ideally it would be answered for every model study.

The scale factors for time are discussed in the following for three important categories of problems: static, diffusion, and dynamic. In a static problem (for example the settlement of a footing on dry sand), the scale factor for time is not important. The rates of application and duration of loading need not be precisely scaled.

In a diffusion problem (such as consolidation, heat flow, or contaminant transport) the problem is governed by a differential equation of the form

$$\partial u/\partial t = c_v \partial^2 u/\partial z^2 \tag{5}$$

In Equation 5, u may represent pore pressure, temperature, or pollutant concentration. t represents time, and c_v is a material property: the coefficient of consolidation or diffusion coefficient. z represents a spatial coordinate that scales like any length dimension. The pore water pressure is a hydrostatic stress, and it follows the previously derived scale law for stress: $u^* = \sigma^* = 1$. By inspection of Equation 5,

$$u^* t^{*-1} = c_v^* u^* L^{*-2} \tag{6}$$

$$t_{\text{dif}}^* = L^{*2} c_v^{*-1} = (N^{-1})^2 c_v^{*-1} = N^{-2} c_v^{*-1} \tag{7}$$

If the same materials are used in model and prototype, c_v will be the same in model and prototype, and $c_v^* = 1$. Therefore, for diffusion problems,

$$t_{\text{dif}}^* = N^{-2} \tag{8}$$

The time required for diffusion processes to occur in the model is N^2 times less in the model and prototype.

Alternatively, the model can be thought of as a simulation of a prototype with a different soil, one with a higher diffusion coefficient. In other words, the diffusion coefficient scales as $c_v^* = N^{-1}$. Fine (impermeable) sand can be thought to represent a coarse sand, or a model with silicon oil as a pore fluid may represent the same soil with water as a pore fluid. If c_v is scaled by a factor of N, the time scale factor for diffusion problems becomes

$$t_{\text{dif}}^* = N^{-1} \tag{9}$$

In dynamic problems it is important that the acceleration of the model increases in the same proportion as the gravitational acceleration. Therefore

$$a^* = g^* = N \tag{10}$$

TABLE 1 SCALE FACTORS FOR CENTRIFUGE MODEL TESTS

Quantity	Symbol	Units	Scale Factor
Length	L	L	N^{-1}
Volume	v	L^3	N^{-3}
Mass	M	M	N^{-3}
Gravity	g	LT^{-2}	N
Force	F	MLT^{-2}	N^{-2}
Stress	σ	$ML^{-1}T^{-2}$	1
Moduli	E	$ML^{-1}T^{-2}$	1
Strength	s	$ML^{-1}T^{-2}$	1
Acceleration	a	LT^{-2}	N
Time (dynamic)	t_{dyn}	T	N^{-1}
Frequency	f	T^{-1}	N
Time (diffusion) *	t_{dif}	T	N^{-1} or N^{-2}

*The diffusion time scale factor depends on whether the diffusion coefficient (e.g., coefficient of consolidation) is scaled. If the same soil is used in model and prototype, $t_{dif}^* = N^{-2}$.

Since acceleration has units of L/t^2,

$$t^{*2} = L^*/a^* = N^{-2} \tag{11}$$

$$t^*_{dyn} = N^{-1} \tag{12}$$

Dynamic events occur N times faster in the model than in the prototype. Clearly, unless the coefficient of consolidation is scaled, we have different time scale factors depending on the type of phenomena that is occurring in the model. In most cases, it is clear that the problem is dominated either by dynamic loading or by diffusion. If it is so clear, a modeler simply chooses the appropriate factor. On the other hand, the liquefaction of permeable soils may result in simultaneous dynamic generation of pore pressures (due to cyclic shear strain) and dissipation of pore water pressure, which is governed by diffusion. In this case, it is necessary to scale the coefficient of consolidation. Table 1 summarizes the centrifuge scaling laws.

Figure 1 shows sketches of the shaker mounted on the small centrifuge at Davis. This figure includes a model of a bridge with a pile foundation. The container is made from aluminum and has Plexiglas side walls that permit viewing of a cross section of the model by photography or video cameras. The shaker is mounted on a swinging platform that hangs downward in earth's gravity and gradually swings up as the centrifugal acceleration is increased. The net g-vector, due to addition of earth's gravity and the radial centrifugal acceleration, remains perpendicular to the platform, so the sample will not spill as the bucket swings up.

When the centrifuge acceleration reaches the desired level, and after the pore pressures in the sample are given sufficient time to come into equilibrium, a simulated earthquake can be triggered. At Davis, this is accomplished by pressing a key on a computer. The computer then sends the desired displacement history in analog form to an electronic servocontroller, which in turn sends command signals to the servovalve. The servocontroller receives feedback from a displacement transducer on the model container and performs corrections to compensate for errors. A typical acceleration time history and the corresponding spectral accelerations for the base motion are shown in Figure 2. This figure shows that the desired motion corresponds very well with the actual achieved base motion. The desired acceleration time history shown in Figure 2 was obtained from measured accelerations at Corralitos during the 1989 Loma Prieta earthquake. These data were integrated twice, filtered, and base line corrected to obtain the desired displacement history.

The models are typically instrumented with accelerometers, strain gauges, pore water pressure transducers, and displacement transducers. The same computer that controls the shaker also records data from the experiments. Sixteen channels can be recorded simultaneously. If desired, additional simulated earthquakes may be triggered before stopping the centrifuge. The additional earthquakes may be scaled versions of the initial motion, or they may be completely different motions, such as a sine wave, El Centro, or San Fernando earthquake simulations.

SEISMIC RESPONSE OF TRANSPORTATION STRUCTURES

Some of the recent transportation-related studies at Davis are briefly outlined: modeling of the response of the Struve Slough Bridge during the Loma Prieta earthquake and liquefaction of stratified level ground.

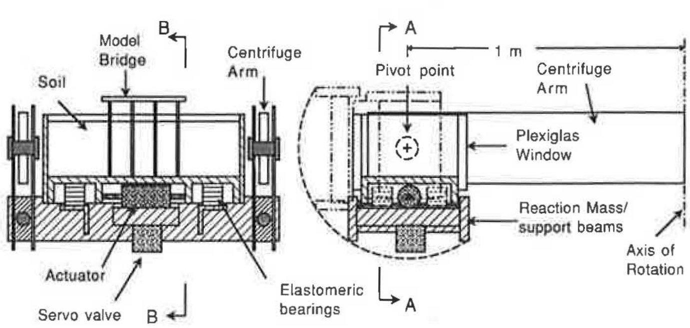

FIGURE 1 Earthquake simulator on the small centrifuge at Davis: *a*, view looking radially inward; *b*, side view showing how the bucket swings up as the centrifuge speed increases.

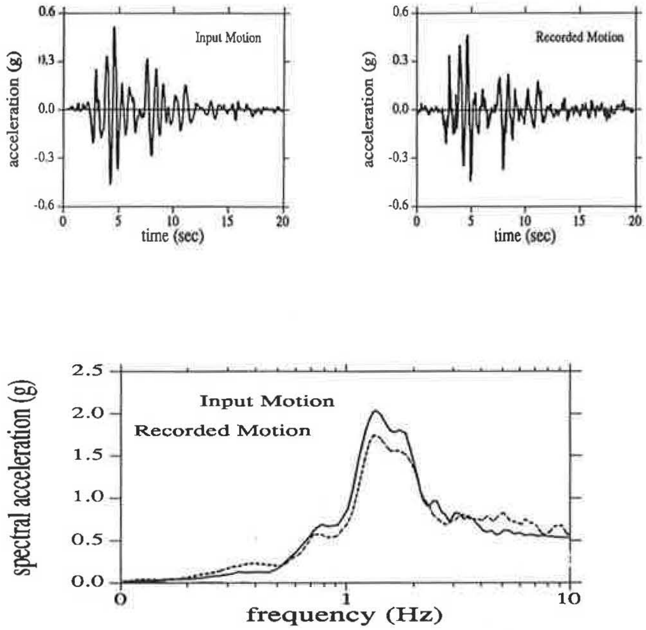

FIGURE 2 Comparison of desired and achieved accelerations: *top left,* desired (input) time history; *top right,* recorded time history; and *bottom,* a comparison of 5 percent damped response spectra.

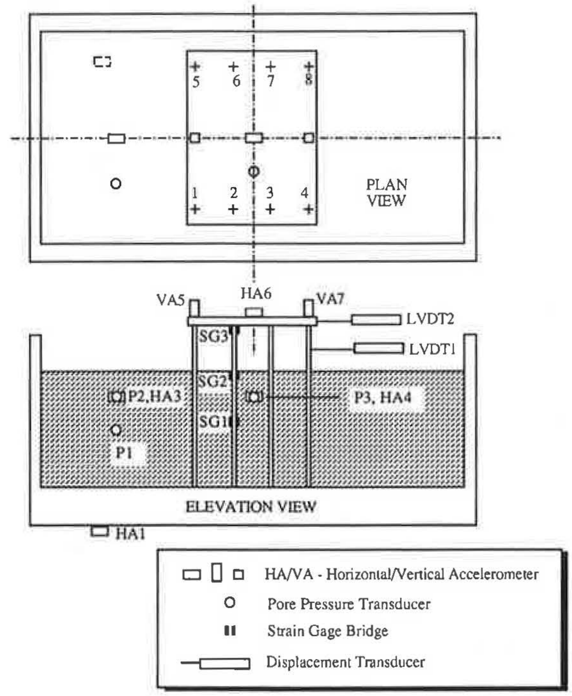

FIGURE 3 Instrument locations for model simulation of Struve Slough Bridge.

Modeling the Failure of the Struve Slough Bridge

The series of tests described in the following demonstrates how the centrifuge can be used to calibrate and develop a method of analyzing a soil-structure interaction problem. Cafe (*5*) describes the research on Struve Slough Bridge in more detail.

Figure 3 shows the model in the centrifuge container. This test was conducted at a centrifuge acceleration of 60 *g*. The spacing of the model piles is 5.1 cm (3.06 m), the diameter is 0.6 cm (0.36 m), and length is 21.3 cm (12.8 m), of which 6 cm (3.6 m) extends above the soil. (The dimensions are given as model dimensions with prototype dimensions in parentheses.) The model container is 56 cm (33.6 m) long and 28 cm (16.8 m) wide. Figure 4 shows a sketch of one bent of the actual bridge. The Struve Slough Bridge is approximately 230 m long, consisting of two separate bridges 10.4 m wide on Highway 1 in Watsonville, California. Each bridge consisted of 22 bents equally spaced at 11.3 m. The skew of the bents was not considered in the model tests. Each bent is supported by four Raymond Can step-tapered piles, which are each extended up to the bridge deck with 0.4-m diameter pile extensions. From the surface downward, the soil profile at midspan consists of 9.1 m of very soft peat with some clay, 4.4 m of soft silty clay with peat, 10.7 m of stiff silty clay, and a layer of medium dense sand with gravel (into which the piles were driven).

Preliminary foundation analyses assuming classical beam on elastic foundation theory were conducted on the prototype piles. It was found that the lateral deflections of the pile below the very soft peat layer were insignificant. It was therefore decided to physically model the piles as being fixed at the base of the peat; the model piles were screwed into an aluminum bar and fixed to the base of the model container. The 9.1 m of peat was the only soil layer included in the model. This soil was collected at the site in disturbed samples and placed in the model at a moisture content of 93 percent.

The model piles were made of 3.2-mm-diameter annealed stainless steel rods and covered in 6.4-mm-diameter soft rubber tubing. This composite pile design was used in order to approximately simulate the correct bending stiffness, moment capacity, and diameter of the prototype piles. The models were instrumented with accelerometers, pore pressure transducers, and displacement transducers. One pile was instrumented with three sets of strain gauges to monitor the bending of the pile at three locations.

In the Loma Prieta earthquake, one of the bridge decks completely collapsed, and some of the broken piles punched through the bridge deck. The other bridge was severely damaged. Failure of the pile extensions at the connection to the bridge deck was obvious, and it appeared that some of the

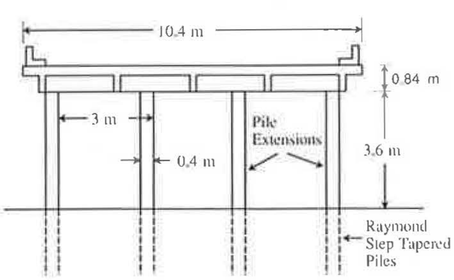

FIGURE 4 Typical cross section of the actual Struve Slough Bridge.

piles may have failed at some depth as well. Large gaps formed around the piles at the ground surface, ranging between about 12 to 25 cm near the middle of the bridge where the peat deposit was the thickest. Similar gaps were also observed to form around the piles in the centrifuge tests.

The analytical model developed to numerically predict the bridge response is shown in Figure 5. The solution was obtained using a linear finite element program called BEAM1DYN. Because of the large spacing of the prototype piles, group effects were neglected. Each pile and extension was modeled as a series of beam elements with a lumped mass. The mass of the bridge deck was attached to the top element of the pile. Viscous damping was introduced as shown at each node. The earthquake motion was introduced at the base of the piles and at springs at each node within the soil. The value of the springs was determined on the basis of an equation provided by Vesic (*6*).

The unique feature of the new procedure is that the input motions of the soil along the pile, $u_s(i)$, are each different. $u_s(i)$ represents the time history of displacement at the *i*th node. The values of $u_s(i)$ were calculated for the free field shear beam using the computer program SHAKE (*7*). A motion of the base of the centrifuge model container obtained from one of the centrifuge model earthquakes was input to a SHAKE analysis of a layer of peat. The analysis provided the acceleration time history at several points within the layer. These accelerations were integrated twice to obtain a displacement record, and after appropriate baseline corrections the displacement history was used as input to BEAM1DYN.

The Young's modulus of the peat (based on a variety of tests) was taken to be 110 kPa, and the Poisson's ratio was assumed to be 0.3. The unit weight of the peat was only 10.3 kN/m^3. Figure 6 compares the displacement of the pile extension relative to the base motion measured in the centrifuge test (LVDT1) and the value predicted by BEAM1DYN using the input motion shown in Figure 2. The peak values of the displacement are reasonably predicted, but the frequency content is not precisely matched. The magnitude of displacement is consistent with the field observation that gaps of 12 to 25 cm formed around the prototype piles after the Loma Prieta earthquake. It appears that the damping in the model test was somewhat larger than that simulated in the prediction. The difference in apparent damping is attributable to nonlinearity, which is present in the experiment, but not in the analysis.

Figure 7 compares the peak positive and negative bending moment distribution along the length of a pile as measured in the centrifuge and predicted using BEAM1DYN. The agreement is remarkable, providing some verification of the proposed analytical model. It should be added, however, that some trial and error selection of parameters, especially for the shear modulus and damping parameters for the peat, was required. The need for trial and error adjustment of the parameters points to the usefulness of the centrifuge for "calibrating" a numerical model.

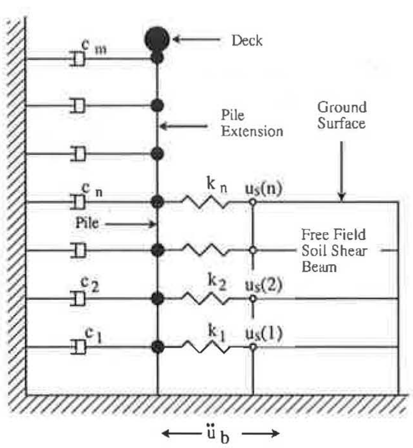

FIGURE 5 Schematic representation of the analytical model used in the computer program BEAM1DYN to predict the response of the Struve Slough Bridge model.

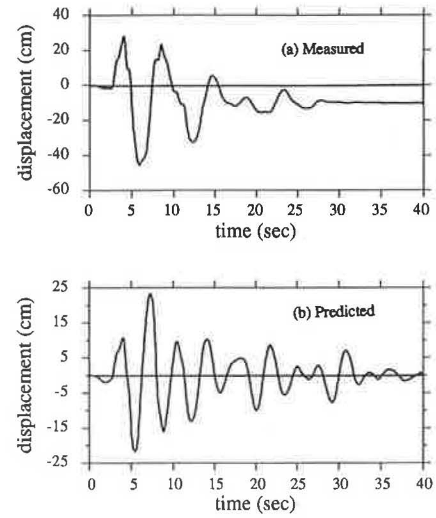

FIGURE 6 Comparison of predicted and measured displacement response: *a*, measured in the centrifuge test by LVDT1; *b*, predicted using BEAM1DYN.

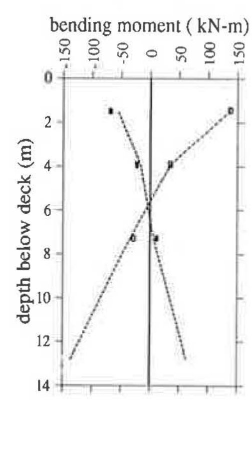

FIGURE 7 Distribution of maximum and minimum bending moment along the length of the pile and pile extension. Experimental data are given by the points, and the predicted moment distribution is represented by the dotted line.

Liquefaction and Sand Boil Mechanisms

Liquefaction and sand boils can cause significant damage to pavements. Such damage was caused by the 1989 Loma Prieta earthquake at the Oakland International Airport, in the Marina District in San Francisco, at the approach to the Oakland Bay Bridge, at the Port of Oakland, and in the city of Santa Cruz.

Centrifuge tests have been conducted to investigate the mechanisms and consequences of liquefaction by many researchers. Some examples of this work are Scott (*8*), Schofield (*1*), and Whitman et al. (*9*). In this paper some recent work involving liquefaction and sand boils in a layered soil is presented. This work is more completely presented by Kutter and Fiegel (*10*) and Fiegel (*11*). The work presented here is part of the author's contribution to VELACS, a collaborative project involving many universities [Arulanandan et al. (*12*)].

The model tested is shown in Figure 8. It consists of a relatively impermeable silica flour (silt) layer overlying a layer of Nevada sand (D_{50} = 0.15 mm, D_r = 60 percent). The layer of silt was thickest around the edges of the sample to prevent leakage along the sides of the model. The silt surface represented a level prototype, but the interface between the silt and sand was sloped to produce the thinnest silt section at the center of the sample. The sample was shaken with a base acceleration history similar to that recorded in the El Centro earthquake but with the acceleration scaled to a peak of 0.65 *g*. During this event, the pore pressures increased to equal the total overburden stress at all locations in the sand and silt.

The excess pore pressures in the sand rapidly dropped off after shaking stopped. This pore pressure dissipation is associated with settlement of the sand. The settlement of the sand results in expulsion of water that collects at the interface since the silt is relatively impermeable. The collection of water at the nonlevel interface produces an unstable situation. This is shown in Figure 9 by the surface displacement contours as recorded by the LVDTs. During shaking, most of the LVDTs record a small settlement, but the contours show that after some time, the center of the sample begins to bulge upward. This bulging can be explained by the fact that the silt layer was thinnest at the center of the sample. The water that collects at the interface flows along the interface to the lightest silt section.

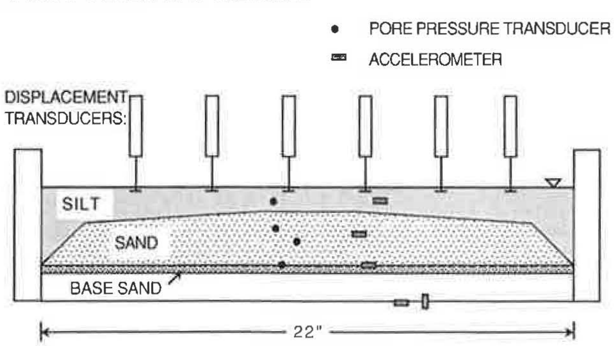

FIGURE 8 Centrifuge model used to study the mechanisms of liquefaction.

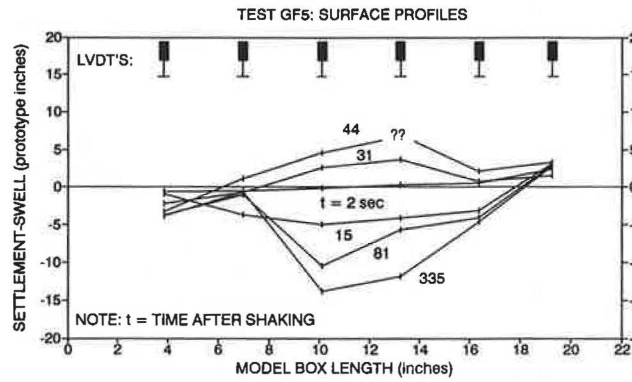

FIGURE 9 Profiles of surface displacement at various times after the beginning of shaking. The surface first settled, then heaved in the middle, and ultimately settled again.

Figure 10 shows the mechanism described. The bulging continues until a crack forms in the overlying silt. Once a crack forms, the water begins to leak through, and if the flow velocities are sufficient, silt and sand in the vicinity of the crack are eroded and carried to the surface in the form of boils.

Figure 11 shows one of the boils observed in the tests. The irregular layer of dark material near the surface of the silt is the initial ground surface. All of the material above this dark line was carried to the surface by the boil. A swirl of sand has penetrated up toward the surface and a layer of sand has been deposited around the mouth of the boil. Because of the process of preparation of the sample, a natural layering is noticeable within the silt. These layers curve downward in the

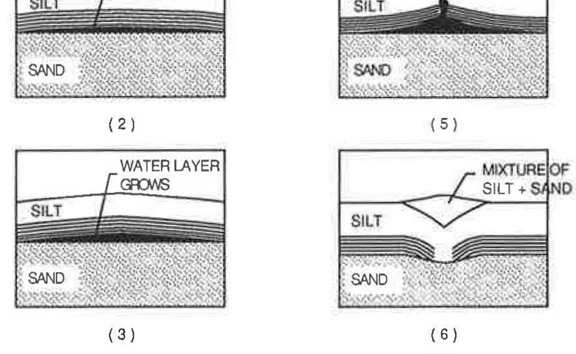

FIGURE 10 Possible sequence of events involving surface heave and eruption of sand boils.

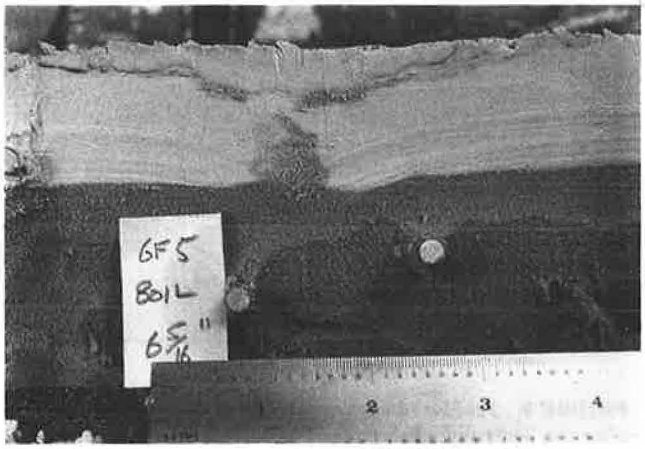

FIGURE 11 Sand boil exposed during excavation after the centrifuge test.

vicinity of the boil because of erosion of underlying silt and sand at the interface by water flowing toward the boil.

CONCLUSIONS

The well-established scaling laws for model testing on a centrifuge have been summarized, and some difficulties have been mentioned. Identical stresses in model and prototype permit the stress-dependent material properties of soils to be accurately simulated.

A new procedure for dynamic analysis of the lateral behavior of piles has been presented and calibrated using centrifuge test data. A mechanism of liquefaction of layered soils is clear after studying a few carefully planned centrifuge model tests. By careful dissection of the sample and consideration of the detailed measurements that are possible, a complex mechanism becomes understandable.

From the examples of centrifuge model tests presented, the centrifuge is shown to be useful in the following ways.

1. Analytical models can be developed on the basis of observed behavior in centrifuge tests.
2. Numerical procedures can be calibrated by comparison of predictions and measurements.
3. Surprising mechanisms, such as local surface heave observed in the liquefaction tests, can be discovered.

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