Lateral Subgrade Modulus of Sands for Deep Foundations

DANIEL O. WONG

The common practice of designing laterally loaded deep foundations is either by means of the lateral subgrade modulus concept or by the lateral load transfer method. The lateral subgrade modulus for sands is a function of several factors including deflection, which itself is usually an unknown. The lateral load transfer method is readily available to analyze laterally loaded deep foundations by using *p-y* curves. However, *p-y* curves are complicated mathematical relations and by no means offer a simple representation of the pile-soil interaction. The concept of the equivalent subgrade modulus for sands, in which the nonlinear pile-soil characteristics are implicitly taken into account, is presented. Relationships of equivalent subgrade modulus versus dimensionless lateral load factor for sands are developed, and a design procedure is proposed. Comparison of solutions by the proposed design procedure and the lateral load transfer method is favorable.

The design of laterally loaded deep foundations typically requires the prediction of lateral deflection and induced bending moment of the foundation elements under applied loads. The analyses can be performed by means of the lateral subgrade modulus concept. The lateral subgrade modulus is a linear relationship of soil pressure and deflection. However, the lateral subgrade modulus is a function of depth for cohesionless soils (*1*). McClelland and Focht (*2*) also showed that soil responses to lateral loading are dependent on depth and pile deflection. A representative value of the lateral subgrade modulus may be assumed for a particular problem (*3*). A more rational approach to analyzing laterally loaded piles is to model the soils by uncoupled, nonlinear load transfer functions, widely known as *p-y* curves. A *p-y* relationship is a mathematical representation of the soil reaction and the lateral pile deflection per unit length at a particular location along the pile. A linear subgrade modulus with respect to depth can be obtained by an iterative procedure using *p-y* data (*4*). Numerical procedures using finite difference or finite element algorithms have been developed to incorporate the *p-y* criteria in the analyses of laterally loaded piles (*5,6*). However, such procedures require extensive numerical procedures such that the linear subgrade modulus method may be preferred in preliminary computations or for relatively simple design cases. Furthermore, a single subgrade modulus for a particular problem usually provides a better overall feeling or understanding of the pile-soil interaction than a series of *p-y* curves. Attempts have been made in the past to develop a simplified method for analyses of laterally loaded piles (*7*). The work presented offers a unique subgrade modulus for sands describing the pile-soil interaction for a specific laterally loaded pile condition.

McBride-Ratcliff and Associates, Inc., 7220 Langtry, Houston, Tex. 77040.

The development of the equivalent subgrade modulus, k_{eq}, for sands under lateral loads is described. The equivalent subgrade modulus is a pile-soil relationship incorporating implicitly the effects of the lateral load and the nonlinearity of pile-soil interaction. A parametric study was conducted to develop the appropriate equivalent subgrade modulus by backcalculating with Broms's equation (*3*) using the required deflection obtained from a numerical model for the laterally loaded pile problem—COM 624 (*6*). Such methodology was used successfully to develop the equivalent subgrade modulus for laterally loaded piles in clays (*8*). Among the parameters investigated in this study are lateral load, pile stiffness, and soil properties. Both submerged and above-water conditions are considered. The design curves of k_{eq} versus the dimensionless lateral load factor for cohesionless soils are subsequently developed and a simple design procedure is proposed. Solutions for maximum pile deflection and maximum bending moment for laterally loaded piles in sands can be readily solved with accuracy comparable with the load transfer method.

BROMS'S EQUATIONS AND NONDIMENSIONAL SOLUTIONS

Presented in this section are the brief synopses of Broms's equations and the nondimensional solutions for solving laterally loaded piles in sands.

Broms's Equations

Broms (*3*) stated that for piles with the dimensionless depth of embedment of ηL larger than 4.0, the magnitude of the lateral deflection at the ground surface is unaffected by a change of the embedment length, L. η is defined as

$$\eta = \left(\frac{k}{EI}\right)^{0.2} \tag{1}$$

where k is the coefficient of subgrade reaction, assumed to be a function of relative density of soil only, and EI is the bending stiffness of the pile.

Terzaghi (*1*) has shown that the horizontal coefficient of subgrade reaction k_h at depth Z, for a pile with diameter B, in sand can be found as

$$k_h = k\frac{Z}{B} \tag{2}$$

Broms recommended that the lateral deflection at the ground surface, y_o, for a free-head long pile ($\eta L > 4.0$) be calculated as

$$y_o = \frac{2.40P}{k^{0.6}(EI)^{0.4}} \tag{3}$$

where P is the applied lateral load at the ground surface.

For a fully restrained long pile, where the slope at the pile head remains zero, the deflection at the ground surface is determined by

$$y_o = \frac{0.93P}{k^{0.6}(EI)^{0.4}} \tag{4}$$

Nondimensional Solutions

Nondimensional solutions for laterally loaded piles require an iterative procedure to achieve convergence of the relative stiffness factor, T, which is the reciprocal of η in Equation 1:

$$T = (EI/k)^{0.2} \tag{5}$$

The lateral deflection, y, and bending moment, M, of the pile can be obtained from the following equations:

$$y = A_y \frac{PT^3}{EI} + B_y \frac{MT^2}{EI} \tag{6}$$

$$\mathrm{M} = A_m PT + B_m M \tag{7}$$

where A_y and B_y are deflection coefficients due to the applied lateral load P and the applied moment M, respectively, and A_m and B_m are moment coefficients due to the applied lateral load P and the applied moment M, respectively.

Depending on the value of the maximum depth coefficient, Z_{max}, the deflection and moment coefficients can be obtained at any depth along the pile. Coefficient charts are available in Matlock and Reese's paper (*4*). Z_{max} is defined as

$$Z_{max} = \frac{L}{T} \tag{8}$$

In the absence of an applied moment at the pile head, the maximum deflection and moment coefficients can be found in Table 1.

Using the maximum coefficients in Table 1, the maximum deflection and maximum moment of piles under the lateral loads can be calculated from Equations 6 and 7, respectively.

Both Broms's equations and the nondimensional solutions describe the soil resistance by a modulus k, which, as mentioned previously, is a function of the depth and magnitude of the lateral load. The estimation of the k value is usually a challenge to practicing engineers but is essential to the accuracy of the solution. This paper presents a representative k value, which implicitly accounts for the effects of pile depth, pile diameter, and the applied lateral loads, to be used directly in the Broms and nondimensional equations.

LOAD TRANSFER RELATIONSHIPS FOR SANDS

Nonlinear lateral load transfer relationships, termed *p-y* curves, are often used to represent the soil responses subjected to lateral loading. A brief description of four procedures to construct *p-y* curves for piles in sands, as presented by Murchison and O'Neill (*9*), is presented in the subsequent paragraphs. Details of the four procedures can be found in their respective references.

Procedure of Reese et al.

Reese et al. (*10*) introduced a *p-y* method based on the results of a series of field tests. A *p-y* curve constructed from this method consists of four segments as shown in Figure 1. The first segment is a linear relationship with a slope of kZ up to a point that can be determined by an empirical relationship, where Z is the depth of interest. k is a soil modulus and can be determined from the standard penetration test blow count. Correlations shown in Figures 2 and 3 are recommended. Figures 2 and 3 are modified by Murchison and O'Neill (*9*) from Gibbs and Holtz (*11*) and Meyer and Reese (*12*), respectively. The second segment is a parabola and terminates at a deflection of $B/60$, where B is the diameter of the pile. The curve continues in a straight line with a slope empirically determined and terminates at a deflection of $3B/80$ with soil resistance reaching an ultimate value, p_u. p_u remains unchanged as deflection increases for the fourth segment in the *p-y* curve. The ultimate soil resistance per unit of depth, p_u, can be calculated as the lesser value of Equations 9 and 10 and modified by an empirical parameter involving the pile, soil, and loading conditions.

$$p_u = \gamma z[B(K_p - K_a) + ZK_p \tan \phi \tan \beta] \tag{9}$$

$$p_u = \gamma BZ[K_p^3 + 2K_o K_p^2 \tan \phi + \tan \phi - K_a] \tag{10}$$

TABLE 1 SUMMARY OF MAXIMUM DEFLECTION AND MOMENT COEFFICIENTS DUE TO LATERAL LOAD ONLY (*4*)

Z_{max}	A_y		A_m	
	Free-head	Restrained-Head (Slope=0.0)	Free-head	Restrained-Head (Slope=0.0)
10	2.435	0.820	0.772	0.930
4	2.445	0.829	0.767	0.930
3	2.723	0.968	0.704	0.970
2.2	4.011	1.268	0.557	1.060

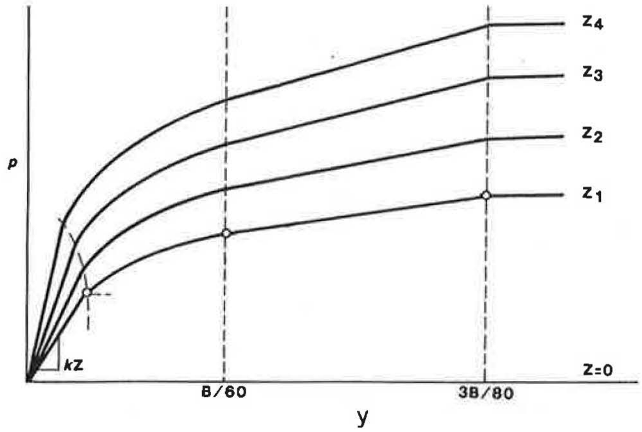

FIGURE 1 Characteristic shape of a family of *p-y* curves for procedure of Reese et al. (*10*).

where γ is the unit weight of soils and K_a and K_p are Rankine active and passive coefficient, respectively. K_o is the at-rest earth pressure coefficient, ϕ is the frictional angle, and β is determined as $45 + \phi/2$.

Bogard and Matlock's Procedure

This method (*13*) called for a modification of the first method by simplifying the calculation of p_u and using nondimensional charts to generate *p-y* curves. p_u can be determined as the minimum of the values given in Equations 11 and 12.

$$p_u = (C_1 Z + C_2 B)\gamma Z \tag{11}$$

$$p_u = C_3 B \gamma Z \tag{12}$$

The parameters C_1, C_2, and C_3 are related to the angle of internal friction of sands. The *p-y* curves are subsequently constructed using normalized charts.

Scott's Procedure

Scott (*14*) idealized the *p-y* curve into two linear segments. The first straight line segment terminates at a resistance value p_k with a slope of kZ. p_k can be determined from design curves of normalized resistance versus angle of internal friction; k can be found from the relationship presented in Figure 3. The second straight line segment starts with p_k with an empirically determined slope of $kZ/4$. There is no ultimate soil resistance value defined in this method.

Murchison and O'Neill Procedure

The *p-y* relationship characterized by this procedure (*9*) is a continuous hyperbolic tangent curve. This method is a reformulation of Parker's recommendation (*15*) on his experimental and analytical study of small diameter piles in sands. Murchison and O'Neill suggested that the characteristic *p-y* curve can be represented as

$$p = \eta A p_u \tanh\left[\left(\frac{kZ}{A\eta P_u}\right)y\right] \tag{13}$$

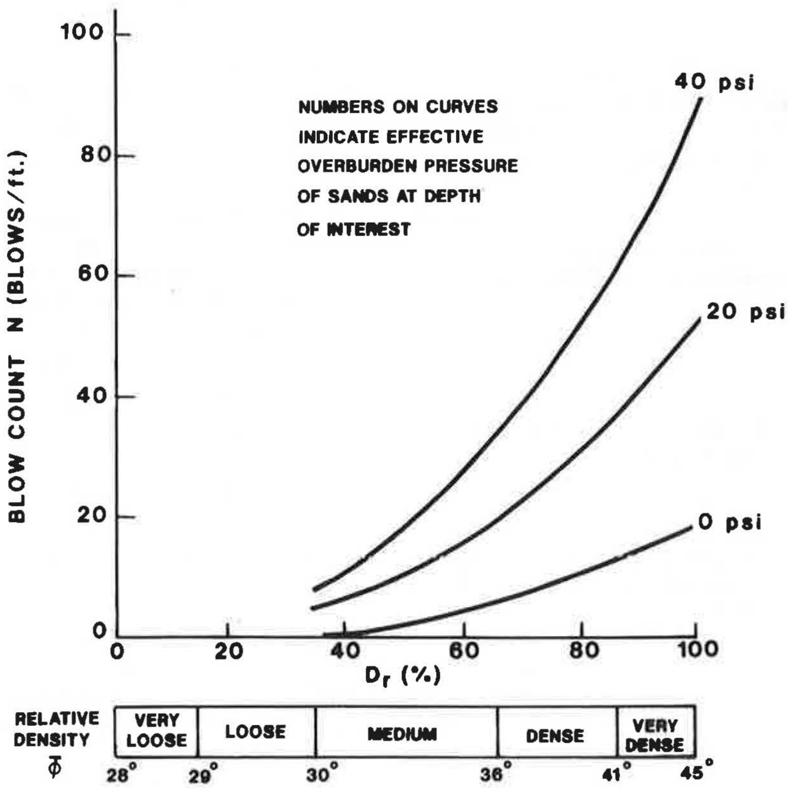

FIGURE 2 SPT blow count versus angle of friction and relative density of sands (*9*).

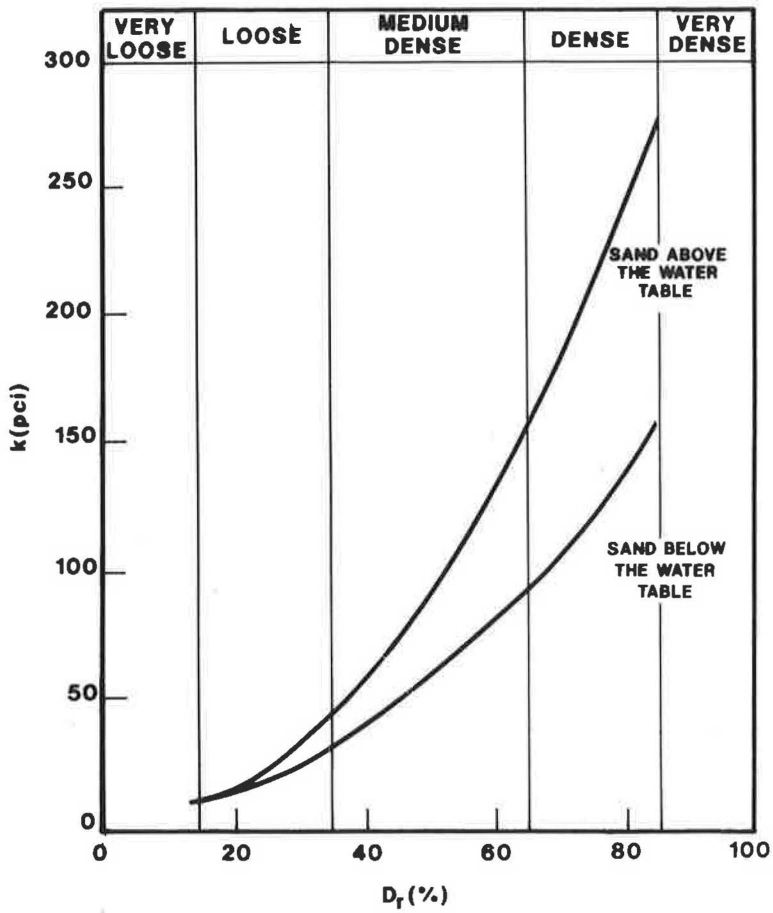

FIGURE 3 *k* versus relative density (*9*).

where p_u is the unmodified ultimate soil resistance found in Equations 9 and 10. η is taken as 1.5 for uniformly tapered piles and 1.0 for circular, prismatic piles. A is a factor related to the diameter of pile, B, and depth Z as

$$A = 3 - 0.8Z/B \geq 9 \quad \text{for static loading} \tag{14}$$

The shape of the *p-y* curve generated from this procedure is similar to that from the procedure of Reese et al. except that it is not a piecewise curve but a continuous analytical function.

PARAMETRIC STUDY

A parametric study was performed on hypothetical problems to determine a representative lateral subgrade modulus, called the equivalent lateral subgrade modulus (k_{eq}), under specific pile-soil conditions. The same methodology was successfully used to develop the lateral equivalent subgrade modulus for piles in clays (*8*). The method involves calculating the lateral deflection of a hypothetical pile, y_o, under specific soil and loading conditions by employing a numerical computer solution COM624 (*6*). The equivalent lateral subgrade modulus, k_{eq}, can be determined by rewriting Equations 3 and 4 as follows:

$$k_{eq} = \frac{4.302\ P^{1.67}}{(EI)^{0.67}\ y_o^{1.67}} \quad \text{for free-head pile} \tag{15}$$

$$k_{eq} = \frac{0.886\ P^{1.67}}{(EI)^{0.67}\ y_o^{1.67}} \quad \text{for restrained-head pile} \tag{16}$$

where P is the applied lateral load and EI is the bending stiffness of pile.

A wide range of subgrade modulus was selected to represent a full spectrum of soil parameters in this study. Table 2 summarizes the selected soil parameters for cohesionless soils above and below the water table. The pile parameters investigated in the study are given in Table 3.

To perform the required numerical analyses using COM624, the lateral pile-soil interaction is modeled by the *p-y* curves, which may be constructed by one of the four procedures described in previous sections. Murchison and O'Neill (*9*) concluded that their proposed procedure was the most accurate of all four procedures. However, the conclusion of accuracy was based on a small data base and the relative comparison of the four procedures. The recent design manual sponsored by the Federal Highway Administration (*16*) recommends the

TABLE 2 SUMMARY OF SOIL PARAMETERS

	Sand Above the Water Table			Sand Below the Water Table		
Frictional Angle ϕ (deg.)	29	33	39	29	33	39
Subgrade Modulus k (pci)	25	95	225	20	60	125

TABLE 3 SUMMARY OF PILE PARAMETERS

Pile Material	Pile Diameter B (in.)	Young's Modulus E (psi)	Moment of Inertia I (in^4)
Concrete	36	3×10^6	82447.9
Steel	12	3×10^7	1017.9

use of the Reese et al. procedure for obtaining *p-y* curves. It appears that this procedure is more widely used and conservative. On the basis of this discussion and the advantage of being internally generated in COM624, the Reese et al. procedure was used to construct *p-y* curves in this parametric study.

EQUIVALENT SUBGRADE MODULUS

From the results of the parametric study described in the previous section, the relationships of the equivalent subgrade modulus, k_{eq}, versus the dimensionless lateral load factor $P\beta/kB^2$ were developed, in which P is the applied lateral load and

$$\beta = (kB/4EI)^{0.25} \quad (17)$$

where k is the subgrade modulus determined from Figure 3. EI is the bending stiffness of the pile, and B is the pile diameter.

Figures 4 and 5 show the relationships of k_{eq} versus the dimensionless lateral load factor for sands above the water table under a free-head and a restrained-head condition, respectively. Figures 6 and 7 show the relationship for sands below the water table under a free-head and a restrained-head condition. The k_{eq} under any particular conditions would not exceed the threshold value of k, which constitutes the initial slopes of the *p-y* curves at any depth Z by the product of kZ.

DESIGN PROCEDURE

The design curves to obtain k_{eq} are shown in Figures 8 and 9 for cohesionless soils above and below the water table, re-

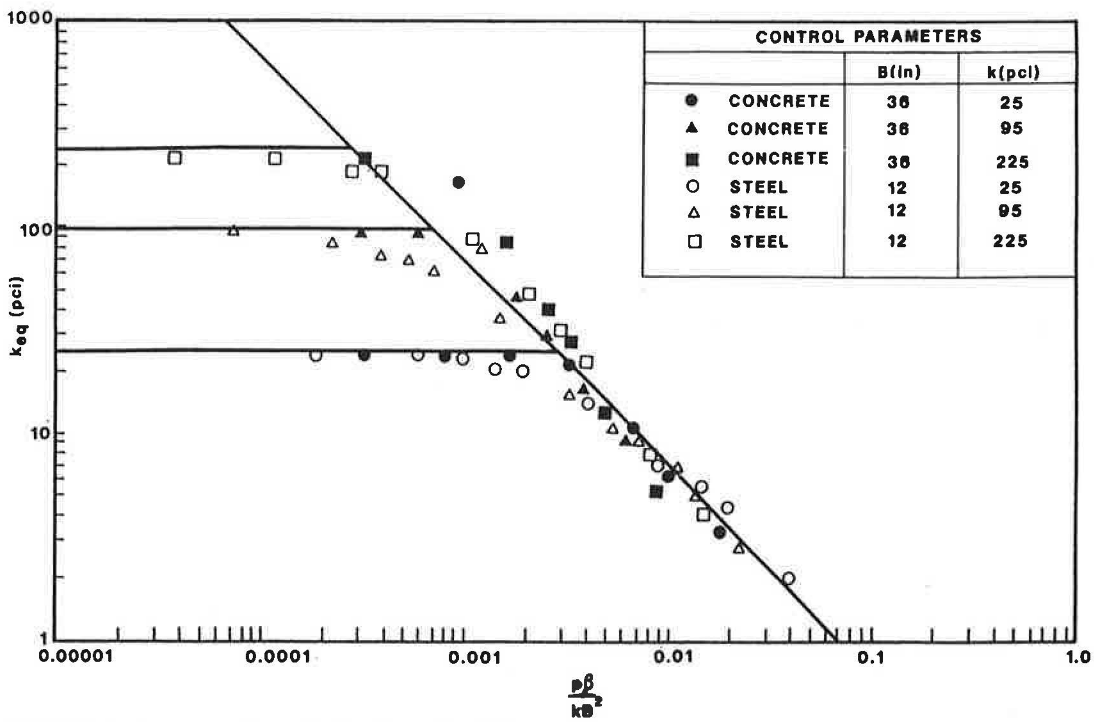

FIGURE 4 k_{eq} **versus dimensionless lateral load factor for sands above the water table, free-head conditioning.**

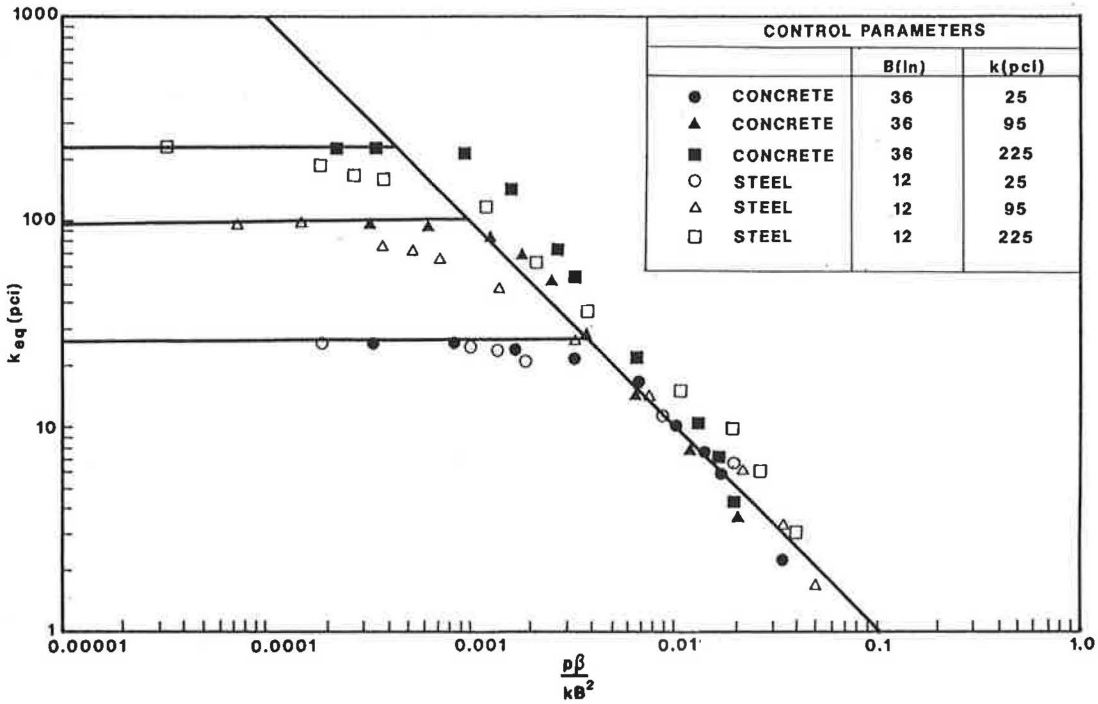

FIGURE 5 k_{eq} **versus dimensionless lateral load factor for sands above the water table, restrained-head conditioning.**

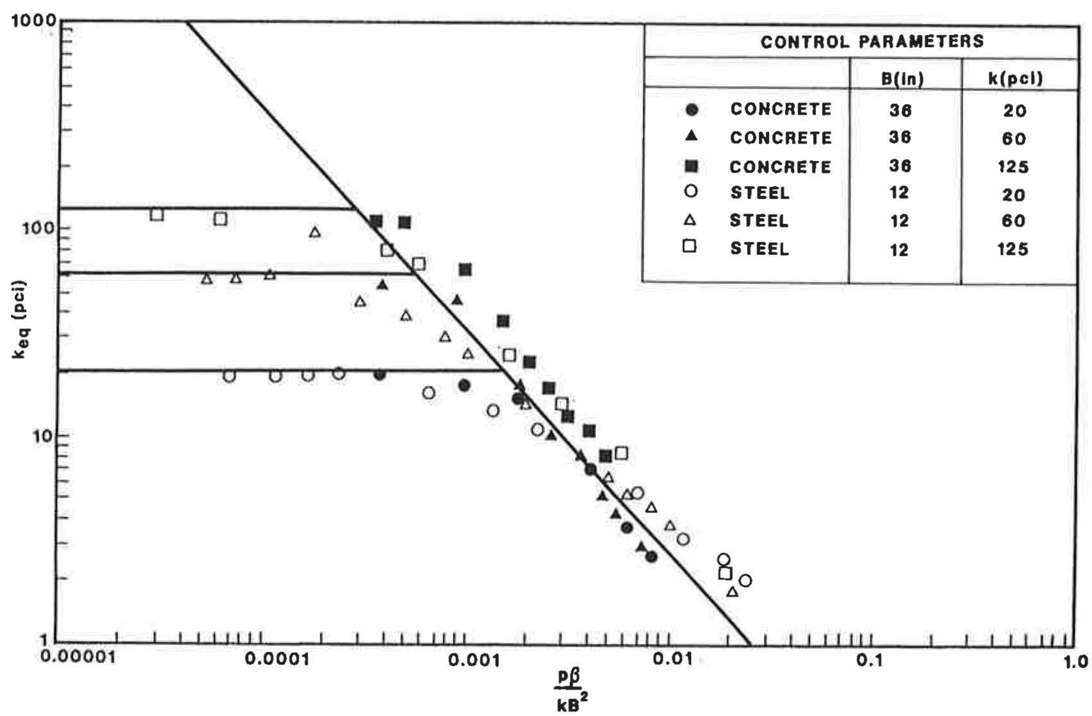

FIGURE 6 k_{eq} **versus dimensionless lateral load factor for sands below the water table, free-head condition.**

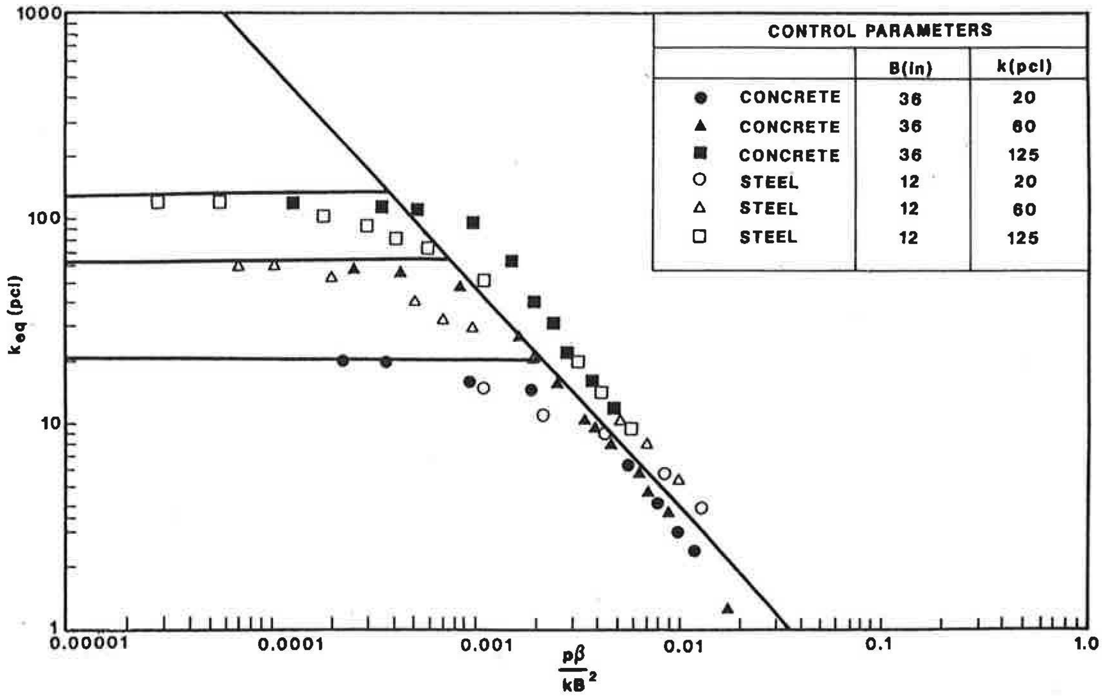

FIGURE 7 k_{eq} **versus dimensionless lateral load factor for sands below the water table, restrained-head condition.**

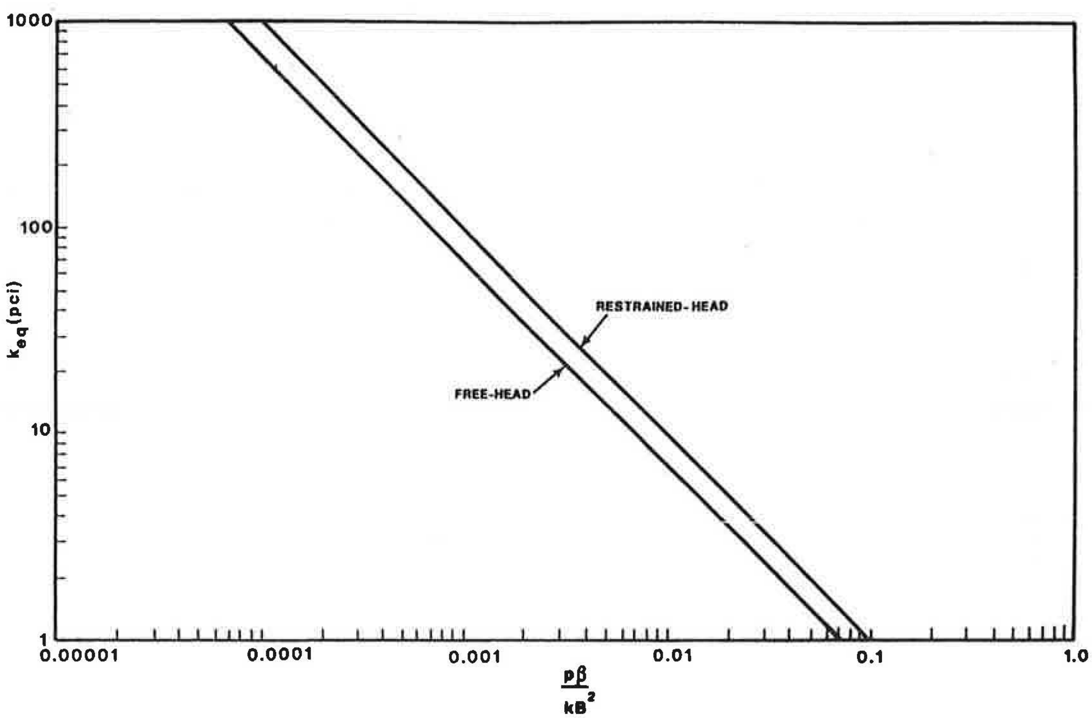

FIGURE 8 Design curves of k_{eq} **versus dimensionless lateral load factor for sands above the water table.**

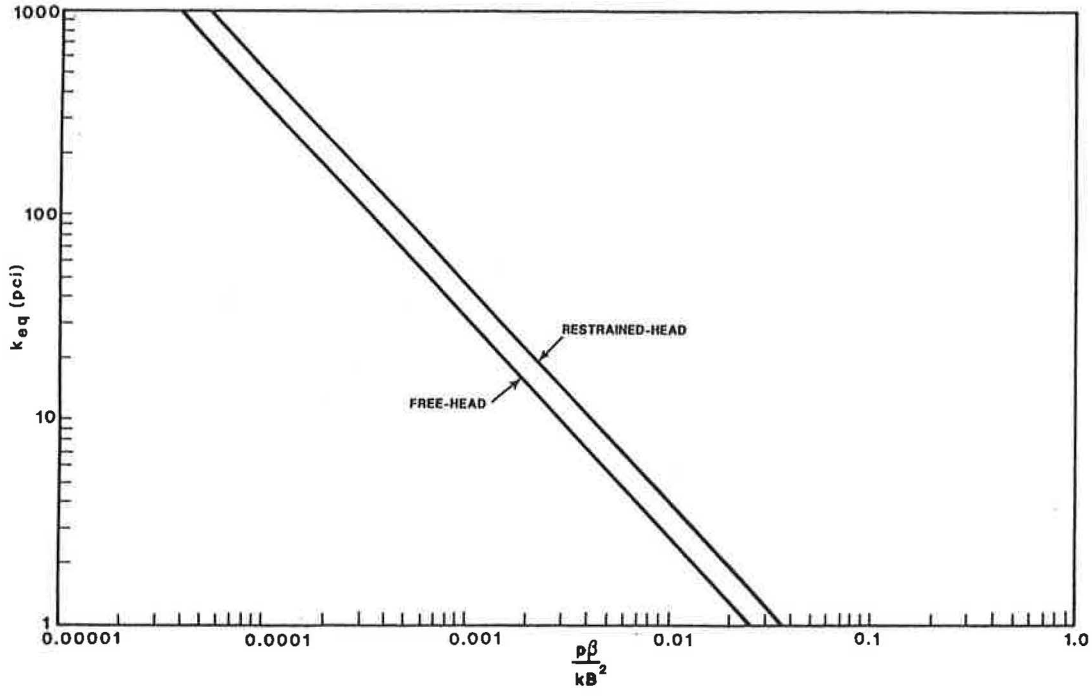

FIGURE 9 Design curves of k_{eq} versus dimensionless lateral load factor for sands below the water table.

spectively. The horizontal lines in Figures 4 through 7 represent the threshold values of k for sands of various relative densities. These lines are omitted in Figures 8 and 9, but the threshold values should be checked before a modulus is chosen from the design curves. The checking mechanism will be further discussed in the following paragraph. A simple design procedure is proposed to analyze laterally loaded piles in sands as follows:

1. Characterize the subsurface soil with a threshold subgrade modulus, k, using the relationships presented in Figures 2 and 3.
2. Obtain coefficient β using Equation 17.
3. Calculate the dimensionless lateral load factor, $P\beta/kB^2$.
4. Determine the equivalent subgrade modulus, k_{eq}, from either Figure 8 or Figure 9. Depending on given subsurface and boundary conditions, check $k_{eq} \leq k$. If $k_{eq} > k$, use $k_{eq} = k$.
5. Calculate the maximum deflection of pile from Equations 3 or 4 by substituting k_{eq} into k.
6. Determine the maximum moment from Equation 7 with all the necessary parameters obtained from Table 1 and Equation 5.

The nondimensional equation (Equation 6) can also be used to determine the maximum deflection of the pile by using the equivalent modulus k_{eq}. However, only Broms's equations (Equations 3 or 4) are considered for the subsequent sections of this paper.

This paragraph demonstrates the simplicity of the proposed design procedure by following Steps 1 to 6 to solve an example problem. Given a pipe pile of 20-in. OD and ⅜ in. in wall thickness driven into submerged sands ($\phi = 30$ degrees), k in Step 1 can be found in Figures 2 and 3 as 30 pci. Obtain β in Equation 17 as 8.186×10^{-3} in.$^{-1}$ using E of 3×10^7 psi and I of 1,113.5 in.^{4} Determine $P\beta/kB^2$ for a lateral load P of 15 kips as 0.01. Obtain a k_{eq} of about 2.8 pci from Figure 9. The maximum deflection and maximum moment are found to be 1.20 in. and 1.2×10^6 in.-lb using Equations 3 and 7, respectively, for free-head conditions.

To examine the effectiveness of the proposed design procedure, a study was conducted to compare results obtained by computer analyses using COM624 and by following the proposed procedure. Three cases were established for the comparison study: (*a*) steel pipe pile 20 in. in diameter and ⅜ in. in wall thickness driven in submerged sands and loaded laterally under free-head condition; (*b*) 24-in.-diameter drilled pier, dry sands, and restrained-head condition; and (*c*) 48-in.-diameter drilled pier, submerged sands, and free-head condition. Table 4 summarizes the important parameters for these three cases. Figures 10 and 11 compare the maximum deflections and maximum moments, respectively, by COM624 and the proposed design method. The comparisons indicate a less than 10 percent discrepancy in results within the range of various controlled parameters used in this study.

CONCLUSIONS

The relationships of equivalent subgrade modulus and dimensionless lateral load factor are established for laterally loaded piles in sands above and below the water table. A simple design procedure using the concept of the equivalent subgrade modulus is proposed to find the maximum deflection

TABLE 4 PARAMETERS FOR COMPARISON STUDY

Case	Pile Material	Pile Diameter B (in.)	Pile-Head Condition	Frictional Angle (deg.)	Soil Condition
A	Steel	20*	Free	30	Submerged
B	Concrete	24	Restrained	35	Dry
C	Concrete	48	Free	36	Submerged

* with 3/8 in. wall thickness

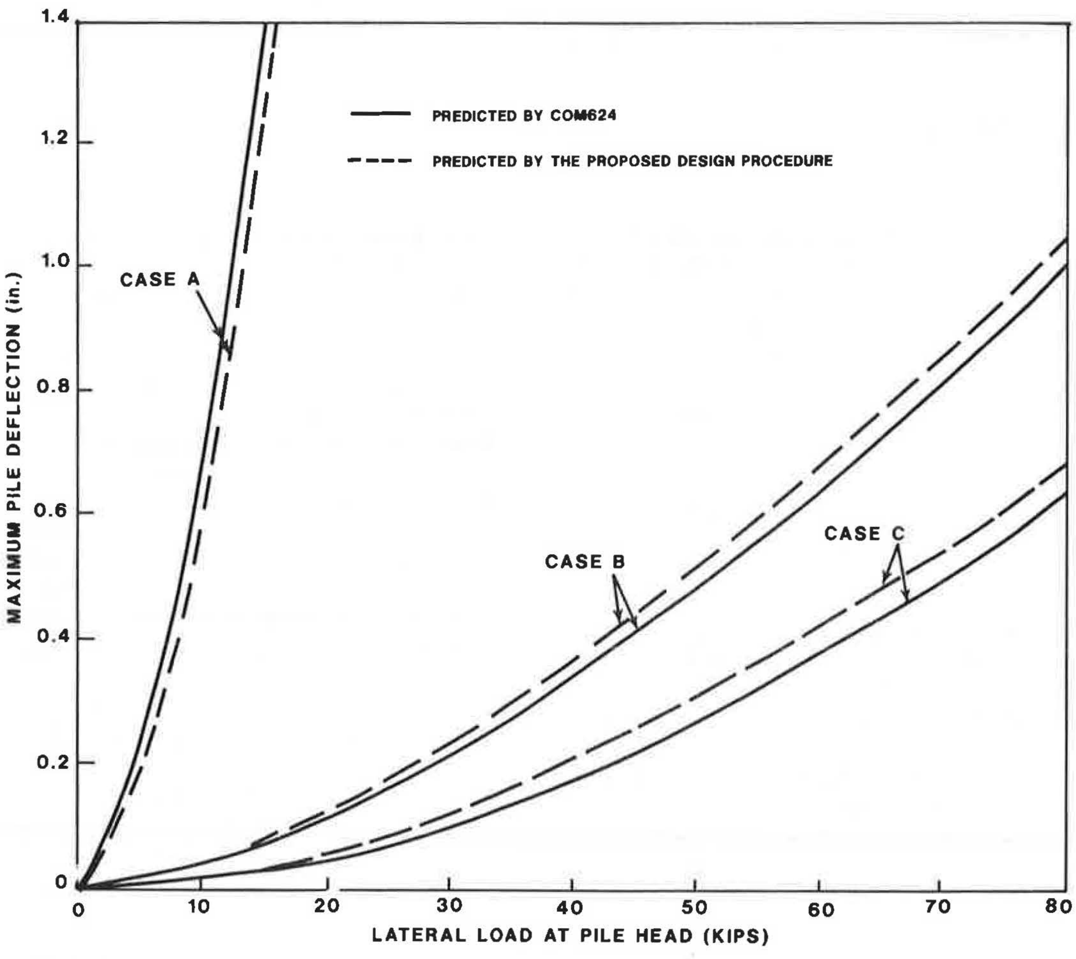

FIGURE 10 Comparison of maximum deflections by COM624 and proposed method.

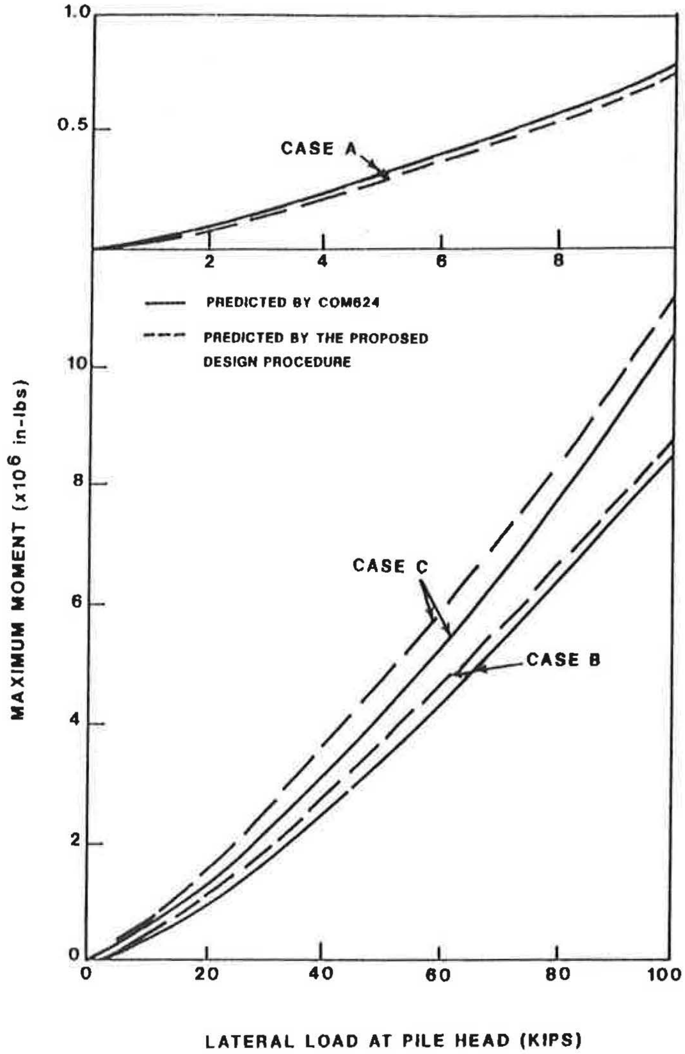

FIGURE 11 Comparison of maximum moments by COM624 and proposed method.

and maximum moment for practical problems. A comparison study indicated that the results obtained by the proposed procedure compared well with those determined by the computer using numerical solution COM624.

LIMITATIONS

The proposed equivalent lateral subgrade modulus and the design procedures were developed on the basis of the range of controlled parameters outlined in Tables 2 and 3. The findings of this study are only applicable for cohesionless soils. Similar results and procedures for cohesive soils can be found in a companion paper (*8*). Potential users of the proposed design curves and procedures are urged to recognize all the limitations.

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